Adaptive Fault Detection Methods Based on PCA Technique

RADHIA FAZAI, OKBA TAOUALI, NASREDDINE BOUGUILA

Laboratory of Automatic Signal and Image Processing, National School of Engineers of Monastir, University of Monastir, 5019, Tunisia

Rue Ibn El Jazzar 5019 Monastir; Tel: + (216) 73 500511, Fax: + (216) 73 500 514, TUNISIA

fazai.radhia@ymail.com; taoualiokba@yahoo.fr, nasreddine.bouguila@gmail.com

Abstract: -For the improvement of reliability, safety and efficiency advanced methods of supervision, fault detection and fault diagnosis become increasingly important for many technical processes. This holds especially for safety related processes like aircraft, trains, automobiles, power plants and chemical plants. The fault detection based upon multivariate statistical projection method such as Principal Component Analysis (PCA) has attracted more and more interest in academic research and engineering practice. The PCA is an appropriate method for the control of the process based on selection of an optimal number of principal components. In this paper we present the design and a comparative study of offline fault detection indices based on PCA method and adaptive fault detection techniques which used the PCA method. These indices are Squared Prediction Error (SPE), Hotelling's Statistic (T^2), Filtred Squared Prediction Error (Filtred SPE) and D_i Index. These indices and the adaptive detection methods are evaluated by handling a numerical example and a Continuous Stirred Tank Reactor (CSTR) benchmark.

Keywords- PCA, SPE, T^2, D_i , onlie fault detection, PCA conventional, SWPCA, RPCA-FOP

1 Introduction

The increasing demand for effective quality, high productivity and safe operation has enhanced research into fault detection and diagnosis methods [1], [2]. Several statistical methods such as Principal Component Analysis PCA [3] and [4], Partial Least Square PLS [5] and more recently Independent Component Analysis ICA [6] have been developed for process monitoring to deal with this challenging problem.

Principal Component Analysis (PCA) is a multivariate statistical method that can be used for process monitoring. The basic strategy of PCA is to extracts linear structure from high dimensional data by finding new principal axes. PCA divides data systematically into two parts, the first part is the data with wide variation and the second part is the data with the least variance, which is noisy. The PCA method is a tool that models the behavior of processes in normal operation. The faults are detected then by comparing the observed behavior and the behavior given by the PCA model or by the generation of the detection indexes [7,8] such as, the squared prediction error SPE [9,10], and the Hotteling's T^2 statistic [11,12]. Since PCA is a projection method to reduce the data, the first step of its usage is devoted to modulation of the process to determine a suitable structure of the PCA model. Dunia and Qin [13] proposed the VRE (Variance of reconstruction error) method to determine the optimal number of components to be used for the construction of the PCA model.

In this paper, the VRE criterion is used to determine the structure of the model. For the fault detection, we present the SPE index, T^2, filtered SPE [14], and the D_i index. We present also a comparative study of the performance of these indices.

The paper is organized as follows: in section 2, the PCA approach and the VRE criterion are presented. The indices SPE, T^2, filtered SPE, and D_i are described in section 3. In section 4, the performances of the fault detection indices are evaluated on a numerical example. The adaptive fault detection methods are detailed in the section 5. Finally section 6 concludes the paper.

2 Principal Component Analysis

2.1 Concept of PCA method

PCA is to replace a family of variables with new variables having maximum variances. The new variables are uncorrelated pairs of variables that are linear combinations of the original variables.

Let at time *k*,

$x(k) = \left[x_1(k), x_2(k), ..., x_m(k)\right]^T \in R^m$ is the vector which contains the m observed system variables, and consider $X = \left[x(1)\ x(2) \ldots x(N)\right]^T \in R^{N\times m}$ the matrix of standardized data that contain N observations collected while the process is operating in normal mode. The PCA determines an optimal (considering covariance criterion) and linear transformation of the matrix *X* as follows:

$$T = XP \text{ and } X = TP^T \quad (1)$$

with $T = \left[t_1, t_2, ..., t_m\right]^T \in R^{N\times m}$,where $t_i\ (i = 1, ..., m)$ are the principal components and the matrix $P = \left[p_1, p_2, \ldots, p_m\right] \in R^{m\times m}$ where the orthogonal vectors p_i are the eigenvectors corresponding to the eigenvalues λ_i that results from the decomposition of the correlation matrix (or covariance) Q of X on eigenvalues and eigenvectors.

$$Q = P\Lambda P^T \text{ and } PP^T = P^T P = I_m \quad (2)$$

with $\Lambda = \text{diag}(\lambda_1 ... \lambda_m)$ is a diagonal matrix where the diagonal elements are ordered in descending order.

