Assembling of the SC Circuit Matrix Based on the Status of Switches

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Abstract: - This paper deals with assembling of the SC circuit matrix based on the status of switches. It is well known matrix assembly process using two-graphs or transformation graphs. However, the matrix can be built only on the basis of the status switches in the SC circuit. This procedure is somewhat simpler than the method of two-graphs. Described method is compared with other methods, too.

Key-Words: - SC circuit, status of switches, nodal charge method, capacitance matrix, four phases of switching.

1 Introduction

It is well known matrix assembly process using two-graphs [1], [2], [3] or transformation graphs [4], [5], [6] describing the circuit at all four stages of switching. The phases are marked as even (with the letters E) and odd (O) and the nodes are numbered to avoid confusion.

2 Problem Formulation

The two-graph method uses a separate denomination of node of voltages and currents, voltage triangle, the square of the current. This method can be simplified as follows.

2.1 Circuit with passive elements only

Circuit containing capacitor C$_1$ and a switched capacitor C, whose circuit diagram is shown in Fig.1 can be described by the set of equations (1) for both phases.

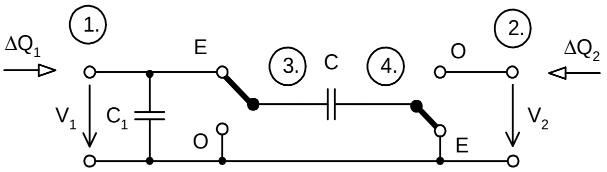


Fig.1 Circuit containing a switched capacitor

$$\begin{aligned}
\Delta Q_{1E}(t) &= (C_1 + C).V_{1E}(t) - C.V_{1O} + C.V_{2O} \\
\Delta Q_{2E}(t) &= 0 \\
\Delta Q_{1O}(t) &= C_1.V_{1O}(t) - C_1.V_{1E} \\
\Delta Q_{2O}(t) &= C.V_{2O}(t) + C.V_{1E}
\end{aligned} \qquad (1)$$

After the Z-transform, a system of equations (1) can be rewritten into the form (2).

$$\begin{aligned}
\Delta Q_{1E} &= (C_1 + C).V_{1E} - z^{-\frac{1}{2}} C.V_{1O} + z^{-\frac{1}{2}} C.V_{2O} \\
\Delta Q_{1O} &= -z^{-\frac{1}{2}} C_1.V_{1E} + C_1.V_{1O} \\
\Delta Q_{2O} &= z^{-\frac{1}{2}} C.V_{1E} + C.V_{2O}
\end{aligned} \qquad (2)$$

This system of equations (2) can be rewritten in next step into the matrix form, where the matrix of the system is (3)

$$\begin{array}{c} \\ \Delta Q_{1E}: \\ \Delta Q_{1O}: \\ \Delta Q_{2O}: \end{array}
\begin{array}{ccc} V_{1E}: & V_{1O}: & V_{2O}: \end{array}$$

$$\begin{array}{r} \Delta Q_{1E}: \\ \Delta Q_{1O}: \\ \Delta Q_{2O}: \end{array}
\left[\begin{array}{c|cc}
C_1 + C & -z^{-\frac{1}{2}} C_1 & z^{-\frac{1}{2}} C \\ \hline
-z^{-\frac{1}{2}} C_1 & C_1 & 0 \\
z^{-\frac{1}{2}} C & 0 & C
\end{array}\right] = \qquad (3)$$

ie. generally (4).

$$= \begin{bmatrix} \mathbf{C}_{\mathbf{EE}} & -z^{-\frac{1}{2}} \mathbf{C}_{\mathbf{EO}} \\ -z^{-\frac{1}{2}} \mathbf{C}_{\mathbf{OE}} & \mathbf{C}_{\mathbf{OO}} \end{bmatrix} \qquad (4)$$

This matrix (3) consists of four partial submatrix, generally $\mathbf{C}_{\mathbf{EE}}$, $\mathbf{C}_{\mathbf{EO}}$, $\mathbf{C}_{\mathbf{OE}}$ and $\mathbf{C}_{\mathbf{OO}}$.

By comparing this matrix (3) with partial schematic diagrams in Fig.2 for both phases the submatrix can be obtained easy as follows, as we can see.

The submatrix which are describing this circuit only in the even phase and in the odd phase are follows:

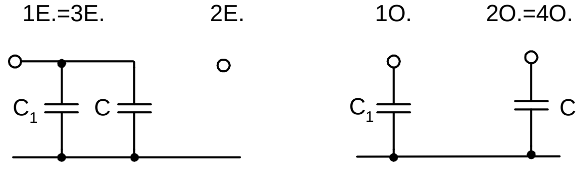


Fig.2 Diagrams of circuit for even and odd phases

$$\begin{array}{cc} & V_{1E} \\ \Delta Q_{1E} & \left[C_1 + C \mid \quad \right] \end{array} = \mathbf{C_{EE}}, \quad \begin{array}{cc} & \begin{array}{cc} V_{1O} & V_{2O} \end{array} \\ \begin{array}{c} \Delta Q_{1O} \\ \Delta Q_{2O} \end{array} & \left[\quad \left| \begin{array}{cc} C_1 & 0 \\ 0 & C \end{array} \right. \right] \end{array} = \mathbf{C_{OO}} \qquad (5)$$

Because charge ΔQ_{1E} is given by equation $\Delta Q_{1E} = (C_1 + C).V_{1E}$ and charge ΔQ_{1E} is in first row which corresponds to the line 1E.=3E., and voltage V_{1E} is in first column which corresponds to the column 1E.=3E., member $C_1 + C$ is in 1E.=3E. row and in the same column. Similarly can be assembled a second matrix (6).

