

Performance Comparison of Spreading Codes in Linear Multi-User Detectors for DS-CDMA System

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Abstract: - Direct Sequence Code Division Multiple Access (DS-CDMA) system is well known wireless technology. This system suffers from MAI (Multiple Access Interference) caused by Direct Sequence users. Multi-User Detection schemes were introduced to detect the users' data in presence of MAI. Linear Multi-user Detectors and conventional Matched Filter (MF) are simulated using gold, PN and even kasami sequences as spreading codes in DS-CDMA system. In this paper odd kasami sequence is proposed. For this, odd kasami sequence of length $L=2^m$ which inclusive of initial bit, The Bit Error Rate (BER) performance of MMSE Detector provides better than Decorrelating detector and conventional Matched filter. Comparative Study shows that the proposed odd kasami sequence is better performed than gold, PN and even kasami sequences in linear Multi-user Detectors and conventional Matched Filter (MF).

Key-words: - Multi-user detection, Matched filter, Decorrelating detector, MMSE, DS-CDMA, PN sequence, gold sequence, even kasami sequence and odd kasami sequence.

1 Introduction

The tremendous increase in demand for wireless services has caused a search for techniques to improve the capacity of current digital communication systems. To bring this vision for future, the current state of wireless technology is necessary for major improvements. Direct sequence code division multiple access (DS-CDMA) system is very popular over last few years. Code Division Multiple Access (CDMA) is one of the several methods of multiplexing wireless users [1]. In CDMA, all users can transmit at the same time. Also, each user is allocated the entire frequency spectrum for transmission; hence, CDMA is also known as spread spectrum communications [2]. The capacity of DS-CDMA system is limited by MAI (Multiple Access Interference) caused by direct sequence users i.e., traditional matched filter detectors are used. To mitigate this problem, Multi-User detection was proposed, which jointly uses the data or information of interfering users to improve the detection performance of desired signal [3-5]. The DS-CDMA Detectors are divided into single-user and Multi-user Detectors. A single user detectors detects the data of one user at a time where as multi-user detectors jointly detects several users'

information [4]. The aim of the detector is to restore the signal, which is corrupted by the channel back to its original form. Multi-user detectors have the potential to significantly improve the performance and capacity of a DS-CDMA system. The multi user detectors classified as optimal and suboptimal detectors.

In the early stages, optimal solutions with best possible performance in Gaussian noise channels have been investigated and developed. Unfortunately, when the number of users increases the complexity of these schemes increases exponentially, this type of detector is not suitable for a practical application. This problem can be reduced by using suboptimal multi-user detection algorithms such as the Decorrelating detector, Minimum mean square error detector (linear detectors) and other sub-optimal detectors (Non-linear detectors). Because of the significant advantages which multi-user detection offers CDMA based wireless systems, in terms of capacity improvements and near-far resistance, all W-CDMA proposals for third generation wireless systems provide a structure to accommodate these promising techniques [3-5].

The paper is organized as follows. In the next section we presented literature about the detectors. In Section 3 Fundamentals of Different PN Sequences or spreading codes are illustrated. Section 4 presents the proposed odd kasami sequence. Section 5 provides some simulation results on the performance of conventional and linear multiuser detectors using different spreading sequences. A summary of the findings is given in the conclusion in section 6.

2 Detectors

The DS-CDMA detectors are classified as conventional single user detector and multiuser detectors.

2.1 Conventional single user detector

The Matched filter is a conventional single user detector. This detector is the simplest suboptimum detector used in DS-CDMA [3]. It follows a single user detection strategy in which each user is treated separately as a signal, while the other users are considered as either interference or noise [6]. It is shown in Figure 1; the matched filter is used to generate sufficient statistics for signal detection.

The baseband received signal is given by

$$r(t) = \sum_{k=1}^K A_k(t)s_k(t)b_k(t) + n(t) \quad (1)$$

Where $A_k(t)$, $s_k(t)$ and $b_k(t)$ are the amplitude, signature code waveform and modulated data of the k^{th} user respectively and $n(t)$ is Additive White Gaussian Noise (AWGN), with a two sided power spectral density of $N_0/2$ W/Hz [6]. The sampled output of the k^{th} matched filter is given by

$$y_k = \int_0^T r(t)s_k(t)dt$$

$$y_k = \int_0^T \left[\sum_{j=1}^k A_j b_j s_j(t) + n(t) \right] s_k(t) dt$$

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \int_0^T s_k(t) s_j(t) dt + \int_0^T s_k(t) n(t) dt$$

Where

$$\rho_{kj} = \int_0^T s_k(t)s_j(t)dt$$

ρ_{kj} is the crosscorrelation of the spreading sequence between the k^{th} and j^{th} user.

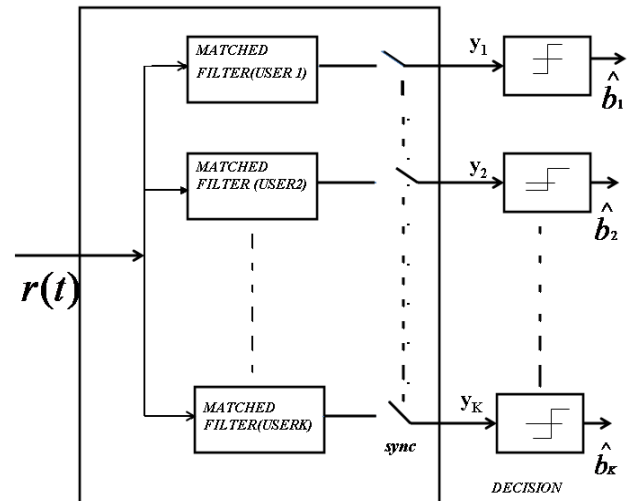


Fig. 1 Matched filter bank.

The decision is made by

$$\hat{b} = \text{sgn}(y_k)$$

The single user matched filter detector takes the MAI as noise and cannot suppress it. In matrix form, the outputs of the matched filter as

$$\mathbf{y} = \mathbf{R}\mathbf{a} + \mathbf{n} \quad (2)$$

Where \mathbf{R} is the normalized crosscorrelation matrix whose diagonal elements are equal to 1 and whose (i,j) elements is equal to the crosscorrelation, $\rho_{i,j}$, $\mathbf{A} = \text{diag}\{A_1, \dots, A_k\}$, $\mathbf{y} = [y_1, \dots, y_k]^T$, $\mathbf{b} = [b_1, \dots, b_k]^T$ and \mathbf{n} is a Gaussian random vector with zero mean and covariance matrix $\sigma^2 \mathbf{R}$ [7].

