Multi-user Detection Based on Weights Selected Particle Filter in Impulsive Noise

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Abstract: Degradation is a major problem for the particle filter algorithm. Although the conventional resampling method can solve the degradation problem, it will make the particles lose diversity. In this paper, we proposed a weights selected particle filter (WSPF) algorithm for adaptive multi-user detection (MUD) in synchronous code division multiple access (CDMA) system. This algorithm assumes the number of particles required to removal the invalid particles, reserve the particles which make great contributions to state estimation. Thus it can mitigate the degradation of the particle filter. And this algorithm is suitable for the non-linear non-Gaussian system. The simulation results show that the bit error rate of the system can be reduced to a large extent. So the proposed algorithm has important reference value.

Key-words: Particle filter, CDMA, Multi-user detection, Weights selected, Non-Gaussian noise

1. Introduction

In the code division multiple access (CDMA) system, multi-user detection is served as one of the key technologies. It is proposed originally by K. S. Schneider in 1979. This method can eliminate the multi-access interference and alleviate the near-far effect effectively. Verdu put forward the best multi-user detection method in 1986[1]. Its performance is best, but its complexity grows exponentially with the increase in the number of users. So this method is difficult to be achieved in engineering. Many researchers put forward different kinds of sub-optimal multi-user detection, such as decorrelation detector and MMSE detector. Their complexity is reduced. But the performance is significant lower than the optimal multi-user detection algorithm.

Multi-user detection based on particle filter is one kind of sub-optimal multi-user detection methods. The particle filter algorithm is an approximate algorithm to Bayesian estimation based on sampling theory. It combines Monte Carlo with Bayesian theory together. When the sample size is large enough, the sample set can description the real posteriori probability density function (PDF) of the state variables. This algorithm has very strong robustness. So it can be applied to non-linear non-Gaussian system. Its accuracy can approximate to the optimal estimation [2]. Degradation is the major drawback of the particle filter algorithm. In this paper, we discuss a new algorithm that weights selected particle filter, which can solve this problem.

Usually, there are some peak pulses which have low probability and high amplitude in the actual system. The noise had a great influence on the transmission of signals. So the research on the system performance in non Gauss noise is very necessary. In this paper Laplace noise and alpha stable noise are simulated non-Gaussian noise. We can prove that the applicability and effectiveness of the particle filter is well in non-Gaussian system by simulation.

2. System model

2.1. CDMA system model

Consider a synchronous CDMA system with K users, symbol interval is T. The received signal is:

$$r(t)=\sum_{k=1}^{K}A_k(t)g_k(t)b_k(t)+n(t)\cdot \tag{1}$$

In the above formula, A_k is amplitude of the k-th signal; $g_k\in\{1,-1\}$ means spread spectrum waveform of the k-th signal, $b_k\in\{1,-1\}$ means the k-th user data;

$n(t)$ is background noise that is selected by different type of the noise.

The matched filter output of the k-th user can be expressed as:

$$y_k = \int_0^T r(t) g_k(t) dt = A_k b_k + \sum_{\substack{i=1 \\ i \neq k}}^{K} \rho_{i,k} A_i b_i + \frac{1}{T_b}\int_0^{T_b} n(t) g_k(t) dt = A_k b_k + MAI_k + z_k \quad (2)$$

In this formula, $\rho_{i,k} = 1/T_b \int_0^{T_b} g_k(t) g_i(t) dt$ is the cross-correlation between the signature waveforms of the i-th user and the k-th user. $A_k b_k$ is signal of the k-th user; MAI_k is multiple access interference (MAI); z_k is noise.

In order to simplify analysis, the received vector can be expressed as matrix.

$$y = RAb + z \quad (3)$$

$y = [y_1, y_2, \ldots, y_K]^T$, $\mathbf{b} = [b_1, b_2 \ldots, b_K]^T$. R is a symmetric correlation matrix with $K \times K$ dimension ($\rho_{i,k} = \rho_{k,i}$). $\mathbf{A} = diag(A_1, A_2 \ldots, A_K)$ is the amplitude of the received signal which is a diagonal matrix. $z = [z_1, z_2, \ldots, z_k]^T$, it is a complex-valued vector with independent real and imaginary components and covariance matrix equal to $\sigma^2 R$.