$$\begin{array}{cc} & 1E. = 3E.: \\ 1E. = 3E.: & \left[C_1 + C \mid \quad \right] \end{array} = \mathbf{C_{EE}},$$

$$\begin{array}{cc} & \begin{array}{cc} 1O.: & 2O. = 4O.: \end{array} \\ \begin{array}{r} 1O.: \\ 2O. = 4O.: \end{array} & \left[\quad \left| \begin{array}{cc} C_1 & 0 \\ 0 & C \end{array} \right. \right] \end{array} = \mathbf{C_{OO}} \qquad (6)$$

Similarly we can also assemble the remaining matrix $\mathbf{C_{EO}}$, $\mathbf{C_{OE}}$ (7), too. However, rows are of opposite phase than columns and their elements must be multiplied by $-z^{-\frac{1}{2}}$.

$$\begin{array}{cc} & \begin{array}{cc} V_{1O}: & V_{2O}: \end{array} \\ \Delta Q_{1E} & \left[\quad \left| \begin{array}{cc} -z^{-\frac{1}{2}}C_1 & z^{-\frac{1}{2}}C \end{array} \right. \right] \end{array} = z^{-\frac{1}{2}}\mathbf{C_{EO}},$$

$$\begin{array}{cc} & V_{1O} \\ \begin{array}{c} \Delta Q_{1O} \\ \Delta Q_{2O} \end{array} & \left[\left. \begin{array}{c} -z^{-\frac{1}{2}}C_1 \\ z^{-\frac{1}{2}}C \end{array} \right| \quad \right] \end{array} = z^{-\frac{1}{2}}\mathbf{C_{OE}} \qquad (7)$$

In the row 1E.=3E. is in the column 1O. member C_1, ie. member simultaneously occurring in nodes 1E.=3E. and 1O. In the row 1E.=3E. is in the column 2O.=4O. member C, ie. member simultaneously occurring in nodes 1E.=3E. and 2O.=4O. Both multiplied by $-z^{-\frac{1}{2}}$, of course. Sign of the C_1 is positive, because from indexes of nodes 1E.=3E. and 1O. one number (ie. 1) at the least is the same (ie. number of node is the same, no phase, the phase may be different). Thus after multiplication by $-z^{-\frac{1}{2}}$ this member is $-z^{-\frac{1}{2}}C_1$. Sign of the C is negative, because from the indexes 1E.=3E. and indexes 2O.=4O. no number is the same. Thus after multiplication by $-z^{-\frac{1}{2}}$ this member will be $-z^{-\frac{1}{2}}(-C) = +z^{-\frac{1}{2}}C$. Similarly can be assembled a second matrix (8), too.

$$\begin{array}{cc} & \begin{array}{cc} 1O.: & 2O. = 4O.: \end{array} \\ 1E. = 3E.: & \left[\quad \left| \begin{array}{cc} -z^{-\frac{1}{2}}C_1 & z^{-\frac{1}{2}}C \end{array} \right. \right] \end{array} = z^{-\frac{1}{2}}\mathbf{C_{EO}},$$

$$\begin{array}{cc} & 1E. = 3E.: \\ \begin{array}{r} 1O.: \\ 2O. = 4O.: \end{array} & \left[\left. \begin{array}{c} -z^{-\frac{1}{2}}C_1 \\ z^{-\frac{1}{2}}C \end{array} \right| \quad \right] \end{array} = z^{-\frac{1}{2}}\mathbf{C_{OE}} \qquad (8)$$

Rules for sign of the members are in Table 1. Sign of member, which is connected between output node of VVT and other node (except ground) is negative already.

Table1. Rules for sign of C

Phase:	the same	different
one number of nodes is the same	$\mathbf{C}$	$-\mathbf{z}^{-\frac{1}{2}}\mathbf{C}$
no number of nodes is the same	$-\mathbf{C}$	$\mathbf{z}^{-\frac{1}{2}}\mathbf{C}$

2.2 Circuit with active elements

Let we are considering circuit containing voltage-to-voltage transducer VVT [1], [2], three capacitors and the source of charge, whose circuit diagram is shown in Fig.3. Its equation is $V_3 = A.V_2$ [7], where A is voltage gain of the VVT.

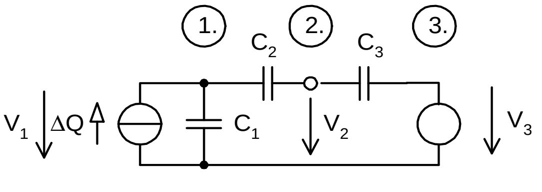


Fig.3 Circuit with a VVT

This circuit can be described by nodal voltage method [1], [2] by the set of equations (9).

$$\begin{aligned} \Delta Q &= C_1.V_1 + C_2.(V_1 - V_2) \\ 0 &= C_2.(V_2 - V_1) + C_3.(V_2 - V_3) \\ V_3 &= A.V_2 \end{aligned} \qquad (9)$$

After overwriting set of equations (9) into following form (10)

$$\begin{aligned} \Delta Q &= (C_1 + C_2).V_1 - C_2.V_2 \\ 0 &= -C_2.V_1 + (C_2 + C_3).V_2 - C_3.V_3 \\ 0 &= A.V_2 - 1.V_3 \end{aligned} \qquad (10)$$

the matrix of the system is (11)

$$\begin{array}{r} \\ 1.: \\ 2.: \\ 3.: \end{array} \begin{array}{c} \begin{array}{ccc} 1.: & 2.: & 3.: \end{array} \\ \begin{bmatrix} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 & -C_3 \\ 0 & A & -1 \end{bmatrix} \end{array} =$$

$$= \begin{array}{c} \begin{array}{cc} 2.: & 3.: \end{array} \\ \left[\begin{array}{ccc} & & \\ & & \\ \hline 0 & A & -1 \end{array}\right] \end{array}_{3.:} + \begin{array}{c} \begin{array}{cc} 1.: & 2.: \end{array} \\ \begin{array}{r} 1.: \\ 2.: \\ \\ \end{array} \left[\begin{array}{ccc} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 & -C_3 \\ \hline & & \end{array}\right] \end{array} =$$

$$= \mathbf{A}_{:} \quad + \quad \mathbf{C}_{\mathbf{O}:} \qquad (11)$$

where matrix $\mathbf{A}_{:}$ contains matrix of the voltage-to-voltage transducer (VVT) only and $\mathbf{C}_{\mathbf{O}:}$ is the remaining part.