Let's consider the *l* first principal components t_i *(i=1,….l)*. It turns out that the rest of the information contained in X can be explained using $\hat{P} = \left[p_1, ..., p_l\right] \in R^{m\times l}$ and represented by $\tilde{T} = X\tilde{P}$. Therefore, the first eigenvectors from the vectorial subspace reduced by the initial data called the principal subspace noted as SP and the *(m-l)* last eigenvectors from the residual subspace noted SR. The matrix P and T can be partitioned as follows:

$$P = \left[\hat{P}\ \ \tilde{P}\right],\ \ T = \left[\hat{T}\ \ \tilde{T}\right] \quad (3)$$

The equation (1) can be written then as follows:

$$X = \hat{T}\hat{P}^T + \tilde{T}\tilde{P}^T = X\hat{P}\hat{P}^T + X\tilde{P}\tilde{P}^T = \hat{X} + E \quad (4)$$

with

$\hat{X} = X\hat{P}\hat{P}^T = X\hat{C},\ E = X\tilde{P}\tilde{P}^T = X\tilde{C}$

$\hat{C} = \hat{P}\hat{P}^T$ and $\tilde{C} = I - \hat{C}$.

$\hat{C}$ and $\tilde{C}$ are the projection matrices respectively in the principal and residual subspaces. They represent the PCA model of the system.

The matrix $\hat{X}$ and E represent respectively the modeled and non-modeled variations of X .

2.2 Determination of the Number of Principal Components

In order to apply the PCA concept in diagnosis, it is important to determine the number of principal components to retain.

For instance, when the number of components to retain is small, we risk losing valuable information which leads to incomplete representation of the process, and therefore have modeling errors that taints the residues causing false alarms.

On the other hand, in the case where the number of components to retain is large, the model will be on-set and may contain measurement noise because it may contain components that are carriers of noise. Further, reduction of the dimension of the residual space causes the non detection of faults.

Therefore, Dunia and Qin suggested determination of the number of components to retain by minimizing the variation of reconstruction error. Reconstruction is estimating a variable of the vector $x(k)$ at a given time denoted as $x_i(k)$ using all the other variables $x_j(k)$ at the same time from the PCA model. The i^{th} variable is reconstructed using the following equation:

$$x_i(k) = G_i x(k) \quad (5)$$

with $G_i^T = \left[\xi_1 ... g_i ... \xi_m\right]$, $g_i^T = \dfrac{\left[c_{-i}^T\ \ 0\ \ c_{+i}^T\right]}{1 - c_{ii}}$ with $c_{ii} < 1$

where:

$\xi_i = \left[0...1...0\right]^T$ is the i^{th} column of the identity matrix.

$c_i^T = [c_{1i}\ c_{2i} \ldots c_{mi}] = \left[c_{-i}^T\ c_{ii}\ c_{+i}^T\right]$. c_i is the i^{th} column of the matrix $\hat{C}$ and the indexes (-i) and (+i) represent respectively the vectors formed by the $(i-1)$ first and $(m-i)$ last elements of the vector c_i.

The variation of the reconstruction error of the i^{th} component $x(k)$ is given by the following equation:

$$\sigma_i(l) = \mathrm{var}(\xi_i^T(x - x_i)) = \frac{\tilde{\xi}_i^T Q \tilde{\xi}_i}{(\tilde{\xi}_i^T \tilde{\xi}_i)^2} \tag{6}$$

with $\tilde{\xi}_i = \hat{C}\xi_i$ and Q is the correlation matrix.

The optimization problem of variables resides therefore in minimizing the variance of reconstruction error σ_i considering the number of principal components l:

$$J(l) = \min_l \sum_{i=1}^{m} \frac{\sigma_i(l)}{\xi_i^T Q \xi_i} \qquad l = 1, \ldots, m-1 \tag{7}$$

Once the model is determined, the faults can be detected.

3 Fault Detection indices

To detect faults, several indices are conventionally used:

3.1 Squared Prediction Error (SPE)

The squared prediction error (SPE) index allows fault detection in the residual subspace.

At time k it is given by:

$$\begin{aligned} SPE(k) &= \tilde{x}(k)^T \tilde{x}(k) = \left\|\tilde{C}\, x(k)\right\|^2 \\ &= \left\|\tilde{t}(k)\right\|^2 = \sum_{j=l+1}^{m} \tilde{t}_j^2(k) \end{aligned} \tag{8}$$

This index is an aggregate indicator that sums residues regardless of their variances. When the systems are no longer linear, residues with high variances carry modeling errors produced by the PCA model. Therefore, they have more effects on the SPE amount compared to the residues with low variances that correspond to linear or quasi-linear redundancy relations. The sensitivity of the SPE indicator to modeling errors causes many false alarms.