R is a symmetric matrix, therefore Colicky factorization can be used. There is a unique lower triangular matrix F such that $R = F^T F$. If we apply F^{-T} to formula (3), can we obtain [3]

$$\overline{y} = F^{-T} y = F^{-T} F^T F A b + F^{-T} z = FAb + \overline{z} \quad (4)$$

We have know that the covariance matrix of $\overline{z}$ is $\sigma^2 I$, where I is the identity matrix. Because the noise becomes independent and identically distributed, white noise. $\overline{y}$ is called the whitened matched filter output. Scalar expression of the received signal as follows:

$$\overline{y_k} = \sum_{l=1}^{K} F_{k,l} a_l b_l + \overline{z_k} \quad (5)$$

The purpose of MUD is to detect users' signals from the output signals of the matched filter.

2.2. Non-Gaussian noise simulation

In order to simplify mathematical analysis, we often assume that the channel noise is Gaussian white noise. However, in some natural and artificial resulting from the impact of impulsive ambient noise case, such as thunder and lightning, all kinds of machine motors, neon signs, etc. do not have Gaussian nature. This kind of noise shows significant peak amplitude. In order to improve the detection performance under the "noise spikes", it is very necessary to establish a more accurate model. The following briefly discusses the non-Gaussian noise [4].

A. Laplace noise

Laplace probability density function (PDF) has an obvious smearing which is the main difference between the Laplace noise and the Gaussian noise. Laplace PDF:

$$p(x) = \frac{1}{\sqrt{2\sigma^2}} \exp(-\sqrt{\frac{2}{\sigma^2}}|x|) \;,\; -\infty < x < +\infty \quad (6)$$

In the formula, σ^2 parameter is variance or power of noise.

B. Alpha noise

Then introduce the Alpha noise which is another type non-Gaussian noise. If the random variable X is subject to the Alpha stable distribution, then its characteristic function is:

$$\phi(u) = \exp\{ jau - \gamma |u|^\alpha [1 + j\beta \operatorname{sgn}(u) \omega(u,\alpha)] \},$$
$$-1 < \beta < 1. \quad (7)$$

$$\omega(u,\alpha) = \begin{cases} \tan(\pi\alpha/2) \cdots\cdots\cdots \alpha \neq 1 \\ (2/\pi)\log|u|, \cdots\cdots \alpha = 1 \end{cases} \quad (8)$$

Where, $\alpha \in (0,2]$, it is named characteristic index which determines the degree of pulse characteristics. α is smaller, the trailing is more thick, and then the pulse characteristics is more obvious. When $\alpha = 2$, this distribution is as same as the Gaussian distribution. β is symmetry parameter which can control the gradient of the distribution. If $\beta = 0$, is it called symmetric α-stable distribution, denoted as $S\alpha S$. When $\alpha = 1$ and $\beta = 0$, this distribution becomes Cauchy distribution. γ is called scattering coefficients. It is dispersion measure about the samples relative to the mean, similar to the variance of the Gaussian distribution. When $a = 0$ and $\gamma = 1$, this distribution is called standard α-stable distribution.

3. Multi-user detection based on standard particle filter algorithm

Particle filter uses samples $\{x_{1:K}^i\}_{i=1}^{N_s}$ and the corresponding weights $\{\omega_{1:K}^i\}_{i=1}^{N_s}$ to express the posteriori PDF $p(b_{1:k} | \overline{y}_{1:k})$. According to the Maximum A Posteriori (MAP) rule, we can get an estimation of the state [2] [5].

