Generic Stochastic Effect Model of Noise in Controlling Source to Electronically Controlled Device

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Abstract: - In this research, the stochastic model which describes the effect of noise in the controlling source to any electronically controlled devices has been proposed. The model is generic since it is applicable to any electronically controlled device either controlled by voltage or current. It is also realistic due to the Nth order approximation and the usage of the Gaussian process for modeling the noise. It can be seen from the verifications using the Monte-Carlo SPICE simulations and the Kolmogorov-Smirnov tests that this model can accurately capture the stochastic behaviour of such devices with as high as 99% confidences. Hence, it has been found to be a convenient tool for the precision analysis along with the high accuracy aimed design of various circuits and systems involving any electronically controlled devices.

Key-Words: - CMOS, electronically controlled device, Gaussian process, Kolmogorov-Smirnov tests, Monte-Carlo SPICE simulations, noise, stochastic process

1 Introduction

As you can see for the title of the paper you must Electronically controlled device either controlled by voltage or current for example, OTA, current conveyor and active resistor, have been found to be applicable in many circuits and systems, for example filters, oscillators and many signal processing/communication circuits. The resulting controlled parameters of these electronically controlled devices for example the transconductance (g_m) of the OTA etc., have been found to be the functions of the controlling parameters either voltages or currents. The desired values of these key parameters can be obtained by adjusting the controlling quantities to the desired values. For each of the electronically controlled devices, the accurate tuning of its key parameter which obviously yields a constant parameter value at the desired one can always be theoretically obtained since the controlling source is assumed to be noise free.

Practically, these controlling sources are noisy. Since noise is a stochastic process, the controlling parameter either voltage or current is also a stochastic process. Hence, for each of the electronically controlled devices, the resulting controlled parameter is also a stochastic process which is randomly varying as time goes by, no longer be constant at the desired value. So, accurate tuning cannot be practically achieved. This is a catastrophic situation for the design of all circuits and systems, particularly those with high precision requirements since the desired performance cannot be guaranteed according to the stochastic nature of the controlled parameters mentioned above.

In [1], the model which captures the stochastic behavior of the resulting inductance of the Operational Transconductance Amplifier (OTA) based inductor, was proposed. Later, the similar model for the monolithic active inductor has been proposed in [2]. It can be seen that the derivation of the model in proposed in [1, 2] have been performed based on the assumption that the bias current which is the controlling parameter under considerations, is uniformly distributed. Obviously, this is practically not the case since noise in electronic circuit is typically modeled with Gaussian process with white power spectral density according to [3]. It can also be seen that the models proposed in [1, 2] have been derived based upon a first order approximation of the stochastic process [4], which is equivalent to the estimation of the whole process by using the data obtained from only one instance of observation. So, these previously proposed models have ceased to be valid in practice. As a result, more realistic models are necessary. Furthermore, the models proposed in both previous research studies are applicable only to specific devices while there exist various electronically controlled devices which the similar models should contribute great deal of conveniences. According to these reasons, a generic and realistic model which is applicable to any electronically controlled devices is obviously necessary.

Hence, in this research, the stochastic effect of noise in the controlling source to the electronically controlled device has been explored, and the resulting model of such effect has been derived. The purpose of this model is to gain more insight in to the cited stochastic effect for a greater chance to obtain the accurate tuning of the electronically controlled devices which yields the good performance of their applications. By using the proposed model, much meaningful statistical information such as mean and variance etc., of the parameter of these devices under the effect of noise can be obtained analytically. Furthermore, the proposed model is a generic one since it is applicable to any electronically controlled device either controlled by voltage or current as will be shown later. It can be seen that the inconvenient case by case analysis for each of these devices and the tedious mathematical works can be avoided by using the proposed model.

The Gaussian process with white power spectral density has been adopted in order to model the distribution of noise. Furthermore, the N^{th} order approximation of the stochastic process [4], which is equivalent to the estimation of the process by using the data obtained from N instances of observation, has also been used. Hence, the proposed model has been found to be a realistic one. The proposed model is capable of accurately capturing the resulting stochastic behavior of any electronically controlled devices with 99% confidence due to the satisfactory results obtained from the model verifications based on the Monte Carlo SPICE simulations of various often cited electronically controlled devices and the corresponding Kolmogorov-Smirnov tests [5, 6].

Hence, the proposed model has been found to be a convenient tool for the precision analysis along with the high accuracy aimed design of various electronically controlled devices based circuits and systems, for example filters, oscillators and active inductors etc. In the following sections, the proposed model, applications and verifications will be subsequently presented followed by the conclusion.

