Improved Performance Model for Web-Based Software Systems

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Abstract: The performance of web-based software systems is one of the most important and complicated consideration. With the help of a proper performance model and an appropriate evaluation algorithm, performance metrics can be predicted. The goal of our work is to introduce and verify an improved multi-tier queueing network model for web-based software systems. In our work, an evaluation and prediction technique applying dominant factors in point of response time and throughput performance metrics has been established and investigated. The Mean-Value Analysis algorithm has been improved to model the behavior of the thread pool. The proposed algorithm can be used for performance prediction. The convergence and limit of the algorithm have been analyzed. The validity of the proposed algorithm and the correctness of the performance prediction have been proven with performance measurements.

Key–Words: Web-based software systems, Queueing model, Mean-Value Analysis, Thread pool, Performance prediction, Convergence

1 Introduction

The performance belongs to the most important factors of information systems, because they face a large number of users, and they must provide high-availability services with low response time, while they guarantee a certain throughput level.

The performance metrics of a system can be predicted at early stages of the development process, with the help of a properly designed performance model and an appropriate evaluation algorithm [1], [2].

In the past few years several methods have been proposed to address this goal. Several from them is based on queueing networks or extended versions of queueing networks [3], [4], [5], [6]. Another group is using Petri-nets or generalized stochastic Petri-nets [7], [8]. As third kind of the approaches the stochastic extension of process algebras, like TIPP (Time Processes and Performability Evaluation) [9], EMPA (Extended Markovian Process Algebra) [10], and PEPA (Performance Evaluation Process Algebra) [11] can be mentioned.

The paper is organized as follows. Section 2 covers backgrounds and related work. Section 3 introduces our enhanced Mean-Value Analysis (MVA) algorithm. Section 4 describes performance prediction with the proposed algorithm. Section 5 determines and proves the convergence and the limit of the original and the enhanced algorithms. Section 6 describes our performance measurements. Section 7 demonstrates the validation of the enhanced algorithm in different versions of ASP.NET environment. Section 8 presents error analysis. Finally, Section 9 reports conclusions.

2 Backgrounds and Related Work

Queueing theory [3], [12], [13] is one of the key analytical modeling techniques used for information system performance analysis. Queueing networks and their extensions (such as queueing Petri nets [14]) are proposed to model web-based information systems [4], [5], [15].

Web-based software systems access some resources while executing requests of the clients, typically several requests arrive at the same time, thus, competitive situation is established for the resources. In case of modelling such situation queueing model-based approaches are widely recognized.

In [5], a queueing model is presented for multi-tier information systems, which are modeled as a network of $M$ queues: $Q_1, ..., Q_M$ illustrated in Fig. 1. Each queue represents an application tier.

A request can take multiple visits to each queue during its overall execution, thus, there are transitions from each queue to its successor and its predecessor as well. Namely, a request from queue $Q_m$ either returns to $Q_{m-1}$ with a certain probability $p_m$, or proceeds to
The input parameters of the algorithm are \( N \) the number of customers, \( M \) the number of tiers, \( Z \) the average user think time, \( V_m \) the visit number and \( S_m \) the average service time for \( Q_m \) \( (1 \leq m \leq M) \). The output parameters are \( \tau \) the throughput, \( R \) the response time, \( R_m \) the response time for \( Q_m \) and \( L_m \) the average length of \( Q_m \).

The initialization is \( L_m = 0 \) \( (1 \leq m \leq M) \). The algorithm introduces the customers into the queueing network one by one \( (1 \leq n \leq N) \). The cycle terminates when all the customers have been entered. The steps of an iteration are the follows:

\[
R_m = V_m \cdot S_m \cdot (1 + L_m), \quad (1)
\]

\[
\tau = \frac{n}{Z + \sum_{m=1}^{M} R_m}, \quad (2)
\]

\[
L_m = \tau \cdot R_m. \quad (3)
\]

Nowadays the Microsoft .NET became one of the most prominent technologies of web-based software systems. Consider that a client is requesting a service from the server, the request goes through several subsystems before it is served (see Fig. 2). Through the settings of the application server the performance can be affected [17].

From the IIS (Internet Information Services), the accepted HTTP connections are placed into a named pipe. This is a global queue between IIS and ASP.NET, where requests are posted from native code to the managed thread pool. From the named pipe, the requests are placed into an application queue, also known as a virtual directory queue. There is one queue for each virtual directory.

The number of requests in these queues increases if the number of available worker and I/O threads falls below the limit specified by \textit{minFreeThreads} property. When the limit is reached, the requests are rejected.

