Analysis of stability for impulsive stochastic fuzzy Cohen-Grossberg neural networks with mixed delays

Qianhong Zhang
Guizhou University of Finance and Economics
Guizhou Key Laboratory of Economics System Simulation
Luchongguan Road 276,
Yunyan District, 550004 Guiyang
P. R. China
zqianhong68@163. com

Jingzhong Liu
Hunan Institute of Technology
School of Computer and Information Science
Henghua Road 18,412002, Hengyang
P.R. China
hnhyls@126.com

Yuanfu Shao
Guilin University of Technology
School of Science
Jiangan Road 12,541004, Guilin
P.R. China
shaoyuanfu@163.com

Abstract: In this paper, the problem of stability analysis for a class of impulsive stochastic fuzzy Cohen-Grossberg neural networks with mixed delays is considered. Based on M-matrix theory and stochastic analysis technique, a sufficient condition is obtained to ensure the existence, uniqueness, and global exponential stability in mean square means of the equilibrium point for the addressed impulsive stochastic fuzzy Cohen-Grossberg neural network with mixed delays. Moreover an illustrative example is given to demonstrate the effectiveness of the results obtained.

Key–Words: Fuzzy Cohen-Grossberg neural networks, Global mean square exponential stability, Mixed delays, Impulses, Ito differential formula

1 Introduction

In recent years, Cohen and Grossberg neural networks [1] have been extensively studied and applied in many different fields such as associative memory, signal processing and some optimization problems. In such applications, it is of prime importance to ensure that the designed neural networks are stable [2]. In practice, due to the finite speeds of the switching and transmission of signals, time delays do exist in a working network and thus should be incorporated into the model equation [3, 4, 6, 7, 8, 9, 10, 11, 12]. In recent years, the dynamical behaviors of Cohen-Grossberg neural networks with constant delays or time-varying delays or distributed delays have been studied, see for example Refs. [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and the references therein.

Most neural networks widely studied and used can be classified as either continuous or discrete. Recently there has been a somewhat a new category of neural networks, which is neither purely continuous-time nor purely discrete-time ones; these are called impulsive neural networks. This third category of neural networks display a combination of both characteristics of continuous-time and discrete-time systems [13, 14, 15, 16, 17, 18].

In addition to the delay and impulsive effects, stochastic effects constitute another source of disturbances or uncertainties in real systems [19, 20, 21]. A lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems or sudden environment switching [21]. Therefore, stochastic perturbations should be taken into account to neural networks. In recent years, the dynamic analysis of stochastic systems (including neural networks) with delays has been an attractive topic for many researchers, and a large number of stability criteria of these systems have been reported, see e.g. Refs. [19, 20, 21] and the references therein.

In this paper, we would like to integrate fuzzy operations into Cohen-Grossberg neural networks. Speaking of fuzzy operations, T. Yang and L. B. Yang [22] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have founded that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs [23, 24, 25, 26, 27, 28, 29]. However, to the best of our knowledge, few author investigated the stability of impulsive stochastic fuzzy Cohen-Grossberg neural networks with mixed delays.

Motivated by the above discussions, in this paper, we consider the following impulsive stochastic fuzzy

Cohen-Grossberg neural networks with mixed delays.

$$\begin{cases} dx_i(t) & = & -a_i(x_i(t))\left[b_i(x_i(t))\right. \\ & & -\sum_{j=1}^{n} c_{ij} f_j(x_j(t-\tau_{ij}(t)))- \\ & & \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} K_{ij}(t-s) g_j(x_j(s)) ds \\ & & -\bigvee_{j=1}^{n} \beta_{ji} \int_{-\infty}^{t} K_{ij}(t-s) \\ & & \times\ g_j(x_j(s)) ds + I_i \left.\right] dt + \sum_{j=1}^{n} \\ & & \sigma_{ij}(x_j(t), x_j(t-\tau_{ij}(t))) d\omega_j(t), \\ & & t \neq t_k, k = 1, 2, \cdots, \\ \Delta_i(x_i(t_k)) & = & J_k(x_i(t_k^-)), i = 1, 2, \cdots, n. \end{cases} \tag{1}$$