If the samples were directly sampled from $p(b_{1:k} | \overline{y}_{1:k})$, every sample has the same weight. However, in practice, $p(b_{1:k} | \overline{y}_{1:k})$ unlikely to be written in the typical form. So the sampling process is very difficult. We often obtain the particles from an importance density function $q(b_{1:k} | \overline{y}_{1:k})$. Then the weights of samples are defined to be[6]:

$$\omega_k^i \propto p(x_{1:k}^i | \overline{y}_{1:k}) / q(x_{1:k}^i | \overline{y}_{1:k}) \qquad i = 1, \cdots, N_s \tag{9}$$

The importance density function can be factorized as follows:

$$q(x_{1:k} | \overline{y}_{1:k}) = q(x_k | x_{1:k-1}, \overline{y}_{1:k}) q(x_{1:k-1} | \overline{y}_{1:k-1}) \tag{10}$$

The posteriori probability density function is expressed as:

$$p(x_{1:k} | \overline{y}_{1:k}) \propto p(\overline{y}_k | x_k) p(x_k | x_{k-1}) p(x_{1:k-1} | \overline{y}_{1:k-1}) \tag{11}$$

By substituting (11) with (9) and (10), we can obtain the importance weights updated formula:

$$\omega_k^i \propto \frac{p(\overline{y}_k | x_k^i) p(x_k^i | x_{k-1}^i) p(x_{1:k-1}^i | \overline{y}_{1:k-1})}{q(x_k^i | x_{1:k-1}^i, \overline{y}_{1:k}) q(x_{1:k-1}^i | \overline{y}_{1:k-1})} = \omega_{k-1}^i \frac{p(\overline{y}_k | x_k^i) p(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{1:k-1}^i, \overline{y}_{1:k})} \tag{12}$$

The standard particle filter algorithm chooses a priori probability density function which is the most easy to achieve as the important density function.

$$q(x_k^i | x_{1:k-1}^i, \overline{y}_{1:k}) = p(x_k^i | x_{k-1}^i) \tag{13}$$

By substituting (12) with (13), the formula is simplified as $\omega_k^i \propto \omega_{k-1}^i p(\overline{y}_k | x_k^i)$. After finishing the calculation for all users, the particles $\{x_{1:K}^i\}_{i=1}^{N_s}$ and the corresponding weights $\{\omega_{1:K}^i\}_{i=1}^{N_s}$ can be used to estimate the posterior PDF. For example, the marginalized posterior probability density function $p(x_k | \overline{y}_{1:k})$ can be expressed as [7]:

$$p(x_k | \overline{y}_{1:K}) \approx \sum_{i=1}^{N_s} \omega_K^i \delta(x_k - x_k^i) \tag{14}$$

$x_k = [x_k^1, x_k^2, \cdots, x_k^{N_s}]^T$, $\omega_k = [\omega_k^1, \omega_k^2, \cdots, \omega_k^{N_s}]^T$. According to the MAP rule, the signal can be expressed as:

$$\hat{b}_k = sign(x_k{}^T \omega_k) \tag{15}$$

A major problem of the particle filter is that the variance of weights will be random increase with the increase of iteration times. In other words, all the particles except for a very few are assigned negligible weights. The result is that a lot of calculation is wasted in the particles whose contribution to the approximation to $p(x_k | \overline{y}_{1:k})$ is almost zero. This problem is called particle degradation. In order to solve this problem, resampling technique was introduced .

Usually, degradation degree of the algorithm is measured by the effective number of particles $N_{eff} = round(1/\sum_{i=1}^{N_s}(\omega_k^i)^2)$. Then we set a threshold $N_{threshold}$. When $N_{eff} < N_{threshold}$, sample N_s times from the posterior PDF to produce new particles $\{x_k^{i^*}\}_{i^*=1}^{N_s}$. The weight of each particle is set to be $\omega_k^j = 1/N_s$.

In summary, the steps of multiuser detection based on particle filter are as follows [8]:

Step 1: Sample for the k-th user, making $x_k^i \sim q(x_k | x_{1:k-1}^i, \overline{y}_{1:k})$.

Step 2: According to formula (12) to calculate the weights of the particles.

