On the Optimum Power Allocation in a Cellular DS/FFH-CDMA System

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Abstract:- The optimization between the average received power and the theoretically achievable average channel capacity per user (in the Shannon sense) of a hybrid direct-sequence/fast frequency hopping code-division multiple-access (DS/FFH-CDMA) cellular system, when operating in a Rayleigh fading environment, is presented. The analysis leads to a novel-closed form expression for the optimal average received power value based on the maximization of the achieved spectral efficiency, estimated in terms of the available average channel capacity per user. Finally, respective numerical results are presented.

Key-Words: Hybrid CDMA systems, Cellular systems, Channel capacity, Rayleigh fading.

1 Introduction

Following the method and the hybrid system described firstly in [1], here, the spectral efficiency of a DS/FFH-CDMA cellular system is evaluated in terms of each user's achievable average channel capacity. The channel capacity expression, establishes an upper bound limit for reliable information transmission over a bandlimited additive white Gaussian noise (AWGN) environment, [2]. When the channel side information (CSI) is not available at the transmitter, the source data is transmitted at a constant rate. Since no CSI is available at the transmitter, data transmission takes place over all fading states including deep fades where the data is lost and hence the effective channel capacity is significantly reduced. In cellular mobile radio, where signal fading is a considerable capacity degradation factor, channel capacity can be estimated in an average sense and used as a figure of merit for system's operation. This average channel capacity formula would indeed provide the true channel capacity, if channel side information were available at the receiver, [3]. However, it must be noticed, that the following analysis does not solve the problem of the capacity region, i.e., the set of information rates at which simultaneously reliable communication of the messages of each user is possible.

The final equation, theoretically derived, to the author's best knowledge, is the first time such expression has been exposed, thus avoiding complex algorithms or lengthy simulations. However, a simulation process must be described analytically, in order to compare with the theoretical results of this paper with previous results, [4-6]. We are still working on this, for a future paper, but results are not yet derived due to complicated system's parameters. Then, the analytical description of a respective simulation process remains, this time, due to complicated system's parameters, an open research problem.

Then, a novel-closed form expression for the optimal average received power, in a Rayleigh fading environment, with respect to the maximization of the achieved spectral efficiency, estimated in terms of the available average channel capacity per user, is derived and respective numerical results are presented. The final expression derived here, can be very useful for the practical design of a DS/FFH-CDMA cellular system, specifically in the power control algorithm applied, and for an initial quantitative analysis.

2 Optimal Received Power in Rayleigh Fading

At first, we consider the twelve co-channel cells, in the first tier, of a cellular DS/FFH-CDMA system, as shown in Fig. 1. In addition, we assume that the cellular hybrid DS/FFH-CDMA system accommodates K of users per cell and the users within each cell can be approximately orthogonalized, [7]. Then, the original transmitted signal is only corrupted by AWGN and co-channel interference (CCI) power. During each frequency hop, a DS signal is transmitted in the form of a spread signal with bandwidth $W_{ds}=G_p \cdot W_s$, where W_s is the signal bandwidth. The totally allocated system's bandwidth W_t is equal to:

$$W_t = M \cdot W_{ds} = M \cdot G_p \cdot W_s \quad (1)$$

where M (M>1) is the number of hops per transmitted bit. In addition, we consider the CCI power resulting only from the first tier of the DS/FFH-CDMA cellular system assuming a cell cluster size equal to twelve and where all base stations' and mobile units' antennas are assumed omnidirectional. Thus, the channel capacity required for error-less transmission of a signal of bandwidth W_{ds} will be given by the Shannon-Hartley theorem [8]:

$$C_{i,DS/FFH} = W_{ds} \cdot \log_2(1 + S_{i,DS/FFH}) \quad (2)$$

where $S_{i,,DS/FFH}$, i=[1,..,12K], is the average signal-to-interference plus noise ratio (SINR) received at the i-th user as it reaches the boundary of a cell. Assuming that in the downlink all mobile units of a certain cell will receive equal average signal power from their cell site, then, for a fourth power law path loss, the average received signal power P_r at the distance r by the i-th user, i=[1,..,12K], will be:

