Characterization of Three-Phase Induction Motor Using Conservative Power Theory (CPT)

BRENO ABREU JR., FLÁVIO A. S. GONÇALVES, HELMO K. M. PAREDES, F. P. MARAFÃO
Group of Automation and Integrating Systems (GASI)
Unesp – Universidade Estadual Paulista
511, Três de Março Av. Sorocaba-SP
BRAZIL
bjr_sp@hotmail.com flavioasg@sorocaba.unesp.br hmorales@sorocaba.unesp.br

Abstract - This paper presents a characterization of three-phase induction motor by means of Conservative Power Theory (CPT). Conservative Power Theory terms are used to determine an equivalent circuit which represents the three-phase induction machine in steady state. The characterized equivalent circuit can be classified in two different types, namely, current source load and voltage source load. In order to validate the approach, a computer simulation of a three-phase induction machine with squirrel cage in several situations of load and voltage feeding were evaluated and compared with the equivalent circuits characterized by CPT terms.

Key-Words: - Conservative power theory, Three-phase induction machine, Characterization

1 Introduction

The Power Quality has been a topic of extensive discussion in recent decades, ranging from regulatory government agencies through energy utilities and some even to end customers. This factor is directly related to the rising demand that the electrical system has been suffering, whether due to new needs arising from the growth of industrial, commercial and farming, and even due to the constant search for technological advances or improvements in quality of life and comfort for the general population [1].

Due to the wide spread in the industrial sector and its contribution in the consumption of all energy generated, the effects of power disturbances in the operation of three-phase induction machines also has been a topic widely discussed in recent decades. Studies demonstrates that disturbances in the power quality such as harmonic distortions, imbalances and variations of short and long term voltage directly influence the operation, behavior, performance and service life of induction motors. [2-5]

The separation of responsibility relative to the power system disturbances has been quite discussed, especially after the introduction of the concepts of distributed grid. Due to particular characteristics of the loads the accounting for each type of disturbances separately and directed to the source or load is necessary [6].

For analysis of these disturbances are used several power theories described in [7-10] without a unanimity on the subject. Alternatively [11-12] proposed the Conservative Power Theory (CPT) where a complete analysis of the physical phenomena related to the voltages and currents, as changes in frequency operating, asymmetry of tension and unbalance loads were provided.

The loads in general (linear or nonlinear) can be characterized in two different types: as "current source" or "voltage source". This characterization is intended to assist the studies related with disturbances caused by these loads in electric power grids and contribute to the areas of electric power pricing and disturbances compensation [11-13].

In this sense, this article discusses the analysis of the behavior of three-phase induction machine (MIT) with squirrel cage rotor type by Conservative Power Theory (CPT) terms to obtain the characterization of its electrical equivalent circuit in steady state.

2 The Three-Phase Induction Machine and the Steady-State Model

Used in most industrial applications, the three-phase induction machine with squirrel cage rotor has robust and simple design characteristics. The rotor consists of short-circuited conducting bars of two rings at its ends. When applied voltage in the terminals stator, induced current appears in the rotor, creating the conjugate. The speed of the rotating field (Synchronous speed) [ω_s] is dependent only on the number of stator poles and frequency [14-15].

$$\omega_s = \frac{4.\pi.f}{n^{\underline{o}}\ of\ poles} \tag{1}$$

Due to the constructive characteristics this machine has slip (s), represented by the relation between synchronous speed and the actual speed (ω_r), then:

$$s = \frac{\omega_s - \omega_r}{\omega_s} \quad (2)$$

The three-phase induction motor can be represented by the equivalent circuit, per phase considering the steady-state as is shown in Fig.1. For this representation, the rotor's parameters were transferred to stator.

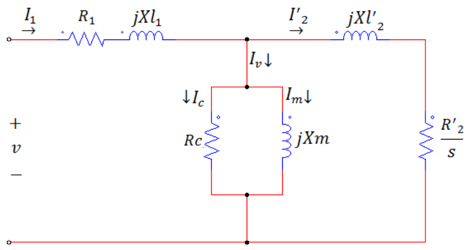

Fig. 1. The three-phase induction motor's per phase equivalent circuit

Where:

v: Voltage per phase at the stator terminals;
R_1: Stator resistance;
jXl_1: Leakage reactance of the stator;
I_1: Stator current;
Rc: Core losses resistance;
jXm: Magnetizing reactance;
I_v: Excitation current responsible for creating the air gap flux;
I_c: Core losses current;
I_m: Magnetizing current;
R'_2: Rotor resistance transferred to stator;
jXl'_2: Rotor leakage reactance transferred to stator;
I'_2: Rotor current transferred to stator;
s: Slip.