The resulting matrix consists of partial (sub)matrixes [8].

The resulting capacitance matrix **C** consists of six following partial (sub)matrixes (12).

$$\mathbf{C} = \left[\begin{array}{c|c} \mathbf{A}_{\mathbf{EE:}} & \\ \hline & \end{array}\right] + \left[\begin{array}{c|c} & \\ \hline & \mathbf{A}_{\mathbf{OO:}} \end{array}\right] + \left[\begin{array}{c|c} \mathbf{C}_{\mathbf{EE:}} & \\ \hline & \end{array}\right] +$$

$$+ \left[\begin{array}{c|c} & \\ \hline & \mathbf{C}_{\mathbf{OO:}} \end{array}\right] + \left[\begin{array}{c|c} & \mathbf{-}\, z^{-\frac{1}{2}}\mathbf{C}_{\mathbf{EO:}} \\ \hline & \end{array}\right] + \left[\begin{array}{c|c} & \\ \hline \mathbf{-}\, z^{-\frac{1}{2}}\mathbf{C}_{\mathbf{OE:}} & \end{array}\right] \qquad (12)$$

Because operational amplifier has not memory, are (sub)matrixes $\mathbf{A}_{:}$ in positions EE and OO only. In other positions EO and OE these (sub)matrixes $\mathbf{A}_{:}$ are not.

Since an operational amplifier in a switched circuit processes high frequencies, the frequency characteristics of its amplification is taken into account by considering either one frequency ω_T

$$A = \frac{A_O.\omega_T}{\omega_T + s.A_O} \qquad (13)$$

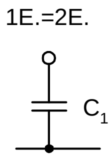
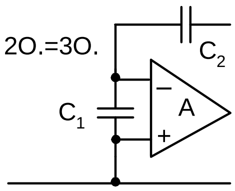
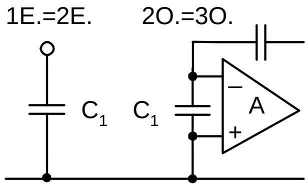
or both frequencies ω_T, ω_2

$$A = \frac{A_O.\omega_T}{\omega_T + s.A_O} . \frac{\omega_2}{\omega_2 + s} \qquad (14)$$

of its diffraction, where A_0 is the maximum value of amplification in an open loop of feedback.

The frequency response can be now calculated by substituting (13) or (14).

Table 2. Partial schematic diagrams and its matrix representation.

<table>
<tr><td>Schematic diagram:</td><td>Corresponding matrix:</td></tr>
<tr><td>
</td><td>$$\begin{array}{c} \begin{array}{c} 1E.=2E.: \end{array} \\ 1E.=2E.: \left[\begin{array}{cc|cc} C_1 & . & . & . \\ : & : & : & : \\ \hline : & : & : & : \\ : & : & : & . \end{array}\right] \end{array}$$</td></tr>
<tr><td>
</td><td>$$\begin{array}{c} 2O.=3O. \\ 2O.=3O.: \left[\begin{array}{cc|cc} . & . & . & . \\ : & : & : & : \\ \hline : & : & C_1 + C_2 & . \\ : & : & : & . \end{array}\right] \end{array}$$</td></tr>
<tr><td>
</td><td>$$\begin{array}{c} 2O.=3O.: \\ 1E.=2E.: \left[\begin{array}{cc|cc} . & . & -z^{-\frac{1}{2}}C_1 & . \\ : & : & : & : \\ \hline : & : & : & : \\ : & : & : & . \end{array}\right] \end{array} , \quad \begin{array}{c} 1E.=2E.: \\ 2O.=3O.: \left[\begin{array}{cc|cc} . & : & . & . \\ : & : & : & : \\ \hline -z^{-\frac{1}{2}}C_1 & : & : & : \\ : & : & : & . \end{array}\right] \end{array}$$</td></tr>
</table>

A summary of relations between the matrix and the partial schematic diagram allow implementation of symbolic analysis [9], [10], is given in Table 2 for an overview.

The capacitor connected from output node of the operational amplifier (i.e. from node VVT) into a different node than the common, has in a column corresponding the index node always a negative sign. This node is always outside the main diagonal of matrix, as is shown from Fig.4 where is schematic diagram of simple circuit, and corresponding matrix (15).

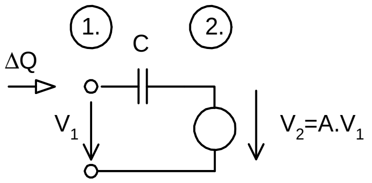


Fig.4 Sign of C is negative in matrix if index of column is the same as index of output node VVT

$$\begin{array}{cc} & \begin{array}{cc} 1.: & 2.: \end{array} \\ \begin{array}{c} 1.: \\ 2.: \end{array} & \left[\begin{array}{cc} C & -C \\ \hline A & -1 \end{array}\right] \end{array} \tag{15}$$

3 Examples

This method whose principle was explained above will now be illustrated by solved two examples.

3.1 Commented solutions in detail

Consider the circuit from Fig.5, containing of four nodes, two capacitors and one operation amplifier.