Algorithm for Matched filter

Step 1: $r(t) = \sum_{k=1}^K A_k(t)s_k(t)b_k(t) + n(t)$

Step 2: $y(t) = \int_0^T r(t)s_k(t)dt$

Step 3: if decision $y(t) > 0$; $\hat{b} = +1$
 Otherwise $\hat{b} = -1$

Step 4: if $\hat{b} \neq b$; error = error+1.

2.2 Multi-User Detection

Multi-User detection deals with the demodulation of digitally modulated signals in the presence of MAI.

A major technological hurdle of CDMA systems is the near / far problem: the bit error rate of the conventional receiver is so sensitive to differences between the received energies of the desired user and interfering users that reliable demodulation is impossible unless stringent power control is exercised [7]. In the early stages, optimal solutions with best possible performance in Gaussian noise channels have been investigated and developed. Unfortunately, the complexity of these schemes increases exponentially with the number of users, which is not suitable for a practical application. This problem has been tackled subsequently and resulted in less complex suboptimal multi-user detection algorithms such as the decorrelating detector, minimum mean square error detector (linear detectors) and other sub-optimal detectors [6]. Multi-user detectors assumed that the receiver has the knowledge of the codes of all users. These detectors can be used only for the uplink transmission. For downlink transmission, a detection scheme is required that needs only the code of desired user. Multi-user detectors have the potential to significantly improve the performance and capacity of a DS-CDMA system [1].

Figure 2, shows the general structure of multi user detection system for detecting each K user's transmitted symbols from the received signal, which consists of a matched filter bank that converts the received continuous time signal to the discrete-time statistics sampled at chip rate without masking any transmitted information relevant to demodulation [6-7].

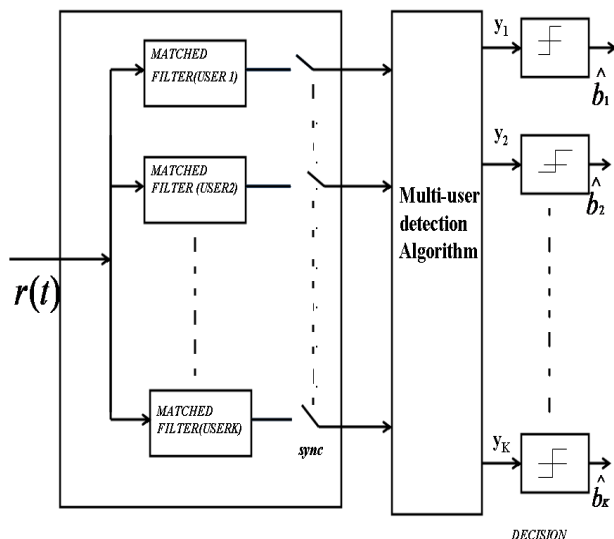


Fig.2 A typical multi-user detector

2.3. Linear Multi-User Detectors

The linear multiuser detectors are basically classified as Decorrelating Detector, and MMSE Detector.

2.3.1 Decorrelating Detector:

Decorrelator is a kind of linear multi-user receiver. The decorrelator has several desirable features. It does not require the knowledge of the users' power, and its performance is independent of the powers of the interfering users. The only requirement is the knowledge of timing which is anyway necessary for the code spreading at the centralized receiver [6].

The decision for the kth user is made based on

$$\begin{aligned} \hat{b} &= \text{sgn}((\mathbf{R}^{-1}\mathbf{y})_k) \\ \hat{b} &= \text{sgn}(\mathbf{R}^{-1}(\mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n})_k) \\ \hat{b} &= \text{sgn}((\mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n})_k) \end{aligned} \quad (3)$$

When the background noise is zero,

$$\hat{b} = \text{sgn}(\mathbf{A}\mathbf{b})$$

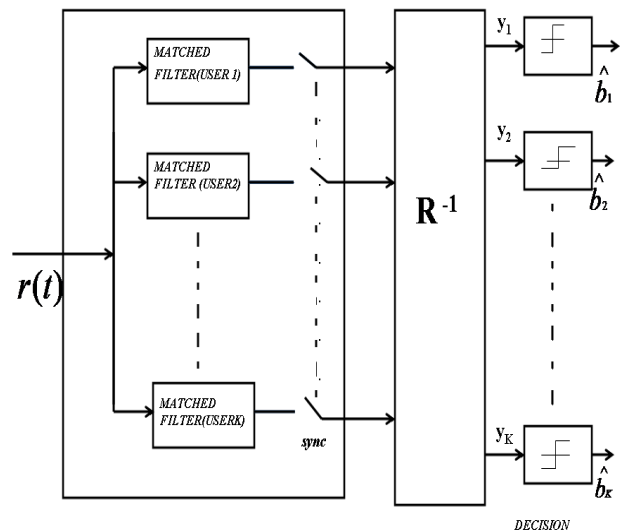


Fig. 3 Decorrelating Detector

Hence, in the absence of background noise the decorrelating detector achieves perfect demodulation unlike the matched filter. It is shown in figure 3. Decorrelating detector can achieve any given performance level in the multi-user environment regardless of the multi-user interference, provided that the desired user is supplied enough power. Thus, it provides a substantial performance or capacity gains over the conventional detector [6-7].

Algorithm for Decorrelating Detector

Step1: $\hat{b}_{dec} = (R^{-1}y(t))$

Step 2: if decision $\hat{b}_{dec} < 0$; $\hat{b} = -1$
 Otherwise $\hat{b} = +1$

Step 3: if $\hat{b} \neq b$; error = error+1.

Otherwise $\hat{b} = +1$

Step 3: if $\hat{b} \neq b$; error = error +1.

2.3.2 Minimum Mean-Squared Error (MMSE) Detector:

In decorrelating detector, the only information required by the detector is the crosscorrelation matrix **R** of the spreading sequences. Recently, there has been considerable interest in linear multi-user detection based on Minimum Mean Square Error (MMSE) criterion [7].