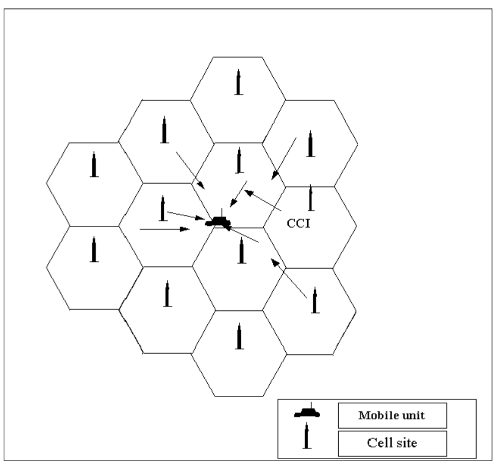

Fig.1. DS/FFH-CDMA cellular system and its interference.

$$P_r = \alpha \cdot r^{-4} \quad (3)$$

where α is a constant factor. Therefore, the SINR received at the mobile unit as it reaches the boundary of a cell, $S_{i,DS/FFH}$, can readily be determined by considering the average CCI power resulting from the eleven co-channel cells of the first dominant tier of interfering cells, [9], i.e.:

$$S_{i,DS/FFH} = \frac{P'_r}{N_0 \cdot W_{ds} + P_h \cdot \left[2K \cdot \alpha \cdot R^{-4} + 3K \cdot \alpha \cdot (2R)^{-4} + 6K \cdot \alpha \cdot (2.633R)^{-4}\right] \cdot \frac{1}{M}} = \frac{\frac{P_r}{M}}{N_0 \cdot W_{ds} + P_h \cdot (2.3123 \cdot K) \cdot \frac{P_r}{M}} \quad (4)$$

since, for a FFH transmission scheme, the CCI power, as seen by a desired signal, originates, on the average, from 1/M of the co-channel users and N_0 is the noise power spectral density of the AWGN. In addition, P'_r is the user's average received signal power, in each of the M frequencies, being equal to:

$$P'_r = \frac{P_r}{M} \quad (5)$$

assuming that, in the FFH case, the totally transmitted signal power is equally shared, by hopping, among the M different carrier frequencies. In eq.(4), P_h is the probability of hit, for the FFH case, approximated by:

$$P_h \cong \frac{1}{M} \quad (6)$$

Thus, eq.(4) can be rewritten in the form:

$$S_{i,\,DS/FFH} = \frac{S}{G_p \cdot M + \frac{1}{M} \cdot (2.3123 \cdot K) \cdot S} \quad (7)$$

where $S=(P_r/N)$ is the average received signal-to-noise ratio (SNR) over signal bandwidth W_s and $N=N_0W_s$ is the AWGN power over signal bandwidth W_s. Following eq.(2), the total channel capacity available to all 12K users, will be given by:

$$C_{DS/FFH} = \sum_{i=1}^{12K} C_{i,DS/FFH} = W_{ds} \cdot \sum_{i=1}^{12K} \log_2(1 + S_{i,DS/FFH}) \quad (8)$$

where $S_{i,DS/FFH}$ is given by eq.(7). Since, in practice, $S_{i,DS/FFH}$, i=[1,..,12K], is well below unity, eq.(8) can be approximated by:

$$C_{DS/FFH} \cong W_{ds} \cdot \log_2(1 + 12 \cdot K \cdot S_{i,DS/FFH}) \quad (9)$$

Assuming that the physical channel of bandwidth W_{ds} is greater than the coherence bandwidth W_{coh} of the Rayleigh fading channel, the maximum number M_{ds} of uncorrelated resolvable paths is approximated by [10]:

$$M_{ds} = [W_{ds} \cdot \Delta] + 1 \approx (G_p \cdot W_s \cdot \Delta) + 1 \quad (10)$$

where Δ is the maximum delay spread of the fading channel and [.] returns the largest integer less than, or equal to, its argument. An M hops per transmitted bit FFH system, can be seen as equivalent to an M-branch maximal-ratio combining (MRC) space diversity system, [11]. Therefore, the average channel capacity per user $\langle C_i \rangle_{DS/FFH,Rayleigh}$, normalized over the total system's bandwidth W_t, is given by:

$$\frac{\langle C_i \rangle_{DS/FFH,Rayleigh}}{W_t} = \frac{\langle C_i \rangle_{DS/FFH,Rayleigh}}{M \cdot W_{ds}} = \int_0^{\infty} \log_2(1+\gamma) \cdot \frac{(\gamma)^{M-1}}{(M-1)!\left(S_{i,DS/FFH}\right)^M} \cdot \exp\left(-\frac{\gamma}{S_{i,DS/FFH}}\right) \cdot d\gamma \quad (11)$$