The system is considered in abnormal operation at time k if:

$$SPE(k) > \delta_\alpha^2 \tag{9}$$

With δ_α^2 is the control limit of $SPE(k)$. For a confidence level α, δ_α^2 is determined theoretically by box [15].

$$\delta_\alpha^2 = g\chi_{h,\alpha}^2 \tag{10}$$

with $g = \frac{\theta_2}{\theta_1}, h = \frac{\theta_1^2}{\theta_2}$ and $\theta_i = \sum_{j=l+1}^{m} \lambda_j^i, i \in \{1, 2\}$ and λ_j is the j^{th} eigenvalue of the matrix Q.

In order to guarantee the detection of fault $d(k)$ by SPE index, a sufficient condition must be added to the projection of the fault in the residual space. You can get this condition using the following equation:

$$SPE(k) = \left\|\tilde{C}^T x(k)\right\|^2 = \left\|\tilde{C}^T x^*(k) + \tilde{C}^T \xi_j\, d(k)\right\|^2 \tag{11}$$

Since $x^*(k)$ is a measurement vector in normal operating mode:

$$\left\|\tilde{C}^T x^*(k)\right\| < \delta_\alpha \tag{12}$$

Since:

$$\left\|\tilde{C}^T x^*(k) + \tilde{C}^T \xi_j\, d(k)\right\| \geq \left\|\tilde{C}^T \xi_j\, d(k)\right\| - \left\|\tilde{C}^T x^*(k)\right\| \tag{13}$$

Consequently:

$$\left\|\tilde{C}^T x^*(k) + \tilde{C}^T \xi_j\, d(k)\right\| \geq \left\|\tilde{C}^T \xi_j\, d(k)\right\| - \delta_\alpha \tag{14}$$

In order to ensure the fault detection, the condition $SPE(k) > \delta_\alpha^2$ must be satisfied. We require then that:

$$\left\|\tilde{C}^T x\right\|^2 \geq (\left\|\tilde{C}^T \xi_j\, d(k)\right\| - \delta_\alpha)^2 > \delta_\alpha^2 \tag{15}$$

The resolution of the inequality (15) yields to:

$$\begin{aligned} &\left\|\tilde{C}^T \xi_j\, d(k)\right\|^2 - 2\delta_\alpha \left\|\tilde{C}^T \xi_j\, d(k)\right\| + \delta_\alpha^2 > \delta_\alpha^2 \\ &\left\|\tilde{C}^T \xi_j\, d(k)\right\|^2 > 2\delta_\alpha \left\|\tilde{C}^T \xi_j\, d(k)\right\| \end{aligned} \tag{16}$$

Consequently:

$$\left\|\tilde{C}^T \xi_j\, d(k)\right\| > 2\delta_\alpha \tag{17}$$

otherwise: $\left|\tilde{d}\right| > 2\delta_\alpha$

where $\tilde{d} = d(k)\left\|\tilde{\xi}_j\right\|$

The equation (2) must be satisfied to guarantee the fault detection.

3.2 Hotelling's T^2Statistic

Unlike the SPE index, the Hotelling's T^2 statistic measures variations of the projections of the observations in the principal subspace. T^2 determined from the first l principal components according to the following equation:

$$T^2(k) = \hat{t}^T(k)\Lambda_l^{-1}\hat{t}(k) = \sum_{j=1}^{l}\frac{\hat{t}_j^2(k)}{\lambda j} \quad (18)$$

where $\Lambda_l = \mathrm{diag}(\lambda_1,...,\lambda_l)$ is the diagonal matrix containing the l largest eigenvalues of the correlation matrix.
The process is considered faulty at time k if:

$$T^2(k) > \chi^2_{l,\alpha} \quad (19)$$

where $\chi^2_{l,\alpha}$ is the chi-2 distribution with degree of freedom equals to l , and confidence level equals to α. To ensure the fault detection using the T^2 indexa condition that considers the fault amplitudes should be added. This condition is determined using the same approach as the SPE index. From equation (18), we get:

$$T^2(k) = \left\|\Lambda_l^{-\frac{1}{2}}\hat{P}^T(x^*(k)+\xi_j d(k))\right\|^2 \quad (20)$$

The following condition guarantees the fault detection:

$$\left\|\Lambda_l^{-\frac{1}{2}}\hat{P}^T\xi_j d(k)\right\| > 2T_\alpha \quad (21)$$

3.3 Filtred SPE Index

In order to improve the fault detection, EWMA (Exponentially Weighted Moving Average) filter may be applied to residues .The expression of the filtered residues is given by [16]:

$$\bar{e}(k) = (I-\beta)\bar{e}(k-1)+\beta e(k) \quad (22)$$

with $\bar{e}(0)=0$ and β is the diagonal matrix given by the following equation:

$$\beta = \gamma I \quad (23)$$

where $0<\gamma<1$ is the forgetting factor.

