Analysis of regimes of voltage regulators with limited capacity voltage sources. Geometrical approach

PENIN ALEXANDR
Institute of Electronic Engineering and Nanotechnologies "D. Ghitu "
Academy of Sciences of Moldova
Academiei str. 3/3, MD-2028, Chisinau,
REPUBLICA MOLDOVA
aapenin@mail.ru http://www.iieti.asm.md/

Abstract: - In power supply systems with limited capacity voltage sources, the limitation of load power is appeared. The systems with PWM boost converters may be the example of ones. But, the boost converters have a nonlinear and two-valued regulation characteristic too. Therefore, it is necessary to correctly determine the regime parameters of the converter relative to maximum permissible load voltage and control pulse width. If such power supply system contains also a quantity of loads with individual regulated voltage converters, then an interference of the converters takes place on the load regimes. Therefore, it is important to carry out the analysis of interference and to obtain relationships for definition of converter parameters. In the present work the results of interpretation of changes or "kinematics" of load regimes are presented on the basis of a conformal and hyperbolic plane.

Key-Words: - limited capacity voltage source, boost converter, regulated characteristic, regime determination, projective transformations, hyperbolic geometry.

1 Introduction

In power supply systems with limited capacity voltage sources, the limitation of load power is appeared. Independent power supply systems with solar array and rechargeable battery may be the example of ones.

Also, it is convenient to apply PWM boost converters in such systems [1]; this makes it possible to use at most the solar array voltage. But, the boost converters have a nonlinear and two-valued regulation characteristic because of inherent losses and internal resistance of the voltage source. In this case, the up-slope direction or the forward branch of its regulation characteristic is used and the down movement of the operating point on the back branch is restrained.

Therefore, it is necessary to correctly determine the regime parameters of the converter relative to maximum permissible load voltage and control pulse width. It will allow estimating, for example, reserves of the control voltage and load voltage, to use this data in calculations of a direct digital control, to carry out some kind of a linearization of the regulation characteristic in a wide range of load voltage change.

If such power supply system contains also a quantity of loads with individual regulated voltage converters, then an interference of the converters takes place on the load regimes.

Therefore, it is important to carry out the analysis of interference and to obtain relationships for definition of converter parameters (at the possible direct coordinated digital control) for the designated load regimes. It will allow excluding the two-valuedness of the regulation characteristics.

In the present work, the results of interpretation of changes or «kinematics» of load regimes are presented on the basis of a conformal and hyperbolic plane [2], [3].

Such behavior of system is similar to an active multi-port network with changeable loads and a demonstration of projective geometry [4].

2 Regimes of voltage regulators with the limited capacity voltage source. Display of conformal and hyperbolic geometry

Let us consider a power supply system with two regulated voltage converters (voltage regulators VR_1, VR_2) and loads R_1, R_2 in Fig.1. Generally, voltage converters with switched tapped secondary windings of transformers or PWM converters can be these regulators. The regulators define voltage transmission coefficient or transformation ratio n_1, n_2. Voltage regulators are connected to the limited capacity supply voltage source U_0. An interference of the regulators on regimes or load voltages U_1, U_2 is observed because of internal resistance R_i . Let us obtain a system of the equations describing behavior or "kinematics" of a circuit at change of parameters n_1, n_2. By definition,

$$n_1 = \frac{U_1}{U}, n_2 = \frac{U_2}{U} . \tag{1}$$

The general power of loads

$$P_L = \frac{U_1^2}{R_1} + \frac{U_2^2}{R_2} = U^2\left(\frac{n_1^2}{R_1} + \frac{n_2^2}{R_2}\right) = \frac{U^2}{R_L},$$

where

$$U = \frac{U_0}{1 + \frac{R_i}{R_L}} = \frac{U_0}{1 + n_1^2\frac{R_i}{R_1} + n_2^2\frac{R_i}{R_2}} . \tag{2}$$

It is possible to put $R_i = R_1 = R_2$ and actual value of the load resistances to consider in the values n_1, n_2.

Then expression (2), taking into account (1), will become

$$\frac{U_0}{U} = 1 + \frac{U_1^2}{U^2} + \frac{U_2^2}{U^2} \text{ or}$$

$$\left(\frac{U_1}{U_0}\right)^2 + \left(\frac{U_2}{U_0}\right)^2 + \left(\frac{U_i}{U_0} - 0.5\right)^2 = 0.25 \tag{3}$$

$$U_i = U_0 - U$$

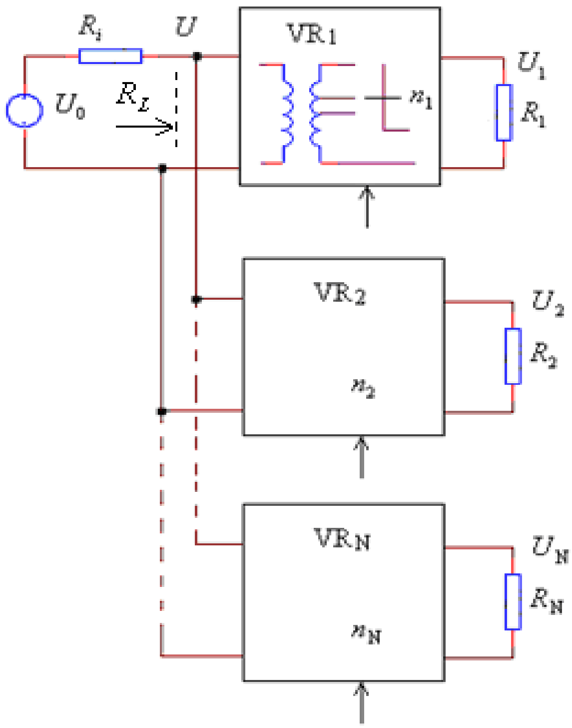

Fig.1. Power supply system with two voltage regulators VR_1, VR_2 and loads R_1, R_2

The obtained expression describes a sphere in the coordinates U_1, U_2, U_i in Fig.2. For simplification of drawing, the axes U_1, U_2 are superposed. Also, we accept $U_0 = 1$.