The total equivalent impedance Ze of equivalent circuit can be expressed as (3)

$$Ze = R_1 + jXl_1 + \frac{\left(\frac{R'_2}{s} + jXl'_2\right).jXm}{\frac{R'_2}{s} + j(Xl'_2 + Xm)} \quad (3)$$

And the motor torque (Te) can be defined as (4).

$$Te = \frac{3.v^2.R'_2}{\omega_s.s.\left(R_1 + \frac{R'_2}{s}\right)^2 + (Xl_1 + Xl'_2)^2} \quad (4)$$

3 Conservative Power Theory in the Three-phase Systems

This section presents the basics concepts required for understanding the application of CPT in three-phase systems. [11-12]

Initially, the quantities are adopted as periodic, with fundamental frequency $f = 1/T$, where T is the period and $\omega = 2\pi f$ is the angular frequency.

The instantaneous active power and the reactive energy are defined as (5) and (6), respectively.

$$P = \langle \underline{v}, \underline{i} \rangle = \sum_{\mu=a}^{m} P_\mu \quad (5)$$

$$Wr = \langle \underline{\hat{v}}, \underline{i} \rangle = \sum_{\mu=a}^{m} Wr_\mu \quad (6)$$

Where: $\underline{v}$ and $\underline{i}$ are the voltage and current vectors, $\underline{\hat{v}}$ corresponds to an unbiased integral of the voltage vector, μ indicates the respective phase in a poly phase system with m phases, being the three-phase systems compound by the phases a, b and c.

3.1 Current Terms

The active current per phase in the three-phase system is represented by (7).

$$i_{a\mu} = \frac{\langle v_\mu, i_\mu \rangle}{\|v_\mu{}^2\|} v_\mu = \frac{P_\mu}{V_\mu{}^2} v_\mu = G_\mu v_\mu \quad (7)$$

Where: G_μ is the conductance per phase, $\|v_\mu\| = V_\mu$ is the Euclidian norm of the voltage per phase. The active current is responsible by the transfer of active power P_μ.

In the three-phase circuits the load may represent an unbalanced behavior, in other words, the equivalent conductance per phase can be different. In order to represent this unbalance the active current can be divided in two components, the balanced active current ($\underline{i}_a^b$) and unbalanced active current ($\underline{i}_a^u$), defined by (8) and (10):

$$\underline{i}_a^b = \frac{\langle \underline{v}, \underline{i} \rangle}{\|\underline{v}^2\|} \underline{v} = \frac{P}{\boldsymbol{V}^2} \underline{v} = G^b \underline{v} \quad (8)$$

$\boldsymbol{V} = \|\underline{v}\|$ represents the Euclidian norm of the combined voltages, as (9):

$$\boldsymbol{V} = \sqrt{v_a{}^2 + v_b{}^2 + v_c{}^2)} \quad (9)$$

The unbalanced active current is defined by the difference between the active current and the balanced active current.

$$\underline{i}_a^u = \underline{i}_a - \underline{i}_a^b \quad (10)$$

The reactive current per phase is responsible to transfer Wr_μ (reactive energy), represented by (11):

$$i_{r\mu} = \frac{\langle \hat{v}_\mu, i_\mu \rangle}{\|\hat{v}_\mu{}^2\|}\hat{v}_\mu = \frac{Wr_\mu}{\hat{V}_\mu{}^2}\hat{v}_\mu = B_\mu \hat{v}_\mu \quad (11)$$

Where: $\hat{V}_\mu$ represents the Euclidean norm of the voltage unbiased integral per phase.

In the same way of active current, the reactive current can be divided in balanced ($\underline{i}_r^b$) and unbalanced ($\underline{i}_r^u$). These terms indicate the reactivity differences in the phases, and can be expressed respectively by (12) and (13):

$$\underline{i}_r^b = \frac{\langle \underline{\hat{v}}, \underline{i} \rangle}{\|\underline{\hat{v}}^2\|}\underline{\hat{v}} = \frac{Wr}{\boldsymbol{V}^2}\underline{\hat{v}} = B^b \underline{\hat{v}} \quad (12)$$

$$\underline{i}_r^u = \underline{i}_r - \underline{i}_r^b \quad (13)$$

The void current ($\underline{i}_v$) represents the current portion not responsible for transmitting active power neither reactive energy (only load's nonlinearities) and it can be expressed by (14):

$$\underline{i}_v = \underline{i} - \underline{i}_a - \underline{i}_r \quad (14)$$