2 The Proposed Model and Applications

In this section, the proposed model along with its examples of applications to OTA, current conveyor and active resistor which are the often cited electronically controlled devices will be subsequently presented.

2.1 The model

Generally, any electronically controlled devices can be modeled by the following block diagram.

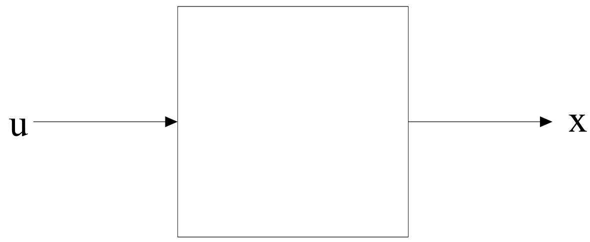


Fig.1Block diagram of any electronically controlled device.

where u and x denote the controlling and controlled parameter respectively. It can be seen that u can be either voltage or current where as x can be various parameters up to the device types for example, x is g_m if the device is a transconductor/OTA and x is R_x if the device is any generation of the current conveyors etc.

Basically, u and x are related by a certain function such that

$$x = f(u) \tag{1}$$

Ideally, noise in controlling sources can be neglected. So, both u and x are deterministic which yields the accurate tuning as mentioned above.

However, this is not the case in practice which the controlling sources are noisy. So, u is interfered by noise which is obviously a stochastic process. By such noise interference u has become a stochastic process, u(t) which yields a stochastic nature in x. As x has become a stochastic process, x(t), (1) has become

$$x(t) = f(u(t)) \tag{2}$$

Naturally, noises in the controlling sources are the Additive White Gaussian Noise (AWGN) [3]. With N^{th} order approximation [4], the Probability Density Function (PDF) of noise is derived by considering its distribution from N in equally distributed instances given by t, $t+\tau,\ldots t+(N-1)\tau$. Such noise can be denoted by n(t). Since noise values corresponding to these instances are independent, the N^{th} order approximated PDF of $n(t)$ can be given by

$$f_n(n(t),t;n(t+\tau),t+\tau,..;n(t+(N-1)\tau,t+(N-1)\tau))$$
$$=\prod_{k=0}^{N-1}\left\{\frac{1}{\sqrt{2\pi}N_{rms}}\exp\left[-\frac{(n(t+k\tau))^2}{2N_{rms}^2}\right]\right\} \quad (3)$$

where $n(t+k\tau)$ and N_{rms} denote the value of noise at any k^{th} instant and the root mean square (rms) value of noise respectively. Of course, n(t) is also stationary beside being white as will be seen later.

Due to the nature of n(t), u(t) can be given by u+n(t) so, (2) has become

$$x(t)=f(u+n(t)) \quad (4)$$

Fortunately, it can be seen that (4) can always be expressed by

$$x(t)=\alpha(f(u))n(t)+(\alpha(f(u))+\beta(f(u))) \quad (5)$$

where both α(f(u)) and β(f(u)) are functions of f(u) and also constant with respected to time, t, so x(t) is obviously a linear function of n(t). According to the theory presented in [7] and the nature of n(t) stated above, x(t) is a Gaussian process. Hence, it can be seen that both α(f(u)) and β(f(u)) must be determined in order for the characterization of x(t).

In order to determine α(f(u)) and β(f(u)), it should be mentioned here that x can be expressed as a function of u follows.

$$x(u)=\alpha(f(u))u+\beta(f(u)) \quad (6)$$

By using the Taylor series expansion [8] around u_0, x(u) can be given by

$$x(u)=f(u_0)+\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right](u-u_0)$$
$$+\frac{1}{2!}\left[\left.\frac{d^2f(u)}{du^2}\right|_{u=u_0}\right](u-u_0) \quad (7)$$
$$+\frac{1}{3!}\left[\left.\frac{d^3f(u)}{du^3}\right|_{u=u_0}\right](u-u_0)+...$$

where u_0 denotes the arbitrary reference value of u. Since both u and u_0 are extremely small due to the currently renowned Low Voltage Low Power (LVLP) design trends, high order terms can be neglected which yields

$$x(u)=f(u_0)+\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right](u-u_0) \quad (8)$$

From (6) and (8), it can be seen that α(f(u)) and β(f(u)) are determined as

$$\alpha(f(u))=\left.\frac{df(u)}{du}\right|_{u=u_0} \quad (9)$$

and

$$\beta(f(u))=f(u_0)-\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right]u_0 \quad (10)$$