The direct and inverse change of variables

$$\left.\begin{aligned} U_1 &= \frac{n_1}{1 + n_1^2 + n_2^2} \\ U_2 &= \frac{n_2}{1 + n_1^2 + n_2^2} \end{aligned}\right\}, \quad \left.\begin{aligned} n_1 &= \frac{U_1}{1 - U_i} \\ n_2 &= \frac{U_2}{1 - U_i} \end{aligned}\right\}. \tag{4}$$

The system of the equations (4) has a simple geometrical sense. The variables n_1, n_2 turn out at the expense of a stereographic projection of points of the sphere from the pole (0,0,1) and define the conformal plane n_1, n_2 in Fig. 2, [5].

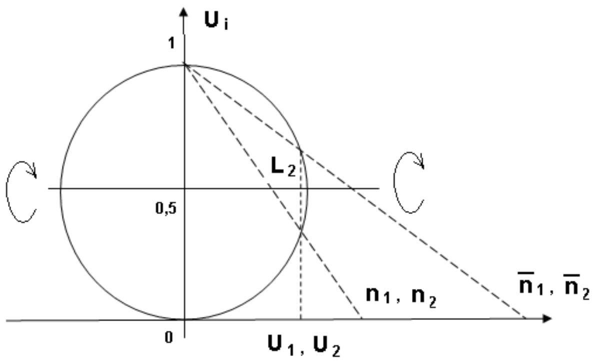

Fig.2. Stereographic projection of the sphere $U_i(U_1, U_2)$ on the plane n_1, n_2

For simplification of drawing, the axes n_1, n_2 are superposed too. Each point of sphere corresponds to one pair of voltage values U_1, U_2 and two pairs of transformation ratio values n_1, n_2, $\bar{n}_1, \bar{n}_2$. And, these values of transformation ratio submit to the inversion property, $n_1 \cdot \bar{n}_1 = 1, n_2 \cdot \bar{n}_2 = 1$. The conformal plane differs from usual (Euclidean) one by presence of the unique infinitely remote point corresponding to the pole of sphere. Also, the conformal plane interprets hyperbolic geometry, which is used for representation of character and interrelation of such parameters of regimes, as voltages U_1, U_2 and ratios n_1, n_2 .

On the plane U_1, U_2, the area of voltage change is defined by the internal area of a circle (by radius 0.5), and which corresponds to the sphere equator, Fig.3,a.

Let, for example, in such power supply system the regime $U_1 = const$ (that is line L_1) is supported at the expense of the n_1, n_2 changes. Then, a circular section L_2 on the sphere and a circle L_3 on the plane n_1, n_2 turn out. The similar family of circles describes this regime as the rotation group of the sphere, as it is shown by arrows in Figs.2, 3. But it can turn out so, that on some step of a switching cycle the working point is passing over the equator and voltage U_2 is going on step-down that is inadmissible.

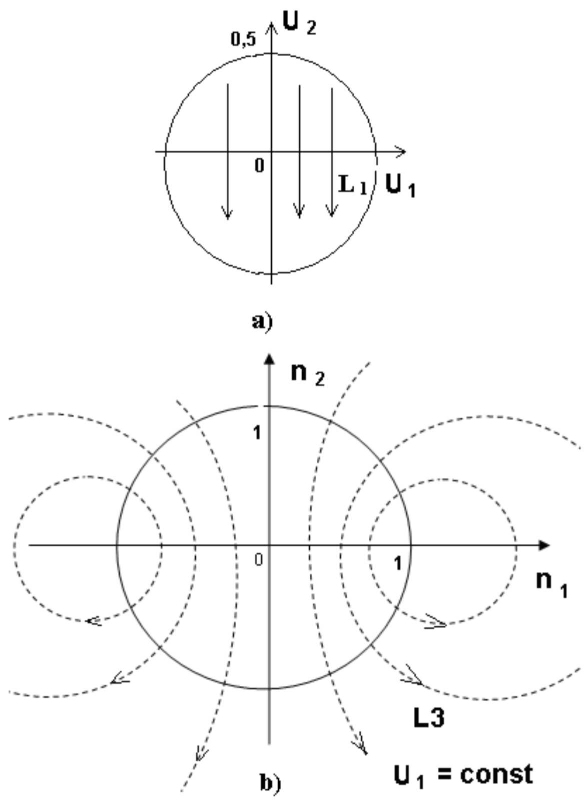

Fig.3. Correspondence the plane U_1, U_2 – a) and conformal plane n_1, n_2 - b) for $U_1 = const$

Therefore, it is better to use such groups of transformations or movements of points in the planes U_1, U_2 and n_1, n_2 , when it is impossible to deduce a working point over the circles, which correspond to equator of sphere by finite switching number.

In this sense, we come to hyperbolic geometry [5]. On the plane U_1, U_2 it is the Beltrami- Klein's model and on the plane n_1, n_2 it is the Poincare's (http://en.wikipedia.org/wiki/Hyperbolic_geometry/) model.

The corresponding circle carries the name of the absolute and defines infinitely remote border.

Let us obtain corresponding expressions of regime parameters and their changes. We put the value $n_2 = 0$. Then, the regime change goes only on axes U_1 and n_1. The conformity of the characteristic points and running point is shown in Fig. 4.