The MMSE receiver is another kind of linear multi-user receiver. It is shown in Figure 4, implements the linear mapping which minimizes the mean-squared error between the actual data and the soft output of the conventional detector, so the decision for the kth user is made based on

$$\hat{b}_k = \text{sgn}\left(\left((\mathbf{R} + \sigma^2 \mathbf{A}^{-2})^{-1} y\right)_k\right) \quad (4)$$

$$\hat{b}_k = \text{sgn}\left(\left((\mathbf{R} + \sigma^2 \mathbf{A}^{-2})^{-1} (\mathbf{R} \mathbf{A} \mathbf{b} + n)\right)_k\right)$$

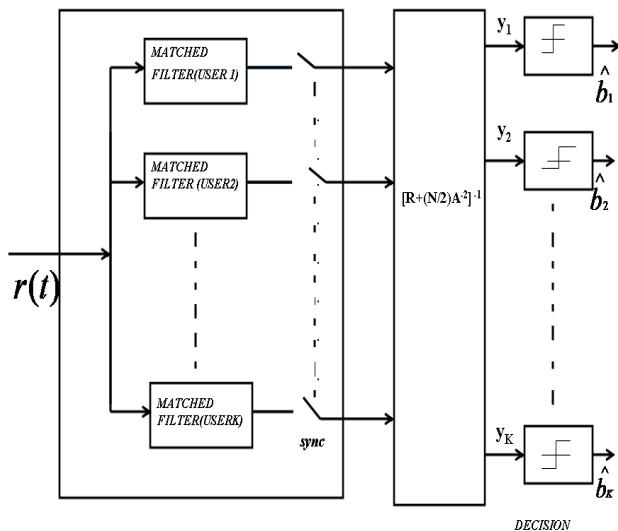


Fig. 4 MMSE linear detector

Algorithm for MMSE

Step1: $\hat{b}_{mmse} = \left(\left((\mathbf{R} + \sigma^2 \mathbf{A}^{-2})^{-1} y(t)\right)\right)$

Step 2: if decision $\hat{b}_{mmse} < 0$; $\hat{b} = -1$

3. Fundamentals of Different PN Sequences

PN sequences are sequence of 1's and 0's where the numbers look like statistically independent and uniformly distributed. As said earlier, they are arranged *random-like*, meaning that it can be generated by mathematically precise rules, but statistically it satisfies the requirements of a truly random sequence in the limiting sense. The PN sequences have the following noise like properties [8]:

- (i) Balance: in each period of maximum length sequence, the no.of 1's is always one more than the no.of 0's. So there must be 2^{m-1} ones and $2^{m-1} - 1$ zeros in a full period of the sequence.
- (ii) Run: the run represents a subsequence of identical symbols (1's or 0's) within one period of sequence. The length of this subsequence is the length of the run. Among the runs of 1's and 0's in each period of a maximum-length sequence. In any PN sequence, 1/2 of the runs have length 1, 1/4 have length 2, 1/8 have length 3, 1/16 have length 4, and so on. For the maximum-length sequence generated by a linear feedback shift register of length m, the total no.of runs is $(L+1)/2$ where $L = 2^m - 1$.
- (iii) Correlation: The autocorrelation function of a maximum-length sequence is periodic. Binary valued and a period $T = NT_c$ where T_c is chip duration. This property is called the Correlation property.

These three properties make PN sequences efficient for speech encryption. However, particularly due to the third property, adjacent bits correlation becomes considerably less, thereby making the PN sequences more effective to be used in systems like CDMA [3].

In DS-CDMA system, the received data should multiplied with the same code or sequence in the receiver for the despreading operation. So the other user codes or sequences in the same frequency band must be uncorrelated with the desired user code. For this reason the DS-CDMA codes or sequences have to be designed so as to posses very low cross-correlation [3-4].

3.1 PN-sequence:

Performance of the PN sequence has been shown better than gold sequence in [7] using conventional matched filter and linear detectors. A Pseudo-random Noise (PN) sequence is defined as coded sequence of ones and zeros with certain auto correlation properties. The Maximal length sequence (*m*-sequence) generator is usually constructed with linear feedback shift registers (LFSR) and exclusive-OR gate. The *m*-sequences are generated by a given shift register of given length with feedback. The feedback function, also called as characteristic polynomial, determines the length and type of the sequence generated.

The length of the generated ML-sequence is $L=2^m-1$. Where L is the ML sequence length, and m is the length of the shift register [7].

Consider the shift register in Figure 5, such a shift register where FF1, FF2, FF3, FF4, FF5 denote the initial binary states, can be represented by a binary polynomial where the coefficients represent whether or not there is a connection to the adder (modulo-2) as $f(x)=1+c_1x+c_2x^2+\dots+c_nx^n$.

The primitive polynomial used here is at degree $m=5$ and the polynomial is $1+x^2+x^5$.

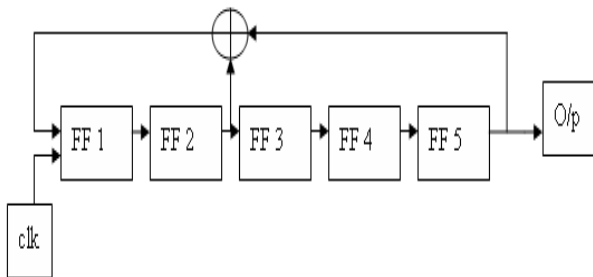


Fig. 5 Generation of m-sequence

3.2 Gold Sequence

Gold sequence is generated by using the preferred pair of *m*-sequences. Both the preferred pair *m*-sequences have the same property as that of the *m*-sequence. Gold codes have bounded small cross-correlations within a set, which is useful when multiple devices are broadcasting in the same range. A set of Gold codes can be generated by Pick two maximum length *m*-sequences of the same length then EX-OR operation will be done between two *m*-sequences [10-11] shown in Figure 6. Here the length of gold sequence is $L=2^m-1$. Where L is the ML sequence length, and m is the length of the shift register and the primitive polynomials are used here is $1+x^2+x^5$ and $1+x^2+x^3+x^4+x^5$

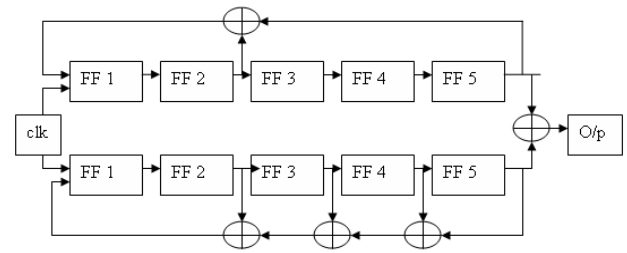


Fig. 6 Generation of Gold Sequence

3.2 Kasami Sequence

So for kasami sequence has been used for even sequence only. The Kasami sequences are a set of sequences that have good cross-correlation properties. There are two different sets of Kasami sequences. One is 'small set' and another one is 'large set'. A procedure similar to that used for generating Gold sequences will generate the 'small set' of Kasami sequences of period $L=2^m-1$, where m is a nonnegative, even integer [8]. In this procedure we begin with an *m*-sequence *a* and we form the sequence *a'* by decimating *a* by $2^{m/2}+1$ [3-4]. Figure 7 shows the kasami Sequence generator for $m=4$ (even). Example: $m=4$, Here the length of sequence is $L=2^4-1=15$ and the primitive polynomial used here is $1+x+x^4$.