With the currents definitions accomplished the total current in a circuit can be defined by (15):

$$\underline{i} = \underline{i}_a^b + \underline{i}_a^u + \underline{i}_r^b + \underline{i}_r^u + \underline{i}_v \quad (15)$$

Due to components orthogonality the total current Euclidean norm is described as (16):

$$\boldsymbol{I} = \sqrt{\boldsymbol{I}_a^{b\,2} + \boldsymbol{I}_a^{u\,2} + \boldsymbol{I}_r^{b\,2} + \boldsymbol{I}_r^{u\,2} + \boldsymbol{I}_v{}^2} \quad (16)$$

Where: $\boldsymbol{I} = \|\underline{i}\|$ is the collective Euclidean norm of the current.

3.2 Power Terms

The apparent power, by definition, is represented as (17):

$$A = \boldsymbol{VI} \quad (17)$$

Where: $\boldsymbol{V}$ and $\boldsymbol{I}$ are the collectives RMS values of the voltage and current.

Thus, from equations (16) and (17) can be seen the apparent power decomposition as (18):

$$A = \sqrt{P^2 + Q^2 + U^2 + D^2} \quad (18)$$

The power terms making up the apparent power are respectively:

- Active power [W]

$$P = \boldsymbol{VI}_a^b \quad (19)$$

- Reactive power [VA]

$$Q = \boldsymbol{VI}_r^b \quad (20)$$

- Unbalance power [VA]

$$U = \boldsymbol{V}\sqrt{\boldsymbol{I}_a^{u\,2} + \boldsymbol{I}_r^{u\,2}} \quad (21)$$

- Distortion power (void) [VA]

$$D = \boldsymbol{VI}_v \quad (22)$$

3.3 Power Factors

Using the power terms definitions accomplished in the preview section, the CPT defines a new concept of power factor [12-13], stated by (23), which incorporates not only the reactivity effects, but also the unbalance and nonlinearity effects.

- **Power Factor**

$$\lambda = \frac{P}{A} = \frac{P}{\sqrt{P^2 + Q^2 + U^2 + D^2}} \quad (23)$$

- **Reactivity Factor:** indicate the presence of energy storages components.

$$\lambda_Q = \frac{Q}{\sqrt{P^2 + Q^2}} \quad (24)$$

- **Unbalance Factor:** indicates the unbalance of the loads in the phases.

$$\lambda_U = \frac{U}{\sqrt{P^2 + Q^2 + U^2}} \quad (25)$$

- **Nonlinearity Factor:** indicates the presence of nonlinear behavior loads

$$\lambda_D = \frac{D}{A} \quad (26)$$

4 Equivalent Circuits

The current terms of CPT can be used to represent the loads behavior [13]. Basically, the load can be divided into two types: current source and voltage source.

4.1 Load Type Current Source

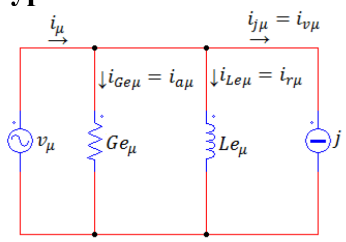

Fig. 2. Per phase equivalent circuit considering load type current source

Fig. 2 shows the equivalent circuit that represents a current source load type from the viewpoint of CPT. The items are represented per phase (μ). The voltage source that feeds the circuit corresponds to v_μ, the total current on the circuit is represented by i_μ, on the equivalent conductance Ge_μ circulates the current $i_{Ge\mu}$ corresponding to active current $i_{a\mu}$, on the equivalent inductance Le_μ circulates the current $i_{Le\mu}$ corresponding to reactive current $i_{r\mu}$ and the harmonic source current j indicates the harmonics distortion presence (nonlinearities) represented by $i_{j\mu}$ corresponding to void current $i_{v\mu}$ (the source

harmonic current shouldn't be represented in case of inexistence of distortion). For the case of current source load type, the reactive energy (Wr_μ) is positive.
In case the voltage source be sinusoidal, by (7) and (11) is possible to determine the equivalent conductance and inductance parameters, given by (27) and (28) respectively.