Consequently, the filtered SPE denoted as $\overline{SPE}$ is given by the following equation:

$$\overline{SPE}(k) = \|\bar{e}(k)\|^2 \quad (24)$$

The process is supposed to be faulty when:

$$\overline{SPE}(k) > \bar{\delta}^2_\alpha \quad (25)$$

where $\bar{\delta}^2_\alpha$ is a control limit of $\overline{SPE}$.

Qin [17] showed that $\bar{\delta}^2_\alpha$ follows χ^2 rule and is related to the non-filtered control limit $\bar{\delta}^2_\alpha$ by the following equation:

$$\bar{\delta}^2_\alpha = \frac{\gamma}{2-\gamma}\delta^2_\alpha \quad (26)$$

3.4 D_i Index

The D_i index is determined using the last principal components using the following relation:

$$D_i(k) = \sum_{j=m-i+1}^{m}\tilde{t}_j^2(k),\ i=1,2,...,(m-1) \quad (27)$$

The process is considered faulty at time k if:

$$D_i(k) > \tau^2_{i,\alpha} \quad (28)$$

The index D_i is indeed an SPE calculated using PCA model that has $(m-i)$ principal components. Thus, the control limits $\tau^2_{i,\alpha}$ can be approximated, for a given confidence level α, by a χ^2 distribution with $h_{(i)}$ degree of freedom:

$$\tau^2_{i,\alpha} = g_{(i)}\chi^2_{h_{(i)},\alpha} \quad (29)$$

The parameters $g_{(i)}$ and $h_{(i)}$ are given by the following equation:

$$g_{(i)} = \frac{\sum_{j=m-i+1}^{m}\lambda_j^2}{\sum_{j=m-i+1}^{m}\lambda_j},\ h_{(i)} = \frac{(\sum_{j=m-i+1}^{m}\lambda_j^2)}{\sum_{j=m-i+1}^{m}\lambda_j^2} \quad (30)$$

Like the SPE index, failure detection is guaranteed if the following condition is satisfied:

$$\left\|\tilde{\xi}_j^{(i)}d(k)\right\| > 2\tau_{i,\alpha} \quad (31)$$

where

$\tilde{\xi}_j^{(i)} = (I-\hat{C}_i)\xi_j$ and $\tilde{\xi}_j^{(i)} = (I-\hat{C}_i)\xi_j$, $\hat{C}_i = \hat{P}^{(i)}\hat{P}^{(i)T}$; $\hat{P}^{(i)}$ is formed by the i last eigenvectors of the matrix Q.

4 Simulation Results

To evaluate the performances of these detection indices, simulations on a numerical example are presented in the following sections.

Consider a static system with seven variables described by the following equations:

$$z_1(k) = u_1(k)+\varepsilon_1(k)$$
$$z_2(k) = u_1(k)+\varepsilon_2(k)$$
$$z_3(k) = u_1(k)+\varepsilon_3(k)$$
$$z_4(k) = 2u_1(k)+u_2(k)+\varepsilon_4(k)$$
$$z_5(k) = u_2(k)+\varepsilon_5(k)$$
$$z_6(k) = u_2(k)+\varepsilon_6(k)$$
$$z_7(k) = 2u_1(k)+3u_2(k)+\varepsilon_7(k)$$

with: ε_i , $i \in \{1,...,7\}$ are random noises with amplitudes that vary from -0.05 to +0.05, u_1 and u_2 represent the inputs of the system.

The inputs have niche forms that have randomly changing amplitudes and durations, and z_i are the outputs of the system with $i \in \{1,...,7\}$.

This system has modeling errors. The system equations show that the variables are correlated since there is a direct redundancy among the variables z_1, z_2 and z_3, and another one between z_5 and z_6. The model is simulated using 500 samples.

Suppose that $z(k) = [z_1(k) \ldots z_m(k)]^T$ is the raw measurement vector and $x(k) = [x_1(k) \ldots x_m(k)]^T$ is the vector of the normalized data.

The application of the PCA allows the identification of the values and eigenvectors of the correlated matrix X. We apply then the VRE criteria to determine the structure of the model.