The thread pool configuration options are the following. According to the connections and limitations, the partition of the .NET thread pool is shown in Fig. 3.

The \textit{maxWorkerThreads} attribute means the maximum number of worker threads, the \textit{maxIOThreads} parameter is the maximum number of I/O threads in the .NET thread pool. These attributes are automatically multiplied by the number of available CPUs.

The \textit{minFreeThreads} attribute limits the number of concurrent requests, because all incoming requests will be queued if the number of available threads in the thread pool falls below the value for this setting. The \textit{minLocalRequestFreeThreads} parameter is similar to \textit{minFreeThreads}, but it is related to requests from localhost (for example a local web service call). These two attributes can be used to prevent deadlocks by ensuring that a thread is available to handle callbacks from pending asynchronous requests.

\[
\tau = \frac{n}{Z + \sum_{m=1}^{M} R_m}, \quad (2)
\]

\[
L_m = \tau \cdot R_m. \quad (3)
\]
Performance metrics are influenced by many factors. Several papers have investigated various configurable parameters, how they affect the performance of web-based information systems. Statistical methods and hypothesis tests are used to retrieve factors influencing the performance. An approach [18] applies analysis of variance, another [19], [20], [21] performs independence test. The thread pool attributes are performance factors, these identified factors must be modelled to improve performance models.

3 Evaluation Algorithm Enhancement Based on Thread Pool

The MVA algorithm can be effectively enhanced by taking into account the behavior of the thread pool, as well.

Consider that the actual request contains CPU as well as I/O (input/output) calls. In case of multiple threads, I/O calls do not block the CPU, because the execution can continue on other non-blocked threads. This enables handling I/O requests and executing CPU instructions simultaneously. Therefore, in the MVA algorithm, to update the queue length a modified form of (3) must be used in the input, output and I/O tiers. For the CPU tiers this equation preserves its original form.

\[ L_{I/O} = \tau \cdot R_{I/O} - \frac{S_{CPU}}{S_{I/O}} \cdot L_{CPU}, \]  

where the CPU index means a CPU tier index and the I/O index corresponds to an I/O tier index from 1 ≤ m ≤ M. Additionally, \( L_{CPU} \) is from the previous iteration step, namely, \( L_{CPU} \) is updated only after this step. After (4), one more checking step is necessary:

\[ \text{if } L_{I/O} < 0 \text{ then } L_{I/O} = 0. \]  

\[ \text{Proof: } \text{Consider that the CPU queue and the I/O queue contain some requests respectively. If a new request arrives, it must wait until each previous request has been handled in the CPU queue. It takes } L_{CPU} \cdot S_{CPU} \text{ time unit to handle each CPU request. During this time the requests residing in the I/O queue can be handled simultaneously, because the CPU and the I/O devices can work in parallel. If an I/O call occurs, the CPU has to wait until the I/O device handles the request. In case of multiple threads the CPU does not have to wait, because the execution can continue in another thread as depicted in Fig. 4. Namely, during this time interval } S_{CPU}/S_{I/O} \cdot L_{CPU} \text{ I/O requests can be handled, which affects that the number of requests waiting in the I/O queue decreases. This difference should be subtracted from the number of requests waiting in the I/O queue calculated by the MVA.} \]

\[ \text{After this equation one more checking step is necessary, because the average length of a queue cannot be negative. Thus, if the obtained } L_{I/O} \text{ value is less than 0, the } L_{I/O} \text{ must be equal to 0.} \]

\[ \text{Our enhancement has a considerable effect on the predicted response time and throughput if the } S_{CPU}/S_{I/O} \text{ coefficient is great (near to 1 or greater than 1). If this coefficient is small (smaller than 1), the output of the MVA and the proposed enhanced MVA are closer (see Fig. 5).} \]

\[ \text{In other parts, since in the difference (4) the} \]
4 Performance Prediction with the Proposed Algorithm

In this section, performance metrics are predicted with our enhanced algorithm.

It is worth decomposing a web application to multiple tiers (presentation, business logic, database layers) as depicted in Fig. 7, and modeling it according to Fig. 1, because the service time of each tier can be very different. The presentation tier (UI) is an output tier and the database tier (DB) corresponds to an input/output tier. Thus, the proposed enhancement must be applied in these tiers. The business logic layer (BLL) corresponds to a CPU tier.

Firstly, the MVA and the proposed enhanced MVA algorithm for closed queueing networks have been implemented with the help of MATLAB.