With (6), (9) and (10), x(t) can be given by

$$x(t)=\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right]n(t)$$
$$+\left\{\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right](u-u_0)+f(u_0)\right\} \quad (11)$$
$$=\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right]n(t)+x$$

where x denotes the noise free value of x(t) which is obviously deterministic. Since x(t) is a Gaussian process as mentioned above, it can be seen from (1), (3) and (11) that mean and variance of x(t) denoted by $\mu_x(t+k\tau)$ and $\sigma_x^2(t+k\tau)$ respectively which are the important key parameters as x(t) is a stochastic process, can be given by

$$\mu_x(t+k\tau)=f(u) \quad (12)$$

and

$$\sigma_x^2(t+k\tau)=\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right]^2N_{rms}^2 \quad (13)$$

Because any sampled instantaneous values of x(t) are independent of each other, the proposed model which capture the stochastic nature of x(t) due to n(t) can be given by using the N^{th} order approximation as follows

$$f_x(x(t),t;x(t+\tau),t+\tau,..;x(t+(N-1)\tau,t+(N-1)\tau)) = \prod_{k=0}^{N-1}\left\{\frac{1}{\sqrt{2\pi}\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right]N_{rms}}\exp\left[-\frac{(x(t+k\tau)-f(u))^2}{2\left[\left.\frac{df(u)}{du}\right|_{u=u_0}\right]^2 N_{rms}^2}\right]\right\} \quad (14)$$

where $x(t+k\tau)$ denotes the sampled instanteneous value of x(t) at any kth instant.

Alternatively, it can be stated that x(t) is a Gaussian process with mean and variance given by (12) and (13) under the effect of noise in the controlling source. For simplicity, the derivative terms in the model can be evaluated by using various available methodologies which simplify their evaluations such as the finite difference based one etc., if f(u) is complicate. Since u_0 is arbitrary, it may be chosen such that it equals to the desired value of u given by the designer, however the existences of the derivative terms must be preserved with such choice of u_0.

Obviously, this model is very realistic since the Gaussian distribution of *n*(*t*) and the Nth order approximation have been adopted. Furthermore, it is highly generic since u can be either voltage or current and x(t) can be the key parameter for any electronically controlled device as can be seen in the following subsections which the applications of the proposed model to various electronically controlled device such as OTA, current conveyor and active resistor, will be demonstrated.

2.2 Application to OTA

In this subsection, the application of the proposed model to a simple CMOS-OTA is demonstrated. The schematic diagram of such simple CMOS-OTA can be depicted as follows [9]

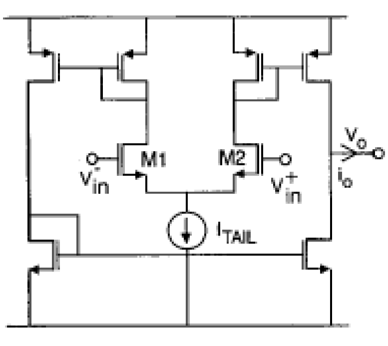


Fig.2 A simple CMOS-OTA [9].

For any OTA, the key parameter is the transconductance. For this OTA, the noise free transconductance can be deterministically given by $\sqrt{\beta I_{TAIL}}$ where β and I_{TAIL} denote the current factor of the transistor and the tail current which is a controlling parameter of the transconductance, respectively. Since the OTA is a current controlled device, noise takes the form of current which $N_{i,rms}$ denotes its rms value.

At this point, it is obviously seen that the proposed model can be applied to this scenario for capturing the stochastic behaviour of the noise affected transconductance by simply letting

$$u = I_{TAIL} \quad (15)$$

$$u_0 = I_{TAIL0} \quad (16)$$

$$x(t+k\tau) = g_m(t+k\tau) \quad (17)$$

and

$$f(u) = \sqrt{\beta I_{TAIL}} \quad (18)$$

where I_{TAIL0} and $g_m(t+k\tau)$ denote the arbitrary reference value of I_{TAIL} and the value of noise affected transconductance at any kth instant respectively. In practice, I_{TAIL} must be set to the value which the designer desired denoted by $I_{desired}$ so I_{TAIL0} can also take this value for simplicity. As a result, the distribution function of the noise affected transconductance for this simple CMOS-OTA which analyically and completely captures its stochastic behaviour, can be conveniently obtained by using the proposed model and simple variable subsitutions as

$$f_{g_m}(g_m(t),t;g_m(t+\tau),t+\tau,..;g_m(t+(N-1)\tau,t+(N-1)\tau)) = \prod_{k=0}^{N-1}\left\{\frac{1}{\sqrt{\frac{\pi\beta}{2I_{desire}}}N_{i,rms}}\exp\left[-\frac{(g_m(t+k\tau)-\sqrt{\beta\cdot I_{desire}})^2}{\frac{\beta}{4I_{desire}}N_{i,rms}^2}\right]\right\} \quad (19)$$