According to the table 1, the minimum of $\sum_{i=1}^{l} \sigma_i(l)$ is obtained when $l = 2$ and the variances of the different variables calculated using two-component model are less than 1 (table2). Consequently, all the variables can be reconstructed and the number of the principal components *l is* equal to two.

Table 1: Variance of of The Reconstruction Error For The Variables

$\sum_{i=1}^{7} \sigma_i(l)$	2.3	0.32	0.67	11.5	47.69	58.6	7.45

We note also from table 1 that the VREs of the variables x_5 and x_6 are higher than the others. We deduce therefore that these variables are less correlated than the others. Consequently, we conclude that the VRE criterion gives an optimal number of the components to identify the PCA model. Once the model is identified, we proceed to the procedure of fault detection. We suppose that the variable x_3 is affected by a simple fault (bias) between the instant 250 and 400 with an amplitude value equals to 22% of the range of variation of this variable.

Table 2: Variance of The Reconstruction Error of Differents Variables For l=2

$\sigma_2(1)$	$\sigma_2(2)$	$\sigma_2(3)$	$\sigma_2(4)$	$\sigma_2(5)$	$\sigma_2(6)$	$\sigma_2(7)$
0.0105	0.0082	0.0088	0.0864	0.1010	0.0955	0.0097

In the figure 1 the evolution of the criterion based on the concept of variation of the reconstruction error as a function of the number of principal components is presented. From the represented curve, the minimum of this criterion is attained when *l* equals two.

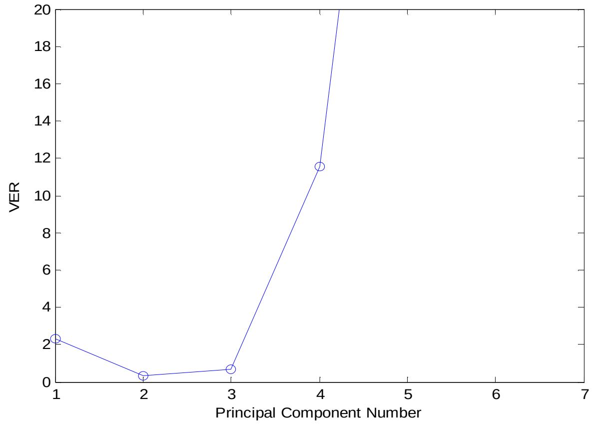

Fig.1 Variances of the reconstruction error for the number of components in the PCA model

In the figure 2 the evolution of the different variables and their estimation is presented. This figure shows that the estimated error is almost zero for all variables except x_7 and x_8 where the estimated values are high. This can be explained by non-linearity of these variables.

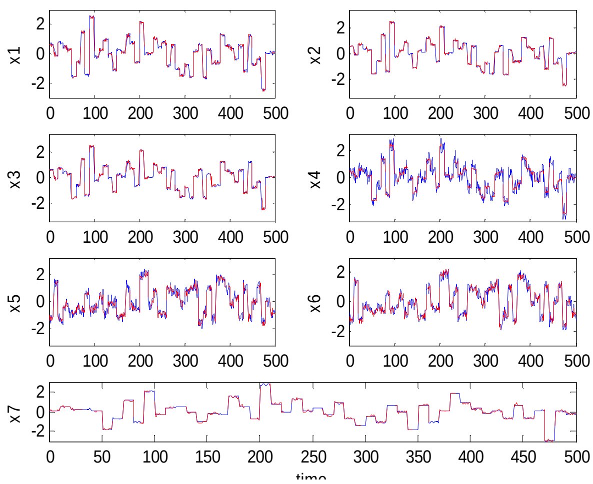

Fig.2 Measurements and estimations of different variables

The performances evaluated for the four indices are:

- The False Alarm Rate (FAR) which expresses the ratio of the violated samples (Those which exceed the detection thresholds) to the faultless data.

$$FAR = \frac{violated\ samples}{faultless\ data}\ \% \qquad (32)$$

- GDR is defined as the ratio between the total time of the detected faults and the total time when the system is not operating properly.

In Figure 3, the evolution of the detection index SPE and the filtered SPE are presented. We remark that the detection using the SPE index causes many false alarms for a confidence level equals to 95%. To reduce these false alarms, we increase the confidence level to 99%. Moreover, we note that the SPE index has undetected errors between the instant 250 and 400. This means that the system is functional despite the presence of a fault. For the filtered SPF index, we notice that the fault is clearly detected with delay.