where n corresponds to the number of units in the neural networks, respectively. For $i = 1, 2, \cdots, n, x_i(t)$ corresponds to the state of the ith neuron. $f_j(\cdot), g_j(\cdot)$ are signal transmission functions. $\tau_{ij}(t)$ corresponds to the transmission delay along the axon of the jth unit from the ith unit and satisfies $0 \leq \tau_{ij}(t) \leq \tau_{ij}(\tau_{ij}$ is a constant). $a_i(x_i(t))$ represents an amplification function at time t. $b_i(x_i(t))$ is an appropriately behaved function at time t such that the solutions of model (1) remain bounded; c_{ij} represents the elements of the feedback template. $I_i = \tilde{I}_i + \bigwedge T_{ij} u_j + \bigvee H_{ij} u_j$. $\alpha_{ij}, \beta_{ij}, T_{ij}$ and H_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation , respectively; u_j denotes the external input of the ith neurons. $\tilde{I}_i$ is the external bias of the $i-$th unit. $K_{ij}(\cdot)$ is the delay kernel function; $\sigma_{ij}(x_j(t), x_j(t-\tau_{ij}(t)))$ is the diffusion coefficient, $\sigma_i = (\sigma_{i1}, \sigma_{i2}, \cdots, \sigma_{in})$: $\omega(t) = (\omega_1(t), \omega_2(t), \cdots, \omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and F_0 contains all P-null sets). $\Delta(x_i(t_k)) = x_i(t_k^+) - x_i(t_k^-)$ is the impulses at moment t_k, the fixed moments of time t_k satisfy $t_1 < t_2 < \cdots, \lim_{k\to+\infty} t_k = +\infty$, and $\min_{2\leq k\leq\infty}\{t_k - t_{k-1}\} > \max_{1\leq i,j\leq n}\{\tau_{ij}\}$.

Remark 1. Model (1) includes the following impulsive Cohen-Grossberg neural network model as a special case:

$$\begin{cases} dx_i(t) & = & -a_i(x_i(t))\left[b_i(x_i(t))\right. \\ & & -\sum_{j=1}^{n} c_{ij} f_j(x_j(t-\tau_{ij}(t))) \\ & & -\bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} K_{ij}(t-s) \\ & & \times g_j(x_j(s)) ds \\ & & -\bigvee_{j=1}^{n} \beta_{ji} \int_{-\infty}^{t} K_{ij}(t-s) \\ & & \times\ g_j(x_j(s)) ds + I_i \left.\right] dt, \\ & & t \neq t_k, k = 1, 2, \cdots, \\ \Delta_i(x_i(t_k)) & = & J_k(x_i(t_k^-)), i = 1, 2, \cdots, n. \end{cases} \tag{2}$$

Since the solution $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ of model (2) is discontinuous at the point t_k , by theory of impulsive differential equations, we assume that $x(t_k) = (x_1(t_k), x_2(t_k), \cdots, x_n(t_k))^T = (x_1(t_k + 0), x_2(t_k+0), \cdots, x_n(t_k+0))^T$. It is clear that, in general, the derivatives $\frac{dx_i(t_k)}{dt}$ don't exist. On the other hand, we can see from the first equation of model (2) that the limits $\frac{dx_i(t_k\mp 0)}{dt}$ exist. According to the above convention, we assume that $\frac{dx_i(t_k)}{dt} = \frac{dx_i(t_k+0)}{dt}$.

Furthermore, if Cohen-Grossberg neural network model with neither impulses nor stochastic effects, model (1) can be transformed to the following special case

$$\begin{aligned} dx_i(t) &= -a_i(x_i(t))\left[b_i(x_i(t))\right. \\ &\quad -\sum_{j=1}^{n} c_{ij} f_j(x_j(t-\tau_{ij}(t))) \\ &\quad -\bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} K_{ij}(t-s) g_j(x_j(s)) ds \\ &\quad -\bigvee_{j=1}^{n} \beta_{ji} \int_{-\infty}^{t} K_{ij}(t-s) g_j(x_j(s)) ds \\ &\quad \left. +I_i\right] dt \end{aligned} \tag{3}$$

For convenience, we introduce several notations. $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ denotes a column vector. $\|x\|$ denotes a vector norm defined by $\|x\| = (\sum_{i=1}^{n} |x_i|^2)^{1/2}$. $C[X, Y]$ denotes the space of continuous mappings from topological space X to topological space Y. Denoted by $C_{F_0}^{b}[(-\infty, 0), R^n]$ the family of all bounded F_0-messurable, $C[(-\infty, 0), R^n]-$valued random variables ϕ, satisfying $\|\phi\|_{L^P} = \sup_{-\infty\leq\theta\leq 0} E\|\phi(\theta)\| < +\infty$, where $E(\cdot)$ denotes the expectation of stochastic process. The initial condition $\phi \in C^b_{F_0}[(-\infty,0),R^n]$. $PC[I,R] = \{\psi : I \to R^n | \psi(t^+) = \psi(t), t \in I, \psi(t^-)$ exist for $t \in (t_0,+\infty), \psi(t^-) = \psi(t)$ for all but points $t_k \in (t_0,+\infty)\}$, where $I \subset R$ is an interval, $\psi(t^+)$ and $\psi(t^-)$ denote the left-hand limit and right-hand limit of the scalar function $\psi(t)$, respectively.