2.3 Application to Current Conveyor

In this subsection, the current conveyor has been focused. The differential pair based CMOS CCCII+ proposed in [10] has been adopted as a candidate current conveyor. Its schematic diagram can be depicted as follows [10]

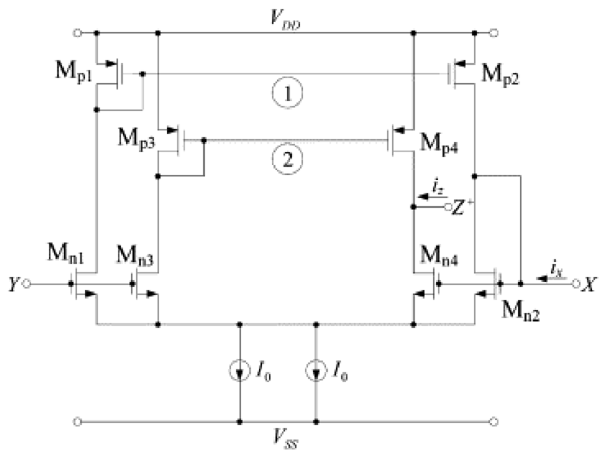


Fig.3 A differential pair based CMOS CCCII+ [10].

In this case, the parasitic resistance at port x is the key parameter. For this CCCII+, the noise free parasitic resistance at port x can be deterministically given by $\frac{1}{2\sqrt{\beta I_0}}$ where I_o denotes the biasing current which is a controlling parameter of the parasitic resistance at port x. Similarly to the above subsection, noise also takes the form of current which $N_{i,rms}$ denotes its rms value.

At this point, it is obviously seen that the proposed model can be applied to this current conveyor case for capturing the stochastic behaviour of the noise affected parasitic resistance at port x by simply letting

$$u = I_o \quad (20)$$

$$u_0 = I_{o0} \quad (21)$$

$$x(t+k\tau) = R_x(t+k\tau) \quad (22)$$

and

$$f(u) = \frac{1}{2\sqrt{\beta I_0}} \quad (23)$$

where I_{o0} and $R_x(t+k\tau)$ denote the arbitrary reference value of I_o and the value of noise affected parasitic resistance at port x for any k^{th} instant respectively. Practically, I_o must be set to the value which the designer desired denoted similarly to the above subsection by $I_{desired}$ so I_{o0} can also take this value for simplicity. As a result, the distribution function of the noise affected parasitic resistance at port x for this CMOS-CCCII+ which analytically and completely captures its stochastic behaviour, can be conveniently obtained by using the proposed model and simple variable subsitutions as

$$f_{R_x}(R_x(t),t;R_x(t+\tau),t+\tau,..;R_x(t+(N-1)\tau,t+(N-1)\tau))$$

$$=\prod_{k=0}^{N-1}\left\{\frac{1}{\sqrt{\frac{\pi}{8\beta\cdot I^3_{desired}}N_{i,rms}}}\exp\left[\frac{\left[R_x(t+k\tau)-\frac{1}{2\sqrt{\beta\cdot I_{desired}}}\right]^2}{\frac{1}{16\beta\cdot I^3_{desired}}N^2_{i,rms}}\right]\right\} \quad (24)$$

It should be mentioned here that this example and the previous one focus on the current controlled device. In the upcoming subsection, the voltage controlled device will be considered.

2.3 Application to active resistor

For this subsection, the application of the proposed model to an active resistor which is voltage controlled has been demonstrated. The CMOS active resistor proposed in [11] has been adopted and its schematic diagram can be depicted as follows [11]

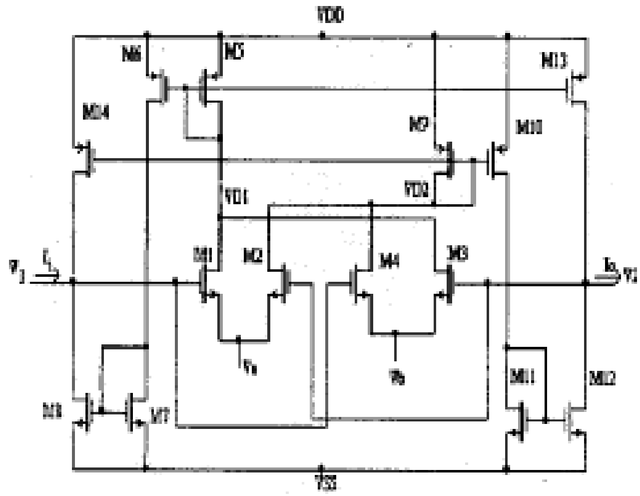


Fig.4 A CMOS active resistor proposed in [11].