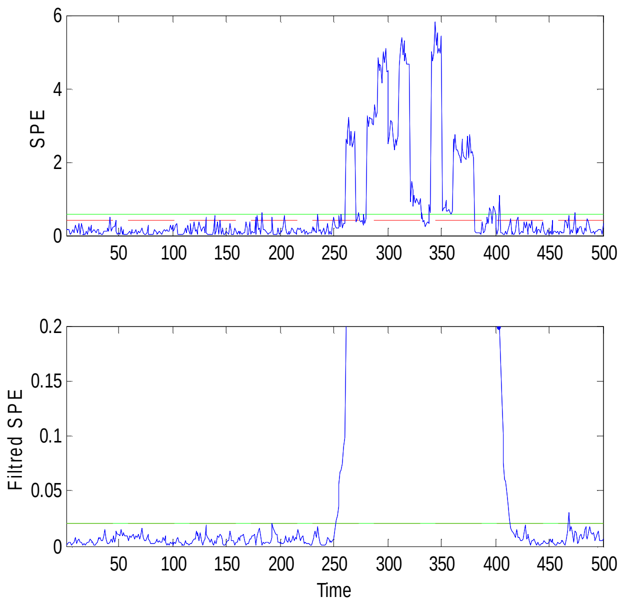

Fig.3 SPE and filtred SPE with a fault on variable x_3

In the figure 4 the evolution of the T^2 index with a fault on the variable x_3 between the instant 250 and 400 is illustrated. This fault is represented by a constant bias of amplitude equal to 22% of x_3. According the figure 4, the fault is not detected and the projection of the variations of the observations in the principal space is masked by the variation of measurements during normal operation.

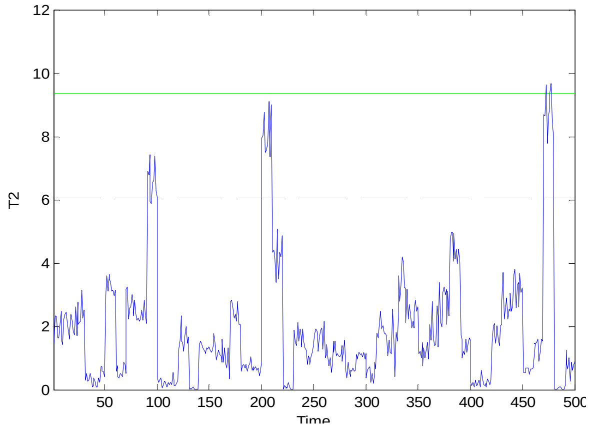

Fig. 4 T^2 with a fault on variable x_3

When the fault satisfies the detectability condition, the fault is guaranteed to be detected. To highlight this fact, a fault is simulated between the instants 250 and 400. The minimum fault amplitude is equal to $d_{\min} = 84.64$.

Figure 5 presents the evolution of the T^2 index with amplitude of fault equals to 84.64. The fault is clearly detected in this case. We can conclude therefore that the Hotelling T^2 index cannot detect fault with low amplitude because the variation of the projection of these fault in the principal subspace can be masked by the variations of measurements in normal operation.

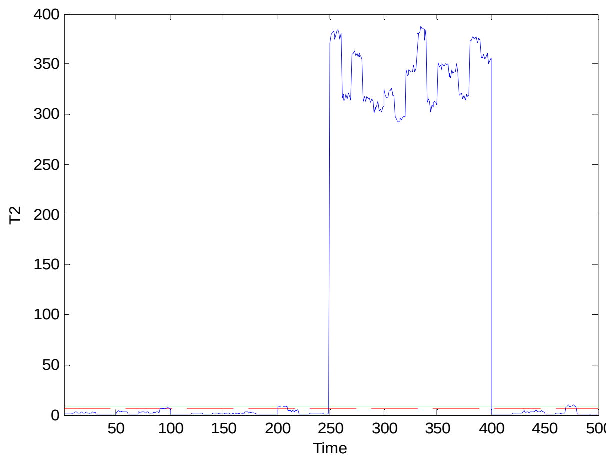

Fig.5 evolution of T^2 with a fault ($d_{\min} = 84,64$) on variable x_3

We add a fault (bias) to the variable x_3 through the previous /example. We apply then the detection procedure using the D_i index. As shown in Figure 6, the fault is detected for $i=1$. Compared to other indices, we notice that the undetected errors and false alarms are very low.

