Error-rate Performance of Selection Combining over Generalized Correlated Weibull Fading Channels

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Abstract: Error-rate performance is studied for selection combining (SC) with arbitrary number of diversity branches over generalized correlated Weibull fading channels. Using a transformation of cumulative distribution function of average branch signal-to-noise ratio (SNR), we present the exact error-rate expressions for coherent and noncoherent modulations in infinite series forms. In particular, we develop asymptotic symbol error rate expressions with closed-form which are valid at high SNR. Presented numerical examples show that taking the first several terms of the series based error-rate expression results in improved error-rate estimation compared to the asymptotic error-rate. These error-rate expressions which can be evaluated efficiently indicate some significant insights into the characteristics of SC over generalized correlated Weibull fading channels. The accuracy of the analytical results is validated by the simulation results.

Key–Words: correlated fading channels; error-rate performance; selection combining; Weibull distribution

1 Introduction

Diversity reception for multi-branch receivers is an effective approach to combat the multipath fading over fading channels. Since Weibull distribution manifests good fit to the radio propagation with experimental results taken around 900 MHz, this distribution was recommended by the IEEE Vehicular Technology Committee for theoretical studies in radio propagation environments [1].

In order to mitigate the fading effect, the fading margin for the link budget analysis needs to be estimated, and spatial diversity receptions are employed. Although a large body of literature has been devoted to study the error-rate performance of various diversity receptions assuming independent Weibull fading [2]-[5], however, antennas at a receiver with multi-branch can be spatially correlated as they are insufficiently separated in practical wireless radio systems [6]. Therefore, the multivariate correlated statistic can be a powerful tool for the accurate performance estimation. For Weibull distribution with constant correlation, reference [7] developed the probability density function (pdf) and the cumulative density function (CDF) of multivariate Weibull distribution in terms of single-integral, but these representations are not applicable for more general correlated Weibull fading channels. The error probability of M-ary phase shift keying (M-PSK) for selection combining (SC) and equal gain combining (EGC) with two diversity branches in the presence of co-channel interference were investigated over the Weibull fading channels with constant correlation [8]. The authors in [9]-[11] also investigated the error-rate performance for dual-branch weibull distribution with constant correlation. For arbitrarily correlated Weibull distribution, reference [12] presented the formulations to investigate the performance for SC and maximum ration combining (MRC) with finite number of branches, but multiple integrations were carried out to calculate the error probability with high computational complexity. A popular multivariate Weibull model proposed in [13] can be generated using correlated Gaussian processes. Based on this model, the joint pdf, CDF, MGF, and the product moments were provided for multivariate Weibull distribution with constant and exponential correlation. However, the infinite series and multiple-integral representations of pdf presented in

[13] have significant complexity for a large number of branches, and precluded its application for such systems. In [14], the single-integral representations of the multivariate pdfs and CDFs for a generalized correlation structure were provided to analyze the outage performance. Although some auhtors have studied the BER performance over Weibull fading channels with independent branches or special correlated branches, to our best knowledge, for the generalized correlated Weibull fading channels with arbitrary number of diversity branches that can unify the aforementioned models, both the asymptotic and exact error-rate analysis for SC with closed-form expressions have not been reported.

The contributions of this paper are as follows. Firstly, we derive a single-integral pdf expression of the instantaneous SNR, and obtain the exact closed-form error-rate expressions for SC with coherent and modulation modulations. Then, in the regime of high SNR, closed-form asymptotic error-rate expressions for SC are developed over the generalized correlated Weibull fading channels with arbitrary number of diversity branches and non-identical fading parameters. Presented numerical results show that taking the first several terms of the series error-rate expressions can result in improved error-rate estimation. All these error-rate expressions can be evaluated efficiently using common mathematical software without resorting to the time-consuming Monte Carlo simulations.