- Equivalent conductance

$$i_{a\mu} = \frac{P_\mu}{V_\mu^{\ 2}} v_\mu = Ge_\mu v_\mu$$

$$\therefore Ge_\mu = \frac{P_\mu}{V_\mu^{\ 2}} \ [\Omega^{-1}] \tag{27}$$

- Equivalent inductance

$$i_{r\mu} = \frac{Wr_\mu}{\hat{V}_\mu^{\ 2}} \hat{v}_\mu = B_{L\mu} \hat{v}_\mu$$

$$\therefore B_{L\mu} = \frac{Wr_\mu}{\hat{V}_\mu^{\ 2}} \tag{28}$$

Thus,

$$Le_\mu = \frac{1}{B_{L\mu}} \ [H] \rightarrow Le_\mu = \frac{\hat{V}_\mu^{\ 2}}{Wr_\mu} \tag{29}$$

4.2 Load Type Voltage Source

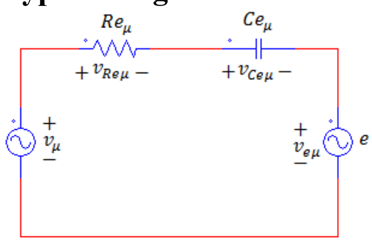

Fig. 3. Per phase equivalent circuit considering load type voltage source

The equivalent circuit Fig. 3 represents a load with voltage source characteristics. By the CPT, the parameters can be determined like per phase equivalent resistance (Re_μ), per phase equivalent capacitance (Ce_μ) and the harmonic voltage source (e), that similarly to the harmonic current source, can be neglected in case of inexistence of distortion. The reactive energy (Wr_μ) is negative for the voltage source equivalent circuit.
As well as developed to the load type current source is possible to determine the equivalent resistance and capacitance parameters to voltage source load type.

- Equivalent resistance

$$v_{Re\mu} = \frac{P_\mu}{I_\mu^{\ 2}} i_\mu = Re_\mu i_\mu$$

$$\therefore Re_\mu = \frac{P_\mu}{I_\mu^{\ 2}} \ [\Omega] \tag{30}$$

- Equivalent capacitance

$$v_{Ce\mu} = -\frac{Wr_\mu}{\hat{I}_\mu^{\ 2}} i_\mu = B_{C\mu} i_\mu \tag{31}$$

Thus,

$$Ce_\mu = \frac{B_{C\mu}}{\omega^2} \therefore Ce_\mu = -\frac{\hat{I}_\mu^{\ 2}}{Wr_\mu} \ [F] \tag{32}$$

5 Characterization of Three-Phase Induction Machine and Simulations Results

The parameters of three-phase induction motor with squirrel cage rotor adopted to accomplish the tests are presented in the table 1 [15].
The three-phase induction motor operation was computationally simulated using the Psim environment and the CPT data factors were calculated through a DLL written in C programming language running embedded in the simulation environment .

Table 1. Parameters of MIT

Line voltage	220 V
Mechanic power	3 HP
Nominal torque	12 N.m
Frequency	60 Hz
Pole numbers	4
Stator resistance	0,435 Ω/per phase
Stator inductive reactance	0,754 Ω/ per phase
Rotor resistance	0,816 Ω/ per phase
Rotor reactance inductive	0,754 Ω/ per phase
Magnetizing reactance	26,13 Ω/ per phase
Rotor moment of inertia	0,089 Kg/m²
Rotor speed	1710 rpm

The others components in the system were implemented by using the available models and their parameterizations. In order to perform the analysis, the motor was connected directly to a three-phase sinusoidal voltage source as shown in fig. 4. The voltage measurement references were acquired through the virtual point formed by three high value resistances in Y connection.
The configurations considered for creating several operating situations of the machine are described in table 2. Basically the selected cases are compounds by the combination of load torque situations, net supply voltages and frequencies, involving their nominal and percentage values.