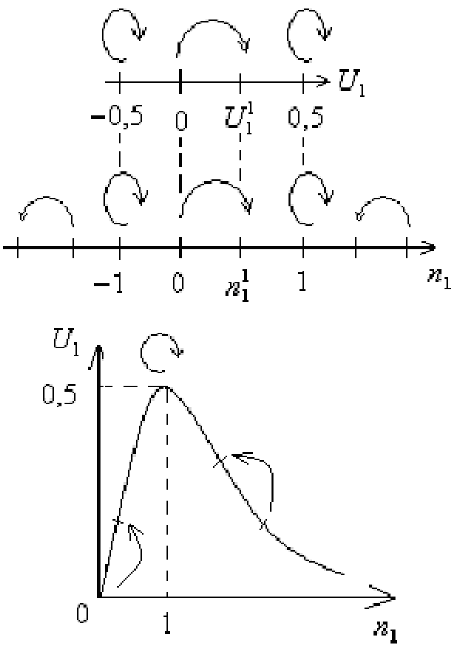

Fig.4. Conformity of the variables U_1, n_1 of the hyperbolic transformations

The used group of transformations should leave motionless the points on the absolute. Therefore, these points are accepted as the base points. The cross- ratio for the initial values of variables U_1 and n_1 relatively to the base points is similar to expression (8) in paper [4] and looks like

$$m_n^1 = (-1\ 0\ n_1^1\ 1\,) = \frac{1+n_1^1}{1-n_1^1},$$

$$m_U^1 = (-0.5\ \ 0\ \ U_1^1\ 0.5\,) = \frac{0.5+U_1^1}{0.5-U_1^1}. \tag{5}$$

Taking into account (4)

$$m_U^1 = \frac{(1+n_1^1)^2}{(1-n_1^1)^2}\ . \tag{6}$$

The expressions (5), (6) lead to identical values if we take a logarithm or use the hyperbolic metrics

$$Ln\, m_U^1 = 2Ln\frac{1+n_1^1}{1-n_1^1} = 2Ln\, m_n^1\ .$$

The cross- ratio, which corresponds to the regime change, has the form

$$m_n^{21} = (-1\, n_1^2\ n_1^1\, 1\,) = \frac{1+n_1^2}{1-n_1^2} : \frac{1+n_1^1}{1-n_1^1} = = \frac{1+\dfrac{n_1^2-n_1^1}{1-n_1^2 n_1^1}}{1-\dfrac{n_1^2-n_1^1}{1-n_1^2 n_1^2}}\ .$$

Then, ***there is a strong reason to introduce the value of transformation ratio change as***

$$n_1^{21} = \frac{n_1^2-n_1^1}{1-n_1^2 n_1^1}\ . \tag{7}$$

Therefore, the subsequent value of transformation ratio

$$n_1^2 = \frac{n_1^1+n_1^{21}}{1+n_1^1 n_1^{21}}\ . \tag{8}$$

Similarly, for the voltage change from U_1^1 to U_1^2 , at once it is possible to write down:

- the voltage change

$$U_1^{21} = \frac{U_1^2-U_1^1}{1-4U_1^2 U_1^1}, \tag{9}$$

- the subsequent value of voltage

$$U_1^2 = \frac{U_1^1+U_1^{21}}{1+4U_1^1 U_1^{21}}\ . \tag{10}$$

Validity of such definitions for changes U_1^{21} and n_1^{21} is confirmed still that for them the initial expression (4) is carried out

$$U_1^{21} = \frac{n_1^{21}}{1+(n_1^{21})^2}.$$

Thus, ***the concrete kind of a circuit imposes the requirements of definition of already system parameters.*** Therefore, arbitrary expressions are excluded.

In the methodical sense, it is useful to notice that expressions (8), (9), by analogy, correspond to the relativistic rule of speed composition in relative movement mechanics. If, for example $U_1^1 = 0.5$, then $U_1^2 = 0.5$ and it not depends from value U_1^{21}. For the considered case $n_2 = 0$ and the dependence (4) becomes simpler

$$U_1 = \frac{n_1}{1 + n_1^2} \ . \tag{11}$$

The plot of this expression is presented in Fig.4. If the voltage source U_0 is powerful enough, then a load voltage can increase beyond all bounds at increase of transformation ratio. The relationships (7), (9) are degenerated in definition of the segment length of the Euclidean geometry

$n_1^{21} = n_1^2 - n_1^1$, $U_1^{21} = U_1^2 - U_1^1$.

As it was mentioned, for the load regime control, it is expedient to use the known groups of transformations. Three types of movements on hyperbolic geometry plane are known, Figs.5, 6.

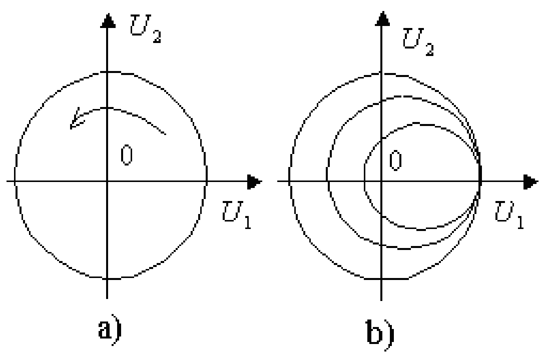

Fig.5. Types of movements on hyperbolic geometry plane: elliptic motion -a), - horocyclic movement -b)

They are: - elliptic motion as the rotation round a point, lying in the center of the absolute; - horocyclic movement round a point, lying on the absolute; - parallel shift or sliding along a straight line, for example, the axis U_1 .

The elliptic motion corresponds to the load constant power regime. The parallel shift along a straight line can be considered as priority regulation on the axis U_1 (from zero to maximum). Thus, the voltage U_2 changes from an initial value to zero. The corresponding transformations are determined, using (10)

$$U_1^2 = \frac{U_1^1 + U_1^{21}}{1 + 4U_1^1 U_1^{21}}\,, \quad U_2^2 = \frac{U_2^1\sqrt{1 - 4U_1^1}}{1 + 4U_1^1 U_1^{21}} \ . \tag{12}$$

In turn, the transformation ratios, corresponding (12), are set by similar group transformation

$$n^2 = \frac{n^1 + n^{21}}{1 + n^1 n^{21}} \ , \tag{13}$$

where the complex value $n = n_1 + jn_2$ (for the power supply system with two loads as the two-dimensional case) is used. The group transformations (12), (13) correspond to Fig.6,a,b.

