Decentralized Adaptive Fuzzy Control for a class of Nonlinear Systems

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Abstract: In this paper, stable direct and indirect decentralized adaptive fuzzy controls are proposed for a class of large-scale nonlinear systems with the strong interconnected. The feedback and adaptive mechanisms for each subsystem depend only upon local measurements to provide asymptotic tracking of a reference trajectory. In both approaches, the proposed controllers are used to approximate the unknown subsystems. In addition, each subsystem is able to adaptively compensate for interconnections without known bounds. Simulation results are given to illustrate the tracking performance of the proposed methods.

Keywords: Decentralized Control, Fuzzy Controller, Interconnected Nonlinear System.

1 Introduction

Decentralized adaptive control systems often arise from various complex situations where there exist physical limitations on information exchange among several systems for which there is insufficient capability to have a single control controller, and due to the physical configuration and high dimensionality of interconnected systems a centralized control is neither economically feasible nor even necessary. Therefore, the decentralized scheme is preferred in control design of large-scale interconnected systems [1], [2], [9]. To control a large-scale system, one essential problem is how to handle the interactions among different systems. Intensive research has been devoted to the observer design for large-scale systems. Uncertainties in a large-scale system require the adaptive decentralized technique, for which many decentralized adaptive schemes have been developed, including the model reference adaptive control [1],[4], and nonlinear control with a special class of interconnections [7]. These approaches focus on stabilisation, where the dynamics of subsystems are assumed to be known or to be linear with a set of unknown parameters. However, in practice, large-scale systems may contain significant uncertainties, and/or with unknown parameters in nonlinear forms and unknown structures.

Fuzzy logic control as one of the most useful approaches for utilizing expert knowledge, has been an active field of research the past decade [8],[11]. Fuzzy logic control is generally applicable to plants that are mathematically poorly modelled and where experienced operators are available for providing qualitative guidance. The most important advantage of fuzzy-logic-control schemes lies in the fact that the developed controllers can deal with increasingly complex systems and controllers without precise knowledge of the model structure of the underlying system. Recently there have been significant research efforts on these issues in fuzzy control system [8],[11],[12] but these approaches work only for large-scale systems with a known or linear dynamics with a set of unknown parameters and bounds interconnections. In practice, however, not all states are usually available.

This paper presents two approaches which can easily tackle the output tracking control problem of a class of large-scale nonlinear system with unknown interconnections bounds. A direct adaptive approach approximates unknown control laws required to stabilize each subsystem, while an indirect adaptive is provided which identifies the isolated subsystem dynamics to produce a stabilizing controller. Both approaches ensure asymptotic tracking using only local measurement.

The organization of this paper is as follows: section 2 describes the problem under investigation; section 3, the direct adaptive decentralized control; while in section 4, we introduce the indirect approach. Experimental results are then used to demonstrate the effectiveness of the proposed approaches is presented in section 5, with a conclusion given in section 6.
2 Problem Formulation
Consider a class of nonlinear interconnected SISO subsystems $S_i$ ($i = 1, 2, \ldots, N$) described as follows:

$$
\begin{align*}
\dot{x}_{i,j} &= x_{i,j} \\
\dot{x}_{i,n} &= f_i(x_i) + g_i(x_i)u_i + \Delta_i(x) \\
y_i &= x_{i,1}
\end{align*}
$$

where $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$, $x_i \in \mathbb{R}^{n_i}$, is the global state vector, $u_i(t) \in \mathbb{R}$ is the control signal input and $y_i \in \mathbb{R}$ is the output of the plant for the subsystem $S_i$. The functions $f_i(.)$ and $g_i(.)$ are unknown and nonlinear, and $\Delta_i(x) \in \mathbb{R}^n$, are interconnection among subsystems unknown ($i = 1, 2, \ldots, N$).

The tracking error for $S_i$ is defined by $e_{i0} = y_{ir} - y_i$. Our objective is to design an adaptive control for each subsystem which will cause the output $y_i$ to track a desired output trajectory $y_{ir}$ ($i.e., e_{i0} \to 0$) in the presence of the strong interconnections using only local measurements.

**Assumption 1:** Let the scalars $q_{ij}$ quantify the strength of the interconnections and the output vector for the $i^{th}$ subsystem be defined by $e_i = [e_{i0}, e_{i1}, \ldots, e_{i(d_i-1)}]^T$; it is assumed that the interconnections satisfy:

$$
|\Delta_i(x)| \leq \sum_{j=1}^{N} q_{ij} \|e_j\|_2
$$

where $\|.\|_2$ is the Euclidean vector norm. This assumption on the interconnections can be satisfied by a variety of decentralized nonlinear systems. For instance, in [10] it is shown to be satisfied for an intervehicle spacing regulation problem in a platoon of an automated highway system.

