A comparison of effectiveness of circular and radial reserve in survivability of symmetrical hierarchical networks

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Abstract: - In this paper we consider survivability of Symmetrical Hierarchical Network (SHN) taking account of destruction of the main radial edge, the circular and radial reserve, and to compare the efficiency of circular and radial reserve.

Key-Words: - Network, Symmetrical, Hierarchical, Survivability, Reserve, Capacity

1 Introduction
This paper is a continuation of [4] dedicated of the analysis of survivability of symmetrical hierarchical flow networks (SHN) under uncertainty conditions. We consider the problem of analysis in nonrigid statement. This means that we take into account the possibility of optimizing the flow distribution after the fault. It is supposed that the remaining capacity will be known. The guaranteed evaluation of the functional capability of SHN assumes finding the worst distribution of the destruction.

In general, network systems do not have a unique efficiency criterion. Here the quality of the network functioning is evaluated based on the completeness of the supply of flow demands users -- pairs.

2 Symmetrical hierarchical network
An arbitrary multicommodity flow network (MFN) [1] is determined by two graphs, physical $G$ and logical $P$ on the same set $V$ of nodes. Edges $r_k$ of graph $G$ mean physical lines of communications between the nodes from $V$, and they are ascribed with non-negative numbers $y_k$, called capacity of edges $r_k, k = 1, \cdots, n$. Edges $p_i, i = 1, \cdots, m$ of graph $P$ correspond to logical connection between certain pairs of nodes (source-sink pairs). This means that there are requests or demands of flow transmission from one node to the other of $p_i$ through the edges of network graph $G$. Thus, each $p_i$ is specified by source-sink pair $v_i^l, v_i^r \in V$ and positive demand $d_i$.

A multicommodity flow network is called symmetrical hierarchical (SHN) if its logical graph has the structure of a star (i.e., the source - sink pairs are given in the form $(v_0, v_i), i \in M = \{1,2,\cdots,m\}$, with the common source $v_0$ and all demands are equal to $d$.

It is known that the physical structure of a star possesses poor properties of survivability. To create an additional circular structure connecting all the sink or extend the capacity of the radial edge to make possible raise the survivability of SHN. Let $c$ be an initial capacity vector of SHN.

Let $z_j$ be the amount flow between nodes $v_0$ and $v_j$ and $f^{i^l}_i$ denote the share of flow $z_j$ between $v_0$ and $v_j$ which passes by edge $(v_0, v_i)$. The variable $f^{i^l}_{m+i}$ and $f^{i^r}_{m+i}$ denote the share of the flow $z_j$ between $v_0$ and $v_j$ that passe by edge $(v_{i-1}, v_i)$ clockwise and counter-clockwise respectively. Let $F(y)$ denote the set of such $f = \{f_i^l\}$.

The flow constraint condition for SHN are
and the capacity constraint for SHN are as follows:

\[
\begin{align*}
\sum_{i=1}^{m} f_{i}^{j} - z_{j} & = 0 \quad \forall j \in M, \\
f_{m+i}^{j} + f_{m+i}^{j} - (f_{m+i}^{j} + f_{m+i}^{j} + f_{m+i}^{j} + f_{m+i}^{j}) & = 0 \quad \forall j \in M, i \not\in \{j, m\}, \\
f_{m+i}^{j} + f_{m+i}^{j} - (f_{m+i}^{j} + f_{m+i}^{j} + f_{m+i}^{j} + f_{m+i}^{j}) & = 0 \quad \forall j \in M \setminus \{m\},
\end{align*}
\]

3 Competitive distribution of flows

Denote the set of all feasible multiflows

\[ Z(y) = \{z \geq 0 \mid \exists f \in F(y) : z = z(f)\}, \]

and

\[ \theta_0 = \theta_0(y) = \max_{f \in F(y)} \min_{i \in M} \frac{z_i(f)}{d}. \quad (1) \]

If \( f^0 \) be the optimal solution of (1) then \( f^0 \) is called competitive distribution of flows.

The value of \( \theta_0 \) shows the measure of effectiveness functioning of SHN.

In the case \( m = 1 \), (1) is well known problem of the flow maximization. For \( m > 1 \) it formalizes the concurrent flow problem. A multiflow achieves demand satisfaction if it ships an amount of each commodity equal to its demand from its source to its sink while obeying the capacity constraint. This corresponds to \( \theta_0 \geq 1 \).

Otherwise, an arbitrary concurrent flow distribution \( f^0 \) [a maximizer in (1)] may be not the best for certain network users.