It is to be compiled his **C** matrix.

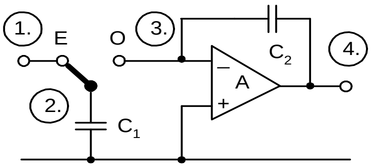


Fig.5 Circuit for example

First it is necessary to draw the schematic diagrams separately for each phase, as is shown in Fig.6.

In the even phase the circuit has three nodes, in odd phase two nodes. Therefore the capacitance matrix **C** will have five rows and five columns, too.

The output of the operational amplifier is connected into node four, therefore into row four (ie. 4E. and 4O.) is necessary write the equation of the amplifier (ie. $V_4 = A.V_3$), but in phases EE and OO only, therefore operational amplifier has not memory.

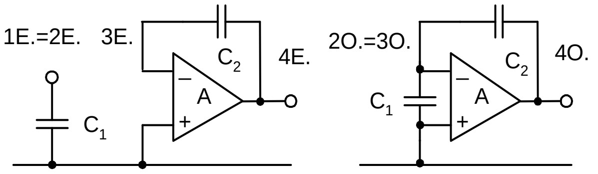


Fig.6 Schematic diagrams separately for both phases

Thus parts of capacitance matrix **C** will be following:

$$\begin{array}{cc} & \begin{array}{cc} 3E: & 4E: \end{array} \\ \begin{array}{c} \\ 4E.: \\ \\ \end{array} & \left[\begin{array}{cc|c} & & \\ A & -1 & \\ \hline & & \\ & & \end{array}\right] \end{array} + \begin{array}{cc} & \begin{array}{cc} 2O=3O: & 4O: \end{array} \\ \begin{array}{c} \\ \\ \\ 4O.: \end{array} & \left[\begin{array}{c|cc} & & \\ & & \\ \hline & & \\ & A & -1 \end{array}\right] \end{array} +$$

This part will be added (sub)matrices for EE and OO phases in next steps:

The members of the matrix are given by indexes of the rows and columns. For example in position row:1E.=2E., column: 1E.=E. is member: C_1, because in the node: 1E=2E capacitor: C_1 is connected:

$$\begin{array}{cc} & 1E.=2E.: \\ 1E.=2E.: & \left[C_1\right] \end{array} \quad . \tag{16}$$

In position row: 3E, column: 4E is member: $-C_2$, because capacitor C_2 is connected between nodes: 3E and: 4E, as we can see.

$$\begin{array}{cc} & 4E.: \\ 3E.: & \left[-C_2\right] \end{array} \tag{17}$$

Thus will be:

$$+ \begin{array}{cc} & \begin{array}{ccc} 1E=2E: & 3E.: & 4E.: \end{array} \\ \begin{array}{c} 1E=2E: \\ 3E: \\ \\ \\ \end{array} & \left[\begin{array}{ccc|c} C_1 & & & \\ & C_2 & -C_2 & \\ \hline & & & \\ & & & \end{array}\right] \end{array} +$$

Similarly we can also assemble the remaining matrix: in position row: 2O.=3O., column: 2O.=3O. is member C_1+C_2, because in node 2O=3O capacitors C_1 and C_2 are connected:

$$\begin{array}{rl} & 2O.=3O.: \\ 2O.=3O.: & \left[C_1+C_2\right] \end{array} \qquad (18)$$

In position row: 2O.=3O., column: 4O. is the member -C_2, because capacitor C_2 is connected between nodes 2O.=3O. and 4O:

$$\begin{array}{rl} & 4E.: \\ 3E.: & \left[-C_2\right] \end{array} \qquad (19)$$

Thus will be:

$$\begin{array}{rc} & \begin{array}{cc} 2O.=3O.: & 4O.: \end{array} \\ +\;\; 2O.=3O.: & \left[\begin{array}{c|cc} & & \\ & & \\ \hline & C_1+C_2 & -C_2 \\ & & \end{array}\right]+ \end{array}$$

Similarly in the matrix $\mathbf{C_{EO}}$, in position: row: 2O.=3O., column: 1E.=2E. is the member $-z^{-\frac{1}{2}}C_1$, because capacitor C_1 is connected between nodes 2O.=3O. and 1E.=2E. The numbers 2E. and 2O. are identical, therefore sign is positive in this case (ie.: +1), as we can see, ie. this member is finally negative: $-z^{-\frac{1}{2}}.(+1).C_1$.

$$\begin{array}{rl} & 1E.=2E.: \\ 2O.=3O.: & \left[-z^{-\frac{1}{2}}C_1\right] \end{array} \qquad (20)$$

In position: row: 2O.=3O., column 3E. is the member $-z^{-\frac{1}{2}}C_2$, because capacitor C_2 is connected between nodes 2O.=3O. and 4E. The numbers 3O. and 3E. are identical, therefore sign is positive (ie.: +1), ie. this member is finally negative: $-z^{-\frac{1}{2}}.(+1).C_2$.

$$\begin{array}{rl} & 3E.: \\ 2O.=3O.: & \left[-z^{-\frac{1}{2}}C_2\right] \end{array} \qquad (21)$$

But in position: row: 2O.=3O., column 4E. is the member $z^{-\frac{1}{2}}C_2$, because capacitor C_2 is connected between nodes 2O.=3O. and 4E. Sign is negative in this case (ie. -1), because from the indexes 1E.=3E. and 2O.=4O. no number is the same as we can see, ie. this member is finally $-z^{-\frac{1}{2}}.(-1).C_2=z^{-\frac{1}{2}}C_2$:

$$\begin{array}{rl} & 4E.: \\ 2O.=3O.: & \left[z^{-\frac{1}{2}}C_2\right] \end{array} \qquad (22)$$

ie. this (sub)matrix will be in following form:

$$\begin{array}{rc} & \begin{array}{ccc} 1E=2E: & 3E.: & 4E.: \end{array} \\ + \;\; 2O.=3O.: & \left[\begin{array}{ccc|c} C_1 & & & \\ & C_2 & -C_2 & \\ \hline -z^{-\frac{1}{2}}C_1 & -z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_2 & \\ & & & \end{array}\right]+ \end{array}$$