where $\langle . \rangle$ indicates average value and $S_{i,DS/FFH}=\langle \gamma \rangle$, given by eq.(7), is the average received SINR in each of the M frequencies where the DS signal is transmitted. Furthermore, if path-diversity reception, provided by a MRC RAKE receiver, is also applied to the DS/FFH-CDMA system, then additional diversity will be achieved. Hence, assuming that the multipath-intensity profile (MIP) has equal path strengths on the average, the SINR after path-diversity applied, in each of the M frequencies, $S_{i,\mathrm{pt},\mathrm{DS/FFH}}$ will be given by [12]:

$$S_{i,\mathrm{pt},\mathrm{DS/FFH}} = M_{ds} \cdot S_{i,\mathrm{DS/FFH}} = = M_{ds} \cdot \frac{S}{G_p \cdot M + \frac{1}{M} \cdot (2.3123K) \cdot S} = M_{ds} \cdot \frac{S}{\frac{W_t}{W_s} + \frac{1}{M} \cdot (2.3123K) \cdot S} \quad (12)$$

where $S_{i,\mathrm{DS/FFH}} = \langle \gamma \rangle$ is the average received SINR in each of the M frequencies in a Rayleigh fading channel (the suffice 'pt' refers to the path-diversity reception applied). Applying directly eq.(12) to eq.(11), $\langle C_i \rangle_{\mathrm{DS/FFH,Rayleigh}}$, normalized over the total system's bandwidth W_t, is rewritten as:

$$\frac{\langle C_i \rangle_{\mathrm{DS/FFH,\,Rayleigh}}}{W_t} = \frac{\langle C_i \rangle_{\mathrm{DS/FFH,\,Rayleigh}}}{M \cdot W_{ds}} = = \int_0^\infty \log_2(1+\gamma) \cdot \frac{(\gamma)^{M-1}}{(M-1)! \cdot (S_{i,\mathrm{pt,\,DS/FFH}})^M} \cdot \exp\left(-\frac{\gamma}{S_{i,\mathrm{pt,\,DS/FFH}}}\right) \cdot d\gamma \quad (13)$$

The problem of maximization of the normalized average channel capacity per user can be stated as follows:

$$\max \int_0^\infty \log_2(1+\gamma) \cdot \frac{(\gamma)^{M-1}}{(M-1)! \cdot (S_{i,\mathrm{pt,DS/FFH}})^M} \cdot \exp\left(-\frac{\gamma}{S_{i,\mathrm{pt,DS/FFH}}}\right) \cdot d\gamma \quad (14)$$

The combined average spread SINR after diversity reception i.e. $S_{i,\mathrm{pt,DS/FFH}} \cdot M_{ds}$, that maximizes eq.(14), equals to 6 dB, [13], i.e.,

$$S_{i,\mathrm{pt,DS/FFH}} \cdot M_{ds} = M_{ds}^2 \cdot S_{i,\mathrm{DS/FFH}} = = M_{ds}^2 \cdot \frac{S}{\frac{W_t}{W_s} + \frac{1}{M} \cdot (2.3123 \cdot K) \cdot S} = 10^{0.6} \quad (15)$$

applying directly eq.(12). Then, using eq.(10), the eq.(15) is rewritten as:

$$M_{ds}^{\ 2} \cdot \frac{\frac{P_r}{N_0 \cdot W_s}}{G_p \cdot M + \frac{1}{M} \cdot (2.3123 \cdot K) \cdot \frac{P_r}{N_0 \cdot W_s}} = 10^{0.6} \quad (16)$$

Then, the optimal received power $P_{r,op}$, (the suffice 'op' refers to the optimal value) can be found directly from eq.(16), as following i.e.:

$$P_{r,op} = \frac{3.9 \cdot M^2 \cdot N_0 \cdot W_s \cdot G_p}{M \cdot M_{ds}^2 - 9.2 \cdot K} \quad (17)$$

The optimal received power $P_{r,op}$ (expressed in Watt (W)), given by eq.(15), is plotted in Figure 2 as function of the number of users per cell K, where the following values are assumed: (i) totally constant allocated system's bandwidth: W_t=10MHz, (ii) signal bandwidth: W_s=30KHz, (iii) number of hops per transmitted bit: M=5, (iv) total multipath spread of the urban Rayleigh fading channel: Δ=3μsec.