Subsystem (1) can be expressed as:

$$
\ddot{\varsigma}_i = \Lambda_{i0}\dot{\varsigma}_i + b_i[f_i(x_i) + g_i(x_i)u_i + \Delta_i(x)]
$$

where

$$
\Lambda_{i0} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \cdots & \cdots & \cdots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix},
\quad b_i = \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix}
$$

and

$$
\dot{\varsigma}_i = [\dot{s}_{i,1}, \dot{s}_{i,2}, \ldots, \dot{s}_{i,d_i}]^T, \quad \varsigma_i = [s_{i,1}, s_{i,2}, \ldots, s_{i,d_i}]^T, \quad s_{i,1} = y_i, \quad s_{i,2} = \dot{y}_i, \ldots, \quad s_{i,d_i} = y_{ir(i-1)}
$$

Assume that the given reference $y_{ir}$ is bounded and have up to $d_i-1$ bounded derivatives. The reference vector is denoted as $Y_{ir} = [y_{ir}, \dot{y}_{ir}, \ddot{y}_{ir}, \ldots, y_{ir(d_i-1)}]^T$.

Define the tracking error of the $i^{th}$ subsystem as $e_{i0} = y_{ir} - y_i$. Then the error vector of the $i^{th}$ subsystem is given by $e_i = [e_{i0}, e_{i1}, e_{i2}, \ldots, e_{i(d_i-1)}]^T$.

It is desired that the output error of the $i^{th}$ subsystem follow

$$
e_i^{(d_i)} + k_{i,d_i-1} e_i^{(d_i-1)} + \ldots + k_{i,0} e_i^{(0)} = 0, \quad \text{here the coefficients are picked so that each:}

$$
\hat{L}_i = s_{i,d_i} + k_{i,d_i-1}s_{i,d_i-1} + \ldots + k_{i,0}
$$

has its roots in the open left-half complex plane (in Hurwitz).

If the subsystem $S_i$ is well known ($g_i(x_i) \neq 0$) and free of external disturbances; ($\Delta_i(x) = 0$), then the primary control should be designed to have the following idealized control law:

$$
u_i^* = \frac{1}{g_i(x_i)}[-f_i(x_i) + K^T e_i + y_{ir}^{(d_i)}]
$$

In spite of the primary control (6) which mathematically cancels the given system and then places it in a stabilizing part, so as to guarantee $\lim_{t \to \infty} e_i = 0$, it is clear that, in practice, an exact cancellation of the given system nonlinearity is theoretically unrealizable and physically impossible. Thus, in this study, the direct adaptive approach implements an adaptive fuzzy system to approximate the idealized control action, and with the indirect approach we approximate the unknown dynamics for each subsystem ($f_i(x_i)$ and $g_i(x_i)$).

3 Direct adaptive fuzzy decentralized control
In this section, a direct adaptive output-feedback fuzzy decentralized controller is designed, with guaranteed stability of the integrated closed loop system.

Assume that in subsystem (1) $g_i(x_i) \neq 0$. The direct adaptive controller is designed as:

$$
u_i = u_i(x_i, \theta_i) = \theta_i^T \phi_i(x_i), \quad \theta_i = [\theta_{i1}^T, \theta_{i2}^T, \ldots, \theta_{im_i}^T]^T
$$

where $u_i(x_i, \theta_i) = \hat{L}_i^T \phi_i(x_i)$, $\theta_i = [\theta_{i1}^T, \theta_{i2}^T, \ldots, \theta_{im_i}^T]^T$ are parameter vectors and $\phi_i(x)$ is a regressor with regressors $\phi_i^l$ ($1 \leq l \leq m_i$, where $m_i$ is the number of rules), which is defined as a fuzzy basis function [5]. The term $a_i(t)e_i^T p_i b_i$ is used to compensate unknown effects from the
interconnections \( p_i \in \mathbb{R}^{d_i \times d_i} \) is a positive definite matrix defined by a Lyapunov matrix equation and \( b_i \in \mathbb{R}^{d_i} \) is a vector), and \( u_{ih} \) is auxiliary control compensation.