In this case, the resulting resistance is the key parameter which its noise free value can be deterministically given by $\frac{1}{\beta V_B}$ where V_B denotes the biasing voltage which is a controlling parameter. It can be seen from Fig.4 that V_B can be applied between two terminals which one of them ties together the sources of M1 and M2 while the other ties the sources of M3 and M4. Since this active resistor is a voltage controlled device, noise takes the form of voltage and the corresponding rms value can be denoted by $N_{v,rms}$.

At this point, it is obviously seen that the proposed model can be applied to this case for capturing the stochastic behaviour of the noise affected parasitic resulting resistance by just letting

$$u = V_B \quad (25)$$

$$u_0 = V_{B0} \quad (26)$$

$$x(t+k\tau) = R(t+k\tau) \quad (27)$$

and

$$f(u) = \frac{1}{\beta V_B} \quad (28)$$

where V_{B0} and $R(t+k\tau)$ denote the arbitrary reference value of V_B and the value of noise affected resulting resistance at any k^{th} instant respectively. In practice, V_B must be set to the value which the designer desired denoted similarly to the above subsection by $V_{desired}$ so V_{B0} can also take this value for simplicity. As a result, the distribution function of the noise affected resulting resistance for this active resistor which analytically and completely captures its stochastic behaviour, can be simply obtained by using the proposed model and the variable subsitutions as

$$f_R(R(t),t;R(t+\tau),t+\tau,..;R(t+(N-1)\tau,t+(N-1)\tau)) = \prod_{k=0}^{N-1}\left\{\frac{1}{\frac{\sqrt{2\pi}}{\beta V_{desired}^2}N_{v,rms}}\exp\left[-\frac{\left[R(t+k\tau)-\frac{1}{\beta V_{desired}}\right]^2}{\frac{1}{(\beta V_{desired}^2)^2}N_{v,rms}^2}\right]\right\} \quad (29)$$

From these subsections, it can be seen that the distribution of the noise affected key parameter for any electronically controlled device which analytically and completely captures its stochastic behaviour as mentioned above, can be simply determined by using the proposed model. Only simple variable substitutions are necessary without the requirement of any tedious mathematical works. It can be seen that the inconvenient case by case analysis can be avioded as any electronically controlled devices can be handled in the similar fashion. This point is also the main contribution of the proposed model.

Before leaving this section, the colour of $x(t)$ which is a worthy of mention stochastic property will be now discussed. The auto correlation function of $x(t)$ denoted by $\varphi_{xx}(\tau)$

$$\varphi_{xx}(\tau) = \overline{x(t)x(t+\tau)} = (f(u))^2 \quad (30)$$

From (23), the power spectral density of $x(t)$ denoted by $\Phi_{xx}(\omega)$ can be given by

$$\Phi_{xx}(\omega) = \Im[\varphi_{xx}(\tau)] = 2\pi\big(f(u)\big)^2\delta(\omega) \quad (31)$$

It can be seen that even though $n(t)$ is white, $x(t)$ is not because $\Phi_{xx}(\omega)$ is not constant with respect to the frequency denoted by ω.

3 The Model Verification

In this section, the verification of the proposed model will now be presented. Before proceed any further, some discussion should be mentioned.

According to (3), it can be seen that the mean and variance of n(t) can be given by

$$\mu(t+k\tau) = 0 \quad (32)$$

and

$$\sigma^2(t+k\tau) = N_{rms}^2 \quad (33)$$

where {k} = {0, 1, 2, 3,…}. It can be seen that $\mu(t+k\tau)$ and $\sigma^2(t+k\tau)$ are time invariant. In the other words, it can be stated that n(t) has time invariant statistical properties. For x(t), both $\mu_x(t+k\tau)$ and $\sigma_x^2(t+k\tau)$ are time invariant as can be seen from (12) and (13). So, x(t) also has time invariant statistical properties.

Since any stochastic process with time invariant statistical properties can be termed as stationary process according to [7], it can be concluded that both n(t) and x(t) are stationary stochastic processed which the verification results obtained from a single instant can represent the results obtained from all instances because the properties of the processes which define the distributions are similar at every instances. According to this property, the model verification will be performed by using the single instant basis which is sufficient for the whole set of instances.

