A New Nonlinear Distortion Correction Approach for Camera-Projector System

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Abstract: This paper presents a new approach allowing a projector to display an undistorted image on an unknown geometry surface. In the distortion correction system, a single digital camera is used to capture the viewer’s perspective of the projection surface, and a computer is used to process the original source image to show an undistorted image from the view position of audience. In the approach, no explicit camera and projector calibrations are needed. The procedure establishes the mapping from camera image to projector image by a well trained neural network (NN). After the mapping has been established, a transform converter table is constructed from the output of NN. With the transform converter table (TCT), image correction process can be realized in real-time, therefore, this approach is available for various applications, such as movies and etc. By using of NN, we do not need to calculate all image pixels for establishing the mapping transformation. The mapping can be obtained correctly by less computation if enough teaching points are provided. The simulation results show that our approach is effective for correcting distortion caused by continuous projective surface and practical enough for most projector presentations.

Key-Words: Projector-camera system, Projective mapping, Distortion correction, Neural network, Computer graphics, Keystone

1. Introduction
The increasing affordability and portability of high quality projectors has generated a surge of interest in projector-camera systems. However a careful aligning to the projection surface or a flat screen or wall are necessary for getting a good projection effect. However, without a careful aligning to the projection surface (wall or screen), the resulting image on the projection surface may appear distorted, or keystoned. If the surface used for projection is not flat completely, a nonlinear distortion will occur caused by nonlinear surface (not in a flat plane). Such distortion is undesirable, for such warping may bring distraction to audiences, and detriments to the interpretation of visual information such as graphs, table and charts and technical drawings. Keystoning can be avoided or reduced by aligning the projection system’s optical axis, so that it is perpendicular to the screen, and ensuring that the image is not rotated with respect to the screen. For fixed projectors that can be mounted from the ceiling and carefully aligned once by some experts with experiences, these constraints are surmountable; however, an alignment at the start of each presentation session must be required for portable projectors. This manual adjusting process is tedious; it can also be impractical to align a portable projector in a manner that eliminates all keystoning effects, since optimal alignment may place the projector in an awkward position (such as in the middle of the audience). If the projection surface is not flat as expected, or we want to project a image on a curved surface, it is impossible to get a non-distorned projecting image by normal alignment. The above reasons motivate the need for a better presentation interface: one that allows arbitrary placement of the projector.

Several works for camera-based automation calibration or correction of projector have recently been proposed. Sukthankar[1] present an automatic keystone correction for camera-assisted presentation interface, which use a digital camera to observe the projected image, some linear projective transforms are constructed by identification process, finally they are...
used to prewarp the image that will be projected to a flat screen. H.Chen[2] also gave some ideas for calibrating camera-projector system, which use a camera homograph tree to construct the linear projective transform that will be used for image prewarp. But most of their works concern a flat surface (a flat screen), their methods may not be effective for a curved surface or unknown geometry surface, which may be met in many cases for a portable projector usage or some special usage.

This paper presents a automatic approach for correcting distortion caused by both keystone and curved projection surface. For such distortion is not linear, the projective transform must be a nonlinear mapping. Neural Network have been used in various application for its excellent learning ability and nonlinear mapping approximation ability. Also there are many researches on neural network, some learning algorithms give good performance in both quick convergence and global stability[4,5,6]. To get the nonlinear projective transform in high accuracy, we adopt a neural network with three layered structure to obtain an approximation of the projective transform that will be used for prewarping the image projected. A digital camera is used for capturing the viewer’s perspective of the projection surface, and the distortion information will be extracted from the camera image. The extracted information will be used for training NN before presentation. The simulation results show that the approach is much effective and practical enough for a continuous curved projective surface.

2. System Calibration
As shown in Fig.1, the system discussed here are: a standard computer, a digital camera, and a projector.

There is no specific constraint on the position or the orientation of the projector and the camera with respect to the projection surface, except that area visible to the camera must be covered by the projector. In detail, the projector can be mounted anywhere, as long as the image falls entirely within the projection surface area. The camera must be mounted such that the projection screen is within its field of view. Therefore, there are three spaces: source image space (that will be projected to the projection surface), physical projection surface space and camera image space.