The remainder of this paper is organized as follows. In Section 2, we present the system model and a brief review on the multivariate Weibull distribution with generalized correlation. In Section 3, using the single-integral representations of pdf and CDF of the Gaussian class Weibull distribution we derive the exact infinite series based error-rate expressions for coherent and noncoherent modulations. In Section 4, we provide the asymptotic error-rate with closed-form expressions. Numerical and simulation results are presented in Section 5. Finally, Section 6 concludes the paper.

2 System and Channel Fading Model

It is assumed that there are L diversity branches experiencing correlated and frequency nonselective fading at the receiver. After demodulation with a matched filter, the signal received at kth branch can be written as

$$y_k = w_k x + n_k, \quad k = 1, ..., L \tag{1}$$

where w_k is the Weibull complex fading coefficient, x is the transmitted signal with average energy E_s, and n_k is the complex Gaussian noise with variance N_0. The fading coefficients w_k's are correlated according to a specific correlation model.

Suppose U_k and V_k are, respectively, Gaussian inphase random variable (RV) and quadrature phase RV with mean zero and variance $1/2$. Then, let $X_k = \left(1 - \lambda_k^2\right)^{\frac{1}{2}} U_k + \lambda_k U_0$ and $Y_k = \left(1 - \lambda_k^2\right)^{\frac{1}{2}} V_k + \lambda_k V_0$, where $-1 < \lambda_k < 1$. It can be proved that X_k and Y_k are two independent Gaussian RVs with mean zero and variance. Then, the Weibull RV W_k with generalized correlation can be written as [14]

$$W_k = \left(\sigma_k \sqrt{X_k^2 + Y_k^2}\right)^{\frac{2}{\beta_k}} \tag{2}$$

where σ_k represents the power scaling factor. The positive fading parameter β_k models the fading severity of the kth branch. The corresponding power correlation coefficient of W_k^2 and W_i^2 can be derived from [13, eq. (15)] as

$$\varrho_{W_k^2 W_i^2} = \frac{\left(1 - \lambda_k^2 \lambda_i^2\right)^{m_i + m_k - 1} {}_2F_1\left(m_k, m_i; 1; \lambda_k^2 \lambda_i^2\right) - 1}{\sqrt{\frac{\Gamma\left(m_k + \frac{2}{\beta_k}\right)}{\Gamma^2(m_k)} - 1}\sqrt{\frac{\Gamma\left(m_i + \frac{2}{\beta_i}\right)}{\Gamma^2(m_i)} - 1}} \tag{3}$$

where $m_k = 1 + 2/\beta_k$, $m_i = 1 + 2/\beta_i$, and ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ denotes the generalized hypergeometric function [15, eq. (9.100)].

When $\beta_k = 2$, eq. (2) can be expressed as $W_k = \sigma_k \sqrt{X_k^2 + Y_k^2}$ which is a Rayleigh RV. In this case, the correlation coefficient between any two complex Gaussian RVs W_k and W_i for $k \neq i$ is given by $\rho = \lambda_k \lambda_i$. When $\beta_k = 1$, eq. (2) can be expressed as an exponential RV . Therefore, the correlated Rayleigh ($\beta_k = 2$) and exponential ($\beta_k = 1$) distributions are two special cases of the correlated Weibull distribution.

Without loss of generality, we normalize the unfaded branch SNR such that $E_s/N_0 = 1$. Using the single-integral form pdf of $\mathbf{W} = [W_1, W_2, ..., W_L]$ given by [14, eq. (16)], one can calculate the average SNR of the kth branch as

$$\overline{\gamma}_k = \mathbf{E}\left[W_k^2\right] = \sigma_k^{\frac{4}{\beta_k}} \Gamma\left(1 + \frac{2}{\beta_k}\right) \tag{4}$$

where $\mathbf{E}[\cdot]$ denotes the expectation operator.

3 Exact Error-rate Analysis

In this section, we develop an exact infinite series error-rate expression for SC with binary coherent modulation and noncoherent modulation. Moreover, we discuss the convergence of the error-rate expression based on an infinite series.