Throughout the paper, we give the following assumptions

(**A1**) $a_i(u)$ is a continuous function and $0 < \underline{a_i} \le a_i(u) < \overline{a_i}(\underline{a_i}$ and $\overline{a_i}$ are constant) for all $u \in R, i = 1,2,\cdots,n$.
(**A2**) The signal transmission functions $f_j(\cdot), g_j(\cdot)(j = 1,2,\cdots,n)$ are Lipschitz continuous on R with Lipschitz constants μ_j and ν_j, namely, for any $u,v \in R, f_j(0) = g_i(0) = 0$ and

$$|f_j(u)-f_j(v)| \le \mu_j|u-v|,\ |g_i(u)-g_i(v)| \le \nu_i|u-v|.$$

(**A3**) $b_i(\cdot) \in C(R,R)$ and there exist positive constants b_i such that

$$\frac{b_i(u)-b_i(v)}{u-v} \ge b_i,\ \forall u \ne v,\ i = 1,2,\cdots,n.$$

(**A4**) The delay kernel $K_{ij} : [0,+\infty) \to [0,+\infty)$ is a real-valued non-negative continuous function and satisfies

$$\int_0^{+\infty} e^{\delta s} K_{ij}(s)ds = r_{ij}(\delta).$$

where $r_{ij}(\delta)$ is continuous function in $[0,\eta), \eta > 0$, and $r_{ij}(0) = 1, i,j = 1,2,\cdots,n$.
(**A5**) There exist non-negative number s_{ij}, w_{ij} such that

$$\sigma_i(u,v)\sigma_i^T(u,v) \le \sum_{j=1}^n s_{ij}u^2 + \sum_{j=1}^n w_{ij}v^2$$

for all $u = (u_1,u_2,\cdots,u_n)^T \in R^n, v = (v_1,v_2,\cdots,v_n)^T \in R^n, i = 1,2,\cdots,n$.

Definition 1 *The equilibrium point $x^* = (x_1^*, x_2, \cdots, x_n^*)^T$ of system (1) is said to be global mean square exponential stable, if there exist positive constants $M \ge 1, \lambda > 0$ such that*

$$E(\|x(t)-x^*\|^2) \le M\|\phi - x^*\|^2_{L^2} e^{-\lambda(t-t_0)},\ t > 0.$$

where $x(t) = (x_1(t),x_2(t),\cdots,x_n(t))^T$ is any solution of model (1) with initial value $x_i(t+s) = \phi_i(s) \in PC((-\infty,0],R), i = 1,2,\cdots,n$.

Definition 2 *A real matrix $A = (a_{ij})_{n\times n}$ is said to be an $M-$matrix if $a_{ij} \le 0 (i,j = 1,2,\cdots,n; i \ne j)$ and successive principal minors of A are positive.*

Lemma 3 *Let Q be an $n \times n$ matrix with non-positive off-diagonal elements, then Q is an M-matrix if and only if one of the following conditions holds:*
(i) There exists a vector $\xi > 0$ such that $\xi^T Q > 0$;
(ii) There exists a vector $\xi > 0$ such that $Q\xi > 0$.

Lemma 4 *[22] Suppose x and y are two states of system (1.1), then we have*

$$\left|\bigwedge_{j=1}^n \alpha_{ij}g_j(x) - \bigwedge_{j=1}^n \alpha_{ij}g_j(y)\right| \le \sum_{j=1}^n |\alpha_{ij}||g_j(x)-g_j(y)|,$$

and

$$\left|\bigvee_{j=1}^n \beta_{ij}g_j(x) - \bigvee_{j=1}^n \beta_{ij}g_j(y)\right| \le \sum_{j=1}^n |\beta_{ij}||g_j(x)-g_j(y)|.$$

Lemma 5 *If $H(x) \in C^0$ satisfies the following conditions:*
(i) $H(x)$ is injective on R^n;
(ii) $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$,
then $H(x)$ is homeomorphism of R^n onto itself.

2 Main results

In this section, we will consider the existence and global mean square exponential stability of system (1).

Theorem 6 *Under condition $(A1) - (A5)$, and $-(Q+T)$ is an $M-$matrix, where*

$$Q = (q_{ij})_{n\times n},\ q_{ij} = \frac{1}{\underline{a_i}} s_{ij}, i \ne j;$$

$$q_{ii} = -2b_i\frac{\underline{\underline{a_i}}}{\overline{a_i}} + \sum_{j=1}^n |c_{ij}|\mu_j + \sum_{j=1}^n (|\alpha_{ij}|+|\beta_{ij}|)\nu_j + \frac{1}{\underline{a_i}} s_{ii},$$

$$T = (t_{ij})_{n\times n}, t_{ij} = |c_{ij}|\mu_j + \frac{1}{\underline{a_i}} w_{ij} + (|\alpha_{ij}|+|\beta_{ij}|)\nu_j.$$

then model (3) has a unique equilibrium $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$. Furthermore, suppose that
(i) $\sigma_{ij}(x_j^*, x_j^*) = 0, i,j = 1,2,\cdots,n$.
(ii) $J_k(x_i(t_k)) = -\gamma_{ik}(x_i(t_k^-) - x_i^*), 0 < \gamma_{ik} < 2, i = 1,2,\cdots,n; k = 1,2,\cdots$.
Then $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ is a unique equilibrium which is globally mean square exponential stable.

Proof: Let $H(x) = (H_1(x), H_2(x), \cdots, H_n(x))^T$, where

$$H_i(x) = -b_i(x_i) + \sum_{j=1}^n c_{ij}f_j(x_j) + \bigwedge_{j=1}^n \alpha_{ij}g_j(x_j) + \bigvee_{j=1}^n \beta_{ij}g_j(x_j) - I_i, i = 1,2,\cdots,n.$$

In the following we will prove $H(x)$ is a homeomorphism of R^n onto itself.