The verification to be presented has been performed in both qualitative and quantitative aspects. The CMOS-OTA [9], differential pair based CMOS CCCII+ [10] and CMOS active resistor [11] have been chosen as the candidate circuits. For the qualitative aspect, the 90 nm technology based parameterized 1st order approximated distributions of the deviation of x(t) from its desired value for each candidate circuit which can be evaluated by using the proposed model, are comparatively plotted against their counterparts for the similar quantities obtained from the Monte Carlo SPICE simulations of these circuits at 90 nm level.

On the other hand, the verification in the quantitative aspects has been done by performing the Kolmogorov-Smirnov tests [5, 6] to the data obtained from the comparative plots. The Kolmogorov-Smirnov tests has been chosen due to its simplicity compared to the classical chi-square test along with its efficiency for the normality test, since the x(t) has been found to be a Gaussian process. According to [6], the Kolmogorov-Smirnov test statistic denoted by *KS*, can be defined from the point of view of this research as follows

$$KS = \max\{|F_x(\Delta x(t),t)|_{device} - F_x(\Delta x(t),t)|_{model}|\} \quad (34)$$

where $\Delta x(t)$, $F_x(\Delta x(t),t)|_{device}$ and $F_x(\Delta x(t),t)|_{model}$ denote the aforementioned deviation of x(t) from its desired value, the Cumulative Distribution Function (CDF) form of the distribution obtained from the Monte-Carlo SPICE simulation results of the candidate devices and the CDF form of the 90 nm based parameterized 1st order approximated distribution functions for each device which can be obtained by using the proposed model respectively.

By using KS, the Kolmogorov-Smirnov test can be performed by comparing KS to the critical value denoted by c which can be defined by different functions of n which denotes the number of samples, according to different desired confidence levels. For example, c can be given according to the confidence level of 99% by [6]

$$c = \frac{1.63}{\sqrt{n}} \quad (35)$$

Of course, different functions for c must be utilized if different confidence levels are desired. The key concept of the Kolmogorov-Smirnov test is that the model is accurate at the specified confidence level if and only if KS ≤ c. At this point, the verification of the proposed model will be presented.

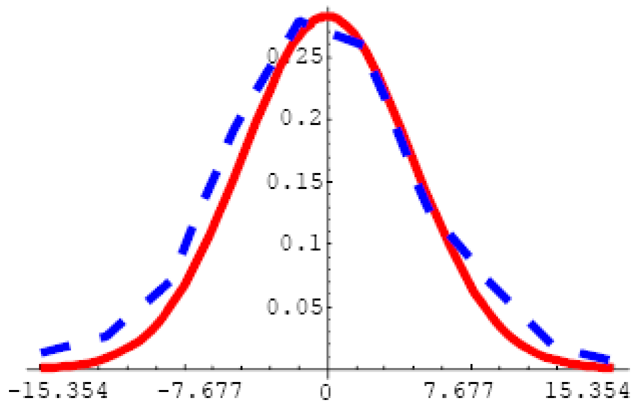
3.1 Verification by using OTA

In this subsection, the model verification by using the OTA will be presented. According to [3], $N_{i,rms}$ is given by 3.33 nA. The comparative plot between the 90 nm based parameterized 1st order approximated distribution of the deviation in transconductance for the CMOS-OTA [9] in Fig. 2 obtained by using the proposed model and the counterpart distribution for the similar quantity obtained from the Monte Carlo SPICE simulation of this circuit at 90 nm level, can be shown in Fig. 5 where such deviation is denoted by $\Delta g_m(t)$ and measured as percentage of the desired value. Of course, $\Delta g_m(t)$ can be mathematically defined in practice where I_{TAIL} is set to $I_{desired}$ as follows

$$\Delta g_m(t) = \frac{g_m(t) - \sqrt{\beta I_{desire}}}{\sqrt{\beta I_{desire}}} \quad (36)$$

where $g_m(t)$ is the transconductance value at any instant. It should be mentioned here that *n* has been chosen to be 100 for this case.

Probability Distribution



Δg_m (t) (%)

Fig.5: 90 nm based parameterized 1st order approximated $\Delta g_m(t)$ distribution obtained from the proposed model (line) v.s. Monte-Carlo SPICE based counterpart obtained from the simulation of CMOS-OTA [9] at 90 nm level (dash)

It can be observed that a strong agreement between the model based distribution and the Monte-Carlo SPICE based counterpart exists. So, it can be seen that the proposed model has high capability to predict the stochastic behavior of the candidate CMOS-OTA according to the cited observed agreement. At this point the verification in the qualitative aspect has been performed.