Secondly, the input values of the algorithms have been estimated in both environments from one-one measurement. The inputs of the script are the number of tiers $M$, the maximum number of customers (simultaneous browser connections), the average user think time $Z$, the average service times $S_m$ and the visit numbers $V_m$ for $Q_m$ ($1 \leq m \leq M$).

During the measurement the number of tiers was constant (three). The maximum number of customers means that the load was characterized as follows: we started from one simultaneous browser connection, then we continued with two, until the maximum number (arbitrary, in this case it is set to 60) had been reached.

Finally, the model has been evaluated to predict performance metrics. The script computes the response time and the throughput up to a maximum number of customers.

**Proposition 2** The proposed algorithm can be applied for performance prediction. The response time and throughput performance metrics can be predicted with the improved algorithm.

**Proof:** Evaluating the queueing model (Fig. 1) with the proposed algorithm, the predicted response time and throughput performance metrics are as follows. The predicted throughput is depicted in Figs. 8 and 9, the predicted response time can be seen in Figs. 10 and 11.

In this case the proposed enhancement has a considerable effect on the predicted response time and throughput, because a $S_{CPU}/S_{I/O}$ coefficient is large, thus high curvatures can be observed in the response time and in the throughput, as well (see Figs. 9, 9, 10 and 11).

5 Convergence and Limit Analysis

In this section, the convergence and the limit of the original and the enhanced algorithms are analyzed and proved.

The first (finite many) elements of a sequence can be eliminated, substituted, modified, etc., the convergence, non-convergence, and the limit of the sequence do not change.
Let $D_m$ denote the service demand for $Q_m$, namely $V_m \cdot S_m = D_m$ ($1 \leq m \leq M$), and $D_{max}$ corresponds to the maximum value of service demands.

If the steps (1), (2), and (3) of the recursive MVA are substituted to each other, the following equation can be derived for throughput:

$$\tau(n) = \frac{\sum_{m=1}^{M} D_m + D_m^2 \tau(n-1) + \ldots + D_m^n \tau(n-1) \tau(n-2) \ldots \tau(1)}{n},$$

in other form:

$$\tau(n) = \frac{n}{\left( \sum_{i=2}^{n} \left( D_1^i + D_2^i + \ldots + D_M^i \right) \prod_{j=1}^{i-1} \tau(n-j) \right)}.$$  \hspace{1cm} (6)\

(7)

**Proposition 3** The throughput sequence of the original algorithm is convergent.

**Proof:** Every bounded and monotonic sequences are convergent. Firstly, the bounded above property of the throughput sequence, secondly, the monotonic increasing property of the throughput sequence will be proved. Finally, the limit of the convergent throughput will be determined.

The bounded above property is proved by mathematical induction. The above bound is $1/D_{max}$.

**Step 1** $\tau(1) < 1/D_{max}$, because

$$\tau(1) = \frac{1}{D_1 + \ldots + D_m \ldots + D_M} < \frac{1}{D_{max}},$$

(8)
Since every $D_i$ (service demands) are positive.

Step 2 Let assume that $\tau(2), \tau(3), ..., \tau(n-1) < 1/D_{\max}$, as well.

Step 3 Let prove that $\tau(n) < 1/D_{\max}$ using the assumption. Proof by contradiction: $\tau(n) \geq 1/D_{\max}$. $\tau(n)$ is determined as (7), thus

$$ nD_{\max} \geq \sum_{i=1}^{n} (D_1^i + ... + D_M^i) \prod_{j=1}^{i-1} \tau(n-j) \quad (9) $$

must be proved. With the assumption, the left side of the inequality can be estimated, which is greater than $n \cdot D_{\max}$, hence this is paradox.

Our result has shown that the throughput sequence is bounded above by $1/D_{\max}$.

The monotonic increasing property is proved direct: $\tau(n-1) < \tau(n)$. Using (7) the following inequality can be derived:

$$ (n-1) \cdot \left( \sum_{i=1}^{n} (D_1^i + ... + D_M^i) \prod_{j=1}^{i-1} \tau(n-j) \right) $$

$$ < n \cdot \left( \sum_{i=1}^{n-1} (D_1^i + ... + D_M^i) \prod_{j=1}^{i-1} \tau(n-j) \right) \quad (10) $$

Using the bound proved previously, the following inequality is true, because each difference is positive except the difference with max, which is zero:

$$ \sum_{i=1}^{n} \sum_{m=1}^{M} D_{m,i} D_{\max}^{n-i} - D_{\max}^{n} > 0. $$

Our result has shown that the throughput sequence is monotonic increasing. Thus, the throughput is bounded above and monotonic increasing, hence the throughput sequence is convergent. □

Proposition 4 The throughput of the original algorithm converges to $1/D_{\max}$.