First, we prove that $H(x)$ is an injective map on R^n. In fact, if there exist $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ and $y = (y_1, y_2, \cdots, y_n)^T \in R^n, x \neq y$, such that $H(x) = H(y)$, then

$$\begin{aligned} b_i(x_i) \quad - \quad & b_i(y_i) \\ = \quad & \sum_{j=1}^{n} c_{ij}(f_j(x_j) - f_j(y_j)) \\ & + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} f_j(y_j) \\ & + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} f_j(y_j) \quad (4) \end{aligned}$$

Multiply both side of (4) by $|x_i - y_i|$, it follows from assumptions $(A2), (A3)$, Lemma 4 and element inequality $2ab \leq a^2 + b^2$ that

$$\begin{aligned} & \left(2b_i - \sum_{j=1}^{n} |c_{ij}|\mu_j - \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)\nu_j\right) |x_i - y_i|^2 \\ \leq \quad & \left(\sum_{j=1}^{n} |c_{ij}|\mu_j + \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)\nu_j\right) |x_j - y_j|^2 \quad (5) \end{aligned}$$

Let $\Upsilon = (\zeta_{ij})_{n\times n}$, where

$$\begin{aligned} \zeta_{ii} \quad = \quad & 2b_i - \sum_{j=1}^{n} |c_{ij}|\mu_j - \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ & -|c_{ii}|\mu_i - (|\alpha_{ij}| + |\beta_{ij}|)\nu_i, \end{aligned}$$

$\zeta_{ij} = -|c_{ij}|\mu_j - (|\alpha_{ij}| + |\beta_{ij}|)\nu_j, i \neq j, i, j = 1, 2, \cdots, n$

Then (5) transforms into the following inequality

$$\Upsilon(|x_1 - y_1|^2, |x_2 - y_2|^2, \cdots, |x_n - y_n|^2)^T \leq 0 \quad (6)$$

Set $-(Q + T) = (\kappa_{ij})_{n\times n}$, noting that $\underline{a_i} \leq \overline{a_i}, s_{ij} > 0, w_{ij} > 0$, we have

$$\kappa_{ij} \leq \zeta_{ij}, \quad i, j = 1, 2, \cdots, n.$$

Since $-(Q + T)$ is an $M-$matrix, Hence Υ is also an $M-$matrix. It follow from (6) that $x_i = y_i, i = 1, 2, \cdots, n$. which is a contradiction. So $H(x)$ is an injective on R^n.

Next we prove that $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$. Since Υ is an $M-$matrix. From Lemma 3, there exists a positive vector $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in R^n$ such that

$$\begin{aligned} & \xi_i \left(2b_i - \sum_{j=1}^{n} |c_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j\right) \\ & - \sum_{j=1}^{n} \xi_j(|c_{ji}|\mu_i + (|\alpha_{ji}| + |\beta_{ji}|)\nu_i) > 0. \end{aligned}$$

for $i = 1, 2, \cdots, n$. We can choose a small $\varrho > 0$ such that

$$\begin{aligned} & \xi_i \left(2b_i - \sum_{j=1}^{n}(|c_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j)\right) \\ & - \sum_{j=1}^{n} \xi_j(|c_{ji}|\mu_i + (|\alpha_{ji}| + |\beta_{ji}|)\nu_i) \geq \varrho > 0. \quad (7) \end{aligned}$$

for $i = 1, 2, \cdots, n$. Let $\widetilde{H}(x) = (\widetilde{H_1}(x), \widetilde{H_2}(x), \cdots, \widetilde{H_n}(x))^T$, where

$$\begin{aligned} \widetilde{H_i}(x) \quad = \quad & -(b_i(x_i) - b_i(0)) + \sum_{j=1}^{n} c_{ij}(f_j(x_j) - f_j(0)) \\ & + \bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(0) \\ & + \bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(0) \quad (8) \end{aligned}$$

From assumptions $(A2)$, $(A3)$, and inequality $2ab \leq a^2 + b^2$, we can get

$$\begin{aligned} \sum_{i=1}^{n} \quad 2 \quad & \xi_i |x_i| \text{sgn}(x_i) \widetilde{H_i}(x) \\ \leq \quad & \sum_{i=1}^{n} \left[\xi_i \left(2b_i - \sum_{j=1}^{n}(|c_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j)\right)\right. \\ & \left. + \sum_{j=1}^{n} \xi_j(|c_{ji}|\mu_i + (|\alpha_{ji}| + |\beta_{ji}|)\nu_i)\right] |x_i|^2 \\ \leq \quad & -\varrho\|x\|^2 \end{aligned}$$