Substituting (7) to (1), and adding \( g_i(x_i)u_i^* \) and then subtracting \( g_i(x_i)u_i^* \) on the right-hand side of (1), we obtain,

\[
y_i^{(d)} = g_i(x_i)u_i(x_i, \Theta_i) - u_i^* + a_i(t)e_i^T p_i b_i + u_{ih} + K_i^T e_i + y_i^{(d)} + \Delta(x)
\]

The error dynamics for the \( i^{th} \) subsystem may be expressed as:

\[
e_i^{(d)} = y_i^{(d)} + g_i(x_i)(u_i^* - u_i(x_i, \Theta_i)) - a_i(t)e_i^T p_i b_i - u_{ih} - K_i^T e_i
\]

Substituting (7) to (1), and adding compensation.

Assumption 2: Define the optimal parameter vectors and fuzzy approximation error as:

\[
\theta^*_i = \arg \min_{\theta_i} \left\{ \max_{\phi_i(x_i)} \sup_{x_i, e_i} \left[ y_i^* - u_i(x_i, \Theta_i) \right] \right\}
\]

where \( \Omega_i = \left\{ \theta_i : \theta_i^T \theta_i \leq M_i \right\} \) is the convex compact set which contains feasible parameter sets for \( \theta^*_i \).

Define the parameter error as \( \Phi_i = \theta^*_i - \theta_i \).

In the analysis to follow, we will use the fact that \( u_i^* - u_i(x_i, \Theta_i) = \Phi_i^T \phi_i(x_i) \), where \( \Phi_i = [\Phi_{i,1}, \Phi_{i,2}, ..., \Phi_{i,m_i}]^T \) are parameter vectors, \( \phi_i(x_i) = [\phi_{i,1}, \phi_{i,2}, ..., \phi_{i,m_i}]^T \) are defined above which is defined as a fuzzy basis function [5].

The dynamics equations of the \( i^{th} \) subsystem can be written as:

\[
\dot{e}_i = \Delta e_i + b_i [g_i(x_i)\Phi_i^T \phi_i(x_i) + \beta_i(x_i)]w_i
\]

and \( w_i \) represent the fuzzy approximation error of the \( i^{th} \) subsystem.

Assumption 2: We assume that, there exists a function \( T_{w_i} (z_i) > 0 \) such that:

\[
|\beta_i(x_i)|w_i | < T_{w_i} (z_i) \quad \forall 1 \leq i \leq N
\]

The direct adaptive fuzzy decentralized control that we have proposed in (7) can be classified as:

\[
\theta_i = \eta_i \text{ Pr of}[\cdot]
\]

where \( \text{Pr of}[\cdot] \) is the projection operator [5]

\[
u_{ih} = T_{w_i} (z_i) \text{ sign}(e_i^T p_i b_i)
\]

\[
d_i = \eta_{d_i} (e_i^T p_i b_i)^2
\]

where \( \eta_i > 0 \), and \( \eta_{d_i} > 0 \) are fixed adaptive gains.

**Theorem 1:** Consider the nonlinear subsystem (1), suppose that assumptions 1-2 are satisfied. If there exists a matrix \( p_i = p_i^T > 0 \) satisfying the Lyapunov equation: \( A_i^T p_i + p_i A_i + Q_i = 0 \) where \( Q_i = Q_i^T > 0 \).

The adaptive fuzzy decentralized controller law is chosen as (7) with parameter adaptation law (13)-(15). Then the proposed fuzzy decentralized control scheme can guarantee that (i) all the variables of the closed-loop system are bounded and (ii) performance tracking is achieved.

**Proof:** consider the following Lyapunov function for the \( i^{th} \) subsystem:

\[
v_i = e_i^T p_i e_i + \frac{1}{\eta_i} \Phi_i^T \Phi_i + \frac{1}{\eta_{d_i}} \Phi_i^T \Phi_i
\]

where \( \Phi_i = \theta_i^* - \theta_i \), \( \Phi_{d_i} = a_i - e_i^* \), \( \tau_i \) well be defined shortly , and each \( p_i \in \mathbb{R}^{d_i \times d_i} \) is a positive definite and symmetric matrix.

Taking the time derivative of \( v_i \), yields

\[
\dot{v}_i = (e_i^T p_i e_i + e_i^T p_i e_i) + \frac{2}{\eta_i} \Phi_i^T \Phi_i + \frac{1}{\eta_{d_i}} \Phi_i^T \Phi_i
\]

Substituting (11) into (17), applying (12)-(14) and choosing \( \zeta_i(t) = e_i^T p_i b_i \) yields

\[
\dot{v}_i = -e_i^T \Phi_i e_i - \tau_i (e_i^T p_i b_i)^2 + 2e_i^T p_i b_i \frac{\Delta(x)}{\tau_i} + (\frac{\Delta(x)}{\tau_i})^2
\]

\[
\dot{v}_i = -e_i^T \Phi_i e_i - \tau_i (e_i^T p_i b_i + \frac{\Delta(x)}{\tau_i})^2
\]

so that if each \( \tau_i > 0 \), we simply obtain

\[
\dot{v}_i \leq - \frac{1}{\tau_i} (\Delta_i(x))^2
\]

Now consider the composite system Lyapunov candidate \( V = \sum_{i=1}^{N} e_i v_i \) where each \( e_i > 0 \), yields

\[
\dot{V} \leq \sum_{i=1}^{N} e_i [-e_i^T \Phi_i e_i + \frac{1}{\tau_i} (\sum_{j=1}^{N} q_{ij} \| e_j \|^2)]
\]

Since

\[
\sum_{j=1}^{N} q_{ij} \| e_j \|^2 = \psi^T X_i
\]

where

\[
\psi = [\| e_1 \|^2, \| e_2 \|^2, ..., \| e_N \|^2]^T
\]
\(\chi_i = [q_{i1}, q_{i2}, ..., q_{iN}]^T\), let \(\lambda_i\) the real part of the eigenvalue of \(Q_i\), (20) may be written as
\[
\dot{V} \leq \sum_{i=1}^{N} e_i^T [-\lambda_i] e_i^T + \frac{1}{\tau_i} \bar{\gamma}^T \chi_i \chi_i^T \bar{\gamma}
\]  
(21)

Define \(K^* = [\tau_1^*, \tau_2^*, ..., \tau_N^*]\). Let \(\tau_i = \tau^* i = 1, 2, ..., N\) for some \(0 < \tau^*\). Define 
\[
D = \text{diag} \{e_1 \lambda_1, e_2 \lambda_2, ..., e_N \lambda_N\}
\]
and 
\[
M = \sum_{i=1}^{N} e_i \chi_i \chi_i^T
\]
so that \(\dot{V} \leq -\gamma^T A \gamma\), where 
\[
A = D - \frac{1}{\tau} M.
\]
Then for some sufficiently large \(\tau^*\) the matrix \(A\) is positive definite. The diagonal dominance property may be established using Gersgorin’s Theorem [3]. Now define 
\[
K^* = \text{arg min}_{\tau^*} \{K^T K^*: A = D - \frac{1}{\tau} M \text{ is positive definite}\}
\]
(22)

There exists sufficiently large \(\tau^*\) such that \(A\), defined by (22), is positive definite, which implies that \(V \in \ell_{\infty}\), and thus \(\|\gamma\|_2 \in \ell_{\infty}\). Also
\[
\int_0^\infty \gamma^T A \gamma dt \leq -\int_0^\infty \dot{V} dt + \text{const}
\]
(23)
so that \(\|\gamma\|_2 \in \ell_2\). Since all of the signals are well defined, we also have \(\dot{e}_i \in \ell_2\) so that 
\[
d / dt \|\gamma\|_2 = e_i^T \dot{e}_i / \|\gamma\|_2 \leq \|\gamma\|_2 \in \ell_{\infty}.
\]
Using Barbalat’s Lemma, we thus establish that 
\[
\lim_{t \to \infty} \|\gamma\|_2 = 0,
\]
thus we are guaranteed asymptotically stable tracking for each of the subsystems.

### 4 Indirect adaptive fuzzy decentralized control

In this section, it is assumed that the function \(f_i(x_i)\) and \(g_i(x_i)\) are unknown. Take a universal fuzzy system \(\hat{f}_i(x_i / \theta_i)\) with \(x_i \in U_{x_i}\) for some compact set \(U_{x_i}\) to approximate the uncertain term \(f_i(x_i)\) where \(\theta_i\) contains the tunable parameters. Here the linearly parameterized fuzzy model [8] is employed in the approximation procedure. Then we replace \(f_i(x_i)\) and \(!g_i(x_i)\) by the fuzzy system 
\[
\hat{f}_i(x_i / \theta_1)\) and 
\[
\hat{g}_i(x_i / \theta_{i2})
\]
respectively, with singleton fuzzifier, center average defuzzifier, and product inference. The fuzzy system \(\hat{f}_i(x_i / \theta_1)\) and \(\hat{g}_i(x_i / \theta_{i2})\) can be expressed as:
\[
\hat{f}_i(x_i / \theta_1) = \theta_1^T \phi_i(x_i)\) and 
\[
\hat{g}_i(x_i / \theta_{i2}) = \theta_{i2}^T \phi_i(x_i)\) for \(i = 1, 2, ..., N\)
(24)