On the other hand, for the quantitative aspect verification, let the confidence level of the test be 99%. In other words, let α = 0.01. c can be defined by using (35) with n = 100 as c = 0.163. Since *KS* has been found from using (34) to be 0.1611 which is smaller than c, the model is accurate at 99% confidence. At this point, the accuracy of the proposed model is now verified in both aspects by the agreement seen in the comparative plot and the satisfactory Kolmogorov-Smirnov test result. This verification result is always valid according to the reason mentioned above.

3.2 Verification by using Current Conveyor

Here, the verification using the current conveyor will be discussed. Similarly to the above subsection, $N_{i,rms}$ is given by 3.33 nA [3]. The comparative plot between 90 nm based parameterized 1st order approximated distribution of the deviation in the x-port parasitic resistance for the differential pair based CMOS CCCII+ [10] in Fig. 3 obtained by using the proposed model and the counterpart distribution of the similar quantity obtained from the Monte Carlo SPICE simulation of this device at 90 nm level, can be shown in Fig. 6 where such deviation is denoted by $\Delta R_x(t)$ and measured as percentage of the desired value. Obviously, $\Delta R_x(t)$ can be mathematically defined in practice where I_o is set to $I_{desired}$ as

$$\Delta R_x(t) = \frac{R_x(t) - \left(2\sqrt{\beta I_{desired}}\right)^{-1}}{\left(2\sqrt{\beta I_{desired}}\right)^{-1}} \tag{37}$$

where $R_x(t)$ denotes the value of parasitic resistance at port x for any value of t and n is chosen to be 100.

A strong agreement between the model based distribution and the Monte Carlo SPICE based one also exists. So, the proposed model has high capability to predict the stochastic behavior of the candidate current conveyor according to the cited strong agreement. At this point the verification in the qualitative aspect has been performed.

On the other hand, for the quantitative aspect verification, also let the confidence level of the test be 99% so, c = 0.163. Since *KS* has been found by using (34) to be 0.1571 which is smaller than c, the model is accurate with 99% confidence. Now, the accuracy of the proposed model is verified in both aspects. Similarly to the above subsection, this verification result is always valid due to the aforementioned reason.

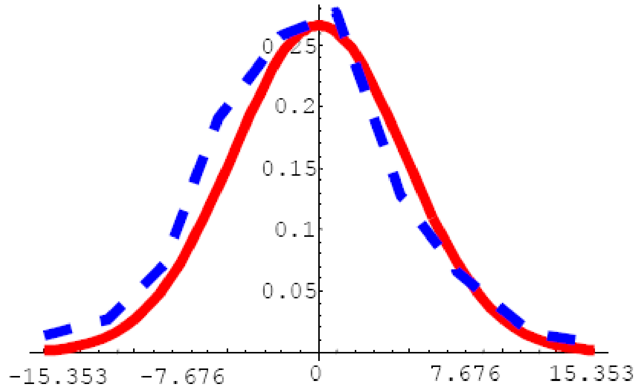


Fig.6: 90 nm based parameterized 1st order approximated $\Delta R_x(t)$ distribution obtained from the proposed model (line) v.s. Monte-Carlo SPICE based counterpart obtained from the simulation of the candidate differential pair based CMOS CCCII+ [10] at 90 nm level (dash)

3.3 Verification by using active resistor

At this point, the verification using the active resistor will be discussed. Also due to [3] $N_{v,rms}$ is given by 3.33 nA. The comparative plot between 90 nm based parameterized 1st order approximated distribution of the active resistance deviation for the CMOS active resistor [11] in Fig. 4 obtained by using the proposed model and the counterpart distribution of the similar quantity obtained from the Monte Carlo SPICE simulation of this circuit at 90 nm level, can be shown in Fig. 7 where such deviation is denoted by $\Delta R(t)$ and measured as percentage of the desired value. Obviously, $\Delta R(t)$ can be mathematically defined in practice where where V_B is set to $V_{desired}$ as

$$\Delta R(t) = \frac{R(t) - \left(\beta V_{desired}\right)^{-1}}{\left(\beta V_{desired}\right)^{-1}} \tag{38}$$

where $R(t)$ denotes the resulting resistance value at any t. Of course, n is also chosen to be 100.

In this case, a strong agreement between model based distribution and the Monte Carlo SPICE based one also exists. So, the proposed model has high capability to predict the stochastic behavior of the candidate active resistor according to the cited strong agreement. At this point the verification in the qualitative aspect has been performed.