We assume the average SNR of kth branch is defined as $\overline{\gamma}_k = g_k\overline{\gamma}$, where g_k is the gain of kth branch w.r.t. the average SNR $\overline{\gamma}$, and $\overline{\gamma} = \left(\prod_{k=1}^{L} \bar{\gamma}_k\right)^{\frac{1}{L}}$. In SC, the branch with the highest instantaneous received SNR is selected for signal detection and the SNR at the reciever can be expressed as

$$\gamma_{\mathrm{SC}} \triangleq \max_{k=1,\ldots,L} \gamma_k. \tag{5}$$

Since γ_{SC} is the maximum of all the instantaneous SNR, the CDF of γ_{SC} can be calculated from

$$\begin{aligned} F_{\gamma_{\mathrm{SC}}}(\gamma) &= \Pr\left[|w_1|^2 \le \gamma, |w_2|^2 \le \gamma, \ldots, |w_L|^2 \le \gamma\right] \\ &= F_{\mathbf{W}}\left(\sqrt{\gamma}, \sqrt{\gamma}, \ldots, \sqrt{\gamma}\right). \end{aligned} \tag{6}$$

where $\Pr[\cdot]$ is the probability operation. Substituting the single-integral CDF of Weibull RVs given by [14, eq. (17)] into (6), the joint CDF of $\mathbf{W}$ can be expressed in single-integral form as

$$\begin{aligned} F_{\gamma_{\mathrm{SC}}}(\gamma) = &\int_0^\infty e^{-t} \prod_{k=1}^{L} \Bigg\{1 - \\ &Q_1\left(\sqrt{\frac{2\lambda_k^2 t}{1-\lambda_k^2}}, \sqrt{\frac{2p_k\gamma^{\frac{\beta_k}{2}}}{\overline{\gamma}^{\frac{\beta_k}{2}}}}\right)\Bigg\} \mathrm{d}t \end{aligned} \tag{7}$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q-function of order one, and $p_k = g_k^{-\frac{\beta_k}{2}}\left(1-\lambda_k^2\right)^{-1}\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)$. Using the infinite series form of generalized Marcum Q-function and the generalized Laguerre polynomial shown in [16], one can rewrite the joint CDF of multivariate Weibull distribution as

$$\begin{aligned} F_{\gamma_{SC}}(\gamma) = &\prod_{k=1}^{L} p_k \sum_{i=0}^{\infty} (-1)^i \int_0^\infty e^{-(1+\Lambda)t} \sum_{j_1+\cdots+j_L=i} \\ &\times \prod_{k=1}^{L} \frac{L_{j_k}^{(0)} p_k^{j_k}\left(\frac{\lambda_k^2 t}{1-\lambda_k^2}\right)}{\Gamma(j_k+2)} \mathrm{d}t \left(\frac{\gamma}{\overline{\gamma}}\right)^{\kappa} \end{aligned} \tag{8}$$

where $\kappa = \frac{B}{2} + \frac{1}{2}\sum_{k=1}^{L}\beta_k j_k$, and $L_i^{(\alpha)}(\cdot)$ denotes the generalized Laguerre polynomial given by [22, eq. (22.3.9)]. We define

$$\begin{aligned} \Psi_i &= (-1)^i \int_0^\infty e^{-(1+\Lambda)t} \prod_{k=1}^{L} \frac{L_{j_k}^{(0)}\left(\frac{\lambda_k^2 t}{1-\lambda_k^2}\right)}{\Gamma(i+2)} p_k^{j_k} \mathrm{d}t \\ &= (-1)^i \prod_{k=1}^{L} \frac{p_k^{j_k}}{\Gamma(j_k+2)} \sum_{l_1=0,\ldots l_L=0}^{j_1\cdots j_L} \\ &\times \frac{\Gamma\left(1+\sum_{k=1}^{L} l_k\right)}{(1+\Lambda)^{1+\sum_{k=1}^{L} l_k}} \prod_{k=1}^{L} \frac{(l_k+1)_{j_k-l_k}}{(j_k-l_k)!\,l_k!}\left(\frac{-\lambda_k^2}{1-\lambda_k^2}\right)^{l_k} \end{aligned} \tag{9}$$