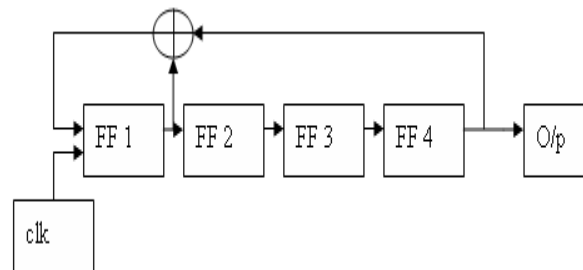


Fig. 7 Generation of even kasami Sequence

$$\begin{array}{ccccccccccccccc}
 a = & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & \uparrow & & & & & \uparrow & & & & & & \uparrow & & & \\
 a' = & 1 & 1 & 0 & & & & & & & & & & & & \\
 b = & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
 a \text{ xor } b = & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0
 \end{array}$$

4. Proposed odd Kasami Sequence

Now, it is proposed kasami sequence for odd sequence also, which is included with initial bit. The Kasami sequence sets are one of the important types of binary sequence sets because of their very low

cross-correlation. Kasami codes are based on PN codes of length of $L = 2^m$ including initial bit. Where m is a nonnegative, odd integer. To generate a kasami sequence, first of all the sequence a' is found by selecting every $2^{(m+1)/2}$ bit of an m -sequence a . the first kasami sequence can be found by adding (modulo-2 addition) the sequences a and a' . By including the sequence a in the set, a set of $2^{(m-1)/2}$ Sequences can be found. For example, for the case of $m=5$, the length of sequence is $L= 2^5 = 32$. we take 32 length PN code and take it's every 8th bit and keep repeating it to find the sequence a' . The first member of the set is found by adding a' with the PN code a that is shown below. The primitive polynomial used here is $1+x^2+x^5$. Figure 8 shows the kasami Sequence generator for $m=5$ (odd).

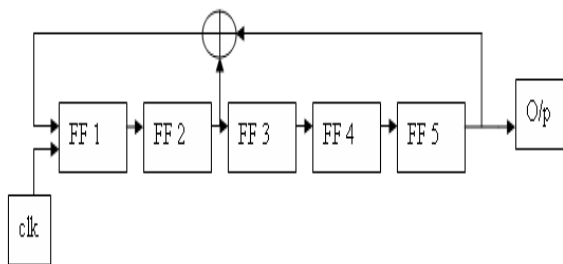


Fig.8 Generation of proposed odd kasami Sequence

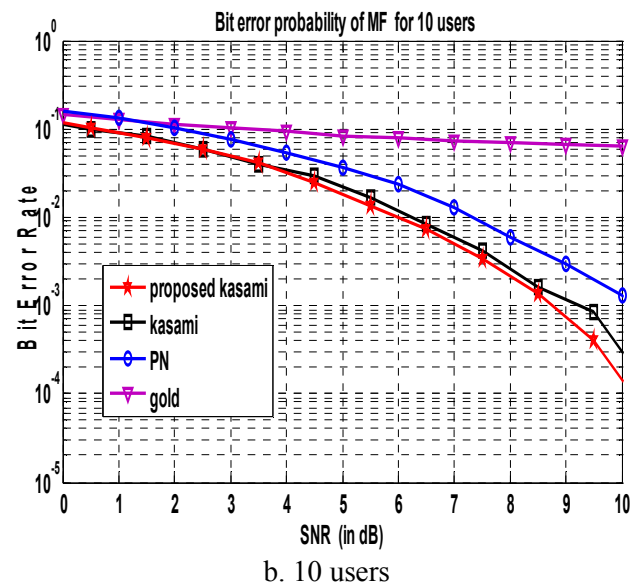
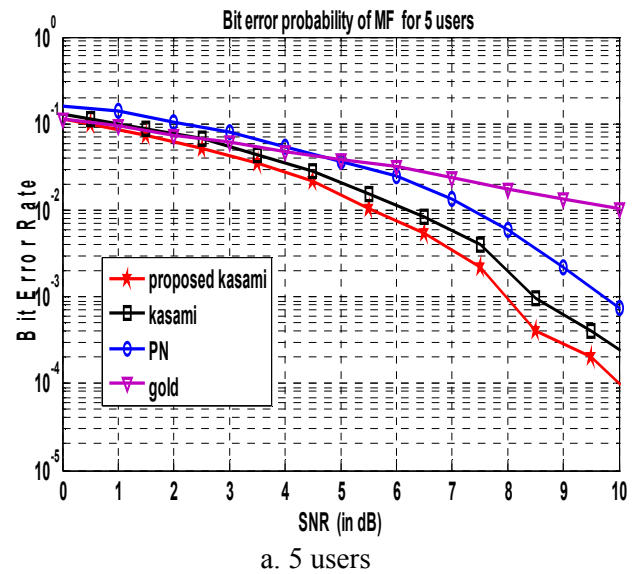
$$\begin{aligned}
 a &= [1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 0] \\
 &\quad \uparrow \qquad \qquad \qquad \qquad \qquad \qquad \uparrow \\
 &\quad 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0] \\
 &\quad \uparrow \qquad \qquad \qquad \qquad \qquad \qquad \uparrow \\
 a' &= 1\ 0\ 0\ 1 \\
 b &= [1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1] \\
 &\quad 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1] \\
 a \text{ xor } b &= [0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 1] \\
 &\quad 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1]
 \end{aligned}$$

5. Simulation Results

Conventional single user Matched Filter (MF), Decorrelating and Minimum Mean-Squared Error (MMSE) Detectors are investigated. In this section a description of the K-user discrete time basic synchronous DS-CDMA model has been used. BPSK modulation technique is used to spread the user information. Different types of spreading sequences (gold, PN, even and odd kasami) are used First of all, the BER performance comparison between gold, PN, even kasami and odd kasami (proposed method) sequences is compared. The

study followed by the performance comparison with increasing number of active users is investigated.