Hence

$$\begin{aligned} \varrho\|x\|^2 \quad \leq \quad & \sum_{i=1}^{n} 2\xi_i |x_i| |\widetilde{H_i}(x)| \\ \leq \quad & 2 \max_{1\leq i\leq n}\{\xi_i\} \sum_{i=1}^{n} |x_i| |\widetilde{H_i}(x)| \\ \leq \quad & 2 \max_{1\leq i\leq n}\{\xi_i\} \|x_i\| \|\widetilde{H_i}(x)\| \end{aligned}$$

That is

$$\varrho\|x\| \leq 2 \max_{1\leq i\leq n}\{\xi_i\} \|\widetilde{H_i}(x)\|$$

Therefore $\|\widetilde{H}(x)\| \to +\infty$ as $\|x\| \to +\infty$, which directly implies that $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$. By Lemma 5, we know that $H(x)$ is a homeomorphism on R^n, hence $H(x) = 0$ has a unique equilibrium point $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T \in R^n$. i. e., Model (3) has a unique equilibrium point x^*. From conditions (i) and (ii) of theorem, we know that x^* is also a unique equilibrium point of model (1).

Set $y_i(t) = x_i(t) - x_i^*$, $\widetilde{\sigma_{ij}}(y_j(t)) = \sigma_{ij}(y_j(t) + x_j^*) - \sigma_{ij}(x_j^*)$, then the first equation of system (1) can be transformed into the following equation

$$\begin{aligned} d \quad & y_i(t) \\ = \quad & -a_i(y_i(t)+x_i^*)\left[b_i(y_i(t)+x_i^*) - b_i(x_i^*)\right. \\ & -\sum_{j=1}^{n} c_{ij}(f_j(y_j(t-\tau_{ij}(t))+x_j^*) - f_j(x_j^*)) \\ & -\left(\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}K_{ij}(t-s)g_j(y_j(s)+x_j^*)ds\right. \\ & \left.-\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}K_{ij}(t-s)g_j(x_j^*)ds\right) \\ & -\left(\bigvee_{j=1}^{n}\beta_{ji}\int_{-\infty}^{t}K_{ij}(t-s)g_j(y_j(s)+x_j^*)ds\right. \\ & \left.\left.-\bigvee_{j=1}^{n}\beta_{ji}\int_{-\infty}^{t}K_{ij}(t-s)g_j(x_j^*)ds\right)\right]dt \\ & +\sum_{j=1}^{n}\widetilde{\sigma_{ij}}(y_j(t), y_j(t-\tau_{ij}(t)))d\omega_j(t), \\ & t \neq t_k, i = 1,2,\cdots,n; k = 1,2,\cdots \qquad (9) \end{aligned}$$

Since $-(Q+T)$ is an M-matrix, there exists $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$ such that $0 < -(Q+T)\xi$, that is

$$\begin{aligned} 0 \quad < \quad & \left[2b_i\frac{\underline{\underline{a_i}}}{\overline{a_i}} - \left(\sum_{j=1}^{n}|c_{ij}|\mu_j + \sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right)\right]\xi_i \\ & -\sum_{j=1}^{n}\left[\frac{1}{\overline{a_i}}s_{ij} + |c_{ij}|\nu_j + \frac{1}{\overline{a_i}}w_{ij}\right. \\ & \left.+(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right]\xi_j, i = 1,2,\cdots,n. \end{aligned}$$

We can choose a small positive number $\varepsilon > 0$ such that,for $i = 1,2,\cdots,n$.

$$\begin{aligned} 0 \quad < \quad & \left[2b_i\frac{\underline{\underline{a_i}}}{\overline{a_i}} - \frac{\varepsilon}{\overline{a_i}} - \left(\sum_{j=1}^{n}|c_{ij}|\mu_j\right.\right. \\ & \left.\left.+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right)\right]\xi_i \\ & -\sum_{j=1}^{n}\left[\frac{1}{\overline{a_i}}s_{ij} + e^{\varepsilon\tau}\left(|c_{ij}|\nu_j + \frac{1}{\overline{a_i}}w_{ij}\right)\right. \\ & \left.+(|\alpha_{ij}|+|\beta_{ij}|)\nu_j r_{ij}(\varepsilon)\right]\xi_j. \qquad (10) \end{aligned}$$

Let

$$u_i(t) = e^{\varepsilon(t-t_0)}|y_i(t)|^2, i = 1,2,\cdots,n.$$

By the Ito differential formula, the stochastic derivative of $u_i(t)$ along (2.6) can be obtained as follows:

$$\begin{aligned} L \quad & u_i(t) \\ = \quad & \varepsilon e^{\varepsilon(t-t_0)}|y_i(t)|^2 + 2e^{\varepsilon(t-t_0)}|y_i(t)| \\ & \times \mathrm{sgn}(y_i(t))\left\{-a_i(y_i(t)+x_i^*)\right. \\ & \left[b_i(y_i(t)+x_i^*) - b_i(x_i^*)\right. \\ & -\sum_{j=1}^{n}c_{ij}(f_j(y_j(t-\tau_{ij}(t))+x_j^*) - f_j(x_j^*)) \\ & -\left(\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}K_{ij}(t-s)g_j(y_j(s)+x_j^*)ds\right. \\ & \left.-\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}K_{ij}(t-s)g_j(x_j^*)ds\right) \\ & -\left(\bigvee_{j=1}^{n}\beta_{ji}\int_{-\infty}^{t}K_{ij}(t-s)g_j(y_j(s)+x_j^*)ds\right. \\ & \left.\left.\left.-\bigvee_{j=1}^{n}\beta_{ji}\int_{-\infty}^{t}K_{ij}(t-s)g_j(x_j^*)ds\right)\right]\right\} \\ & +e^{\varepsilon(t-t_0)}\widetilde{\sigma}_i\widetilde{\sigma}_i^T \end{aligned}$$