where $\Lambda = \sum_{k=1}^{L} \frac{\lambda_k^2}{1-\lambda_k^2}$. Then, the CDF of γ_{SC} can be rewritten as

$$F_{\gamma_{\mathrm{SC}}}(\gamma) = \prod_{k=1}^{L} p_k \sum_{i=0}^{\infty} \sum_{j_1+\cdots+j_L=i} \Psi_i \left(\frac{\gamma}{\overline{\gamma}}\right)^{\frac{B}{2}+\frac{1}{2}\sum_{k=1}^{L}\beta_k j_k}. \tag{10}$$

Let $B = \sum_{k=1}^{L} \beta_k$, the corresponding pdf of γ_{SC} for SC can be obtained by differentiating (10) as

$$\begin{aligned} f_{\gamma_{\mathrm{SC}}}(\gamma) = &\prod_{k=1}^{L} p_k \sum_{i=0}^{\infty} \sum_{j_1+\cdots+j_L=i} \left(\frac{B}{2} + \frac{1}{2}\sum_{k=1}^{L}\beta_k j_k\right) \\ &\times \frac{1}{\overline{\gamma}} \Psi_i \left(\frac{\gamma}{\overline{\gamma}}\right)^{\frac{B}{2}+\frac{1}{2}\sum_{k=1}^{L}\beta_k j_k - 1}. \end{aligned} \tag{11}$$

3.1 Error-rate for binary coherent modulation

The conditional error probability for binary coherent modulations is given by $P_{e,c}(\gamma) = pQ\left(\sqrt{q\gamma}\right)$, where p and q can determine the underlying modulation format, e.g., binary shift keying (BPSK) for $p = 1$, and $q = 2$. Using an alternative expression for the Gaussian Q-function, one can obtain the average error-rate for BPSK as

$$\begin{aligned} P_e^c &= \int_0^\infty Q\left(\sqrt{q\gamma}\right) f_{\gamma_{\mathrm{SC}}}(\gamma)\, d\gamma \\ &= \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} F_{\gamma_{\mathrm{SC}}}\left(\frac{t^2}{q}\right) \mathrm{d}t. \end{aligned} \tag{12}$$

The infinite series form of error-rate for the SC can be evaluated by substituting (10) into (12) as

$$\begin{aligned} P_e^c = &\frac{1}{2\sqrt{\pi}} \prod_{k=1}^{L} p_k \sum_{i=0}^{\infty} \sum_{j_1+\cdots+j_L=i} \Psi_i \\ &\times \left(\frac{2}{q\overline{\gamma}}\right)^{\delta} \Gamma\left(\frac{B + \sum_{k=1}^{L}\beta_k j_k + 1}{2}\right). \end{aligned} \tag{13}$$

where $\delta = \frac{B}{2} + \frac{1}{2}\sum_{k=1}^{L}\beta_k j_k$.

At asymptotically high-SNR ($\overline{\gamma} \to \infty$), the error-rate given in (13) is dominated by the first term. The leading coefficient Ψ_0 can be simplified as

$$\Psi_0 = \frac{1}{\Lambda + 1}. \tag{14}$$

We define a correlation coefficient matrix as

$$\mathbf{M} = \begin{pmatrix} 1 & \lambda_1\lambda_2 & \cdots & \lambda_1\lambda_L \\ \lambda_1\lambda_2 & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1\lambda_L & \cdots & \cdots & 1 \end{pmatrix}, \tag{15}$$