The function \( \theta^\gamma_0(c) \) denotes the guaranteed level of demand satisfaction depending on reserve capacity \( c \) and demands \( d \) and define as follows:

\[ \theta^\gamma_0(c) = \min_{y \in Y_\gamma(c)} \theta_0(y) = \min_{y \in Y_\gamma(c)} \min_{f \in F(y)} \frac{z_i(f)}{d}. \]

where

\[ Y_\gamma(c) = \{y \geq 0\mid \sum_{i=1}^{2m} y_i = (1 - \gamma) \sum_{i=1}^{2m} c_i, \quad y \leq c\}, \]

and \( Z(y) \) is the set of all feasible multiflows \( z = (z_1, \cdots, z_m) \) in the network with capacity vector \( y \).

Here \( \gamma \in (0,1) \) is a parameter which characterizes the power of network destruction.

The Survivability of SHN is defined by \( \theta^\gamma_0(c) \).

4 Comparative survivability of SHN with radial and circular reserve

In this paper we consider survivability of SHN taking account of destruction of the main radial edge, the circular and radial reserve, and to compare their efficiency.

Denote by \( \gamma(c) \) the smallest value \( \gamma \) that is enough for loss of connectivity of SHN with initial capacity vector \( c \), i.e.

\[ \gamma(c) = \min_{\theta^\gamma_0(c) = 0} \{\gamma \in (0,1)\mid \theta^\gamma_0(c) = 0\} = \min_{\theta^\gamma_0(c) = 0} \{\gamma \in (0,1)\mid \exists y \in Y_\gamma(c) : \theta_0(y) = 0\}. \]

For SHN with capacity vector \( c = (d + t, \cdots, d + t, 0, \cdots, 0) \) (i.e. SHN with radial reserve) we have \( \gamma(c) = \frac{1}{m} \), and for SHN with capacity vector \( c = (d, \cdots, d, t, \cdots, t) \), (i.e. SHN with circular reserve) \( \gamma = \frac{d + 2t}{m(d + t)} \).

Lemma 1: Let \( c = (d + t, \cdots, d + t, 0, \cdots, 0) \) be the capacity vectors of SHN with radial reserve, then

\[ \theta^\gamma_0(c) = \begin{cases} 
\frac{(d + t)(1 - m\gamma)}{d} & \text{if } \gamma < \frac{1}{m}, \\
0 & \text{if } \gamma \geq \frac{1}{m}.
\end{cases} \]

Lemma 2: Let \( c = (d, \cdots, d, t, \cdots, t) \) be the capacity vectors of SHN with circular reserve, and reserve value \( t = \frac{md}{8} \). Then
Lemma 3: Let \( c' = (d, \ldots, d, t, \ldots, t) \), \( c'' = (d + t, \ldots, d + t, 0, \ldots, 0) \) be the capacity vectors of SHN with circular and radial reserve correspondingly, with reserve value \( t = \frac{md}{8} \). Then we have

\[
\theta_{\gamma'}^e(c') = \begin{cases} 
1 - \gamma(1 + \frac{m}{8}), & \text{if } \gamma \leq \gamma^*, \\
1 + (\frac{1}{4} - \gamma)m - \frac{1}{8}m^2\gamma, & \text{if } \gamma^* < \gamma < \gamma^*, \\
0, & \text{if } \gamma \geq \gamma^*, 
\end{cases}
\]

where

\[
\gamma^* = \frac{2m}{(m-1)(m+8)}, \quad \gamma = \frac{2m+8}{m(m+8)}.
\]

In the next lemma the effectiveness of circular and radial reserve be compared in survivability of symmetrical hierarchical networks.

|\[ \theta_{\gamma'}^e(c') < \theta_{\gamma'}^e(c''), \quad \text{if } \gamma \leq \hat{\gamma}, \]
|\[ \theta_{\gamma'}^e(c') = \theta_{\gamma'}^e(c''), \quad \text{if } \gamma = \hat{\gamma}, \]
|\[ \theta_{\gamma'}^e(c') > \theta_{\gamma'}^e(c''), \quad \text{if } \hat{\gamma} < \gamma < \gamma^*, \]

where

\[
\hat{\gamma} = \frac{m}{(m-1)(m+8)}, \quad \gamma = \frac{2m+8}{m(m+8)}.
\]

5 Conclusion

Lemma 3 follows that, for small \( \gamma \) to create an additional circular structure connecting all the sink is better to make possible raise the survivability of SHN than extend the capacity of the radial edges, and for big \( \gamma \) vise versa.(see fig. 1).

References:


[4] Ahmadi M.B., Guarantee of survivability of symmetrical hierarchical networks, Third Moscow international conference on operations research (Moscow, April 4-6, 2001).