for $i = 1,2,\cdots,n; t_{k-1} < t < t_k, k = 1,2,\cdots$. Applying assumptions $(A1)-(A3)$, $(A5)$, and Lemma 4, we can get

$$\begin{aligned} L \quad & u_i(t) \\ \leq \quad & \varepsilon e^{\varepsilon(t-t_0)}|y_i(t)|^2 + 2e^{\varepsilon(t-t_0)}|y_i(t)|\mathrm{sgn}(y_i(t)) \\ & \left[-\underline{a_i}b_i|y_i(t)| + \overline{a_i}\sum_{j=1}^{n}|c_{ij}|\mu_j|y_j(t-\tau_{ij}(t))|\right. \\ & +\overline{a_i}\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|) \\ & \left.\times\int_{-\infty}^{t}K_{ij}(t-s)|y_j(s)|\nu_j ds\right] + e^{\varepsilon(t-t_0)} \\ & \times\left[\sum_{j=1}^{n}s_{ij}y_j^2(t) + \sum_{j=1}^{n}w_{ij}y_j^2(t-\tau_{ij}(t))\right] \quad (11) \end{aligned}$$

By applying inequality $2ab \leq a^2 + b^2$, it follows that

$$\begin{aligned} L \quad & u_i(t) \\ \leq \quad & \varepsilon u_i(t) - 2\underline{a_i}b_i u_i(t) \\ & +\overline{a_i}\left[\sum_{j=1}^{n}|c_{ij}|\mu_j u_i(t)\right. \\ & +\sum_{j=1}^{n}|c_{ij}|\mu_j e^{\varepsilon\tau_{ij}}u_j(t-\tau_{ij}(t)) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j u_i(t)+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\\
&\times\int_{-\infty}^{t}e^{\varepsilon(t-s)}K_{ij}(t-s)u_j(s)ds\Big]\\
&+\sum_{j=1}^{n}s_{ij}u_j(t)+\sum_{j=1}^{n}w_{ij}e^{\varepsilon\tau_{ij}}u_j(t-\tau_{ij}(t))\\
\le\ & \overline{a_i}\left\{\left[-2b_i\frac{\underline{a_i}}{\overline{\overline{a_i}}}+\frac{\varepsilon}{\underline{a_i}}+\left(\sum_{j=1}^{n}|c_{ij}|\mu_j\right.\right.\right.\\
&\left.\left.+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right)\right]u_i(t)+\sum_{j=1}^{n}\frac{1}{\underline{a_i}}s_{ij}u_j(t)\\
&+e^{\varepsilon\tau}\sum_{j=1}^{n}\left((|\alpha_{ij}|+|\beta_{ij}|)\nu_j+\frac{1}{\underline{a_i}}w_{ij}\right)\\
&\times u_j(t-\tau_{ij}(t))+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\\
&\left.\times\int_{-\infty}^{t}e^{\varepsilon(t-s)}K_{ij}(t-s)u_j(s)ds\right\}
\end{aligned}
$$

for $i=1,2,\cdots,n; t_{k-1}<t<t_k, k=1,2,\cdots$. Furthermore, we have

$$
\begin{aligned}
D^+\ & (Eu_i(t))\\
\le\ & \overline{a_i}\left\{\left[-2b_i\frac{\underline{a_i}}{\overline{\overline{a_i}}}+\frac{\varepsilon}{\underline{a_i}}+\left(\sum_{j=1}^{n}|c_{ij}|\mu_j\right.\right.\right.\\
&\left.\left.+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right)\right]Eu_i(t)\\
&+\sum_{j=1}^{n}\frac{1}{\underline{a_i}}s_{ij}Eu_j(t)+e^{\varepsilon\tau}\sum_{j=1}^{n}(((|\alpha_{ij}|+|\beta_{ij}|)\nu_j\\
&+\frac{1}{\underline{a_i}}w_{ij}\Big)Eu_j(t-\tau_{ij}(t))+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\\
&\left.\times\nu_j\int_{-\infty}^{t}e^{\varepsilon(t-s)}K_{ij}(t-s)Eu_j(s)ds\right\} \qquad (12)
\end{aligned}
$$