Similarly the remaining matrix $\mathbf{C_{OE}}$ is following:

$$\begin{array}{rc} & \begin{array}{cc} 2O.=3O.: & 4O.: \end{array} \\ \begin{array}{r} 1E.=2E.: \\ 3E.: \\ + \\ \vdots \end{array} & \left[\begin{array}{c|cc} & -z^{-\frac{1}{2}}C_1 & \\ & -z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_2 \\ & & \\ \hline & & \\ & & \end{array}\right]= \end{array}$$

where member in row: 1E.=2E. and column: 2O.=3O. is: $-z^{-\frac{1}{2}}C_1$ (23)

$$\begin{array}{rl} & 2O.=3O.: \\ 1E.=2E.: & \left[-z^{-\frac{1}{2}}C_1\right] \end{array} \qquad (23)$$

because capacitor C_1 is connected between nodes: 2O.=3O. and: 1E.=2E.

As we can see, the numbers: 2 in members: 2E. and 2O. are identical, therefore sign is positive (ie.: +1), ie. this member will be negative finally: $-z^{-\frac{1}{2}}.(+1).C_1$.

Thus the resulting capacitance matrix will be in following form (24).

$$
= \begin{array}{r}
 \\
1E=2E: \\
3E: \\
 \\
2O.=3O.: \\
 \\
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1E=2E: & 3E.: & 4E.: & 2O.=3O.: & 4O.: \end{array} \\
\left[\begin{array}{ccc|cc}
C_1 & 0 & 0 & -z^{-\frac{1}{2}}C_1 & 0 \\
0 & C_2 & -C_2 & -z^{-\frac{1}{2}}C_1 & z^{-\frac{1}{2}}C_2 \\
0 & A & -1 & 0 & 0 \\
\hline
-z^{-\frac{1}{2}}C_1 & -z^{-\frac{1}{2}}C_1 & z^{-\frac{1}{2}}C_2 & C_1+C_2 & -C_2 \\
0 & 0 & 0 & A & -1
\end{array}\right]
\end{array}
\qquad (24)
$$

3.2 Common solution without comment

Previous solution has been very extensively commented.

This can lead to the assumption that the described method is too laborious. Thus, the next example will be solved without comment.

Consider the circuit from Fig.7, containing from five nodes, two capacitors, two switched capacitors and one operational amplifier with finite amplification A.

Schematic diagrams separately for each phase are shown in Fig.8.

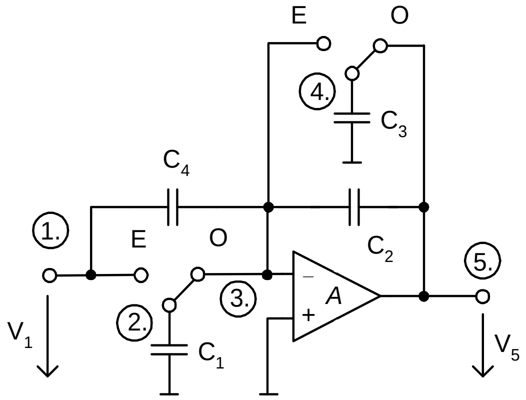
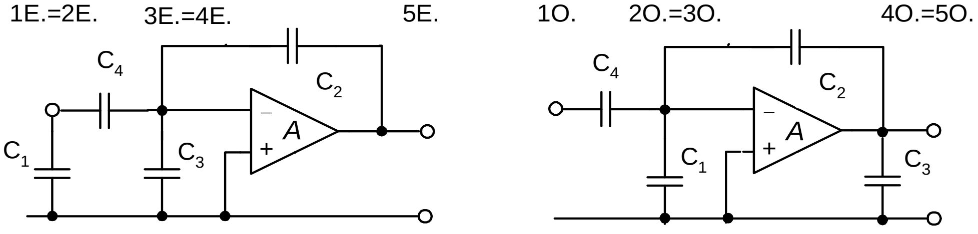


Fig.7 Schematic diagram to example



E-phase: O-phase:

Fig. 8 Partial schematic diagrams

Partial matrixes are following:

$$
\begin{array}{r}
 \\
1E.=2E.: \\
+3E.=4E.: \\
 \\
\end{array}
\begin{array}{c}
\begin{array}{ccc} 1E.=2E.: & 3E.=4E.: & 5E.: \end{array} \\
\left[\begin{array}{ccc}
C_1+C_4 & -C_4 & 0 \\
-C_4 & C_2+C_3+C_4 & -C_2 \\
\hline
0 & 0 & 0
\end{array}\right]+
\end{array}
$$

$$
\begin{array}{r}
 \\
1O.: \\
+2O.=3O.: \\
 \\
\end{array}
\begin{array}{c}
\begin{array}{ccc} 1O.: & 2O.=3O.: & 4O.=5O.: \end{array} \\
\left[\begin{array}{ccc}
C_1+C_4 & -C_4 & 0 \\
-C_4 & C_1+C_2+C_4 & -C_2 \\
\hline
0 & 0 & 0
\end{array}\right]+
\end{array}
$$

$$
\begin{array}{r}
 \\
1O.: \\
+(-z^{-\frac{1}{2}})2O.=3O.: \\
 \\
\end{array}
\begin{array}{c}
\begin{array}{ccc} 1E.=2E.: & 3E.=4E.: & 5E.: \end{array} \\
\left[\begin{array}{ccc}
C_1+C_4 & -C_4 & 0 \\
-C_4 & C_2+C_4 & -C_2 \\
\hline
0 & 0 & 0
\end{array}\right]+
\end{array}
$$