Figure 9 to 11 show the BER performances of Conventional single user Matched Filter (MF), Decorrelating and MMSE Detectors using gold, PN, even kasami and proposed odd kasami sequences for 5, 10, 15, 20 and 25 users. The simulation result shows the odd kasami sequence provides better performance compared to gold, PN sequence and also even kasami sequence. Using this odd kasami sequence these detectors are compared and giving better performance for MMSE detector for all the users and shown in figure 12.



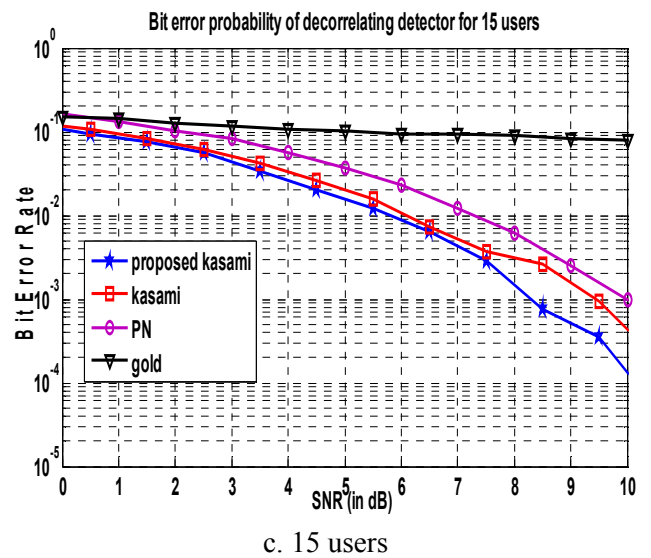
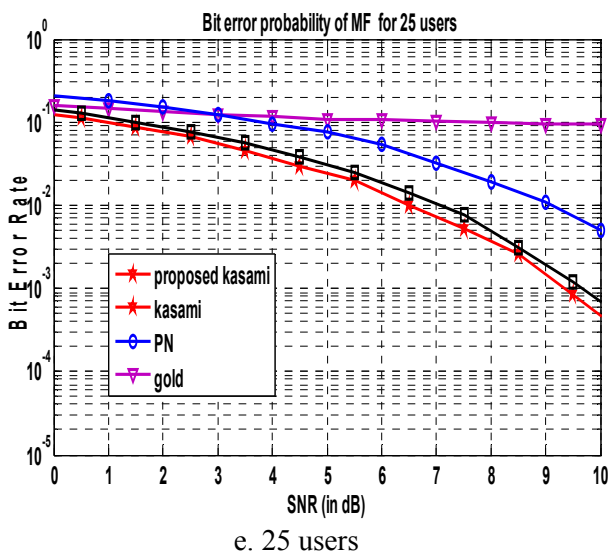
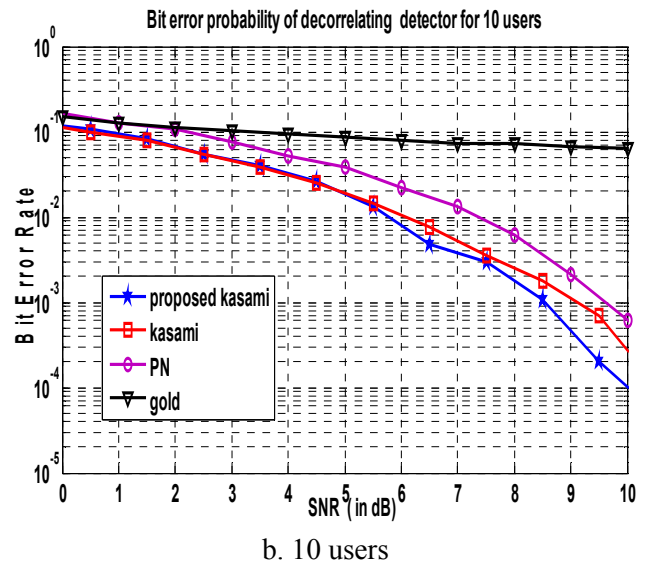
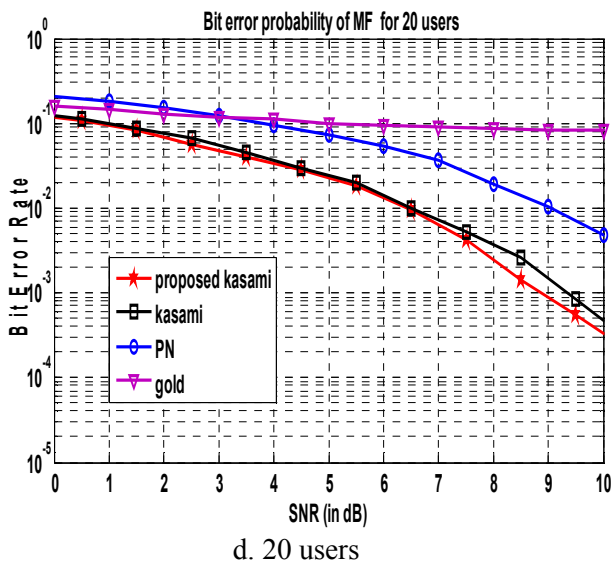
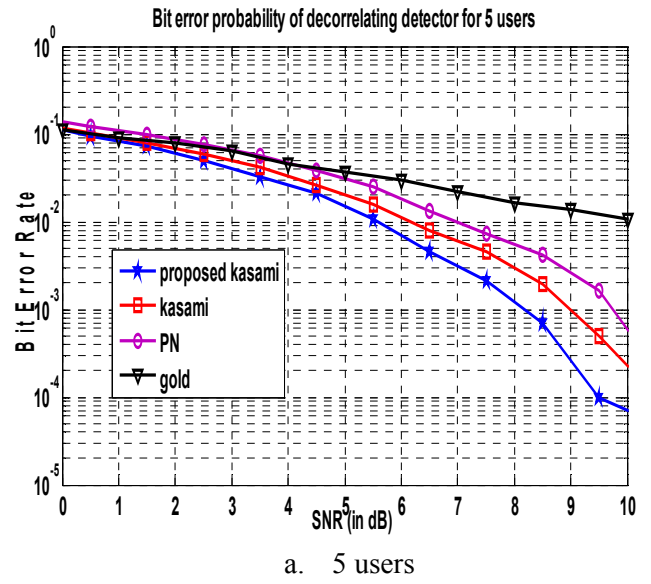
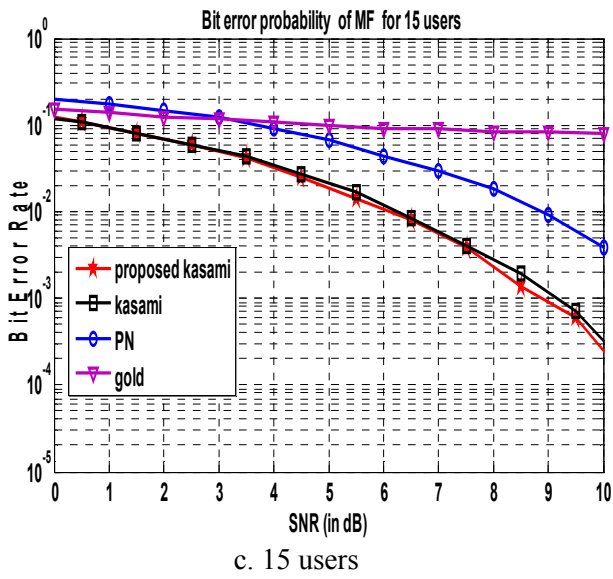