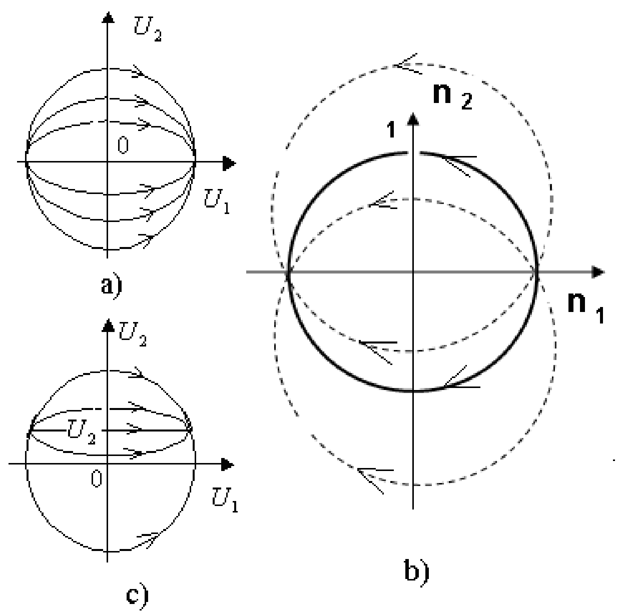

Fig.6. Types of parallel shift along the line U_1. Case $U_2 = 0$: trajectories on the plane $U_1 U_2$ - a), and on the plane $n_1 n_2$ - b); case $U_2 \neq 0$ - c)

If the parallel shift goes along a straight line $U_2 \neq 0$, the voltage U_2 stabilization regime turns out, Fig.6,c.

At increase of number of converters with loads (a connection of N'th converter is shown by a dotted line in Fig.1), the dimension of geometrical model by (3), (4) is increased

$$\left(\frac{U_1}{U_0}\right)^2+\left(\frac{U_2}{U_0}\right)^2+\ldots$$

$$\ldots+\left(\frac{U_N}{U_0}\right)^2+\left(\frac{U_i}{U_0}-0.5\right)^2=0.25,$$

$$\left.\begin{array}{l} U_1=\dfrac{n_1}{1+n_1^2+n_2^2+\ldots+n_N^2} \\ U_2=\dfrac{n_2}{1+n_1^2+n_2^2+\ldots+n_N^2} \\ \quad . \quad . \quad . \\ U_N=\dfrac{n_N}{1+n_1^2+n_2^2+\ldots+n_N^2} \end{array}\right\}, \qquad \left.\begin{array}{l} n_1=\dfrac{U_1}{1-U_i} \\ n_2=\dfrac{U_2}{1-U_i} \\ \quad . \quad . \quad . \\ n_N=\dfrac{U_N}{1-U_i} \end{array}\right\}.$$

But in the present paper, the problem of in-depth study of application of such group transformations in practical problems is not put.

3 Boost converter

Let us consider a boost voltage *PWM* converter in Fig.7. The static regulated characteristic for a continuous current mode of choke L with loss resistance R by width t and pulse period T

$$U_L=U_0\frac{\dfrac{T}{T-t}}{1+\dfrac{R}{R_L}\left(\dfrac{T}{T-t}\right)^2}=U_0\frac{n}{1+\sigma^2 n^2}. \qquad (14)$$

We can add the loss resistance R to the internal resistance of the voltage source U_0.

Then, it is possible to introduce the voltage U on the input of already idealized boost converter. Therefore, the value

$$n=\frac{U_L}{U}\geq 1$$

is the transformation ratio and voltage $U=\dfrac{U_0}{1+n^2\sigma^2}$.

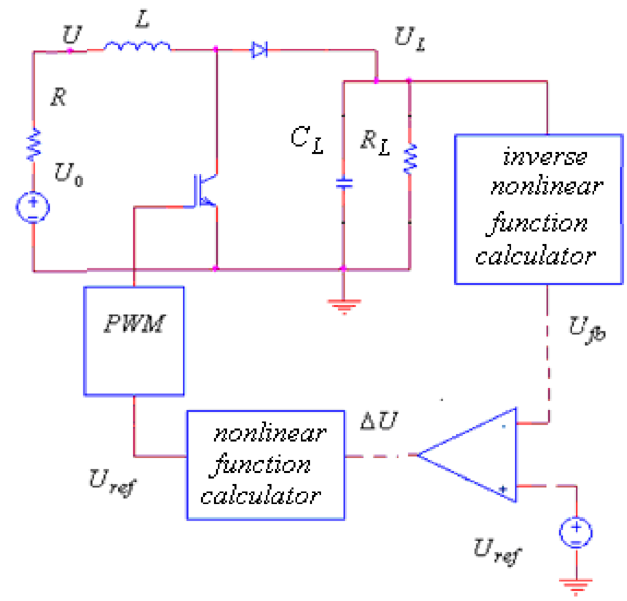

Fig.7. Boost voltage converter with linearization function calculators of control voltage ΔU and feedback voltage U_{fb}

Using expression of the regulated characteristic (14), we exclude a value n. Then, we obtain the equation of a circle or an ellipse

$$\sigma^2U_H^2+U^2-UU_0=0.$$

The plot of this ellipse is given in Fig.8.

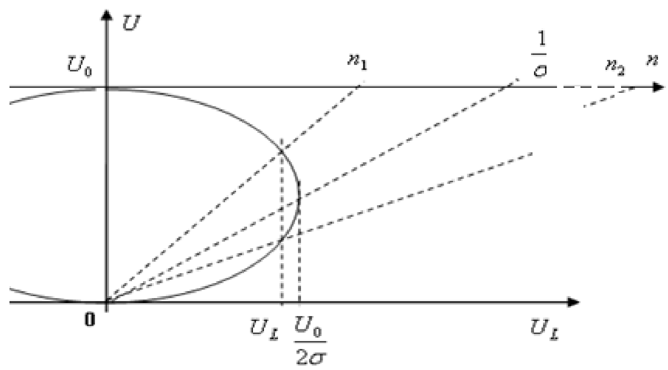

Fig.8. Geometrical model of the regulated characteristic of a boost converter

The variable n turns out at the expense of a stereographic projection of points of the circle from the bottom pole $(0,0)$. The maximum variable values

$$U_{LM} = \pm\frac{U_0}{2\sigma}, \ \ n_M = \pm\frac{1}{\sigma}.$$

The regime change or the load voltage regulation is defined by a group hyperbolic transformation, which consecutively, by steps, translates an initial point U_L^1 in a point U_L^2 and so on.