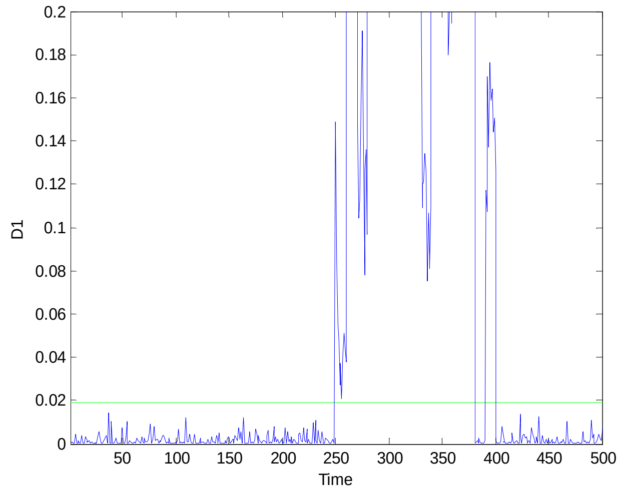

Fig.6 D_i with a fault on variable x_3

From tables 3 and table 4, we note that the index D_1 has a low FAR and good GDR.

Table 3: FAR and GDR of different indices for $\alpha = 99\%$

	SPE	Filtred SPE	T^2	D_1
FAR(%)	2.38	15.2	2.9	1.39
GDR(%)	72.19	98.68	0	93.38

Table 4: FAR and GDR of different indices for $\alpha = 95\%$

	SPE	Filtred SPE	T^2	D_1
FAR(%)	15.69	15.25	17.1	13.73
GDR(%)	79.47	98.68	0	93.38

We conclude that D_1 has the best performance since, unlike other indices, it detects very low amplitudes.

5 Online fault detection methods

Despite its great success, PCA with its original form is not able to cope with time-varying systems that operate at deferent conditions and modes. Furthermore, when conventional PCA is used to monitor such a process, an excessive rate of false alarms and missing detection may occur. Therefore, an adaptation strategy for PCA algorithms is recommended. To overcome this problem, the Conventional PCA [18], the Sliding Window PCA (SWPCA) [19-20] and the Recursive Principal Component Analysis based on First Order Perturbation (RPCA-FOP) [21] have been proposed in recent years.

5.1 Conventional PCA

Based on new measurements that represent the normal process operation, conventional PCA is used to update the PCA model for online fault detection. The following algorithm is adopted.

1- Construct an initial PCA model using initial identification data
2- Collect a next testing sample and scale it using current scaling parameters.
3- Evaluate the monitoring index for the scaled testing sample, if the control limit is not exceeded, the new measurement is considered normal. So, it will be used to update the PCA model.
4- Recalculate the eigenvalues and the eigenvectors of the updated correlation matrix
5- Repeat from step 2

5.2 Sliding window principal component analysis (SWPCA)

The basic idea of SWPCA method consists in moving a window along data in real time allowing the algorithm to operate online in time-varying environment. A sliding window technique is applied by removing the oldest sample and adding the newly available one. A detailed algorithm for the implementation of the SWPCA is presented as follows [22]:

1- Construct an initial PCA model using initial identification data
2- Collect a new input sample *x*. Scale it using a current mean and a standard deviation.
3- Determine the monitoring index, if the control limit is not exceeded, the measurement x is considered normal. So, it will be used to update the PCA model.
4- Slide the training data window by concatenating it in the measurement matrix and deleting the oldest one.

5- Recalculate the eigen-value decomposition of the new correlation matrix to update the PCA model
6- Repeat from step 2

5.3 Recursive Principal Component Analysis based on First Order Perturbation (RPCA-FOP)

The principle of the RPCA-FOP method for fault detection consist that the recursive computation of the eigen-values and eigenvectors is based on perturbation analysis of the correlation matrix.

5.4 Simulations

The process is a Continuous Stirred Tank Reactor CSTR, a dynamic non-linear system, used for the conduct of the chemical reactions [23], [24] so that two reactants 1 and 2, with concentration C_{b1} and C_{b2} and feed w_1 and w_2, respectively, are mixed to provide a final product with a concentration C_b. The physical equations describing the process are:

$$\frac{dh(t)}{dt} = w_1(t) + w_2(t) - 0.2\sqrt{h(t)}. \quad (33)$$

$$\frac{dC_b(t)}{dt} = \left(C_{b1}(t) - C_b(t)\right)\frac{w_1(t)}{h(t)} + \left(C_{b2}(t) - C_b(t)\right)\frac{w_2(t)}{h(t)} - \frac{k_1 \cdot C_b(t)}{\left(1 + k_2 \cdot C_b(t)\right)^2}.$$

Where $h(t)$ is the height of the mixture in the reactor. k_1 and k_2 are the consuming reactant rate assumed to be constant. The temperature in the reactor is constant and equal to the ambient temperature. A diagram of this reactor is given in Figure 7.