Table 2. Set of Cases for Simulation

Case	Torque	Voltage	Frequency
1.1	No load (null)	Nominal	Nominal
1.2		80% of nominal	Nominal
1.3		50% of nominal	Nominal
1.4		30% of nominal	Nominal
1.5		Nominal	80% of Nominal
1.6		80% of nominal	80% of Nominal
1.7		50% of nominal	80% of Nominal
1.8		30% of nominal	80% of Nominal
2.1	50% of nominal	Nominal	Nominal
2.2		80% of nominal	Nominal
2.3		50% of nominal	Nominal
2.4		30% of nominal	Nominal
2.5		Nominal	80% of Nominal
2.6		80% of nominal	80% of Nominal
2.7		50% of nominal	80% of Nominal
2.8		30% of nominal	80% of Nominal
3.1	Nominal	Nominal	Nominal
3.2		80% of nominal	Nominal
3.3		50% of nominal	Nominal
3.4		30% of nominal	Nominal
3.5		Nominal	80% of Nominal
3.6		80% of nominal	80% of Nominal
3.7		50% of nominal	80% of Nominal
3.8		30% of nominal	80% of Nominal

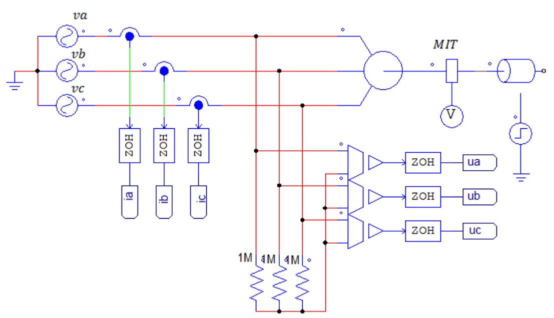

Fig. 4. Three-phase induction motor connected to power source.

For each case, the machine was simulated and characterized through CPT rates. The per phase equivalent impedance calculation for each operating point depends of totals conductance and inductance values.

Per phase equivalent impedance depends of total active power and reactive energy, which vary according to changes in the applied voltage, torque and frequency. Thus, for each operation point, per phase equivalent impedance has a different value. Using the characterization data from CPT terms, the equivalent circuit was modeled and simulated, as presented in fig. 5.

The steps to accomplish the characterization are described as follow.

Step 1) Determination of Load Type: current source or voltage source

The three phase induction motor is determined to be a load with behavior of current source type, because the reactive power measurement is always positive.

It is important to note there is not necessary the representation of the source j in the evaluated cases, because the power supply used in the evaluated cases is purely sinusoidal and the characterized load has a behavior of the linear type.

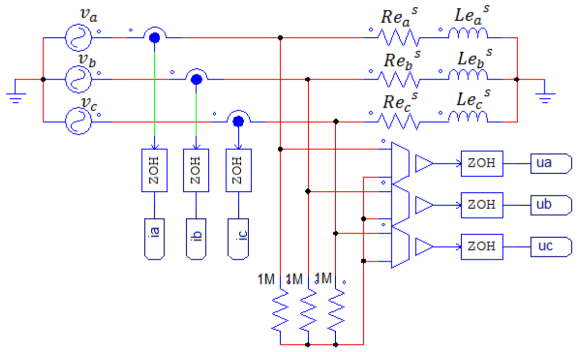

Fig. 5. Equivalent circuit of three-phase induction motor

Step 2) Determination of equivalent conductance and inductance per phase

Using the equations (27) and (28) and results concerning the computational simulations for each case, where the active power and reactive energy must be divided by 3, the conductance and inductance can be calculated using (33) and (34) for the parallel equivalent circuit (indicated as p).

$$Ge_{\mu}{}^{p} = \frac{P_{\mu}}{V_{\mu}{}^{2}} \tag{33}$$

$$Le_{\mu}{}^{p} = \frac{\hat{V}_{\mu}{}^{2}}{Wr_{\mu}} \tag{34}$$

Step 3) Transformation of parallel association in series association

Using the values of conductance and inductance calculated in step 2, the series circuit components per phase can be calculated by (35) and (36) (indicated as s).

$$Ze_{\mu}{}^{p} = \frac{Re_{\mu}{}^{p} Xle_{\mu}{}^{p}}{\sqrt{Re_{\mu}{}^{p2} + Xle_{\mu}{}^{p2}}} \tag{35}$$

$$\varphi_{\mu}{}^{p} = arctang\,\frac{Re_{\mu}{}^{p}}{Xle_{\mu}{}^{p}} \tag{36}$$

$$Re_{\mu}{}^{s} = Ze_{\mu}{}^{p}\cos(\varphi_{\mu}{}^{p}) \tag{37}$$

$$Xle_{\mu}{}^{s} = Ze_{\mu}{}^{p}\,\mathrm{sen}(\varphi_{\mu}{}^{p}) \tag{38}$$

The Tables 3, 4 and 5 show the CPT rates calculated using results from the computational simulation of the system considering the machine dynamic model and the characterized circuits in the cases presented in the Table 2.