Set

$$h_0=\frac{\|\phi-x^*\|_{L^2}^2}{\min_{1\le i\le n}\{\xi_i\}}.$$

then $s\in(-\infty,t_0]$, we have

$$
\begin{aligned}
Eu_i(s) &= e^{\varepsilon(s-t_0)}E|y_i(s)|^2\\
&\le E|y_i(s)|^2=E|\phi_i(s-t_0)-x_i^*|^2\\
&\le \|\phi-x^*\|_{L^2}^2\le\xi_i h_0. \qquad (13)
\end{aligned}
$$

In the following, we will use the mathematical induction to prove that,$i=1,2,\cdots,n;k=1,2,\cdots,$

$$Eu_i(t)\le\xi_i h_0,\ t_{k-1}\le t<t_k, \qquad (14)$$

when $k=1$, let us prove that

$$Eu_i(t)\le\xi_i h_0,\ t_0\le t<t_1,\ i=1,2,\cdots,n. \qquad (15)$$

In fact, if (15) is not true, then there exist some i_0 and $t^*\in[t_0,t_1)$ such that, for $t\in(-\infty,t^*), j=1,2,\cdots,n.$

$$Eu_{i_0}(t^*)=\xi_{i_0}h_0,\ D^+Ex_{i_0}(t^*)\ge0,\ Eu_j(t)\le\xi_j h_0, \qquad (16)$$

From (12) and (16), we can get

$$
\begin{aligned}
D^+\ & (Eu_{i_0}(t^*))\\
\le\ & \overline{a_{i_0}}\left\{\left[-2b_{i_0}\frac{\underline{a_{i_0}}}{\overline{\overline{a_{i_0}}}}+\frac{\varepsilon}{\underline{a_{i_0}}}+\left(\sum_{j=1}^{n}|c_{i_0j}|\mu_j\right.\right.\right.\\
&\left.\left.+\sum_{j=1}^{n}(|\alpha_{i_0j}|+|\beta_{i_0j}|)\nu_j\right)\right]\xi_0+\sum_{j=1}^{n}\left[\frac{1}{\underline{a_{i_0}}}s_{i_0j}\right.\\
&+e^{\varepsilon\tau}\left((|\alpha_{i_0j}|+|\beta_{i_0j}|)\nu_j+\frac{1}{\underline{a_{i_0}}}w_{i_0j}\right)\\
&+(|\alpha_{i_0j}|+|\beta_{ij}|)\nu_j r_{i_0j}(\varepsilon)]\,\xi_j\}\,h_0 \qquad (17)
\end{aligned}
$$

It follows from (10) and (17) that

$$D^+(Eu_{i_0}(t^*))<0.$$

which is a contradiction. So (15) is true. Suppose that the inequalities, for $i=1,2,\cdots,n;k=1,2,\cdots,$

$$Eu_i(t)\le\xi_i h_0,\ t_{k-1}\le t<t_k, \qquad (18)$$

hold for $k=1,2,\cdots,m$. From condition (ii) of this theorem, we have

$$
\begin{aligned}
|x_i(t_k)-x_i^*| &= |x_i(t_k^-)+J_k(x_i(t_k^-))-x_i^*|\\
&= |1-\gamma_{ik}||x_i(t_k^-)-x_i^*|\\
&\le |x_i(t_k^-)-x_i^*|
\end{aligned}
$$

for $i=1,2,\cdots,n;k=1,2,\cdots$. Therefore

$$u_i(t_k)\le u_i(t_k^-),\ i=1,2,\cdots,n;k=1,2,\cdots.$$

Furthermore, we can get

$$Eu_i(t_k)\le Eu_i(t_k^-),\ i=1,2,\cdots,n;k=1,2,\cdots. \qquad (19)$$

It follows from (18) and (19) that

$$Eu_i(t_m)\le Eu_i(t_m^-)<\xi_i h_0,\ i=1,2,\cdots,n. \qquad (20)$$

This, together with both (13), (18) and (20), lead to

$$Eu_i(t)\le\xi_i h_0,\ t\in(-\infty,t_m],\ i=1,2,\cdots,n. \qquad (21)$$

It is similar to the proof of (14), we can prove that

$$Eu_i(t) \le \xi_i h_0,\ t \in [t_m, t_{m+1}),\ i = 1, 2, \cdots, n. \tag{22}$$

By mathematical induction, we can conclude that (14) holds. Hence

$$E|x_i(t) - x_i^*|^2 \le \xi_i h_0 e^{-\varepsilon(t-t_0)}, t \ge t_0,\ i = 1, 2, \cdots, n.$$

So $E\|x(t) - x^*\|^2 \le M\|\phi - x^*\|_{L^2}^2 e^{-\varepsilon(t-t_0)},\ t \ge t_0$. This means that the unique equilibrium point x^* of model (1) is globally mean square exponential stable. The proof is completed. □

Remark 2 *If we don't consider fuzzy AND and fuzzy OR operations in system (1), then system (1) becomes traditional impulsive stochastic Cohen-Grossberg neural networks with mixed delays. it is clear that Theorem 6 [21] is corollary of Theorem 6. Therefore our results generalize the known results.*