Fig. 9 BER performance of matched filter using spreading sequences.

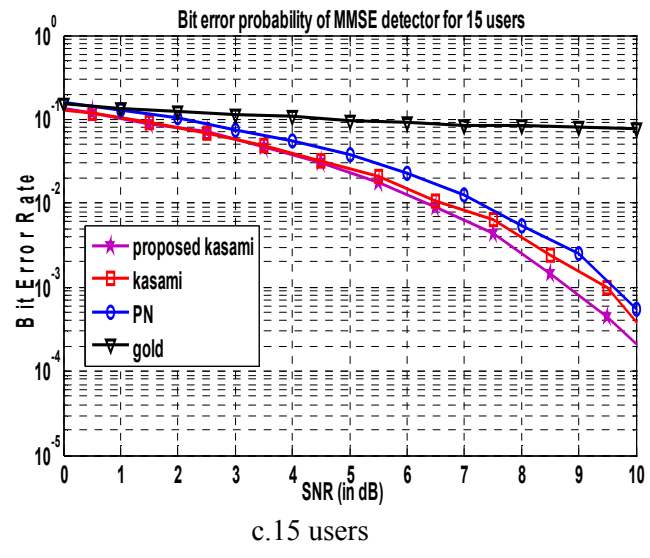
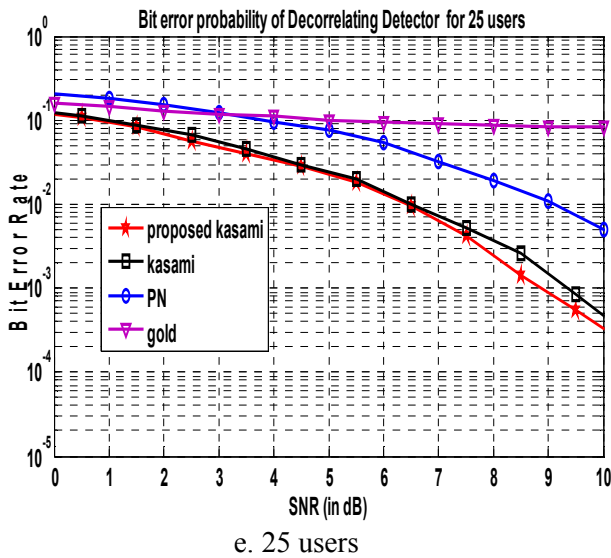
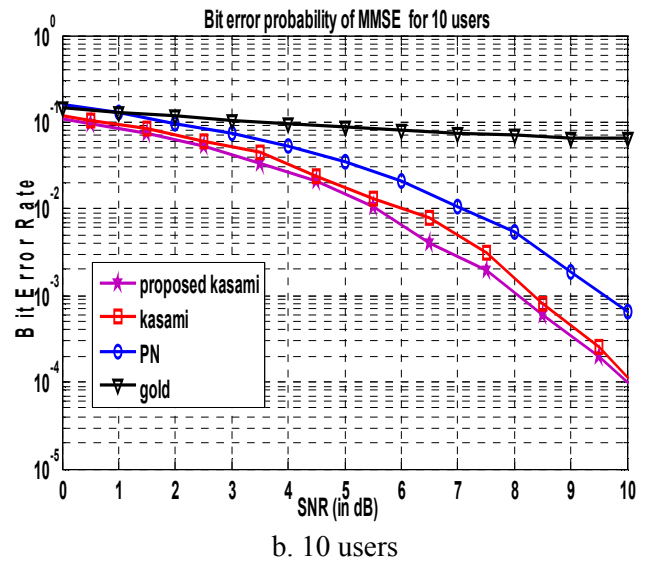
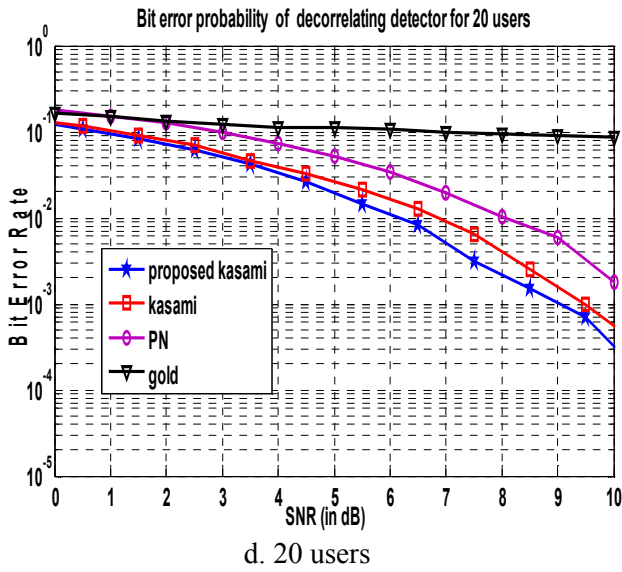
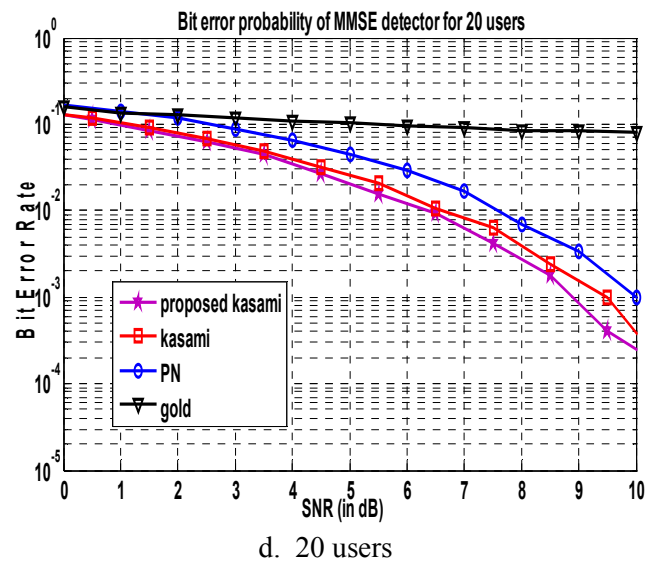
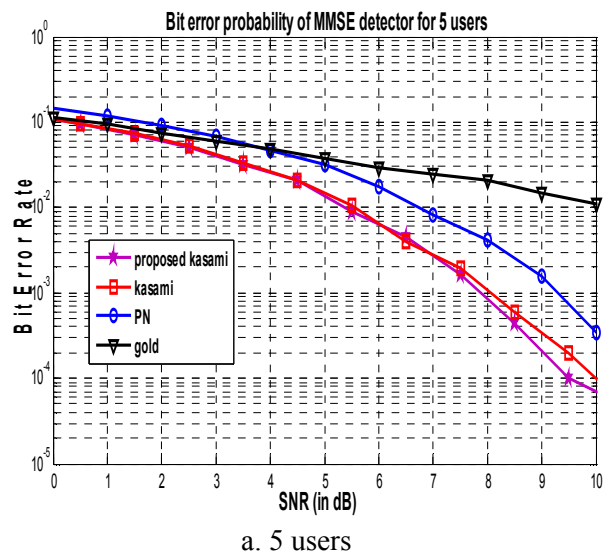
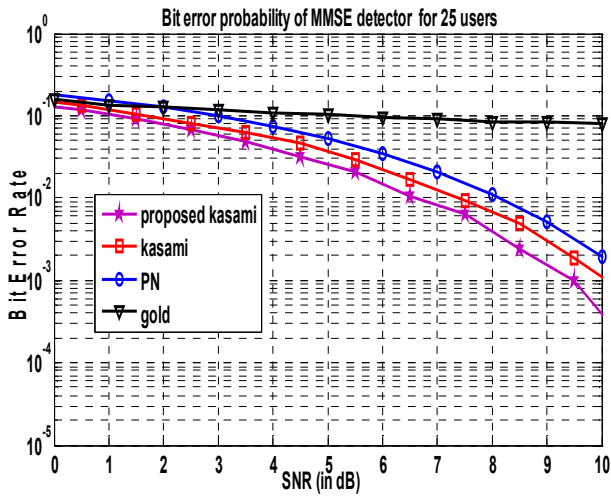