To implement an effective distortion correction, the projective transform mapping between the source image space to the camera image space must be determined. If the projection surface space is flat completely, the projective transform from source image space to the projective surface space and the projective transform from the projective surface space to the camera image space are linear mapping both. such transform mapping can be shown in the following.

\[
\begin{bmatrix}
wx \\
wy \\
w
\end{bmatrix}
= \begin{bmatrix}
p1 & p2 & p3 \\
p4 & p5 & p6 \\
p7 & p8 & p9
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
1
\end{bmatrix}
\]

where \((x, y)\) and \((X, Y)\) are corresponding points in two 2-D spaces of reference, and \(p = (p1 \ldots p9)^T\) are parameters specifying the homographies. Because the projective transform from the source image space to the camera image space can be derived by simple matrix calculation, the projective transform is a linear mapping. In this case, the parameters contained in the mapping can be identified by using some point correspondences in the spaces. Here, we assume that the projection surface is a smooth curved surface with unknown geometry, it is easy to know that the projective transform from the source image space to the camera image space must be a nonlinear mapping, and the linear approach can not be used for solving the problem. In general, the nonlinear mapping between pixels in the source image space \((x, y)\) and the camera image space \((u, v)\) can be shown as

\[
x = P_x(u, v) \quad y = P_y(u, v)
\]

where \(P_x\) and \(P_y\) are nonlinear functions which can not be determined by only parameters.

It is well known that Neural Network has been used in various applications for its excellent learning ability and nonlinear mapping approximation ability. To get
the nonlinear projective transform in a high accuracy, we adopt a neural network to obtain an approximation of the projective transform that will be used for pre-warping the image projected. To training the NN, we select the algorithm by [4] that provides a both fast convergence and good global stability. After training with some teaching data, the NN may give a correct approximation of the projective transform.

Assume that a group of NN training data for getting the above mapping are extracted from one or more calibration images by some image recognition approaches. The data \( P_i \in N^4 \) are shown as

\[
P_i = \{u_i, v_i; x_i, y_i\} \quad \text{where} \quad 1 \leq i \leq K
\]

Our aim is to construct a training NN to obtain an approximation of the space mapping. To measure the approximation result in quantity, a cost function \( J_{NN} \) as follow is defined as following.

\[
J_N = \sum_{i=1}^{K} (\|x_i - \hat{P}_x(u_i, v_i)\| + \|y_i - \hat{P}_y(u_i, v_i)\|)
\]

The source image may be projected to an arbitrary quadrilateral with curved sides in the projection image space as shown in Fig.2. Since the pre-warped image can only be displayed within the area of the bounded source image space, the corrected image must lie within the bounds of the quadrilateral. Also the projected image should be as large as possible for a better efficiency. This is equivalent to finding the largest rectangle with appropriate aspect ratio within the projection image space, this process is easy to implement.

With the desired size and location of the corrected image, we will construct a transform converter table with well trained NN. The size of TCT will be affected by the resolution of the projector and the available rectangle area in the camera space. In our approach, we will use the TCT instead of the trained NN to pursue a quick image transformation calculation in the stage of image distortion correction. Assume the image size is \( M \times N \), the TCT \( T_c = \{T_x, T_y\} \in N^{M \times N \times 2} \) can be generated as follows:

**step 1**

Calculate a possible rectangular area in projection surface, if necessary, the ratio of length and height should be considered for showing graphics or photos.

**step 2**

Based on the available resolution provided by the projector, determine the pixels numbers in horizontal and vertical direction for the rectangular area obtained in previous step. Assume that the rectangular area is filled by \( M \times N \) pixels matrix, and the rectangular area is determined by two top coordinates \((u_{left}, v_{top})\) and \((u_{right}, v_{bottom})\).

**step 3**

For \( 1 \leq i \leq M \) and \( 1 \leq j \leq N \), calculate \( T_x(i, j) = \hat{P}_x(u_i, v_j) \) and \( T_y(i, j) = \hat{P}_y(u_i, v_j) \) iteratively, where

\[
u_i = u_{left} + \frac{u_{right} - u_{left}}{M - 1} (i - 1)
\]

\[
v_j = v_{top} + \frac{v_{top} - v_{bottom}}{N - 1} (j - 1)
\]

\( T_x(i, j) \) and \( T_y(i, j) \) are the elements of TCT, \( \hat{P}_x \) and \( \hat{P}_y \) are the approximation of the nonlinear projective transform \( P_x \) and \( P_y \) by well trained NN.

3. Distortion Correction

Based on the nonlinear TCT \( T_c \), which represents the projective transform between source image space and camera space, we can correct the distortion by pre-warping the source image \( I_s \in N^{M \times N} \), so that its
projection image may be shown as a rectangular image on the camera space. Therefore, the projected image will be watched as a rectangular from the camera position. The prewarped source image \( I_w \in N^{M \times N} \) is generated from source image \( I_s \) as followings.

**step 1**

Create a new table as \( T_{trans} = \{ T_c, T_s \} \), where \( T_s \in B^{M \times N} \) is used to save information to indicate the status of the scanned elements. For 1 ≤ \( i \) ≤ \( M \) and 1 ≤ \( j \) ≤ \( N \), let \( I_w(i,j) = 0 \) and \( T_s(i,j) = FALSE \). Also set \( i = 1 \) and \( j = 1 \).