The conformity or transformation of the characteristic points and running points of different variables is shown in Fig.9.

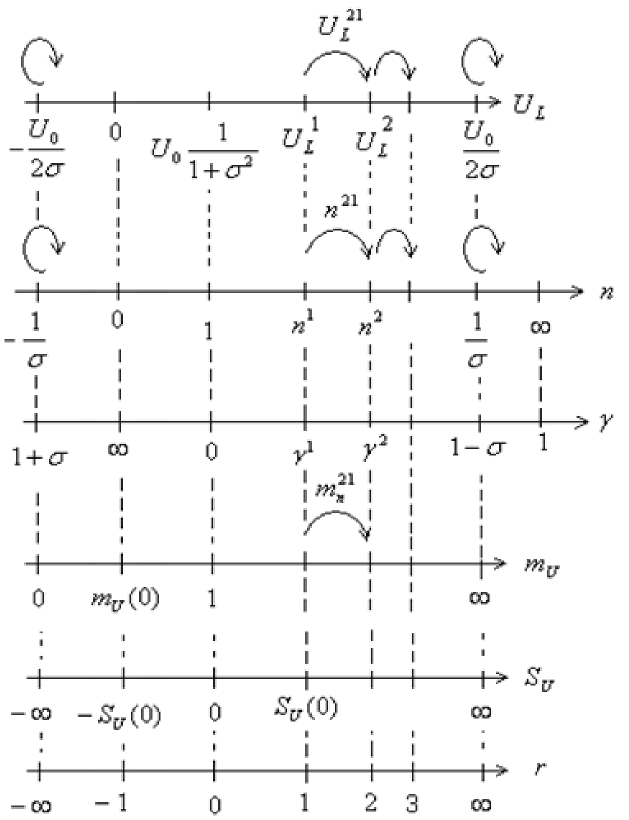

Fig.9. Regime change as a group transformation and conformity of the characteristic and running points of variables U_L, n, γ

Such transformation possesses an invariant; it is the cross ratio of four points. We determine necessary points of characteristic regimes. These are two points of the maximum voltage and the reference or starting point, when

$$U_{L0} = \frac{U_0}{1+\sigma^2}, n = 1, \gamma = 0.$$

For the initial values n^1, U_L^1, the corresponding cross ratios is similar to (5)

$$m_n^1 = (-\frac{1}{\sigma} \ \ n^1 \ 1 \ \ \frac{1}{\sigma}) = \frac{\frac{1}{\sigma}+n^1}{\frac{1}{\sigma}-n^1}\cdot\frac{1-\sigma}{1+\sigma}, \qquad (15)$$

$$m_U^1 = (-\frac{U_0}{2\sigma} \ \ U_L^1 \ \ \frac{U_0}{1+\sigma^2} \ \ \frac{U_0}{2\sigma}) = = \frac{\frac{U_0}{2\sigma}+U_L^1}{\frac{U_0}{2\sigma}-U_L^1}\cdot\frac{(1-\sigma)^2}{(1+\sigma)^2} = (m_n^1)^2 \qquad . \qquad (16)$$

The values γ and n are connected among themselves by the fractionally – linear expression

$$\gamma = \frac{n-1}{n}.$$

Then, it is possible to express the cross ratio (15) by value γ

$$m_\gamma^1 = m_n^1 = (\ (1+\sigma) \ \ \gamma^1 \ \ 0 \ \ (1-\sigma) \ ) = = \frac{(1+\sigma)-\gamma^1}{(1-\sigma)-\gamma^1}\cdot\frac{1-\sigma}{1+\sigma} \qquad . \qquad (17)$$

The expressions (15), (16) also lead to identical values if we take a logarithm similarly to (6)

$$S^1 = Ln \ m_U^1 = 2Ln \ m_n^1 \ .$$

Then hyperbolic distance

$$S^1 = Ln\frac{\frac{U_0}{2\sigma}+U_L}{\frac{U_0}{2\sigma}-U_L} + 2Ln\frac{1-\sigma}{1+\sigma}.$$

In particular, for the reference point the hyperbolic distance is equal to zero.

Besides considered three points of characteristic regimes, there is still the fourth one, the scale point $U=0, n=0, \gamma=\infty$. This scale point should be considered too.

The cross ratio and hyperbolic distance for the scale point

$$m_U(0)=\left(\frac{1-\sigma}{1+\sigma}\right)^2<1 ,$$

$$S\ (0)=Ln\ m_U(0)=2Ln\frac{1-\sigma}{1+\sigma}<0 .$$

The corresponding hyperbolic distance will be positive for inverse value of the cross ratio.