Proof: Let $A$ denote the limit of the throughput ($n \to \infty$), namely $\tau(n), \tau(n-1), ... = A$. Thus, the (7) can be expressed as (divided both side of the equation by $A$):

$$ 1 = \frac{n}{\sum_{i=1}^{n} (D_1^i + ... + D_M^i) A^i}, \quad (11) $$

Let $A$ equals to $1/D_{\max}$, then the right side of the equation is:

$$ \sum_{i=1}^{n} \left( \frac{D_1^i}{D_{\max}^i} + ... + \frac{D_M^i}{D_{\max}^i} \right)^i \quad (12) $$

If $D_k \neq D_{\max}$, then $\left( \frac{D_k}{D_{\max}} \right)^n \to 0$. If $D_k = D_{\max}$, then $\sum_{i=1}^{n} \left( \frac{D_k}{D_{\max}} \right)^i = \sum_{i=1}^{n} \left( \frac{D_{\max}}{D_{\max}} \right)^i = \sum_{i=1}^{n} 1^i = n$. Thus the denominator is $n$. Therefore the right side of the (11) is 1, and the left side is 1, too.

In other words, the limit of the throughput in the original algorithm is truly $1/D_{\max}$. □

Proposition 5 The throughput of the enhanced algorithm converges to $1/D_{\max}$.

Proof: If the steps (1), (2), (3) in CPU tiers, and (4) in I/O tiers of the enhanced recursive MVA algorithm are substituted to each other, in addition, let $A$ denote the limit of the throughput ($n \to \infty$), namely $\tau(n), \tau(n-1), ... = A$, the following equation can be derived for the limit of throughput (divided both side of the equation by $A$):

$$ 1 = \frac{n}{\sum_{i=1}^{n} (D_1^i + ... + D_M^i) A^i - \sum_{i=1}^{n} \sum_{j=1}^{n} (D_1^j + ... + D_M^j) A^j \vartheta(n-i)} = \frac{n}{\sum_{i=1}^{n} \sum_{j=1}^{n} (D_1^j + ... + D_M^j) A^j \vartheta(n-i)} \quad (13) $$

where $1, 2, ..., M \neq CPU$ means, that only I/O tiers are in the second sum in the denominator, furthermore $\vartheta(n-i)$ corresponds to the enhancement in the queue length, namely, $\vartheta(n-i) = S_{CPU} / S_{I/O} \cdot L_{CPU}(n-i-1)$.

If in (13) there are only CPU tiers, i.e. the enhancement must not be applied, the negative sum is 0, hence the original form (11) can be derived.

Let $A$ equals to $1/D_{\max}$, then the right side of the equation are analyzed. The first sum in the denominator is $n$ (see the proof of Proposition 4).

The second sum in the denominator is $0$. The prove is the following. Two cases must be distinguished.

Firstly, if the $D_{\max}$ is in a CPU tier, the product of service demands and limit $- (D_1^i + ... + D_M^i) A^i$, where $A = 1/D_{\max}$ is 0, because only I/O service demands are in the sum, and the maximum service demand is in a CPU tier, thus in case of $\forall k$, $\left( \frac{D_k}{D_{\max}} \right)^n \to 0$. If each coefficient of the $\vartheta$ is 0, then the entire second sum is 0.

Secondly, if the $D_{\max}$ is in an I/O tier, the $\vartheta$ is 0, because $\vartheta$ can be expressed similar as $\sum_{i=1}^{n} (D_1^i + ... + D_M^i) A^i$, but only CPU tiers are in the sum, since $\vartheta(n-i) = S_{CPU} / S_{I/O} \cdot L_{CPU}(n-i-1)$. In addition the maximum service demand is in an I/O tier, thus in case of $\forall k$, $\left( \frac{D_k}{D_{\max}} \right)^n \to 0$. If each coefficient of the product of service demands and limit $- (D_1^i + ... + D_M^i) A^i$ is 0, then the entire second sum is 0.

Thus the denominator is $n$. Therefore the right side of the (13) is 1, and the left side is 1, too.
The convergence of the original and the enhanced algorithms

In other words, the limit of the throughput in the enhanced algorithm is truly $1/D_{\text{max}}$. □

Our results have shown that the throughput sequence of the original algorithm is bounded above and monotonic increasing, thus the throughput is convergent, in addition the limit of the throughput in the original and enhanced algorithms is the inverse value of the maximum service demand.