The reactive energy results obtained using the dynamic machine model keep always positive for all evaluated cases. Thus, it is possible to confirm that three-phase induction motor has current source behavior.

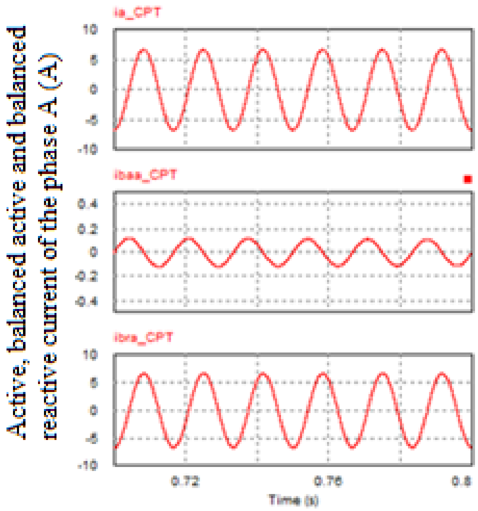

Fig. 6. Motor's steady state current to case1.1 (Null torque, nominal voltage and frequency)

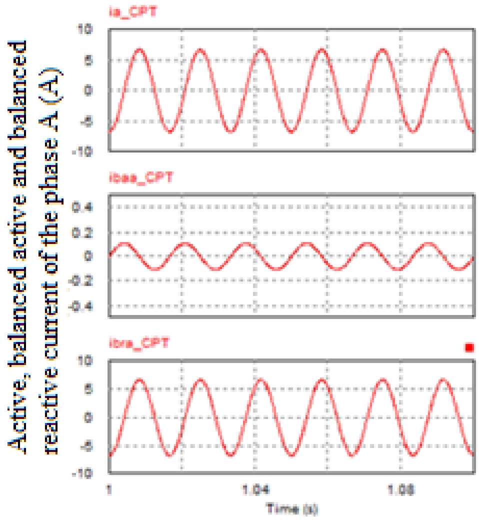

Fig. 7. Steady state current of the characterized equivalent circuit to case 1.1

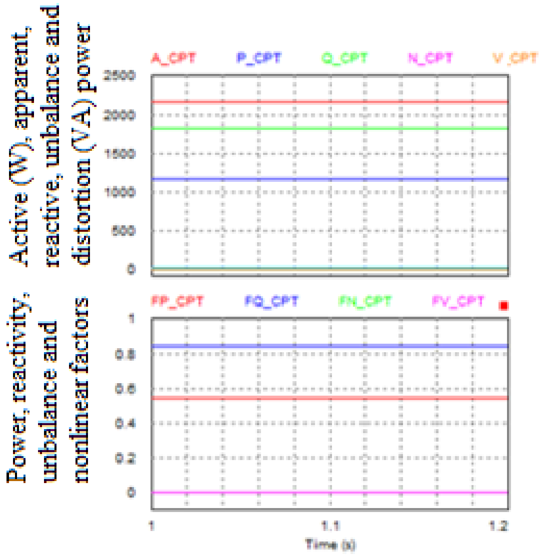

Fig. 8. Powers and CPT factors to the motor (50% of torque, nominal voltage and frequency)

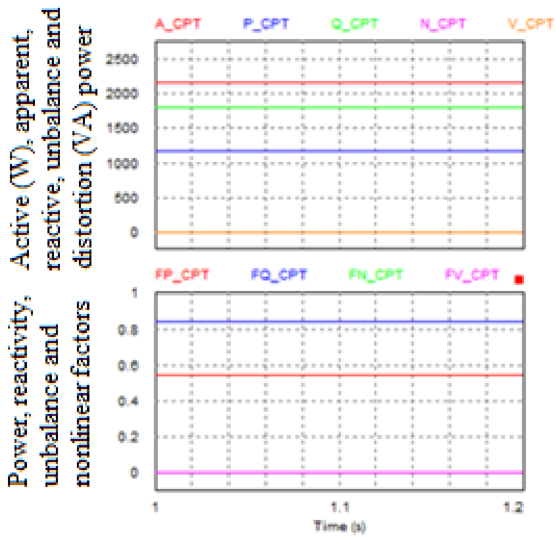

Fig. 9. Powers and CPT factors to the characterized equivalent circuit motor (50% of torque, nominal voltage and frequency)

Table 3. Values of CPT rates for Machine Model and for Characterized Circuit, considering situation of null torque