Further, it is natural to introduce the normalized hyperbolic distance for a running regime (the index «1» is lowered), using the obtained scale

$$r\ =\frac{S}{S\ (0)}=\frac{Ln\dfrac{\dfrac{U_0}{2\sigma}+U_L}{\dfrac{U_0}{2\sigma}-U_L}}{2Ln\dfrac{1+\sigma}{1-\sigma}}-1 . \tag{18}$$

Thus, the normalized distance considers the all characteristic points. The inverse expression is

$$U_L=\frac{U_0}{2\sigma}\,\frac{\left(\dfrac{1+\sigma}{1-\sigma}\right)^{2(r+1)}-1}{\left(\dfrac{1+\sigma}{1-\sigma}\right)^{2(r+1)}+1} . \tag{19}$$

For regime change $n^1\to n^2$, we have

$$m_n^{21}=(-\frac{1}{\sigma}\ n^2\ n^1\ \frac{1}{\sigma})=\frac{\dfrac{1}{\sigma}+n^2}{\dfrac{1}{\sigma}-n^2}:\frac{\dfrac{1}{\sigma}+n^1}{\dfrac{1}{\sigma}-n^1}=$$
$$=\frac{\dfrac{1}{\sigma}+\dfrac{n^2-n^1}{1-\sigma^2\ n^2n^1}}{\dfrac{1}{\sigma}-\dfrac{n^2-n^1}{1-\sigma^2\ n^2n^1}} .$$

It is possible to introduce, similarly to (7)

$$n^{21}=\frac{n^2-n^1}{1-\sigma^2\ n^2n^1} . \tag{20}$$

Then

$$m_n^{21}=\frac{\dfrac{1}{\sigma}+n^{21}}{\dfrac{1}{\sigma}-n^{21}} . \tag{21}$$

The subsequent value

$$n^2=\frac{n^1+n^{21}}{1+\sigma^2\ n^1n^{21}} . \tag{22}$$

Similarly, for the voltage change $U_L^1\to U_L^2$ at once it is possible to write down

$$\left.\begin{aligned} m_U^{21}&=\frac{\dfrac{U_0}{2\sigma}+U_L^{21}}{\dfrac{U_0}{2\sigma}-U_L^{21}}\\ U_L^{21}&=\frac{U_L^2-U_L^1}{1-4\sigma^2\dfrac{U_L^2U_L^1}{U_0{}^2}}\\ U_L^2&=\frac{U_L^{21}+U_L^1}{1+4\sigma^2\dfrac{U_L^{21}U_L^1}{U_0{}^2}} \end{aligned}\right\} \tag{23}$$

Consider the change m_γ^{21}. Using (17), we have

$$m_\gamma^{21}=m_\gamma^2\div m_\gamma^1=$$
$$=\frac{(1+\sigma)-\gamma^2}{(1-\sigma)-\gamma^2}\div\frac{(1+\sigma)-\gamma^1}{(1-\sigma)-\gamma^1}=\frac{1-\sigma\gamma^{21}}{1+\sigma\gamma^{21}} ,$$

where the change

$$\gamma^{21}=\frac{\gamma^1-\gamma^2}{1-\sigma^2+\gamma^1\gamma^2-(\gamma^1+\gamma^2)},\ n^{21}=\gamma^{21} . \tag{24}$$

Thus, mutually coordinated system of all regime parameters turns out.

4 Examples

Let converter with concrete values of the elements $U_0 = 25V$, $\sigma_1 = 0.08$ is given. The regulated characteristic, according to (14), has a characteristic form, Fig.10.

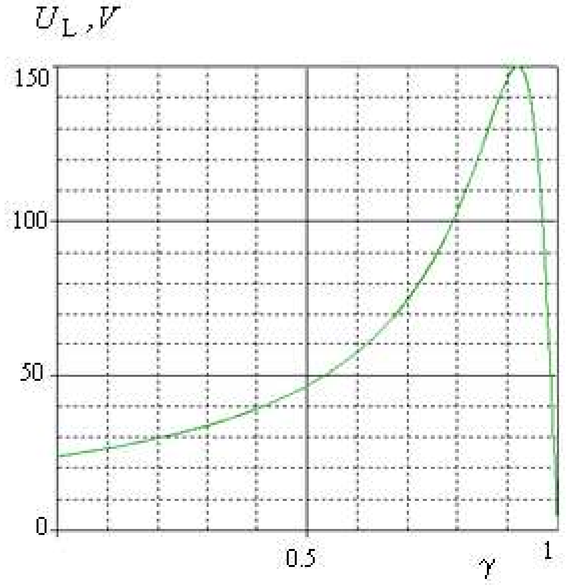

Fig.10. Regulated characteristic of the boost converter

Example1. Let $U_L^1 = 48.49V$ for the initial regime. Define normalized distance for this regime.

We find the characteristic values of all regime parameters, Fig.11

$$U_{LM} = \pm\frac{U_0}{2\sigma} = \pm 156.25V\,, \quad n_M = \pm\frac{1}{\sigma} = \pm 12.5\,,$$

$$\gamma_M = \frac{n_M - 1}{n_M} = 1 - \sigma = 0.92\,.$$

For $n_1 = 1,$ we have

$$U_L = 25\frac{1}{1 + 0.08^2} = 24.841V\,.$$

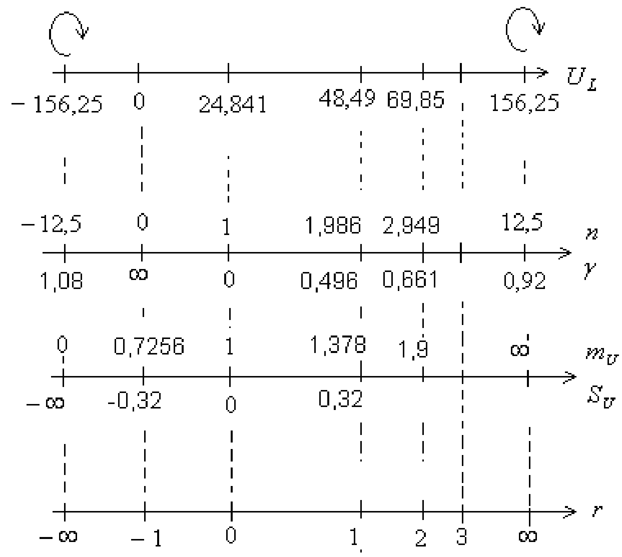

Fig.11. Example of regime change and conformity of points of the axes U_L, n , γ of the boost converter