**step 2**

If \( T_s(i,j) = FALSE \), then find out all adjacent elements \( T_c(m,n) \) of element \( T_c(i,j) \), all elements \( T_c(m,n) \) satisfy \( T_c(m,n) = T_c(i,j) \). For all such adjacent element, set \( T_c(m,n) = TRUE \) and \( T_s(i,j) = TRUE \). Record the indexes \( (m_k, n_k) \) of all the adjacent elements found here, where 1 ≤ \( k \) ≤ \( R \).

**step 3**

If \( T_s(i,j) = TRUE \), then increase \( i \) or \( j \), then go to step 2.

**step 4**

Based on the above elements information, compute \( I_w(T_s(i,j), T_p(i,j)) \) as following, if \( R = 0 \) then

\[
I_w(T_x(i,j), T_y(i,j)) = I_s(i,j)
\]

else

\[
I_w(T_x(i,j), T_y(i,j)) = \left( I_s(i,j) + \sum_{k=1}^{R} I_s(m_k, n_k) \right) / (R + 1)
\]

**step 5**

Go to Setp 2 to continue the above operations iteratively for 1 ≤ \( i \) ≤ \( M \) and 1 ≤ \( j \) ≤ \( N \).

Since the computed image pixel coordinates are real-valued, it may not exactly correspond to a dispersed pixel in the source image space. Here a technique that use bilinear interpolation[7]. The application image is embedded in a temporary image of black pixels to ensure that only pixels within the corrected image are illuminated. By the above operations, the final image shown in camera view will appears undistorted, even with a misalignment of projector or curved projection surface as shown in Fig.3 conceptually. To measure the distortion correction errors, Assuming \((u_i,v_j)\) is the projected pixel with our distortion correction for the pixel \( I_{ij} \) in the source image, and \((\hat{u}_i, \hat{v}_j)\) is the position that \( I_{ij} \) is correctly projected to the camera image space, with \((u_i, v_j)\) and \((\hat{u}_i, \hat{v}_j)\), we can construct an Euclidean distance error \( E(i,j) \) on the camera image space as

\[
E(i,j) = (u_i - \hat{u}_i)^2 + (v_j - \hat{v}_j)^2
\]

To confirm the performance of the nonlinear correction, We defined a quadratic cost function as follows with the error \( E(i,j) \),

\[
J = \sum_{i=1}^{M} \sum_{j=1}^{N} E(i,j)
\]

By checking the value of the error cost function, we can evaluate how the distortion correction accomplished by the NN and TCT. The calibration and correction operations can be done iteratively until obtaining a satisfactory correction level.

**Figure 3**: Appropriately Pre-warped Image Appears Rectilinear in Projection Image Space or Camera Image Space

**4. Simulation and Discussion**

In order to evaluate the nonlinear distortion correction ability, we made a simple simulator on the platform of Matlab, which generates some type of misalignment of projector and a curved projective surface. We considered a simulation system shown in Fig.4. Here about 20x16 calibration points were used to calibrate the system, these position information is also used for training the NN that will give an approximation for
Figure 4: the Simulation System Layout

the projective transform between the projector space and camera space. The NN used here has three layers, which middle layer has 40 to 50 neurons. To training the NN, we adopt Bayesian regularization approach, which updates the weight and bias values according to Levenberg-Marquardt optimization. It minimizes a combination of squared errors and weights, and then determines the correct combination so as to produce a network which generalizes well.

Here we chose some typical curved surfaces such as plane and quadratic surface for simulation tests. The some simulation results for several typical surface are shown in Fig.5 to Fig.7.

Figure 5: Simulation result for a curved surface: case 1

The above experiment results show that a very good distortion correction is achieved by the approach present here. To confirm the distortion correct performance, some distortion correction results for some sample images are shown in Fig.8 and Fig.9.

We confirmed that our approach can deal with a variety of distortion caused by projector configurations, misalignment situations, and curved projection surface with unknown geometry. In our approach, most time consuming operation is training the NN for non-linear mapping in system calibration phase, but with the advances of computation ability of PC, this will not be a drawback of our approach. If a neural network hardware can be used, the computation time is be reduced greatly.
5. Conclusions
In this paper, we described a new nonlinear distortion correction approach for projector-camera system. In our approach, a neural network is used to approximate the nonlinear projective transform mapping. Because a transform converter table is introduced and constructed by a well trained NN to implement a real-time prewarping operation, we obtain a both high distortion correction accuracy and a fast computation which is important for real-time processing without special hardware support in our approach. Simulation results show that the approach is effective and practical enough for a continuous curved projective surface. We will implement the approach presented in this paper to a real system to confirm its performance.

References: