Steady State Analysis of Self-Excited Induction Generator using Phasor-Diagram Based Iterative Model

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Abstract: – Induction generators in self excited mode are found to be suitable for remote and windy locations. Prior to installation there is a need to predict the behaviour of machine under all possible operating conditions. This is possible through steady state modeling of such generators. Steady state analysis of self excited induction generator (SEIG) needs an estimation of generated frequency and magnetizing reactance under all possible operating conditions. So far most of the researchers have used loop impedance, nodal admittance or iterative techniques to determine the steady state performance of such machines. In this paper a new model based upon phasor diagram of induction generator has been proposed to analyze the behaviour of self excited induction generator. Modeling results in a third order equation in generated frequency and a simple expression for magnetizing reactance. Complete mathematical analysis to derive the different expressions is presented here. Computed results have been compared with experimental results on test machines. Closeness between the two proves the validity of proposed modeling.

Keywords: - Non-conventional sources, Renewable energy, Steady state analysis, Self-excited induction generator (SEIG), Wind energy.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>per unit frequency</td>
</tr>
<tr>
<td>b</td>
<td>per unit speed</td>
</tr>
<tr>
<td>C</td>
<td>excitation capacitance per phase</td>
</tr>
<tr>
<td>E1</td>
<td>air gap voltage per phase at rated frequency</td>
</tr>
<tr>
<td>Err1</td>
<td>error in successive values of generated frequency</td>
</tr>
<tr>
<td>Err2</td>
<td>error in successive values of magnetizing reactance per phase</td>
</tr>
<tr>
<td>Err3</td>
<td>error in successive values of stator voltage per phase</td>
</tr>
<tr>
<td>I1</td>
<td>stator current per phase</td>
</tr>
<tr>
<td>I2</td>
<td>rotor current per phase, referred to stator</td>
</tr>
<tr>
<td>IC</td>
<td>capacitor current per phase</td>
</tr>
<tr>
<td>IL</td>
<td>load current per phase</td>
</tr>
<tr>
<td>Irc</td>
<td>core loss current per phase</td>
</tr>
<tr>
<td>Im</td>
<td>magnetizing current per phase</td>
</tr>
<tr>
<td>R</td>
<td>load resistance per phase</td>
</tr>
<tr>
<td>R1</td>
<td>stator resistance per phase</td>
</tr>
<tr>
<td>Rc</td>
<td>core loss resistance per phase</td>
</tr>
<tr>
<td>R2</td>
<td>rotor resistance per phase, referred to stator</td>
</tr>
<tr>
<td>V1</td>
<td>stator voltage per phase</td>
</tr>
<tr>
<td>X</td>
<td>load reactance per phase</td>
</tr>
<tr>
<td>X1</td>
<td>stator reactance per phase</td>
</tr>
</tbody>
</table>
$X_2$ rotor reactance per phase, referred to stator

$X_C$ capacitive reactance due to $C$ at rated frequency

$X_m$ magnetizing reactance per phase at rated frequency

1. Introduction

A rapid increase in power demand and continuous depletion of fossil fuels has diverted the attention of scientists from conventional energy sources to non-conventional energy sources. Wind energy is emerging as a potential source among various non conventional energy sources. Most of the countries across the world are promoting such wind energy generating units.

Induction generators with cage rotor are found to be most suitable for wind energy conversion due to their advantages such as simple and rugged construction, low cost and no need of synchronization with existing grid. These machines can be operated in grid connected as well as in self-excited mode. Induction generator in self-excited mode is found to be capable to generate the power even in the absence of power grid. This makes it to be most useful machine for the remote windy locations.

Various methodologies adopted for the analysis of SEIG by researchers [1-9] are:

- Loop impedance technique
- Nodal admittance technique
- Iterative technique

Above methods require either lengthy derivations or solution of nonlinear equations. In this paper an attempt has been made to estimate the generated frequency and magnetizing reactance for self-excited induction generator using a new strategy based upon phasor diagram of the machine. Proposed modeling results in the third order equation in generated frequency and a simple expression for magnetizing reactance. Comparison of computed and experimental results on test machines confirms the validity of proposed modeling.

2. Steady-State Analysis

The steady-state operation of the self-excited generator may be analyzed by using the equivalent circuit representation as shown in Fig.1. In this circuit all parameters are assumed to be constant except magnetizing reactance.

![Fig.1 Equivalent circuit representation.](image)

Analysis of Fig. 1 in the absence of any power source & with ‘$V_i$ & $E_i$’ as potential of node 1 & 2 gives.

$$
\frac{E_i}{jX_m} + \frac{aE_1}{R_c} + \frac{E_1 - \frac{V_i}{a}}{Z_1} + \frac{E_i}{R_2} = 0
\quad (1)
$$

Where, $Z_1 = \frac{(R_j/a)}{a} + jX_j$

$$
\frac{E_1 - \frac{V_i}{a}}{Z_1} + \frac{V_i}{a} + \frac{a}{a} = 0
\quad (2)
$$

Equation (1) and (2) gives;

$$
\frac{E_1}{jX_m} + \frac{aE_1}{R_c} + \frac{V_i}{a} + \frac{V_i}{a} + \frac{E_i}{a - b} = 0
\quad (3)
$$
Separation of real and imaginary parts of (3), results in the following;

\[
\frac{E_1 R_2}{a-b} + \frac{a E_1}{R_c} + \frac{V_t}{R} = 0 \quad (4)
\]

and

\[
-\frac{E_1 X_2}{a-b} + \frac{a V_t}{X_c} = 0 \quad (5)
\]

Simplification of (4) gives a simple expression in \(a\) as;

\[
A_3 a^3 + A_2 a^2 + A_1 a + A_0 = 0 \quad (6)
\]

Where

\[
A_3 = R X_2^2 E_1
\]

\[
A_2 = -2 b R X_2^2 E_1 + V_t R_c X_2^2
\]

\[
A_1 = E_1 R R_2^2 + R E_1 R R_2^2 + R_c R E_1 R R_2 - 2 V_t R_c b X_2^2
\]

\[
A_0 = V_t R_c R_2^2 - R_c R E_1 R R_2 b + V_t R_c b^2 X_2^2
\]

Solution of (6) gives the generated frequency for known values of operating speed and load resistance. Further exclusion of core loss branch leads to a quadratic equation in unknown frequency, in contrast to higher order polynomial equation in ‘\(a\)’ as obtained by other research persons.

Equation (5) gives the unknown magnetizing reactance as;

\[
X_m = \frac{E_1}{a \omega C V_t - \frac{E_1 X_2}{\left(\frac{R_2}{a-b}\right)^2 + X_2^2}} \quad (7)
\]

Analysis of phasor diagram of induction generator as shown in Fig.2 gives;

\[
V_t = a_0 \sqrt{E_1^2 - E_{1y}^2} - I_1 R_1 \cos \theta + a_0 I_1 X_1 \sin \theta
\]

\[
\text{---------}(8)
\]

Fig.2. Phasor diagram of induction generator

Where

\[
E_{1y} = \frac{I_1 R_1 \sin \theta}{a} + I_1 X_1 \cos \theta
\]

\[
I_1 = \sqrt{I_{L}^2 + I_{C}^2}
\]

\(E_{1y}\) is resolved component of \(E_1\) along an axis perpendicular to terminal voltage.

3. Iteration Technique

Generated voltage and frequency for SEIG can be estimated using the following proposed iteration technique.

Step1. Computation of initial values of generated frequency and magnetizing reactance using following expression;
Step 2. Computation of air gap voltage, \( E_I \) from magnetization characteristics (see Appendix-1 and Appendix-2).

Step 3. Computation of modified values of stator voltage from (8).

Step 4. Estimation of generated frequency ‘\( \alpha \)’ and magnetizing reactance ‘\( X_m \)’ from (6) and (7) after using the modified values of \( E_I \) and \( V_t \).

Step 5. Comparison of the new value of generated frequency ‘\( \alpha \)’ with previous value i.e. \( a_0 \) as used in step 1.

If \( Err_1 = |\alpha - a_0|/\varepsilon \), Where \( \varepsilon = 0.000001 \), \( \alpha \) is treated as generated frequency and modified value of \( V_t \) may be treated as terminal voltage for SEIG.

\[
X_{m0} = \frac{1}{a_0} \frac{X_c}{X_c - R_s^2 + X_s^2(a_0 - b)^2}
\]

Initial values as;

\[
E_{10} = V_{t0} = 1 \text{pu}, \quad a_0 = 0.9999b
\]

If it is not so, process may be repeated by replacing ‘\( a_0 \)’ with ‘\( \alpha \)’ until difference in the successive values for generated frequency comes out to be \( \varepsilon \).

It is also possible to compute the generated voltage and frequency by comparing the successive values of \( X_m \) and \( V_t \) obtained after each iteration. For such comparison iterative procedure terminates only if following expressions are satisfied.

\[
Err_2 = |X_m - X_{m0}|/\varepsilon
\]

\[
Err_3 = |V_t - V_{t0}|/\varepsilon
\]

4. Results and Discussions

Modeling proposed in the paper is found to be useful for estimation of generated frequency and magnetizing reactance of SEIG. Further, inclusion of core loss branch makes the analysis more realistic.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>( Z_{pu} )</th>
<th>Experimental Results</th>
<th>Simulated Results using Phasor Diagram Based Analysis (PDA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( V_t )</td>
<td>( Err_1 )</td>
</tr>
<tr>
<td>1</td>
<td>2.3632</td>
<td>0.9820</td>
<td>0.9078</td>
</tr>
<tr>
<td>2</td>
<td>2.5004</td>
<td>0.9830</td>
<td>0.9210</td>
</tr>
<tr>
<td>3</td>
<td>2.6628</td>
<td>0.9856</td>
<td>0.9326</td>
</tr>
<tr>
<td>4</td>
<td>2.8312</td>
<td>0.9864</td>
<td>0.9684</td>
</tr>
<tr>
<td>5</td>
<td>3.0247</td>
<td>0.9870</td>
<td>0.9868</td>
</tr>
<tr>
<td>6</td>
<td>3.5013</td>
<td>0.9892</td>
<td>1.0131</td>
</tr>
<tr>
<td>7</td>
<td>4.4193</td>
<td>0.9922</td>
<td>1.0578</td>
</tr>
<tr>
<td>8</td>
<td>6.1056</td>
<td>0.9960</td>
<td>1.0921</td>
</tr>
<tr>
<td>9</td>
<td>6.6759</td>
<td>0.9968</td>
<td>1.1000</td>
</tr>
<tr>
<td>10</td>
<td>7.1224</td>
<td>0.9984</td>
<td>1.1052</td>
</tr>
</tbody>
</table>

\( b = 1.0133, C_{pu} = 0.8448 \)
Table 1 shows the comparison of experimental and simulated results on Machine-1 [Appendix-1] using iterative process as explained in section-3. It is observed that irrespective of error function (in terms of $a/X_m/V_y$) the final values for generated voltage and frequency turns out to be same up to 3rd digit after decimal.

However minimum numbers of iterations are required for final results in case error function is in terms of frequency. Table 2 shows the simulated results for Machine-2. The closeness between the experimental and simulated results as shown proves the validity of modeling adopted. Simulated results may be obtained for any type of load connected across the machine provided equations (6) and (7) are modified as given in appendix-2. Fig 3 and Fig 4 shows the variation of generated frequency and terminal voltage with load for different load power factors. Unity power factor load seems to be justified for better frequency and voltage regulation.

Fig. 3. Variation of generated frequency with load.

Fig. 4 Variation of stator voltage with load.
TABLE 2: COMPARISON OF EXPERIMENTAL AND SIMULATED RESULTS (MACHINE-2)

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Experimental Results</th>
<th>Simulated Results using Phasor Diagram Based Analysis (PDA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>1.</td>
<td>0.9533</td>
<td>0.9438</td>
</tr>
<tr>
<td>2.</td>
<td>0.9780</td>
<td>0.9660</td>
</tr>
<tr>
<td>3.</td>
<td>0.9986</td>
<td>0.9870</td>
</tr>
<tr>
<td>4.</td>
<td>1.0106</td>
<td>0.9984</td>
</tr>
<tr>
<td>5.</td>
<td>1.0286</td>
<td>1.0148</td>
</tr>
<tr>
<td>6.</td>
<td>1.0466</td>
<td>1.0338</td>
</tr>
<tr>
<td>7.</td>
<td>1.0640</td>
<td>1.0508</td>
</tr>
<tr>
<td>8.</td>
<td>0.8533</td>
<td>0.8434</td>
</tr>
<tr>
<td>9.</td>
<td>0.8806</td>
<td>0.8700</td>
</tr>
<tr>
<td>10.</td>
<td>0.9020</td>
<td>0.8934</td>
</tr>
<tr>
<td>11.</td>
<td>0.9266</td>
<td>0.9182</td>
</tr>
<tr>
<td>12.</td>
<td>0.9600</td>
<td>0.9480</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper attempt has been made to estimate the steady-state performance of self-excited induction generator using a new iterative technique approach based upon phasor diagram of the machine. The proposed technique has not been used by any other research person so far. Iterative technique is found to be very simple and effective. Simulated results are verified using experimental results on two test machines with different ratings. Close agreement between simulated and experimental results proves the validity of proposed modeling.

In future this research work may be extended to estimate the excitation capacitance and operating speed to obtain a constant voltage constant frequency...
operation. This may be helpful to promote the applications of self excited induction generators in remote and windy areas. In turn it may be helpful to preserve the conventional fuels which are likely to be finished with time.

Appendix 1

The details of the induction Machine-1 used to obtain the experimental results are;

- Specifications
  3-phase, 4-pole, 50 Hz, star connected, squirrel cage induction machine
  750W/1HP, 380 V, 1.9 A

- Parameters
  The equivalent circuit parameters for the machine in pu are
  \[ R_1 = 0.0823, R_2 = 0.0696, X_1 = X_2 = 0.0766 \]

- Base values
  Base voltage = 219.3 V
  Base current = 1.9 A
  Base impedance = 115.4 Ω
  Base frequency = 50 Hz
  Base speed = 1500 rpm

- Air gap voltage
  The variation of magnetizing reactance with air gap voltage at rated frequency for the induction machine is as given below.
  \[ X_m(169.2) \quad E_1 = 512.69 - 2.13X_m \]
  \[ 179.42)X_m \geq 169.20 \quad E_1 = 891.66 - 4.37X_m \]
  \[ 184.46)X_m \geq 179.42 \quad E_1 = 785.79 - 3.78X_m \]
  \[ X_m \geq 184.46 \quad E_1 = 0 \]

Appendix 2

The details of the induction Machine-2 used to obtain the experimental results are;

- Specifications
  3-phase, 4-pole, 50 Hz, delta connected, squirrel cage induction machine
  2.2kW/3HP, 230 V, 8.6 A

- Parameters
  The equivalent circuit parameters for the machine in pu are
  \[ R_1 = 0.0723, R_2 = 0.0379, X_1 = X_2 = 0.1047 \]

- Base values
  Base voltage = 230 V
  Base current = 4.96 A
  Base impedance = 46.32 Ω
  Base frequency = 50 Hz
  Base speed = 1500 rpm

- Air gap voltage
  The variation of magnetizing reactance with air gap voltage at rated frequency for the induction machine is as given below.
  \[ X_m(82.292) \quad E_1 = 344.411 - 1.61X_m \]
  \[ 95.569)X_m \geq 82.292 \quad E_1 = 465.12 - 3.077X_m \]
  \[ 108.00)X_m \geq 95.569 \quad E_1 = 579.897 - 4.278X_m \]
  \[ X_m \geq 108.00 \quad E_1 = 0 \]

Appendix-III

Case I: For R load (excluding \( R_c \))

Expression in \( a \) can be written as;
\[ A_2 a^2 + A_1 a + A_0 = 0 \]

Where
\[ A_2 = V_i X_2^2 \]
\[ A_1 = E_i R R_2 - 2b V_i X_2^2 \]
\[ A_0 = b^2 V_i X_2^2 - b E_i R R_2 + V_i R_2^2 \]

The expression of magnetizing reactance can be written as;
\[ X_m = \frac{E_1}{a \omega C V_i - \frac{E_1 X_1}{\left( \frac{R_2}{a-b} \right)^2 + X_2^2}} \]
Here stator current, $I_1$ expression is same as written in the IV section.

**Case II: For RL load (including $R_c$)**

Expression in $a$ can be written as:

$$A_5a^5 + A_4a^4 + A_3a^3 + A_2a^2 + A_1a + A_0 = 0$$

Where:

$$A_5 = E_iX^2X_2^2$$

$$A_4 = -2E_i bX^2X_2^2$$

$$A_3 = E_i R_c X_2^2 + E_i R^2 X_2^2 + X^2 R_c^2 E_i + E_i X^2 X_2^2$$

$$A_2 = -E_i R_c R_c bX^2 - 2bX_2^2 + V_e R R_c X_2^2$$

$$A_1 = E_i R_c R_c R^2 + E_i R^2 R_c^2 + E_i b^2 X^2 R^2 - 2V_e R R_c b X_2^2$$

$$A_0 = -E_i R_c R_c R^2 b + V_e R R_c R_c^2 + V_e R R_c b^2 X^2$$

$$X = R \sqrt{\left( \frac{1}{pf} \right)^2} - 1$$

The expression of magnetizing reactance can be written as:

$$X_m = \frac{E_i}{a \omega CV_i - \frac{E_i X_2}{\left( \frac{R_c}{a-b} \right)^2 + X_2^2} - \frac{V_e X}{\left( \frac{R}{a} \right)^2 + X^2}}$$

**Case III: For RL load (excluding $R_c$)**

Expression in $a$ can be written as:

$$A_5a^5 + A_4a^4 + A_3a^3 + A_2a^2 + A_1a + A_0 = 0$$

Where;

$$A_5 = E_i R^2 X^2$$

$$A_4 = V_e R R_c X_2^2 - 2b E_i R^2 X_2^2$$

$$A_3 = -2b V_e R R_c X_2^2 + E_i R^2 R_c^2 + b^2 E_i X^2 X_2^2 + E_i R R_c R_c^2$$

$$A_2 = V_e R R_c R_c^2 + b^2 V_e R R_c X_2^2 - 2b E_i X^2 X_2^2 - E_i R R_c R_c^2 b$$

The expression of magnetizing reactance can be written as;

$$X = R \sqrt{\left( \frac{1}{pf} \right)^2} - 1$$

Here stator current, $I_1$ expression for case III and case IV, is modified and written as;

$$I_1 = \sqrt{(I_c - I_L \sin \phi)^2 + (I_c \cos \phi)^2}$$

**Case IV: For RC load (including $R_c$)**

Expression in $a$ can be written as;

$$A_5a^5 + A_4a^4 + A_3a^3 + A_2a^2 + A_1a + A_0 = 0$$

Where;

$$A_4 = V_e R R_c X_2^2 - 2b E_i R^2 X_2^2$$

$$A_3 = -2b V_e R R_c X_2^2 + E_i R^2 R_c^2 + b^2 E_i X^2 X_2^2 + E_i R R_c R_c^2$$

$$A_2 = V_e R R_c R_c^2 + b^2 V_e R R_c X_2^2 - 2b E_i X^2 X_2^2 - E_i R R_c R_c^2 b$$
\[ A_1 = E_i X_{se}^2 R_c^2 + b^2 E_i X_{se}^2 X_2^2 + E_i R_c X_{se}^2 \]

\[ A_0 = -E_i R_c X_{se}^2 b \]

\[ X_{se} = aR \sqrt{\left( \frac{1}{pf} \right)^2 - 1} \]

The expression of magnetizing reactance can be written as:

\[ X_m = \frac{E_1}{aV_i - \frac{E_1 X_2}{X_c} \left( \frac{R_2}{a-b} \right)^2 + X_2^2 + \left( aR \right)^2 + X_{se}^2} \]

**Case V: For RC load (excluding \( R_c \))**

Expression in \( a \) can be written as:

\[ A_4 a^4 + A_3 a^3 + A_2 a^2 + A_1 a + A_0 = 0 \]

Where;

\[ A_4 = V_i R X_2 \]

\[ A_3 = -2V_i R X_2^2 + E_i R_c R^2 \]

\[ A_2 = V_i R R_2 + b^2 V_i R X_2^2 - E_i R_c b R^2 \]

\[ A_1 = E_i R_c X_{se}^2 \]

\[ A_0 = -bE_i R_c X_{se}^2 \]

**References**