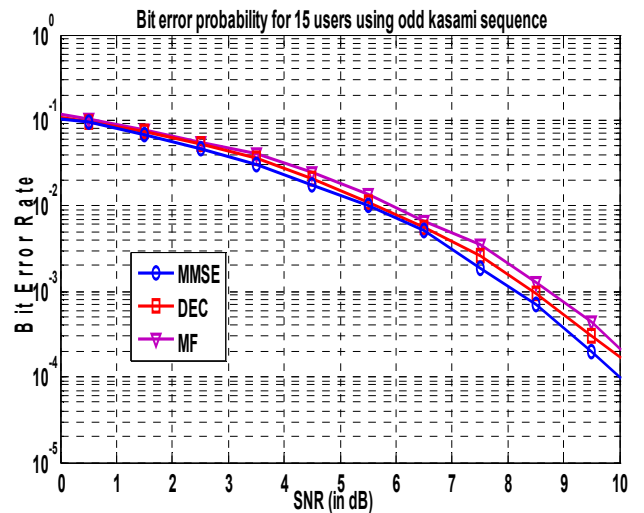
Fig. 10 BER performance of decorrelating detector using spreading sequences.



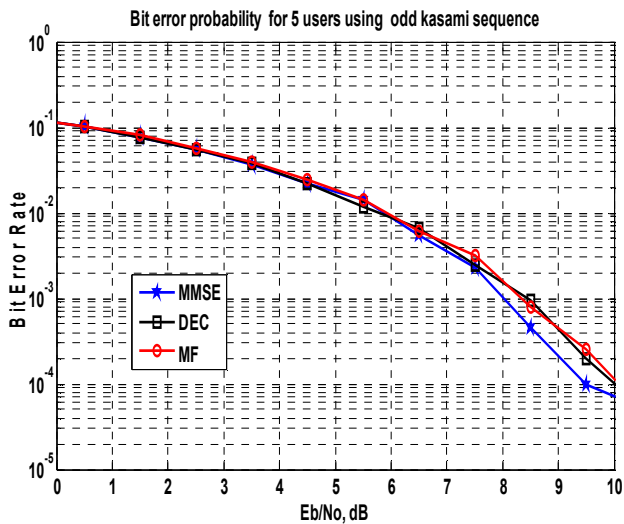


e. 25 users

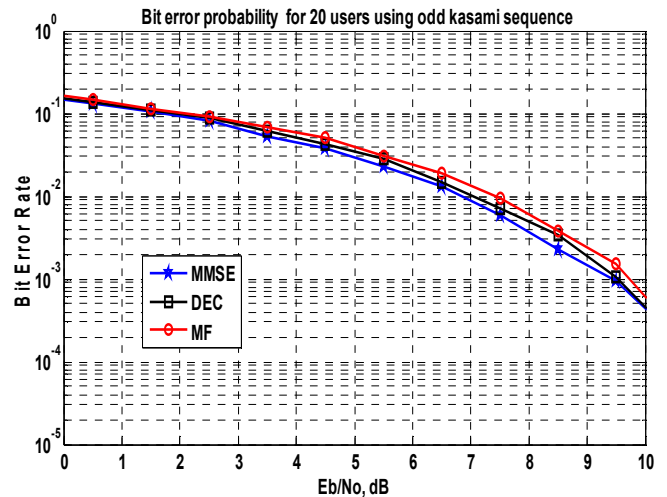
Fig. 11 BER performance of MMSE detector using spreading sequences.