$$
\begin{array}{r}
 \\
1E.=2E.: \\
+(-z^{-\frac{1}{2}})3E.=4E.: \\
 \\
\end{array}
\begin{array}{c}
\begin{array}{ccc} 1O.: & 2O.=3O.: & 4O.=5O.: \end{array} \\
\left[\begin{array}{ccc}
C_1+C_4 & -C_4 & 0 \\
-C_4 & C_2+C_4 & -C_2 \\
\hline
0 & 0 & 0
\end{array}\right]+
\end{array}
$$

$$+\begin{array}{r}\\ \\ \\ 5E.:\end{array}\!\!\begin{array}{ccc}\multicolumn{2}{c}{3E.=4E.:} & 5E.:\\ \left[\begin{array}{ccc}0 & 0 & 0\\ 0 & 0 & 0\\ \hline 0 & A & -1\end{array}\right.&&\left.\right]\end{array} + \begin{array}{r}\\ \\ \\ 4O.=5O.:\end{array}\!\!\left[\begin{array}{ccc}\multicolumn{2}{c}{2O.=4O.:} & 4O.=5O.:\\ 0 & 0 & 0\\ 0 & 0 & 0\\ \hline 0 & A & -1\end{array}\right] =$$

$$= \begin{array}{r|cccccc} & 1E.=2E.: & 3E.=4E.: & 5E.: & 1O.: & 2O.=3O. & 4O.=5O.:\\ \hline 1E.=2E.: & C_1+C_4 & -C_4 & 0 & -z^{-\frac{1}{2}}(C_1+C_4) & z^{-\frac{1}{2}}C_4 & 0\\ 3E.=4E.: & -C_4 & C_1+C_2+C_4 & -C_2 & z^{-\frac{1}{2}}C_4 & -z^{-\frac{1}{2}}(C_2+C_4) & z^{-\frac{1}{2}}C_2\\ & 0 & A & -1 & 0 & 0 & 0\\ \cline{2-7} 1O.: & -z^{-\frac{1}{2}}(C_1+C_4) & z^{-\frac{1}{2}}C_4 & 0 & C_1+C_4 & -C_4 & 0\\ 2O.=3O.: & z^{-\frac{1}{2}}C_4 & -z^{-\frac{1}{2}}(C_2+C_4) & z^{-\frac{1}{2}}C_2 & -C_4 & C_2+C_3+C_4 & -C_2\\ & 0 & 0 & 0 & 0 & A & -1\end{array} \tag{25}$$

4 Compared with others methods

Method described above will now be compared with two common methods.

4.1 Compared with two-graphs method

For the purpose of comparison with the two-graph method [1], [2] will now solved the same example. In this case, the circuit is described by the matrix (26)

$$\begin{array}{c|cc} & \mathbf{V_E} & \mathbf{V_O}\\ \hline \mathbf{Q_E} & \mathbf{A_E} & \mathbf{B_E}\\ \mathbf{Q_O} & \mathbf{B_O} & z\mathbf{A_O}\end{array} \tag{26}$$

where $\mathbf{A_E}$, $\mathbf{A_E}$, $\mathbf{A_E}$ and $\mathbf{A_E}$ are submatrices.

Consider the same circuit ie. circuit from Fig.9 with two capacitors, two switched capacitors and operational amplifier with finite amplification A.

The circuit has five nodes, the numbers of nodes are in the circle.

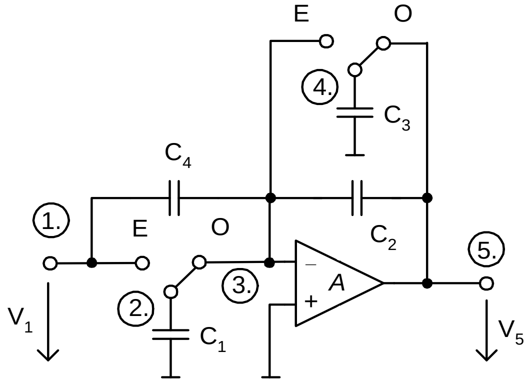
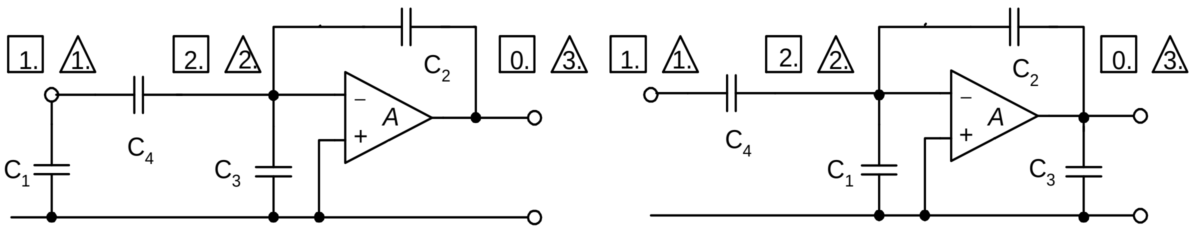


Fig.9 Schematic diagram for comparison

Schematic diagrams separately for each phase of this circuit are shown in Fig.10. The numbers of nodes in square are for the charge, in the triangle for the voltage.