The speed of the converge can be seen in Fig. 12. The original algorithm converge to $1/D_{\text{max}}$ (from below) very quickly (after a few customers). The enhanced algorithm converge to $1/D_{\text{max}}$ (from above) very slowly. If the proposed $S_{\text{CPU}}/S_{I/O}$ coefficient is small, then the enhanced algorithm is closer to the original algorithm, because (4) converges to the (3) depicted in Fig. 5, as well.

6 Performance Measurements

Three-tier ASP.NET web applications have been implemented in different ASP.NET environments (Fig. 7).

The web server of our web application was Internet Information Services (IIS) 6.0. The server runs on a 2.8 GHz Intel Pentium 4 processor with Hyper-Threading technology enabled. It had 1GB of system memory; the operating system was Windows Server 2003 with Service Pack 1.

There were two environments. In the first environment the application server was ASP.NET 1.1 runtime environment, the database management system was Microsoft SQL Server 2000 with Service Pack 3. In the second environment ASP.NET 2.0 and Microsoft SQL Server 2005 with Service Pack 1 were used.

7 Experimental Validation

Finally, the proposed algorithm are validated and the correctness of the performance prediction with the proposed algorithm are verified in different ASP.NET environments.

A typical web application has been tested with concurrent user sessions, comparing the observed and predicted values in order to validate the proposed algorithm.

While the number of simultaneous browser connections varied, the average response time and throughput were measured in the two environments. As illustrated in Fig. 13, the new version of ASP.NET runtime environment and SQL Server database management system are slightly faster than the older version, but the character of the curve shapes are very...
The results correspond to the common shape of the response time and throughput performance metrics (see Fig. 13). Increasing the number of concurrent clients, the response time grows, since the queue limits are enough large, thus, all of the requests can be successfully served.

**Proposition 6** With performance measurements, the correctness of the proposed algorithm is validated, the correctness of the performance prediction with the proposed algorithm is verified.

**Proof:** Our proposed algorithm was experimentally validated by comparing the observed, the predicted values with MVA, and the predicted values with our enhanced MVA.

Our results (depicted in Figs. 14, 15, 16 and 17) have shown that the output of the proposed algorithm approximates the measured values much better than...
8 Error Analysis

Finally, error is analyzed to verify the correctness of the performance prediction with the proposed algorithms. Two methods are applied: the average absolute error function and the error histogram.

The average absolute error function is defined as follows:

\[
error_{MVA-obs} = \sum_{i=1}^{n} \left| RT_{MVA_i} - RT_{obs_i} \right| / n,
\]

(14)

\[
error_{EnhMVA-obs} = \sum_{i=1}^{n} \left| RT_{EnhMVA_i} - RT_{obs_i} \right| / n,
\]

(15)

where \( RT \) means response time, \( MVA \) index corresponds to the MVA evaluation algorithm, \( EnhMVA \) index is enhanced MVA algorithm, \( obs \) index corresponds to the observed values, and \( n \) is the maximum number of customers, namely, the number of measurements.

Proposition 7 The average absolute error of the proposed algorithm is less than the average absolute error of the original algorithm.

Proof: The results of the average absolute error function are presented in Table 1. It can be seen that the error of the enhanced algorithm is substantially less in both environments than the error of original MVA algorithm.

Table 1: The results of the average absolute error function

<table>
<thead>
<tr>
<th></th>
<th>ASP.NET 1.1</th>
<th>ASP.NET 2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVA</td>
<td>340.4833</td>
<td>309.4667</td>
</tr>
<tr>
<td>Enhanced MVA</td>
<td>62.8500</td>
<td>38.5333</td>
</tr>
</tbody>
</table>

The error histograms used 10 bins are depicted in Figure 18 and 19. In case of the original MVA algorithm, the histograms are Gaussian in shape. With the enhanced algorithm, the most of the errors is in the first bin, and it continuously decreases.

The error analysis has verified the correctness of the performance prediction with the proposed algorithm, namely, the enhanced algorithm predicts the performance metrics much more accurately than the original MVA algorithm.
9 Conclusions

In this paper the Mean-Value Analysis algorithm has been improved to model the behavior of the thread pool. The proposed algorithm can be applied for performance prediction. In addition, the convergence and limit of the algorithms have been investigated.

The paper introduces and verifies a multi-tier queueing network model to model web-based software systems. The model can be used for performance prediction in ASP.NET environments. The validity of the model, the correctness of the proposed evaluation algorithm, furthermore, the correctness of the performance prediction have been proven with performance measurements.

In order to illustrate the practical applications of the results, the proposed method has been applied in real systems. This method facilitates the efficient performance prediction of web-based software systems.

References:

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