and one can obtain the asymptotic error-rate by applying (14) as

$$P_e^{c-a} \approx \frac{2^{\frac{B}{2}-1}\Gamma\left(\frac{B+1}{2}\right)}{\sqrt{\pi}q^{\frac{B}{2}}\Delta} \prod_{k=1}^{L} \left[\frac{\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)}{\overline{\gamma}^{\frac{\beta_k}{2}}} \right]. \tag{16}$$

where $\Delta = \left(1 + \sum_{k=1}^{L} \frac{\lambda_k^2}{1-\lambda_k^2}\right) \prod_{k=1}^{L} \left(1-\lambda_k^2\right)$ represents the determinant of $\mathbf{M}$.

3.2 Error-rate for binary noncoherent modulation

For binary noncoherent modulations, the conditional error-rate is defined as $P_{e,nc}(\gamma) = p\exp(-q\gamma)$ [19], where p and q can also denotes the noncoherent modulation form. For instance, $p = q = 1/2$ for binary noncoherent frequency shift keying (BNCFSK), $p = 1/2$ and $q = 1$ for binary differential phase shift keying (BDPSK).

Substituting (10) into $P_{e,nc}(\gamma)$, one can obtain the infinite series form error-rate expression for binary noncoherent modulations as

$$P_e^{nc} = \frac{1}{2}\prod_{k=1}^{L} p_k \sum_{i=0}^{\infty} \sum_{j_1+\cdots+j_L=i} \frac{\Psi_i \Gamma(\delta+1)}{(q\overline{\gamma})^{\delta}} \tag{17}$$

Similarly, as $\overline{\gamma} \to \infty$ the error-rate of coherent modulations can be approximated to

$$P_e^{nc-a} \approx \frac{\Gamma\left(\frac{B}{2}+1\right)}{2q^{\frac{B}{2}}\Delta} \prod_{k=1}^{L} \left[\frac{\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)}{\overline{\gamma}^{\frac{\beta_k}{2}}} \right]. \tag{18}$$

3.3 Convergence analysis

We now consider the convergence of error-rate in (10) for a sufficiently large value of $\overline{\gamma}$. The single-integral form of coefficient Ψ_i is expressed as

$$\Psi_i = (-1)^i \int_0^{\infty} \exp\left(-(\Lambda+1)t\right) \times \sum_{j_1+\cdots+j_L=i} \prod_{k=1}^{L} \frac{L_{j_k}^{(0)}\left(\frac{\lambda_k^2 t}{1-\lambda_k^2}\right)}{\Gamma(2+j_k)} p_k^{j_k} \mathrm{d}t. \tag{19}$$

Using the inequality $\left|L_{j_k}^{(0)}\left(\frac{\lambda_k^2 t}{1-\lambda_k^2}\right)\right| \le \exp\left(\frac{\lambda_k^2 t}{2(1-\lambda_k^2)}\right)$ [22, eq. (22.14.13)], one obtains an upper bound of Ψ_i as

$$\begin{aligned} |\Psi_i| &\le \frac{2}{\Lambda+2} \sum_{j_1+\ldots+j_L=i} \prod_{k=1}^{L} \frac{p_k^{j_k}}{\Gamma(j_k+2)} \\ &\le \frac{2}{\Lambda+2} \sum_{j_1+\ldots+j_L=i} \prod_{k=1}^{L} \frac{p_k^{j_k}}{\Gamma(j_k+1)} \\ &= \frac{2}{\Lambda+2} \frac{\left(\sum_{k=1}^{L} p_k\right)^i}{\Gamma(i+1)}. \end{aligned} \tag{20}$$