E-phase: O-phase:

Fig. 10 Partial schematic diagrams

In the two-graph method, two tables must be now prepared. Therefore matrix formulation required assembling following four tables for even and odd phases:

Table for $\mathbf{A_E}$ matrix:

Element		C_1	C_2	C_3	C_4	$\frac{VVT}{inp.}$	$\frac{VVT}{out.}$
$Q_E - gr.$	*from*	1	2	2	1	2	0
	to	0	0	0	2	0	0
$V_E - gr.$	*from*	1	2	2	1	2	3
	to	0	3	0	2	0	0

,

Table for $\mathbf{A_O}$ matrix:

Element		C_1	C_2	C_3	C_4	$\frac{VVT}{inp.}$	$\frac{VVT}{out.}$
$Q_o - gr.$	*from*	2	2	0	1	2	0
	to	0	0	0	2	0	0
$V_O - gr.$	*from*	2	2	3	1	2	3
	to	0	3	0	2	0	0

The matrixes based on these tables are (27), (28).

$$\mathbf{A_E} = \begin{array}{c|ccc|} & 1.\Delta & 2.\Delta & 3.\Delta \\ \boxed{1.} & C_1+C_4 & -C_4 & 0 \\ \boxed{2.} & -C_4 & C_2+C_3+C_4 & -C_2 \\ \cline{2-4} & 0 & A & -1 \end{array}, \tag{27}$$

$$\mathbf{A_O} = \begin{array}{c|ccc|} & 1.\Delta & 2.\Delta & 3.\Delta \\ \boxed{1.} & C_1+C_4 & -C_4 & 0 \\ \boxed{2.} & -C_4 & C_1+C_2+C_4 & -C_2 \\ \cline{2-4} & 0 & A & -1 \end{array} \tag{28}$$

Table for $\mathbf{B_E}$ matrix:

Element		C_1	C_2	C_3	C_4	$\frac{VVT}{inp.}$	$\frac{VVT}{out.}$
$Q_E - gr.$	*from*	1	2	2	1	2	0
	to	0	0	0	2	0	0
$V_O - gr.$	*from*	2	2	3	1	2	3
	to	0	3	0	2	0	0

,

Table for $\mathbf{B_O}$ matrix:

Element		C_1	C_2	C_3	C_4	$\frac{VVT}{inp.}$	$\frac{VVT}{out.}$
$Q_o - gr.$	*from*	2	2	0	1	2	0
	to	0	0	0	2	0	0
$V_E - gr.$	*from*	1	2	2	1	2	3
	to	0	3	0	2	0	0

The matrixes based on these tables are (29), (30).

$$\mathbf{A_E} = \begin{array}{c|ccc|} & 1.\Delta & 2.\Delta & 3.\Delta \\ \boxed{1.} & C_1+C_4 & -C_4 & 0 \\ \boxed{2.} & -C_4 & C_2+C_3+C_4 & -C_2 \\ \cline{2-4} & 0 & 0 & 0 \end{array}, \tag{29}$$

$$\mathbf{A_O} = \begin{array}{c|ccc|} & 1.\Delta & 2.\Delta & 3.\Delta \\ \boxed{1.} & C_1+C_4 & -C_4 & 0 \\ \boxed{2.} & -C_4 & C_1+C_2+C_4 & -C_2 \\ \cline{2-4} & 0 & 0 & 0 \end{array} \tag{30}$$

The resulting matrix $\mathbf{C}$ is after substituting into (26) in following form (31).

$$\mathbf{C} = \left[\begin{array}{ccc|ccc} C_1+C_4 & -C_4 & 0 & C_1+C_4 & C_4 & 0 \\ -C_4 & C_1+C_2+C_4 & -C_2 & C_4 & C_2+C_4 & C_2 \\ 0 & A & -1 & 0 & 0 & 0 \\ \hline C_1+C_4 & C_4 & 0 & z(C_1+C_4) & -C_4 & 0 \\ C_4 & C_2+C_4 & C_2 & -zC_4 & z(C_2+C_3+C_4) & -zC_2 \\ 0 & 0 & 0 & 0 & zA & -z \end{array}\right] \tag{31}$$

4.2 Compared with full matrix method

For the purpose of comparison with the full matrix method [11], [7] will now solved the same example from the Fig.9. The circuit has five nodes and so its capacitance $\tilde{\mathbf{C}}_{\mathbf{O}}$ matrix (32) will have them, too.

$$\tilde{\mathbf{C}}_{\mathbf{O}} = \begin{array}{c|ccccc|} & 1.: & 2.: & 3.: & 4.: & 5.: \\ \hline 1.: & C_4 & 0 & -C_4 & 0 & 0 \\ 2.: & 0 & C_1 & 0 & 0 & 0 \\ 3.: & -C_4 & 0 & C_2+C_4 & 0 & -C_2 \\ 4.: & 0 & 0 & 0 & C_3 & 0 \\ 5.: & 0 & 0 & -C_2 & 0 & C_2 \end{array} \qquad (32)$$

But in matrix last row is replaced by equation of the VVT, i.e. $V_5 = A.V_3$. Thus the capacitance matrix $\mathbf{C}_{\mathbf{O}}$ is in form (33).

$$\mathbf{C}_{\mathbf{O}} = \begin{array}{c|ccccc|} & 1.: & 2.: & 3.: & 4.: & 5.: \\ \hline 1.: & C_4 & 0 & -C_4 & 0 & 0 \\ 2.: & 0 & C_1 & 0 & 0 & 0 \\ 3.: & -C_4 & 0 & C_2+C_4 & 0 & -C_2 \\ 4.: & 0 & 0 & 0 & C_3 & 0 \\ 5.: & 0 & 0 & A & 0 & -1 \end{array} \qquad (33)$$

Thus the resulting capacitance matrix $\mathbf{C}$ containing all phases of switching will then have the following form (34), matrix consists of ten rows and ten columns. This matrix is somewhat unclear, as we can see. Thus matrix (34) is in last step reduced by closing the switches into its final form (35). Final matrix (35) consists from six rows and six columns, as we can see.