Step 3: Normalize the weights according to formula (18).

Step 4: Resample for the particles.

Step 5: According to formula (15) to estimate the signals of the k-th user.

Step 6: Turn to step1, and estimate the signals of the next user.

4. Multi-user detection based on weights selected particle filter

In order to solve sample depletion problem, the WSPF algorithm is introduced in this chapter. The basic idea of the WSPF algorithm as follows: If we needed N_s particles to estimate the users' signals, we sample $N_p (N_p > N_s)$ particles and calculate the corresponding weights of the particles. At last we choose N_s particles which have the largest weights to estimate the users' data. As a result the selected particles maintain the diversity.

The steps of multi-user detection based on WSPF are as follows [2] [9]:

Step 1: Sample for the k-th user, making $x_k^i \sim q(x_k \mid x_{1:k-1}^i, \bar{y}_{1:k})$.

Step 2: Calculate the weights of the particles according to formula (12).

Step 3: Sort the particles in accordance with the weights value, choose the first N_s particles.

Step 4: According to formula (18) to normalize the weights of the N_s particles.

Step 5: According to formula (15) to estimate the signals of the k-th user.

Step 6: Restore the selected particles, then normalize the weights of all the N_p particles.

Step 7: Turn to step1, and estimate the signals of the next user.

In this algorithm, all particles join in the particle update. The advantage of this algorithm is that each particle is statistically independent. In other words, the diversity of the particles set is improved.

5. Simulation results and conclusions

Analyze the performance of the multiuser detection based on weights selected particle filter algorithm by simulation. Consider a synchronous CDMA system with 8 users, 50000 information bits, 31-bit gold spread-spectrum code and the user power partial value is 10. Channel noises include Gaussian noise, Laplace noise and Alpha stable noise. Three kinds of noise variance are all 2, and Alpha stable noise parameters: $\alpha = 1.8$, $\beta = 0$, $\gamma = 1$, $a = 0$. The range of signal to noise ratio (SNR) for all users is -4~10 dB. Set $N_p = 80, N_s = 50$.

Figure1: When the signals are interfered by the additive Gaussian noise, it is clear that the performance of the WSPF algorithm is better than the PF algorithm. The research results have important reference value for the research of MUD system.

Figure2: This figure analyzes the error code performance of WSPF detection aiming to Gaussian noise, Alpha stable noise and Laplace noise. It can be seen from the Figure 2 that the error code performance of the Gaussian noises and the Laplace noise are almost the same. The error code performance of the Alpha stable noises slightly weakened. It is due to the true power of the Alpha stable noise is not 2γ. But, in the simulation, we assume that his power is 2γ. The result also proves that the applicability of the weight selection particle filter algorithm is well in the non-Gaussian system. In other words, this algorithm has a strong robustness. Therefore, this algorithm has practical reference value.

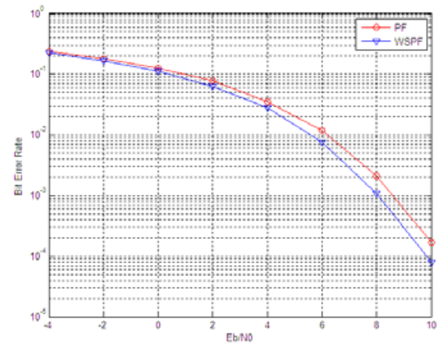

Figure 1. **The BER of PF and WSPF detection**

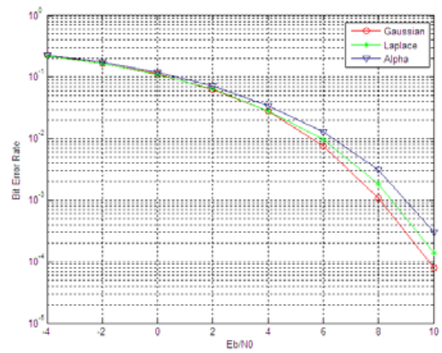

Figure 2. **BER of WSPF detection under three kinds of noise**

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