We can infer from (13) that the convergence region for error-rate depends on $\overline{\gamma}$. However, finding the region of convergence of the error-rate directly from (13) is complicated because of the existence of Ψ_i with in the nested summations. For the sake of simplicity, we can find a subset of convergence regions by using the upper bound of Ψ_i. It is assumed that $\beta_1 \leqslant \cdots \leqslant \beta_L$. Then, an upper bound of error-rate in (13) for coherent modulations can be written as

$$\begin{aligned} P_e^{c-u} \leqslant& \frac{1}{\sqrt{\pi}(\Lambda+2)} \left(\frac{2}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}L} \prod_{k=1}^{L} p_k \\ &\times \sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{\beta_L L+\beta_L i+1}{2}\right)}{\Gamma(i+1)} \left(\frac{2}{q\overline{\gamma}}\right)^{\frac{\beta_L i}{2}} \eta^i \end{aligned} \tag{21}$$

where $\eta = \sum_{k=1}^{L} p_k$, and the infinite series based error-rate expression converges when $\left(\frac{2}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}} \eta < 1$.

In the same manner, the upper bound of error-rate in (17) for noncoherent modulations can be obtained as

$$\begin{aligned} P_e^{nc-u} \leqslant& \frac{1}{\sqrt{\pi}(\Lambda+2)} \left(\frac{2}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}L} \prod_{k=1}^{L} p_k \\ &\times \sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{\beta_L L+\beta_L i+1}{2}\right)}{\Gamma(i+1)} \left(\frac{1}{q\overline{\gamma}}\right)^{\frac{\beta_L i}{2}} \eta^i \end{aligned} \tag{22}$$

and (22) converges when $\left(\frac{1}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}} \eta < 1$.

We can conclude that the requirement $\left(\frac{2}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}} \eta < 1$ or $\left(\frac{1}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}} \eta < 1$ is satisfied provided that $\overline{\gamma}$ is sufficiently high. For instance, for branches experiencing equally correlated Rayleigh fading, equal average branch SNR, and BPSK coherent modulation, the convergence is achieved when $\overline{\gamma} > \frac{L}{1-\sqrt{\varrho}}$, where $\varrho = \lambda^4$ denotes the corresponding correlation coefficient between the Weibull RVs in (3). We can also infer that the minimum required $\overline{\gamma}$ to guarantee the convergence of error-rate expressions will increase as the number of branches and correlation coefficient increase which is verified by numerical results. Finally, when the value of fading parameter β increases, the infinite series expression in (13) will converge at a higher value of $\overline{\gamma}$. This fact will be verified by numerical results in Section 5.

4 Asymptotic Error Rate Analysis

To the best of our knowledge, the asymptotic error probability for different diversity receptions over generalized correlated Weibull fading channels has not been presented in the literature. In what follows, we investigate the asymptotic symbol error rate (SER) for SC in the high SNR regime.

Let γ be the instantaneous SNR at the receiver. The characteristics of the pdf of γ near the origin determine the error rate performance in the regime of high SNR [17]-[20]. As the exponential function and the modified Bessel function of the first kind are included in the single-integral pdf of $\mathbf{W} = [W_1, W_2, ..., W_L]$ given by [14, eq. (16)], using the first-order Taylor series expansion of the exponential function near the origin, i.e., $\lim_{x\to 0} \exp(x) = 1 + o(1)$, where $o(\cdot)$ denotes an infinitely small quantity, and the series expansion of the zeroth order modified Bessel function of the first kind near the origin, i.e., $\lim_{x\to 0} I_0(x) = 1 + o(1)$, one can derive the joint pdf of $\mathbf{W}$ as

$$f_{\mathbf{W}}(w_1, ..., w_L) = \frac{1}{\Delta}\prod_{k=1}^{L}\frac{\beta_k}{\sigma_k^2}\left\{w_k^{\beta_k-1} + o\left(w_k^{\beta_k-1}\right)\right\}. \quad (23)$$

From (23), we conclude that in the regime of high SNR, the the joint pdf of $\mathbf{W}$ for correlated branches can be expressed by scaling the pdf of $\mathbf{W}$ for independent branches with a factor Δ.