CASE	1.1		1.2		1.3		1.4		1.5		1.6		1.7		1.8	
	Motor	**Circuit**	**Motor**	**Circuit**	**Motor**	**Circuit**	**Motor**	**Circuit**	**Motor**	**Circuit**	**Motor**	**Circuit**	**Motor**	**Circuit**	**Motor**	**Circuit**
A [VA]	1807,581	1806,479	1156,851	1157,488	451,891	452,251	162,681	162,766	2259,125	2259,917	1445,840	1446,428	564,781	565,249	203,321	203,341
P [W]	29,306	29,435	18,772	18,797	7,735	7,746	2,685	2,688	45,729	45,769	29,266	29,292	11,432	11,442	4,116	4,119
Q [VA]	1807,343	1806,232	1156,675	1157,334	451,824	452,184	162,659	162,744	2258,662	2259,453	1445,542	1446,131	564,665	565,133	203,279	203,299
D [VA]	0,000	0,083	0,000	0,002	0,002	0,001	0,000	0,000	0,051	0,051	0,030	0,032	0,012	0,012	0,004	0,004
U [VA]	0,000	0,214	0,005	0,148	0,031	0,033	0,001	0,000	0,051	0,052	0,030	0,034	0,012	0,013	0,004	0,004
W [J]	4,793	4,790	3,067	3,069	1,198	1,199	0,431	0,431	7,488	7,490	4,792	4,794	1,872	1,873	0,673	0,673
λ_P	0,016	0,016	0,016	0,016	0,017	0,017	0,016	0,016	0,020	0,020	0,020	0,020	0,020	0,020	0,020	0,020
λ_Q	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999	0,999

Table 4. Values of CPT rates for machine model and for characterized circuit, considering situation of 50% nominal torque

CASE	2.1		2.2		2.3		2.4		2.5		2.6		2.7		2.8	
	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit
A [VA]	2159,149	2159,883	1666,577	1667,079	1337,151	1337,436	2263,825	2266,341	2439,539	2440,083	1725,539	1726,123	1134,790	1135,010	1148,387	1148,810
P [W]	1172,749	1172,629	1169,951	1169,864	1195,013	1194,987	1412,062	1411,489	958,169	958,065	946,524	946,433	950,972	950,967	1036,053	1036,138
Q [VA]	1812,894	1813,845	1186,895	1187,673	599,931	600,617	1769,458	1773,132	2243,492	2244,128	1442,766	1443,524	619,192	619,603	495,367	496,166
D [VA]	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,055	0,055	0,039	0,039	0,025	0,025	0,026	0,026
U [VA]	0,000	0,000	0,020	0,000	0,000	0,000	0,000	0,000	0,054	0,054	0,038	0,038	0,025	0,025	0,026	0,025
W [J]	4,808	4,810	3,148	3,150	1,591	1,593	4,693	4,702	7,437	7,439	4,783	4,785	2,052	2,050	1,642	1,644
λ_P	0,543	0,542	0,702	0,701	0,893	0,893	0,623	0,622	0,392	0,392	0,548	0,548	0,838	0,837	0,902	0,901
λ_Q	0,839	0,839	0,712	0,712	0,448	0,449	0,781	0,782	0,919	0,919	0,836	0,836	0,545	0,545	0,431	0,431

Table 5. Values of CPT rates for Machine Model and for Characterized Circuit, considering situation of Nominal Torque

CASE	3.1		3.2		3.3		3.4		3.5		3.6		3.7	
	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit	Motor	Circuit
A [VA]	3021,611	3021,510	2731,551	2731,987	2875,931	2876,261	2263,825	2265,609	2954,590	2955,072	2421,715	2422,153	2174,653	2174,966
P [W]	2343,715	2343,611	2367,049	2366,963	2558,224	2558,121	1412,062	1411,752	1887,888	1887,728	1891,755	1891,661	1979,031	1979,009
Q [VA]	1907,127	1907,094	1363,389	1364,272	1313,225	1314,874	1769,458	1771,988	2272,768	2273,527	1511,939	1512,758	901,416	902,218
D [VA]	0,000	0,000	0,011	0,001	0,001	0,001	0,000	0,000	0,066	0,066	0,001	0,054	0,049	0,049
U [VA]	0,000	0,000	0,134	0,001	0,001	0,001	0,000	0,000	0,065	0,065	0,054	0,053	0,048	0,048
W [J]	5,058	5,058	3,616	3,618	3,483	3,487	4,693	4,699	7,534	7,537	5,012	5,015	2,988	2,991
λ_P	0,775	0,775	0,866	0,866	0,889	0,889	0,623	0,623	0,638	0,638	0,781	0,780	0,910	0,909
λ_Q	0,631	0,631	0,499	0,499	0,456	0,457	0,781	0,782	0,769	0,769	0,624	0,624	0,414	0,414