where \(\theta_{ik} = [\theta_{ik1}, \theta_{ik2}, ..., \theta_{ikm}] \in \mathbb{R}_{m_i}\) is a parameter vector (\(k = 1, 2\)) and \(\phi_i(x_i) = [\phi_i(x_i), \phi_i^2(x_i), ..., \phi_i^{m_i}(x_i)] \in \mathbb{R}_{m_i}\) is a regressive vector with the regressor \(\phi_i(x_i)\) defined as,
\[
\phi_i(x_i) = \prod_{j=1}^{n_i} \mu_{F_{i,j}}(x_j)
\]
(25)
where \(\mu_{F_{i,j}}(\cdot)\) is the membership functions for 
\[1 \leq l \leq m_i\] (\(m_i\) is the number of rules) and \(1 \leq j \leq n_i\) in this paper, we present the decentralized adaptive fuzzy controller defined as 
\[
u_i = u_{ji} + \frac{1}{\hat{g}_i(x_i / \theta_{i2})} (-\hat{f}_i(x_i / \theta_1) + K_i^T e_i + y_{ij}(d_i) + a_i(t) e_i^T p_i b_i / 2 + u_{hi})
\]
(26)
The decentralized fuzzy controller \(u_{ji}\) is constructed from the following \(\prod_{k=1}^{d_i} n_{i,k}\) rules:
\[
R_i^{h_{i-1:k}}: \text{if } e_i \text{is } A_{i1}^{(k)} \text{ and } e_{i2} \text{is } A_{i2}^{(k)} \text{ and } ...
\]
and \(e_{i,d} \text{is } A_{i,d}^{(k)}\), then \(R_i^{h_{i-1:k}}\) is \(C_i^{h_{i-1:k}}\)
(27)
where \(n_{i,k}\) define the number of fuzzy sets \(A_{i,k}^{(k)}\) in \(U_{i,k}\) (\(1 \leq k \leq n_{i,k}\) and \(1 \leq k \leq d_i\)) such that for any \(e_i \in U_{i,k}\), there exists a fuzzy set \(A_{i,j}^{(k)}\) so that the memberships function \(\mu_{A_{i,j}^{(k)} (e_i)} \neq 0\). The centres of these fuzzy sets are adapted by the proposed low which will be defined below.

According to the universal approximation theorem [6], there exist optimal approximation parameters \(\theta_1^*\) and \(\theta_{i2}^*\) such that \(\hat{f}_i(x_i / \theta_1^*)\) and \(\hat{g}_i(x_i / \theta_{i2}^*)\) can, respectively, approximate \(f_i(x_i)\) and \(g_i(x_i)\) as best as possible. Define the optimal parameter vectors and fuzzy approximation errors
is given by the $\eta$ and $\theta$. The centre $= \theta$ is positive constant.

Throughout this section we need the following assumptions:

**Assumption 3:** There exists a positive function $\omega > M_{\omega} x_i$ such that

$$|f_i(x_i) + \Delta f_i(x_i)| \leq M_{\omega} x_i \quad \forall 1 \leq i \leq N$$

(31)

Substituting (26.) to (1), the tracking error dynamics can be written as

$$\dot{e}_i = \lambda e_i + b_i [f_i(x_i) - \hat{f}_i(x_i / \theta_i)]$$

$$+ (g_i(x_i) - \hat{g}_i(x_i / \theta_i)) u_i$$

$$- a_i(t) e_i^T p_i b_i / 2 - u_{ih} - \Delta_i(x)$$

(32)

where

$$\Lambda_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -K_{i0} & K_{i1} & \cdots & K_{id_i} & 1 \end{bmatrix}$$