For the Weibull CDF, we utilize the series expansion of the first order Marcum Q-function [21, eq. (8)] included in the single-integral CDF of $\mathbf{W}$ given by [14, eq. (16)] near the origin and derive an approximate Weibull CDF of $\mathbf{W}$ near the origin as

$$\begin{aligned} F_{\mathbf{W}}(w_1, ..., w_L) &= \frac{1}{\Delta\prod\limits_{k=1}^{L}\sigma_k^2}\prod_{k=1}^{L}\left\{w_k^{\beta_k} + o\left(w_k^{\beta_k}\right)\right\} \\ &= \frac{1}{\Delta}\prod_{k=1}^{L} g_k(w_k) \end{aligned} \quad (24)$$

where $g_k(w_k) = \left(w_k^{\beta_k} + o\left(w_k^{\beta_k}\right)\right)/\sigma_k^2$. We observe that the approximate CDF in (24) has a product form and it resembles the joint CDF of independent RVs. Thus, we may infer that the asymptotic error probability over the correlated Weibull channels at high SNR tends to have similar behaviour as that of the independent Weibull channels.

As the characteristics of the pdf of γ near the origin can determine the error probability when the average branch SNR approaches to infinity, the pdf of γ is calculated as $f(\gamma) = \alpha\gamma^t + o(\gamma^t)$, where α and t are the shape parameters related to the coding gain and diversity gain. The corresponding asymptotic SER for coherent modulation and noncoherent modulation are given by [18]

$$P_{e-c}^{asym} = \frac{2^t\alpha\Gamma\left(t+\frac{3}{2}\right)p}{\sqrt{\pi}(t+1)q^{t+1}} + o\left(\overline{\gamma}^{t+1}\right) \quad (25)$$

and

$$P_{e-nc}^{asym} = \frac{\alpha\Gamma(t+1)p}{q^{t+1}} + o\left(\overline{\gamma}^{t+1}\right), \quad (26)$$

respectively. Therefore, it is straightforward to estimate the asymptotic SER for different modulation once the values of shape parameters α and t are known.

Substituting (24) and (4) into (6), one obtains the first-order approximation of the CDF of γ_{SC} as

$$F_{\gamma_{\mathrm{SC}}}(\gamma) = \frac{1}{\Delta}\prod_{k=1}^{L}\left[\frac{\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)}{\overline{\gamma}_k^{\frac{\beta_k}{2}}}\right]\gamma^{\frac{B}{2}} + o\left(\gamma^{\frac{B}{2}}\right) \quad (27)$$

where the pdf of γ_{SC} at high SNR can be expressed by taking a derivative with respect to (27) as

$$f_{\gamma_{SC}} = \alpha\gamma^t + o\left(\gamma^t\right) \quad (28)$$

where

$$\alpha = \frac{B}{2\Delta}\prod_{k=1}^{L}\left[\frac{\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)}{\overline{\gamma}_k^{\frac{\beta_k}{2}}}\right] \quad (29)$$

and

$$t = \frac{\sum_{k=1}^{L}\beta_k}{2} - 1. \quad (30)$$

Therefore, substituting (29) and (30) to (25), one obtains the asymptotic SER with closed-form expression for coherent modulations as

$$P_{e-c}^{asym} = \frac{2^{\frac{B}{2}-1}\Gamma\left(\frac{B+1}{2}\right)p}{\sqrt{\pi}q^{\frac{B}{2}}\Delta}\prod_{k=1}^{L}\left[\frac{\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)}{\overline{\gamma}_k^{\frac{\beta_k}{2}}}\right]. \quad (31)$$