We find the cross ratio for the initial regime from (15, 16)

$$m_U^1 = \frac{156.25 + 48.49}{156.25 - 48.49}\cdot(0.852)^2 = 1.378\,,$$

$$m_n^1 = \sqrt{m_U^1} = \sqrt{1.378} = 1.174\,.$$

Then, the corresponding value n^1 can be calculated from the inverse formula to (15)

$$n^1 = \frac{1}{\sigma}\cdot\frac{m_n^1\cdot\frac{1+\sigma}{1-\sigma} - 1}{m_n^1\cdot\frac{1+\sigma}{1-\sigma} + 1} = 1.986\ .$$

In turn, we have

$$\gamma^1 = 0.496\,, \quad S_U^1 = Ln1.378 = 0.320\,.$$

The cross ratio and distance for a scale point of the regulator are

$$m_U(0) = \left(\frac{1-\sigma}{1+\sigma}\right)^2 = 0.725,$$

$$, S_U(0) = Ln\, m_U(0) = 2Ln\frac{1-\sigma}{1+\sigma} = -0.320.$$

Then, the normalized distance for the initial regime, $r^1 = 1$.

Example 2. The regime of the converter has changed on the value $r^{21} = 1$. It is necessary to define its actual regime parameters.

The distance for this regime

$$r_1^2 = r_1^1 + r_1^{21} = 2.$$

Then

$$U_L^2 = 156.25 \cdot \frac{1.174^{2(r+1)} - 1}{1.174^{2(r+1)} + 1} = = 156.25 \cdot \frac{2.618 - 1}{2.618 + 1} = 69.85V.$$

This value can be obtained by another way, using the change U_L^{21} and the expression (23). In this case, the change $U_l^{21} = 24.841V$ (is equal to scale value). Therefore,

$$U_l^2 = \frac{24.841 + 48.49}{1 + 4 \cdot 0.0064 \dfrac{24.841 \cdot 48.49}{25^2}} = 69.88V.$$

Similarly, the change $n^{21} = 1$.

Then, by (22)

$$n^2 = \frac{1.986 + 1}{1 + 0.0064 \cdot 1.986 \cdot 1} = 2.949, \ \gamma^2 = 0.661.$$

The values U_L, γ, n for the further steps are shown in Fig. 11. It is visible, that ***each time the actual voltage change is reduced and can not reach the maximum value.***

Example 3. Let it is necessary the regime with the initial value $U_L^1 = 48.49V$ to change to value $U_L^N = 69.85V$ consistently by small steps. The consecutive reduction of change step reduces, in turn, undesirable transients. Let the number of steps is $N = 5$. It is necessary to find the values U_L^i, n^i on each step i.

We find a hyperbolic distance, corresponding to the initial (a zero step) regime and final (the fifth step) one. According to the example 2, this distance is $r^{50} = 1$. For five steps, the regime change (as a hyperbolic distance) is equal $\Delta r = 0.2$.

The value (or length) of the first step is $r^1 = r^0 + r^{10} = 1 + 0.2 = 1.2$.

The first step voltage according to (19) is

$$U_L^1 = 156.25\, \frac{1.174^{2 \cdot 2.2} - 1}{1.174^{2 \cdot 2.2} + 1} = 52.96V,$$

$$n^1 = 2.183$$

The voltage change on the first step according to (23) is

$$U_L^{10} = \frac{52.963 - 48.49}{1 - 4 \cdot 0.0064 \dfrac{52.963 \cdot 48.49}{25^2}} = 5V.$$

The changes on following steps also are equal to this value. The transformer ratio change on the first step according to (20) is

$$n^{10} = \frac{2.183 - 1.986}{1 - 0.0064 \cdot 2.183 \cdot 1.986} = 0.202.$$

For subsequent steps, values n, U_L are obtained via the recurrent relationships (22, 23), since the changes on the following steps keep their values

$$n^2 = \frac{2.183 + 0.2029}{1 + 0.0064 \cdot 2.183 \cdot 0.2029} = 2.379,$$

$$U_L^2 = \frac{52.963 + 5}{1 + 4 \cdot 0.0064 \dfrac{52.963 \cdot 5}{25^2}} = 57.34V,$$

$n^3 = 2.574, n^4 = 2.768, n^5 = 2.96$,

$U_L^3 = 61.617V, U_L^4 = 65.78V, U_L^5 = 69.84V$.

It is visibly, as such actual changes of the values n, U_L are being decreased.

Example 4. We will consider possibility of linearization of a regulated characteristic.

Let us express γ via the r similarly to (19)

$$\gamma = 1 - \sigma \frac{\left(\frac{1+\sigma}{1-\sigma}\right)^{r+1} + 1}{\left(\frac{1+\sigma}{1-\sigma}\right)^{r+1} - 1} . \quad (25)$$

It turns out, that the value r is equally expressed via γ and U_L. It is possible to interpret as some kind of linearization of the dependence $U_L(\gamma)$.

Further, it is possible to accept that the value of γ is equal to the regulating voltage U_{reg} of the PWM and, as hyperbolic distance r, we accept the input voltage ΔU of the nonlinear function calculator (it is introduced before PWM), Fig.12.

Expressing r via U_L according to (18) (inverse nonlinear function calculator), we obtain the value closed to the voltage ΔU, that is the feedback voltage U_{fb}, Fig. 13.

For acknowledgment of linearization of the actual regulating characteristic, the results of ORCAD modeling of the boost converter with

$$R = 0.256\Omega, R_L = 40\Omega, C_L = 10\mu F, L = 250\mu H$$

are presented in Fig. 14. The switching period of PWM is $T = 50\ \mu S$.

The Fig.14 shows parameters and forms of all voltage plots via time t.