Table 6. Operation points for Null Torque

CASE	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
Ze [Ω]	0,436+j26,886	0,435+j26,866	0,459+j26,859	0,443+j26,866	0,435+j21,498	0,435+j21,497	0,434+j21,488	0,435+j21,504
V [V]	127,729	101,823	63,639	38,183	127,729	101,824	63,639	38,183
Frequency (Hz)	60,000	60,000	60,000	60,000	48,000	48,000	48,000	48,000
Torque [N.m]	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Speed [rpm]	1800,000	1800,000	1799,956	1799,985	1440,118	1440,118	1440,118	1440,118
S [%]	0,000	0,000	0,00002	0,00001	0,200	0,200	0,200	0,200

Table 7. Operation points for 50% of Nominal Torque

CASE	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8
Ze [Ω]	12,216+j18,895	13,093+j13,291	8,117+j4,079	1,202+j1,509	7,82+j18,316	9,88+j15,068	8,969+j5,843	3,434+j1,644
V [V]	127,279	101,823	63,639	38,183	127,279	101,823	63,638	38,183
Frequency (Hz)	60,000	60,000	60,000	60,000	48,000	48,000	48,000	48,000
Torque [N.m]	6,000	6,000	6,000	0,000	6,000	6,000	6,000	6,000
Speed [rpm]	1763,006	1741,378	1637,873	0,006	1416,552	1402,916	1339,862	1078,476
S [%]	0,021	0,033	0,090	1,000	0,213	0,221	0,256	0,401

Table 8. Operation points for Nominal Torque

CASE	3.1	3.2	3.3	3.4	3.5	3.6	3.7
Ze [Ω]	12,476+j10,151	9,864+j5,684	3,757+j1,93	1,202+j1,509	10,506+j12,652	10,029+j8,019	5,083+j2,317
V [V]	127,279	101,823	63,639	38,183	127,279	101,823	63,639
Frequency (Hz)	60,000	60,000	60,000	60,000	48,000	48,000	48,000
Torque [N.m]	12,000	12,000	12,000	4,787	11,999	12,000	12,000
Speed [rpm]	1724,081	1677,209	1389,830	0,008	1392,209	1363,331	1214,589
S [%]	0,042	0,068	0,228	1,000	0,227	0,243	0,325

Therefore, it was performed the method described in the section 4 to obtain a three-phase circuit that characterize the induction motor as a current source load type. Furthermore, the comparison among the CPT indices obtained by the two models allows noticing the effectiveness of the proposed method, where can be easily observed the correlation with a small relative difference.

Figures 6 and 7 show current waveforms derived by CPT in steady state for the machine and for characterized equivalent circuit considering the case 1.1, respectively. The same way, Figures 8 and 9 shows the power waveforms derived by CPT in steady state for both situations considering the case 2.1.

Based on the acquired results derived through the CPT, it can be noted that the residual currents and the nonlinearity factors are null, and load behavior is balanced, due to the absence of current unbalance and null unbalance factor. Therefore, unbalanced currents are not shown because they are zero.
Tables 6, 7 and 8 show the derived operating points and the characterized equivalents impedances for each case. The equivalent impedance varies due to the operating point determined by the load torque, machine parameters and power supply conditions.

6 Conclusion

This paper presented the application of Conservative Power Theory to perform a three-phase induction motor characterization in steady state. The approach adopted a squirrel cage induction motor in several operating situations as test system. An equivalent circuit was inferred by means of CPT terms, and for the three-phase induction machine operating as motor, the resultant circuit has a current source behavior.
The analyses were based in result from computational simulation of twenty four situations of motor operation, where load torque and supply system to the determination of CPT factors. Depending on the obtained CPT factors, the characterization was made considering the systems as a current source load type, because the average reactive energy is always positive.
Considering the simulations results of the dynamic circuits and characterized circuits through the CPT, differences are negligible, in the steady state operation, proving the effectiveness of the utilization of CPT to the characterization of load in the evaluated conditions.
Relative to the magnetic saturation effects can be verified their performance both torque as the speed, therefore, indirectly the analyses by the CPT can be extended to represent this effects.

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