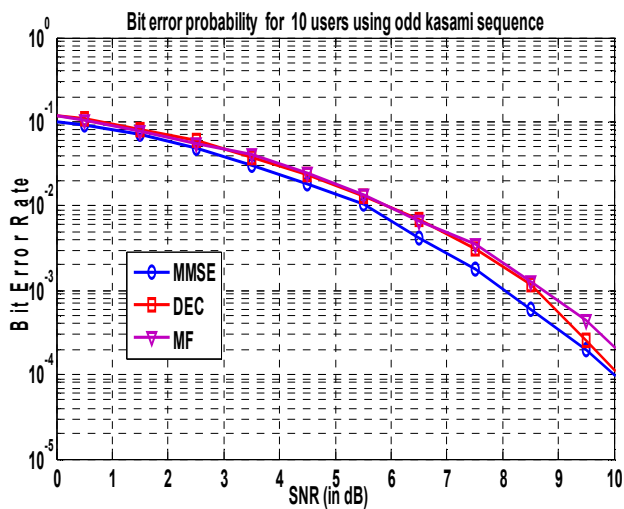
c. 15 users



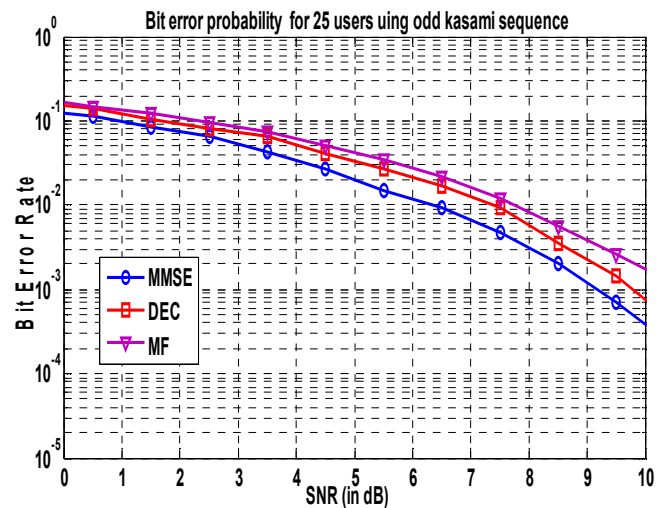
a. 5 users



d. 20 users

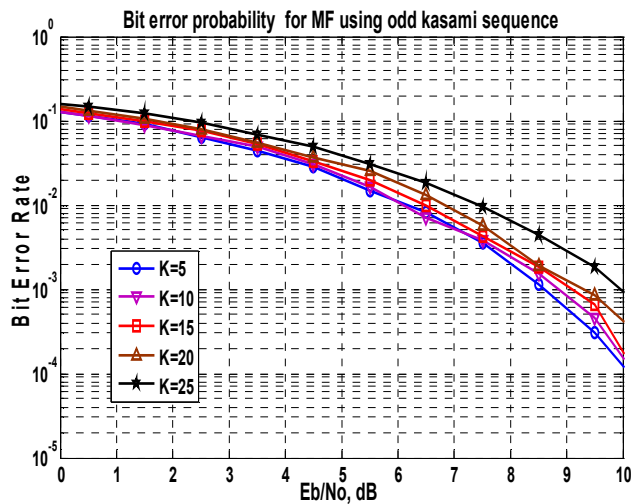


b. 10 users

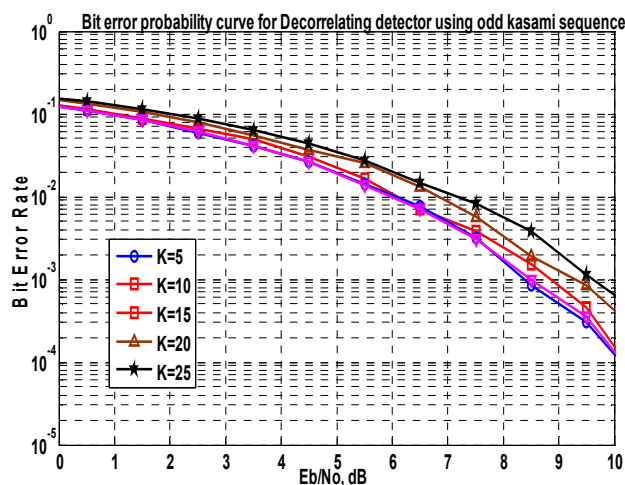


e. 25 users

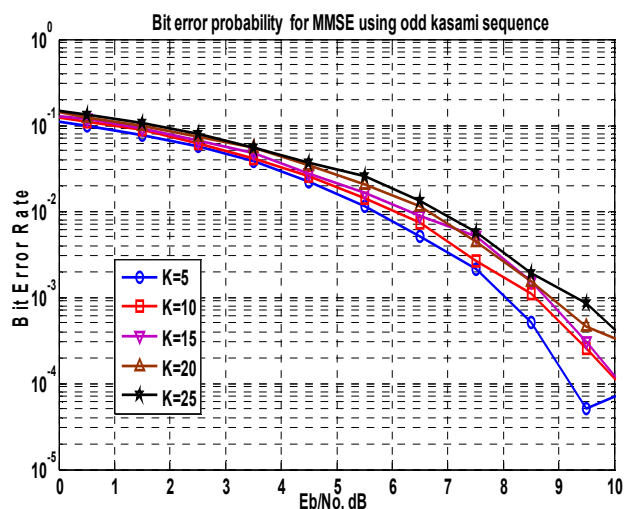
Fig. 12 BER performance matched filter, decorrelating detector and MMSE detector using odd kasami sequence.



a. Matched filter detector



b. Decorrelating detector



c. MMSE detector

Fig. 13 BER performance using proposed odd kasami sequence.

In Fig 13. As number of users increasing the BER performance all these detectors have been plotted. At higher number of users the performance

is poor compared to with lower number of users, because of background noise is adding up.

Finally proposed odd kasami sequence is giving better performance compared to even kasami and pn sequence.

6. Conclusions

The bit error rate reduces for proposed odd kasami sequence in comparison with PN and even kasami sequence of conventional single user and linear multiuser detectors. Multiuser detection technique is the efficient technique in digital signal processing. It is used to overcome limitations poses by multiple access interference, which significantly limiting the performance and capacity of conventional DS-CDMA. The linear multi-user detectors perform better than the conventional matched filter. MMSE detector generally performs better than the decorrelating detector and matched filter because it takes the background noise into account. As the number of users increases the performance is degraded.

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