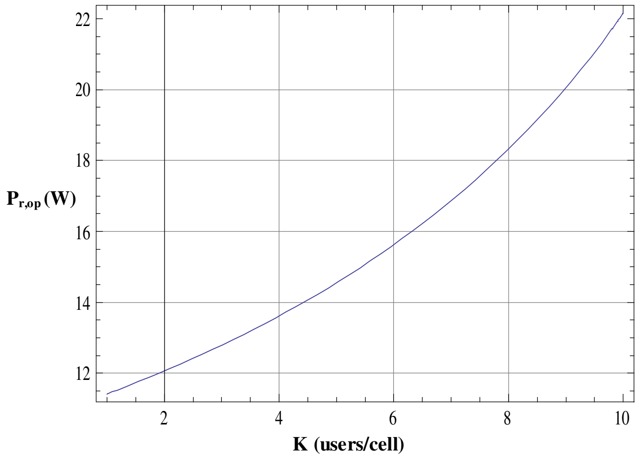

Fig.2. Optimal received power $P_{r,op}$ in a DS/FFH-CDMA cellular system versus the number K of users per cell in a Rayleigh fading environment.

As it can be seen directly from Figure 2, the required value of optimal received power $P_{r,op}$ is increased as the number of users per cell K is increased, indicating that when the number of users per cell increases, and consequently the CCI power increases respectively, the required optimal received power $P_{r,op}$ must be increased respectively in order to minimize the impact of the CCI power on the system's performance.

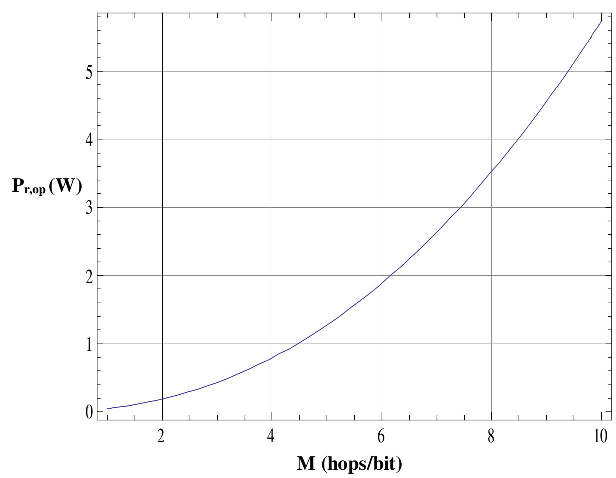

Fig.3. Optimal received power $P_{r,op}$ in a DS/FFH-CDMA cellular system versus the number M of hops per transmitted bit in a Rayleigh fading environment.

In addition in Figure 3, the optimal received power $P_{r,op}$, (expressed in Watt (W)), is plotted as function of the number M of hops per transmitted bit, for: S=30dB, W_t=10MHz, W_s=30KHz, Δ=3μsec and K=10 users per cell as an indicative value (in real cellular systems the actual number K of users per cell is of the order of 50). As it can be seen directly from Figure 3, the required value of optimal received power $P_{r,op}$ is increased as the number M of hops per transmitted bit increased, indicating that although an increased value of number M of hops per transmitted bit provides increased inherent diversity potential, the CCI power is still sufficient and then, an increased value of the received power $P_{r,op}$ is needed finally, in order to minimize the effect of the CCI in the system's performance.

3 Conclusion

In this paper, we estimate the optimal average received power of a cellular DS/FFH-CDMA system, operating in a Rayleigh fading environment, which maximizes the achieved spectral efficiency, in terms of the average channel capacity available to each user, when additional path-diversity reception is applied. It is derived, without applying complex theoretical algorithms or lengthy simulations, a novel general expression, which relates the optimal average received power with all system's parameters. It must be noticed that, similar expressions have been recently derived in [14], but concerning the optimization of different system's parameter. The final expression derived can be useful for the initial practical design of a DS/FFH-CDMA cellular system in a Rayleigh fading environment. However, a simulation process must be described analytically, in a future paper, in order to compare with the theoretical results of this paper.

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