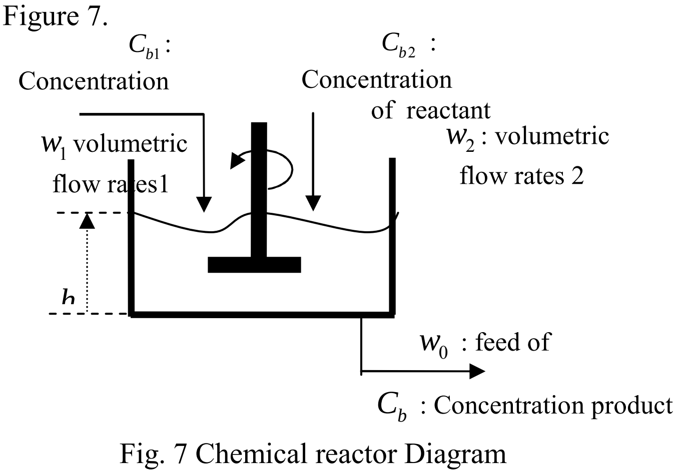

Fig. 7 Chemical reactor Diagram

The variables used to monitor the CSTR process are respectively w_1, w_2, C_{b1}, C_{b2} and C_b

We have collected 3000 data observations from the process.

We have used 1000 observations in order to construct an initial PCA model.

To evaluate the performances of the online detection methods, three types of faults are considered. A bias fault affected the w_1 variable, a drift fault affected the C_{b1} and a normally-distributed noise injected in the C_{b2} variable.

The description of all types of faults is presented in Faults in Table 5.

Table 5: Sensor fault description

Case	Faults description		
	Affected variable	Fault description	Fault time
Bias	w_1	$d_1 = 10\%$	2000-2500
Drift	C_{b1}	$d_2 = 9 \cdot 10^{-5} k - 0.$	2000-2500
Normally-distributed noise	C_{b2}	$d_3 = N(0, 0.1)$	2000-2500

The evaluated performances are respectively, the false alarm rate (FAR), the good detection rate (GDR) and the computing time (CT).

According the table 6, we remark that the RPCA - FOP method is better than the other methods especially in term of computing time.

Table 6: Performances of the three online faults detection methods

Approach	FAR (95%)	GDR (95%)	GDR (99%)	CT(s)
RPCA-FOP	0.06	100	91.24	13.02
SWPCA	0.08	100	96.88	17.07
Conventional PCA	1.02	91	89.45	21.67

In the figures 8, 9 and 10, the SPE index based on the RPCA-FOP method is presented 10. According these figures, we remark that the injected fault is detected in both case of detection threshold.

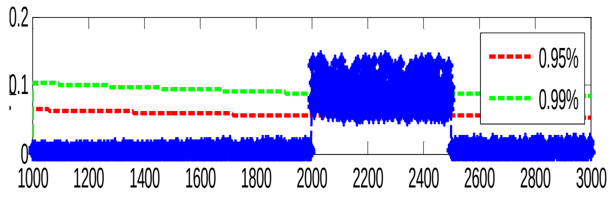

Fig. 8 Online fault detection result in the case of a bias fault

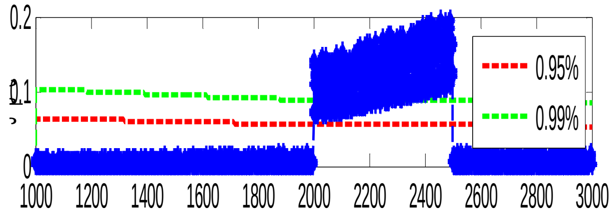

Fig.9 Online fault detection result in the case of a drift fault

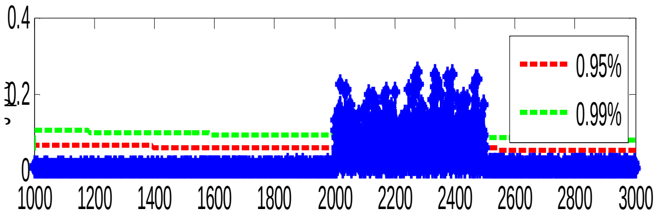

Fig.10 Online fault detection result in the case of a normally-distributed noise.

6 Conclusions

In this paper, we have presented the design and a comparative study of offline fault detection indices based on PCA method. We have presented the four indices of fault detection and we have compared their performance through a numerical example. Then adaptive fault detection techniques based on the PCA method are presented. It has been shown that the RPCA-FOP has better performances than the other methods especially in terms of average computation time. These algorithms have been tested on Benchmark CSTR and the results were satisfactory.

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