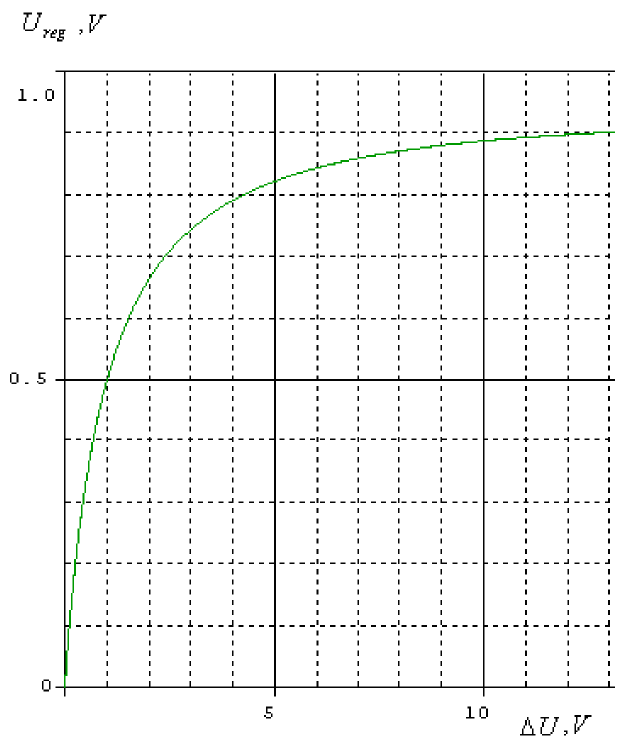

Fig.12. Nonlinear function calculator characteristic

The input voltage ΔU changes under the linear law. Output voltage U_{reg} of the nonlinear function calculator changes under the nonlinear law.

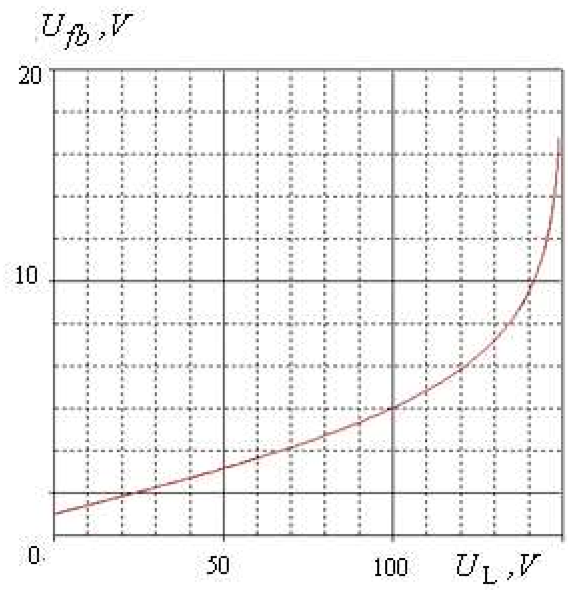

Fig.13. Inverse nonlinear function calculator characteristic

The load voltage U_L changes also under the nonlinear law and feedback voltage U_{fb} changes strictly linearly. It is possible to note the natural restriction of the values U_{reg} and U_L at sufficiently great value of the voltage ΔU .

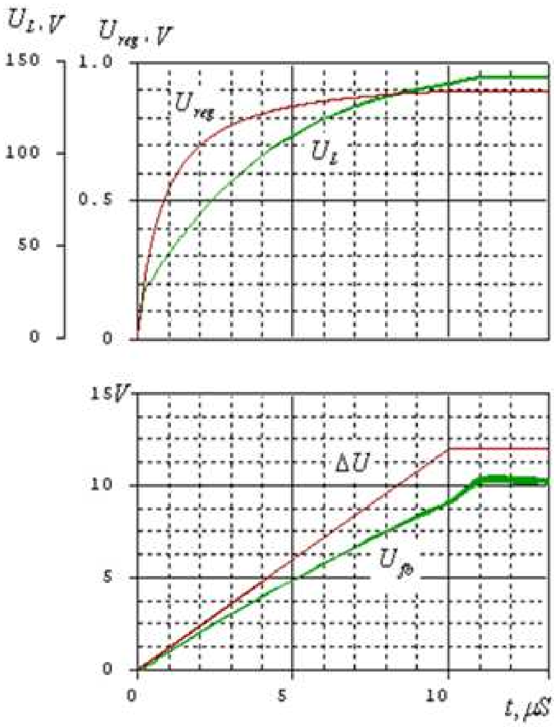

Fig. 14. Results of modeling of the boost converter with the linearized regulated characteristic.

If to use a feedback closed loop (it is shown by a dashed line in Fig.7), the obtained voltage of a error amplifier will be the voltage ΔU .

Such method of load voltage regulation is elaborated (Penin A., Patent MD, n.4067, BOPI, No.8, 2010).

5 Conclusions

Geometrical interpretation of a simplified model of a multichannel power supply system allows proving definition of the operating regime parameters.

The invariant property between a load voltage and transformation ratio allows linearization of the regulated characteristic of a boost converter.

Results can be useful to creation of both separate power supplies and modular power supply systems - organization of the coordinated work of the constituent converters for the specified load regimes.

References:

1. Tamer T.N. Khatib, Mohamed A., Amin N., and Sopian K., An efficient maximum power point tracking controller for photovoltaic systems using new boost converter design and improved control algorithm, *WSEAS Transactions on power systems,* Vol.5, No. 2, 2010, pp. 53- 63. Available at http://www.worldses.org/journals/power/power-2010.htm

2. Penin A. A., Linear- fractional relations in the problems of analysis of resistive circuits with variable parameters, *Electrichestvo,* No.11, 1999, pp. 32-44 (in Russian)

3. Penin A. A., Regimes analysis of the voltage pulse generators on the basic of the invariance property of the control characteristics, *Problems of the regional energetics,* No.1, 2009, (in Russian). Available at http://ieasm.webart.md/contents_ru/?volume_id=21

4. Penin A.A., About the definition of parameters and regimes of active two-port networks with variable loads on the basis of projective geometry, *WSEAS Transactions on circuits and systems* , Vol.10, No.5, 2011, pp. 157-172. Available at http://www.worldses.org/journals/circuits/circuits-2011.htm

5. Coxeter H. S. M., *Introduction to geometry,* New York – London, J.Wiley & Sons, Inc. 1961.