On the other hand, for the quantitative aspect verification, also let the confidence level of the test be 99% so, $c = 0.163$. Since *KS* has been found from the application of (34) to be 0.13901 which is smaller than c, the model is accurate with 99% confidence. Now, the accuracy of the proposed model is now verified in both aspects. Obviously, this verification result is always valid according to the aforementioned reason.

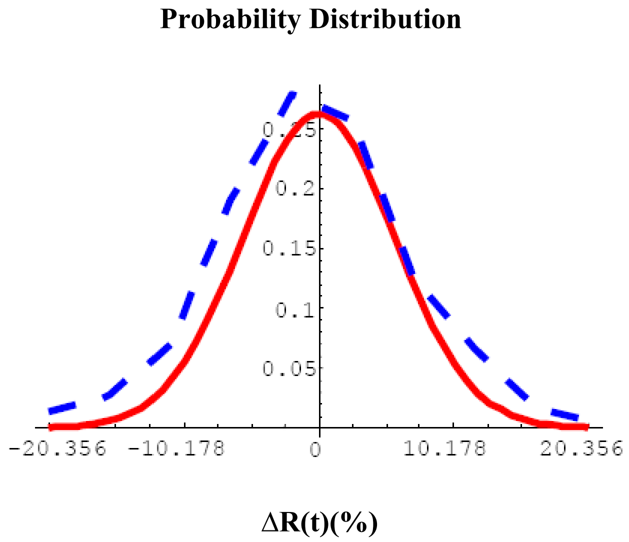


Fig.7: 90 nm based parameterized 1st order approximated ΔR(t) distribution obtained from the proposed model (line) v.s. Monte-Carlo SPICE based counterpart obtained from the simulation of the candidate CMOS active resistor [11] at 90 nm level (dash)

From these verifications, it can be seen that the proposed model can accurately capture the stochastic behavior of both current and voltage controlled electronic devices with very high level of confidences given by 99%. This point is also the virtue of this model.

4 Conclusion

In this research, the generic stochastic model which captures the effect on the resulting controlled parameter of any electronically controlled device due to noise in the controlling source has been derived. This model has been found to be a realistic one, due to the usage of Gaussian distribution of noise and N^{th} order approximation of stochastic process. According to the proposed satisfactory verification results based on Monte Carlo SPICE simulations and Kolmogorov-Smirnof tests, the model can accurately capture the resulting stochastic behavior of any noise affected electronically controlled devices with sufficiently high confidences given by 99%.

Furthermore, the proposed model is applicable to any electronically controlled device either controlled by voltage or current. Hence, it can be referred to as a generic model which makes the inconvenient case by case analysis for each device and the related tedious mathematical works become avoidable. So, it contributes a great deal of convenience to the analysis and design of any electronically controlled devices based application.

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References:

[1] R. Banchuin, R. Chisricharoen and B. Chipipop, The Stochastic Based Inductance Model of the OTA-Based Inductor, *Proceeding of the 2009 International Symposium on Intelligent Signal Processing and Communucation Systems*, 2009, pp.284-287.

[2] R. Banchuin and R. Chisricharoen, The Stochastic Based Model of Monolithic Active Inductor, *Proceeding of the 2010 International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology*, 2010, pp.1-5.

[3] R. Stephen and R. Muro, Characterization of Gaussian Noise Source, *Proceeding of the 2008 Design Conference*, 2008

[4] A.H. Haddad, *Probabilistic Systems and Random Signals,* Prentice Hall, 2006

[5] T. Altiok and B. Melamed, *Simulation Modeling and Analysis with ARENA* Academic Press, 2007

[6] S.A. Klugman, H.H. Panjer H.H and G.E. Willmot G.E, *Loss Models: From Data to Decisions*, John Wiley and Sons, 2008

[7] A. Gelb and W.E. Van de Velde , *Multiple Input Describing Function and Nonlinear System Design*, McGraw-Hill, 1968

[8] A. Jeffry, *Essentials Engineering Mathematics*, Chapman & Hall/CRC, 2005

[9] E. Sanchez-Sinencio and J. Silva-Martinez, CMOS transconductance amplifiers, architectures and active filters: a tutorial, *IEE Proceedings-Circuits, Devices and Systems*, Vol.147, No.1, 2000, pp. 3-12.

[10] R. Chaisricharoen, B. Chipipop and B. Sirinaovakul, *CMOS CCCII:* Structures, characteristics, and considerations, *AEU - International Journal of Electronics and*

Communications, Vol.64, Issue 6, 2010, pp. 540-557.

[11] S.A. Mahmoud, Low voltage CMOS floating resistor, *Proceeding of the 2004 International Conference on Electrical, Electronic and Computer Engineering*, 2004, pp.453-456.