$$\left[\begin{array}{ccccc|ccccc} C_4 & 0 & -C_4 & 0 & 0 & -z^{-\frac{1}{2}}C_4 & 0 & z^{-\frac{1}{2}}C_4 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 & -z^{-\frac{1}{2}}C_1 & 0 & 0 & 0 \\ -C_4 & 0 & C_2+C_4 & 0 & -C_2 & z^{-\frac{1}{2}}C_4 & 0 & z^{-\frac{1}{2}}(C_2+C_4) & 0 & z^{-\frac{1}{2}}C_2 \\ 0 & 0 & 0 & C_3 & 0 & 0 & 0 & 0 & z^{-\frac{1}{2}}C_3 & 0 \\ 0 & 0 & A & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ \hline -z^{-\frac{1}{2}}C_4 & 0 & z^{-\frac{1}{2}}C_4 & 0 & 0 & C_4 & 0 & -C_4 & 0 & 0 \\ 0 & -z^{-\frac{1}{2}}C_1 & 0 & 0 & 0 & 0 & C_1 & 0 & 0 & 0 \\ z^{-\frac{1}{2}}C_4 & 0 & z^{-\frac{1}{2}}(C_2+C_4) & 0 & z^{-\frac{1}{2}}C_2 & -C_4 & 0 & C_2+C_4 & 0 & -C_2 \\ 0 & 0 & 0 & z^{-\frac{1}{2}}C_3 & 0 & 0 & 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & 0 & -1 \end{array}\right] \qquad (34)$$

The last rows in $\mathbf{C}_{\mathbf{OE}}$ and $\mathbf{C}_{\mathbf{EO}}$ matrix consists of all zeros, because an operational amplifier has not memory (i.e. transfer between phases E, O is equal zero, too).

As we can see, the resulting matrices (25) and (35) are identical.

$$\left[\begin{array}{ccc|ccc} C_1+C_4 & -C_4 & 0 & -z^{-\frac{1}{2}}(C_1+C_4) & z^{-\frac{1}{2}}C_4 & 0 \\ -C_4 & C_1+C_2+C_4 & -C_2 & z^{-\frac{1}{2}}C_4 & -z^{-\frac{1}{2}}(C_2+C_4) & z^{-\frac{1}{2}}C_2 \\ 0 & A & -1 & 0 & 0 & 0 \\ \hline -z^{-\frac{1}{2}}(C_1+C_4) & z^{-\frac{1}{2}}C_4 & 0 & C_1+C_4 & -C_4 & 0 \\ z^{-\frac{1}{2}}C_4 & -z^{-\frac{1}{2}}(C_2+C_4) & z^{-\frac{1}{2}}C_2 & -C_4 & C_2+C_3+C_4 & -C_2 \\ 0 & 0 & 0 & 0 & A & -1 \end{array}\right] \qquad (35)$$

Table 3 Comparison table

Step	The two-graphs method:	Evaluation state of switches:	Full-matrix method:
1.	Drawing a circuit diagram	Drawing a circuit diagram	Drawing a circuit diagram
2.	Drawing a circuit diagram for each phase with dual numbering nodes	Drawing a circuit diagram for each phase	Writing of the capacitance matrix
3.	Assembly four tables for each of the four phases	Assembly of four matrices for each of the four phases	Replacing row by equation of the VVT
4.	Assembly of four matrices for each of the four phases	Assembly resulting matrix	Assembly resulting capacitance matrix
5.	Assembly resulting matrix	-	Reduction by closing switch
Note:	Nodes have two different numbers in each phase, one for the charge other for voltage.	Nodes have one number	Nodes have one number

5 Conclusions from comparison

The comparison of the three methods is illustrated in Table 3. Individual steps of solution are described in this table in three columns.

As we can see (from Fig.10), the method of two graphs requires two different type numbers of nodes in each phase, too, one for the charge (in square), and other for voltage (in triangle). Therefore this way becomes these schemas a somewhat complicated.

The method of evaluation state of switches requires only one type of node numbers (Fig.8).

While the two-graph method requires redrawing schematic diagrams, full-matrix then repeated rewriting matrices.

6 Conclusion

Proposed method described above, i.e. evaluation of the status of switches, based on the general coordinate transformation method, described in [7], etc. In this case, the parameters of the elements appear in the resulting matrix on the positions given by identification of the nodes of the elements with the nodes of the circuit, as is shown in Fig.11.

Circuit consists of two elements G, whose matrix is (32)

$$\begin{array}{c|cc} & \tilde{1}.: & \tilde{2}.: \\ \tilde{1}.: & G & -G \\ \tilde{2}.: & -G & G \end{array} \qquad (32)$$

and resulting matrix of this circuit which is obtained

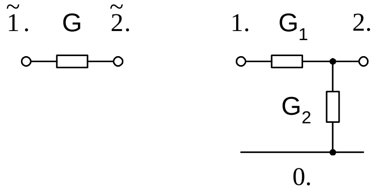


Fig. 11 Coordinate transformation method

through transformations of coordinates, is (33)

$$\begin{array}{c|cc} & 1.(\tilde{1}.): & 2..(\tilde{2}.),(\tilde{\tilde{1}}.): \\ 1.(\tilde{1}.): & G_1 & -G_1 \\ 2..(\tilde{2}.),(\tilde{\tilde{1}}.): & -G_1 & G_1+G_2 \end{array} \qquad (33)$$

where index $\underset{\cdot}{\sim}$ includes an element G_1 and index $\underset{\cdot}{\approx}$ includes an element G_2. The numbers are indexes of nodes.

This general method is applied to the switching circuits. In position i.g. $2..(\tilde{2}.),(\tilde{\tilde{1}}.)$ in above described method can be member i.g. $3E. = 4E.$.

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