(33)

from (24) and (30), (31) can be written as

$$\dot{e}_i = \lambda e_i + b_i [\Phi_{i1}^T \varphi_i(x_i)] + \Phi_{i2}^T \varphi_i(x_i) u_i + \Delta f_i(x_i) + \Delta g_i(x_i)$$

$$- a_i(t) e_i^T p_i b_i / 2 - u_{ih} - \Delta_i(x)$$

(34)

where the parameter error vectors are defined as

$$\Phi_{i1} = \theta_i^* - \theta_{i1}, \Phi_{i2} = \theta_i^* - \theta_{i2}$$

The following update laws are now defined for the decentralized indirect adaptive controller:

$$\dot{\theta}_{i1} = \eta_{i1} e_i^T p_i b_i \varphi_i(x_i)$$

(35)

$$\dot{\theta}_{i2} = \eta_{i2} e_i^T p_i b_i \varphi_i(x_i) u_i$$

(36)

$$u_{ih} = -M_{\omega} x_i \text{sign}(e_i^T p_i b_i)$$

(37)

$$\dot{a}_i = \eta_{ai} (e_i^T p_i b_i)^2$$

(38)

The vector

$$\Gamma_i^{l_1, \ldots, l_{d_i}} = (\Gamma_i^{l_1}, \Gamma_i^{l_2}, \ldots, \Gamma_i^{l_{d_i}})^T$$

is the centre average of $C_i^{l_1, \ldots, l_{d_i}}$. We define $\Gamma_i = (\Gamma_i^{l_1}, \Gamma_i^{l_2}, \ldots, \Gamma_i^{l_{d_i}})$ as the collection of $\Gamma_i^{l_1, \ldots, l_{d_i}}$'s for $l_1 = 1, 2, \ldots, M_{l_{d_i}}$; $l_{d_i} = 1, 2, \ldots, 1$. The centre of the $i^{th}$ set $C_i^{l_1, \ldots, l_{d_i}}$ is given by the proposed equation

$$\Gamma_i^{l_1, \ldots, l_{d_i}} = \begin{cases} \Gamma_i^{l_1, \ldots, l_{d_i}} \text{ if } \max_{j=1,2,\ldots,\eta} \mu_{a_j}(e_j) = 0 \\
\sum_{j=1,2,\ldots,\eta} e_j \mu_{a_j}(e_j) \text{ otherwise} \end{cases}$$

(40)

$\sigma_i$ is a positive constant.

**Theorem 2:** Consider the nonlinear subsystem (1), with the assumptions 1 and 3 are satisfied. If there exists a matrix $p_i = p_i^T > 0$ satisfying the Lyapunov equation:

$$A_i^T p_i + p_i A_i + Q_i = 0 \quad \text{where } Q_i = Q_i^T > 0.$$Then the proposed control (26) with adaptation laws (35-38) will ensure that, for $i = 1, 2, \ldots, N$, (i) all the variables of the closed-loop system are bounded and (ii) performance tracking is achieved.

**Proof:** take the error dynamic equation (34), and consider the Lyapunov function candidate

$$V_i = e_i^T p_i e_i + \frac{1}{\eta_{i1}} \Phi_{i1}^T \Phi_{i1} + \frac{1}{\eta_{i2}} \Phi_{i2}^T \Phi_{i2} + \frac{1}{2 \eta_{a_i}} \Phi_{ai}^T \Phi_{ai}$$

(41)

where $\Phi_{i1} = \theta_i^* - \theta_{i1}, \Phi_{i2} = \theta_i^* - \theta_{i2}, \Phi_{ai} = a_i - \epsilon_i^*$, $\epsilon_i^*$ well be defined shortly, and each $p_i \in N_{d_i}$ is a positive definite and symmetric matrix.
The time derivative of \( v_i \) along the error trajectory (41)
\[
\dot{v}_i = \dot{e}_i^T p_i e_i + e_i^T p_i \dot{e}_i + \frac{2}{\eta_i} \Phi_{i1}^T \Phi_i
\]
\[
+ \frac{2}{\eta_i} \Phi_{i2}^T \Phi_{i2} + \frac{1}{\eta_i} \Phi_{ai}^T \Phi_{ai}
\]  
(42)

Substituting (34) into (42) and choosing \( \zeta_i(t) = e_i^T p_i b_i \) we obtain:
\[
\dot{v}_i = e_i^T (\Lambda_i^T p_i + p_i \Lambda_i) e_i
\]
\[
+ 2e_i^T p_i b_i \Phi_{i1}^T \Phi_i(x_i) + \Phi_{i2}^T \Phi_i(x_i) u_i
\]
\[
+ \Delta_i^T(x_i) + \Delta g_i(x_i) u_i - a_i(t) e_i^T p_i b_i
\]
\[
- u_{ib} - \Delta_i(x_i)) + \frac{2}{\eta_i} \Phi_{i1}^T \Phi_i
\]
\[
+ \frac{2}{\eta_i} \Phi_{i2}^T \Phi_{i2} + \frac{1}{\eta_i} \Phi_{ai}^T \Phi_{ai}
\]  
(43)