3 An example

Example Consider the following impulsive stochastic fuzzy neural networks with time-varying delays and distributed delays

$$\begin{cases} dx_1(t) &= -(3+\cos x_i(t))\,[11x_1(t) \\ & +0.1f_1(x_1(t-\tau_{11}(t))) \\ & +0.7f_1(x_2(t-\tau_{12}(t))) + I_1 \\ & +\bigwedge_{j=1}^{2}\alpha_{1j}\int_{-\infty}^{t}K_{1j}(t-s)f_j(y_j(s))ds \\ & +\bigvee_{j=1}^{2}\beta_{1j}\int_{-\infty}^{t}K_{1j}(t-s)f_j(y_j(s))ds) \\ & +\bigwedge_{j=1}^{2}T_{1j}u_j + \bigvee_{j=1}^{2}H_{1j}u_j\Big]\,dt \\ & +\sigma_{11}(x_1(t), x_1(t-\tau_{11}(t)))d\omega_1 \\ & +\sigma_{12}(x_2(t), x_2(t-\tau_{12}(t)))d\omega_2, t \ne t_k \\ dx_2(t) &= -(2+\sin x_2(t))\,[17x_2(t) \\ & -0.6f_2(x_1(t-\tau_{21}(t))) \\ & +0.3f_2(x_2(t-\tau_{21}(t))) + I_2 \\ & +\bigwedge_{j=1}^{2}\alpha_{2j}\int_{-\infty}^{t}K_{2j}(t-s)f_j(y_j(s))ds \\ & +\bigvee_{j=1}^{2}\beta_{2j}\int_{-\infty}^{t}K_{2j}(t-s)f_j(y_j(s))ds) \\ & +\bigwedge_{j=1}^{2}T_{2j}u_j + \bigvee_{j=1}^{2}H_{2j}u_j\Big]\,dt \\ & +\sigma_{21}(x_1(t), x_1(t-\tau_{21}(t)))d\omega_1 \\ & +\sigma_{22}(x_2(t), x_2(t-\tau_{22}(t)))d\omega_2, t \ne t_k \\ \Delta x_1(t_k)) &= -(1+0.3\sin(1+k^2)x_1(t_k^-) \\ \Delta x_2(t_k)) &= -(1+0.6\sin(1+k)x_2(t_k^-) \end{cases} \tag{23}$$

where $t_0 = 0, t_k = t_{k-1} + 0.2k, k = 1, 2, \cdots$, and

$$f_i(r) = g_i(r) = \frac{1}{2}(|r+1| - |r-1|),$$

$$\tau_{ij}(t) = 0.3|\sin t| + 0.1, K_{ij}(t) = te^{-t}, i, j = 1, 2.$$

$$\alpha_{11} = \frac{5}{3}, \alpha_{21} = \frac{1}{3}, \alpha_{12} = -\frac{1}{4}, \alpha_{22} = \frac{3}{4};$$

$$\beta_{11} = \frac{1}{3}, \beta_{21} = \frac{2}{3}, \beta_{12} = -\frac{1}{4}, \beta_{22} = \frac{3}{4};$$

$$\sigma_{11}(x,y) = 0.2x - 0.1y, \sigma_{12}(x,y) = 0.3x + 0.1y,$$

$$\sigma_{21}(x,y) = 0.1x + 0.2y, \sigma_{22}(x,y) = 0.2x + 0.1y,$$

Let $T_{ij} = H_{ij} = S_{ij} = L_{ij} = u_i = u_j = 1, I_i = 2(i, j = 1, 2)$.

Obviously, model (23) satisfies assumptions $(A1) - (A4)$ with

$$\underline{a_1} = 2, \overline{a_1} = 4, \underline{a_2} = 1, \overline{a_2} = 3,$$

$$b_1 = 11, b_2 = 17, \mu_i = \nu_i = 1(i = 1, 2).$$

It can be easily checked that the assumption $(A5)$ is satisfied with

$$s_{11} = 0.15, s_{12} = 0.02, s_{21} = 0.3, s_{22} = 0.09,$$

$$w_{11} = 0.02, w_{12} = 0.04, w_{21} = 0.06, w_{22} = 0.03.$$

It is easy to compute

$$Q = \begin{pmatrix} -7.6625 & 0.005 \\ 0.1 & -7.9 \end{pmatrix}, \ T = \begin{pmatrix} 2.105 & 1.21 \\ 1.62 & 1.81 \end{pmatrix}$$

and

$$-(Q + T) = \begin{pmatrix} 5.5575 & -1.215 \\ -1.72 & 6.09 \end{pmatrix}$$

is an $M-$matrix. Clearly, all conditions of Theorem 6 are satisfied. Thus model (23) has a unique equilibrium point x^* which is globally mean square exponential stable.

4 Conclusion

In this paper, we have studied the existence, uniqueness and mean square exponential stability of the equilibrium point for impulsive stochastic fuzzy Cohen-Grossberg neural networks with mixed delays. Some sufficient conditions set up here are easily verified and these conditions are correlated with parameters and time delays of the system (1). The obtained criteria can be applied to design globally mean square exponential stable fuzzy Cohen-Grossberg neural networks.

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