Applying parameters adaptation laws (35)-(38), yields
\[
\dot{v}_i = \frac{1}{2} e_i^T Q e_i - 2e_i^T p_i b_i \Lambda_i(x_i)
\]
\[
- \tau_i^*(e_i^T p_i b_i)^2
\]  
(44)

\[
\leq \frac{1}{2} e_i^T Q e_i + \frac{1}{\tau_i} (\Lambda_i(x_i))^2
\]

now consider the composite system Lyapunov candidate
\[
V = \sum_{i=1}^{N} \phi_i v_i
given, where each \( \phi_i > 0 \). Taking the derivative of \( V \) gives
\[
\dot{V} \leq \sum_{i=1}^{N} \phi_i [-\lambda_i \psi_i + \frac{1}{\tau_i} \psi_i^T X_i \dot{X}_i \psi_i]
\]  
(45)

where \( \lambda_i \) is the real part of the eigenvalue of \( Q_i \).

Define \( K^* = [\tau_1^*, \tau_2^*, \ldots, \tau_N^*] \in \mathbb{R}^N \).

Let \( D = \text{diag}(\phi_1 Q_1, \phi_2 Q_2, \ldots, \phi_N Q_N) \) and
\[
M = \sum_{i=1}^{N} \phi_i X_i \dot{X}_i^T, \text{ so that } \dot{V} \leq -\psi^T A \psi, \text{ where}
\]
\[
A = D - \frac{1}{\tau_i^*} M. \text{ Then for some sufficiently large } \tau_i^* > 0, \text{ the matrix } A \text{ is positive definite. The remainder of the theorem (2) follows as for the direct adaptive case.}

5 Simulation results

A double-inverted pendulum connected by a spring can be considered as the simplified example of the large-scale system. Each pendulum may be positioned by a torque input \( u_i \) applied by a servomotor at its base. It is assumed that both \( \phi_i \) and \( \dot{\phi}_i \) (angular position and rate) are available to the \( i^{th} \) controller for \( i = 1, 2 \). Fig. 1.

![Fig.1. Two inverted pendulums connected by a spring.](image)

Consider a double-inverted pendulum model \([10]\). The equations which describe the motion of the pendulums are defined by
\[
\dot{x}_{11} = x_{12}
\]
\[
\dot{x}_{12} = \left( \frac{m_1 g r}{j_1} - \frac{k r^2}{4 j_1} \right) \sin(x_{11}) + \frac{k r}{2 j_1} (l - b)
\]
\[
+ \frac{u_1}{j_1} + \frac{k r^2}{4 j_1} \sin(x_{21})
\]  
(46)

\[
\dot{x}_{21} = x_{22}
\]
\[
\dot{x}_{22} = \left( \frac{m_2 g r}{j_2} - \frac{k r^2}{4 j_2} \right) \sin(x_{21}) + \frac{k r}{2 j_2} (l - b)
\]
\[
+ \frac{u_2}{j_2} + \frac{k r^2}{4 j_2} \sin(x_{12})
\]  
(47)

where \( x_{11} = \phi_1 \) and \( x_{21} = \phi_2 \) are the angular displacements of the pendulums from vertical. The parameters \( m_1 = 2 \text{ kg} \) and \( m_2 = 2.5 \text{ kg} \) are the pendulum end masses, \( j_1 = 0.5 \text{ kg} \) and \( j_2 = 0.625 \text{ kg} \) are the moments of inertia, the constant of connecting spring is \( k = 100N/m \), the pendulum height is \( r = 0.5 \text{ m} \), the natural length of the spring is \( l = 0.5 \text{ m} \) and the gravitational acceleration is \( g = 9.81 \text{ m/s}^2 \).

The distance between the pendulum hinges is defined as \( b = 0.4 \text{ m} \) (with \( b < l \) in this example, so that the pendulum links repel each other when both are in the upright position (Fig 1).

In (45) and (47)

\[
f_1(x_{11}) = \left( \frac{m_1 g r}{j_1} - \frac{k r^2}{4 j_1} \right) \sin(x_{11}) \right), \quad g_1(x_{11}) = 1/j_1 ;
\]

\[
f_2(x_{21}) = \left( \frac{m_2 g r}{j_2} - \frac{k r^2}{4 j_2} \right) \sin(x_{21}) \right), \quad g_2(x_{21}) = 1/j_2
\]
\[ \Delta_1(x) = \frac{kr}{2j_1} (l - b) + \frac{kr^2}{4j_1} \sin(x_{21}), \]
\[ \Delta_2(x) = \frac{kr}{2j_2} (l - b) + \frac{kr^2}{4j_2} \sin(x_{12}), \]

the motion equations fit the format of system (1). Here we will attempt to drive the angular positions to zero, so that \( e_i = -\phi_i \) (i.e., \( y_{1r} = y_{2r} = 0 \)) for \( i = 1, 2 \)

To construct the fuzzy approximators \( u_i(x_j, \theta_j) \) in (9) and \( \hat{\theta}_ij(x_i / \theta_j) \), \( \hat{\theta}_{ij}(x_i / \theta_{ij}) \) in (24), we define three fuzzy sets for component of each \( \xi_1 = (x_{11}, x_{12}) \) and \( \xi_2 = (x_{21}, x_{22}) \) with labels \( A_{x_{ij}}^1, A_{x_{ij}}^2, A_{x_{ij}}^3, A_{x_{ij}}^4, \) and \( A_{x_{ij}}^5 \) characterized by:

\[ \mu_{A_{x_{ij}}^1}(x_{ij}) = \exp(-(x_{ij} + 0.8)^2) \]
\[ \mu_{A_{x_{ij}}^2}(x_{ij}) = \exp(-(x_{ij} + 0.4)^2) \]
\[ \mu_{A_{x_{ij}}^3}(x_{ij}) = \exp(-x_{ij}^2) \]
\[ \mu_{A_{x_{ij}}^4}(x_{ij}) = \exp(-x_{ij} - 0.4)^2 \]
\[ \mu_{A_{x_{ij}}^5}(x_{ij}) = \exp(-x_{ij} - 0.8)^2 \]

with \( n_i, j = 5, j = 1, 2 \) and \( i = 1, 2 \).

Defining 25 fuzzy rules, in the following linguistic description:

\[ R_i^{(j)} : \text{if } x_{i1} \text{ is } A_{x_{ij}}^1 \text{ and } x_{i2} \text{ is } A_{x_{ij}}^2 \text{ then } y_i' \text{ is } C_i^j \]

Denoting \( D_i = \sum_{k=1}^{25} \prod_{k=1}^{2} \mu_{A_{x_{ik}}}(x_{ik}) \),
\[ \varphi_i(x_i) = [\varphi_{i1}(x_i) / D_1, \varphi_{i2}(x_i) / D_1, \ldots, \varphi_{i25}(x_i) / D_1]^T \]

we can construct the fuzzy system (7) and (24) respectively, as follows: we choose \( \eta_i = 0.01, \eta_{ai} = 0.001 \) and \( \eta_i = 0.001, \eta_{ai} = 0.0001, \eta_{ai} = 0.15, \sigma_i = 0.15, \sigma_i = 0.01, \sigma_i = 0.001 \), \( Q_i = \text{diag}(10, 10) \), and each \( A_i \) so that 

\[ \Lambda^T \Lambda = \varsigma^2 + 4\varsigma + 4 \] has roots at \((-2, -2)\).

Choose the initial conditions to be the same for both direct and indirect approaches in the simulations:

\( (x_{11}, x_{12}, x_{21}, x_{22})^T = (1,1,1,1)^T \), \( \theta_{11} = \theta_{12} = 0.25x_{11} \) and \( \Gamma_i = [-1 -0.75 -0.5 -0.25 0.25 0.5 0.75 1] \). For the direct approach, the simulation results are shown in Fig 2. The simulation results are given in Fig 3. Both direct and indirect fuzzy decentralized controllers achieve good performance, as can be seen from the simulation results.

Fig. 2, Control of the pendulums using the proposed direct adaptive decentralized technique.

Fig. 3, Control of the pendulums using the proposed indirect adaptive decentralized technique.

**Conclusion**

In the course of this paper, we have presented an adaptive output-feedback fuzzy decentralized control for a class of large-scale nonlinear systems. In both, direct and indirect adaptive proposed design methods, fuzzy logic systems are used to estimate the part the decentralized adaptive fuzzy controller and unknown nonlinear dynamics
without knowing bounds of interconnections. Furthermore, the stability of nonlinear interconnected systems is also guaranteed and ensures asymptotic tracking using only local measurement. The proposed approaches are simple without complex algorithms. Simulations have shown that the proposed controls methodology is effective, with guaranteed stability and satisfying performance.

References