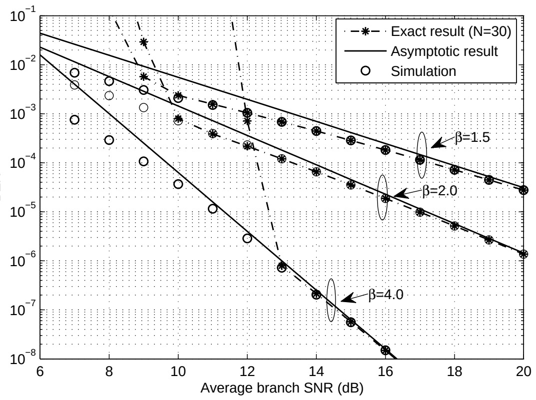

Figure 1: Exact error-rate with truncation (N=30) and asymptotic error-rate for coherent BPSK with triple-branch SC over generalized correlated Weibull fading channels ($[\lambda_1, \lambda_2, \lambda_3] = [0.6, 0.4, 0.9]$).

Simliarly, the asymptotic SER for coherent modulations is written in closed-form as

$$P_{e-nc}^{asym} = \frac{\Gamma\left(\frac{B}{2}+1\right)p}{q^{\frac{B}{2}}\Delta} \prod_{k=1}^{L} \left[\frac{\Gamma^{\frac{\beta_k}{2}}\left(1+\frac{2}{\beta_k}\right)}{\overline{\gamma}^{\frac{\beta_k}{2}}} \right]. \qquad (32)$$

We can observe that (31) and (32) match the approximate error rate result in (16) for $p = 1$ and (18) for $p = 1/2$, respectively. Besides, it is found that the asymptotic error-rate can be expressed by scaling the error probability over independent branches with a factor Δ.

5 Numerical Results

In this section, using the analytical expressions developed above, we present selected numerical results on bit-error rate (BER) performance. For simplicity, we assume that each branch has identical average S-NR with equal fading parameter. For the generalized correlated Weibull fading channels, let $[\lambda_1, \lambda_2, \lambda_3] = [0.6, 0.4, 0.9]$ denote the correlation parameters for triple diversity branches.

We have discussed in Section 3 that the infinite series based error-rate does not converge over the entire SNR region, but it is accurate enough with large SNR values, and is meaningless with small SNR values. This is confirmed by the error probability curves shown in Figures 1 and 2 for coherent BPSK modulation and noncoherent BDPSK modulation with different values of fading parameter ($\beta = 1.5, 2.0$ and 4.0), respectively. From the two figures, we observe that the infinite series based exact error-rate evaluated with $N = 30$ terms match the simulated error-rate at high SNR. We further observe that for $\beta = 1.5, 3.0$ and 4.0, the series error-rate expression for BPSK with coherent modulation starts to converge at average branch SNR values of 10 dB, 11 dB and 13 dB, respectively, which are in agreement with the convergence requirement, $\left(\frac{2}{q\overline{\gamma}}\right)^{\frac{\beta_L}{2}} \eta < 1$. Furthermore, as the fading parameter β increases, the asymptotic BERs tend to agree with the exact analysis at a lower value of $\overline{\gamma}$.

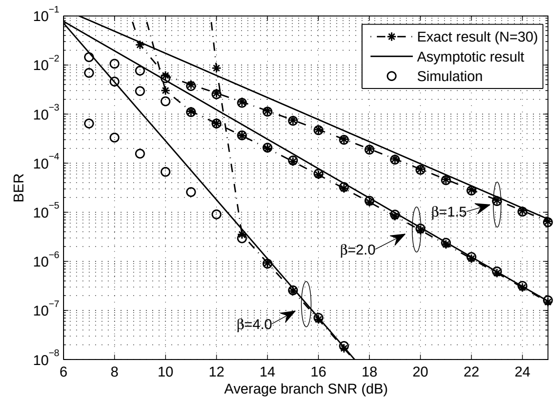

Figure 2: Exact error-rate with truncation (N=30) and asymptotic error-rate for noncoherent BDPSK with triple-branch SC over generalized correlated Weibull fading channels ($[\lambda_1, \lambda_2, \lambda_3] = [0.6, 0.4, 0.9]$).

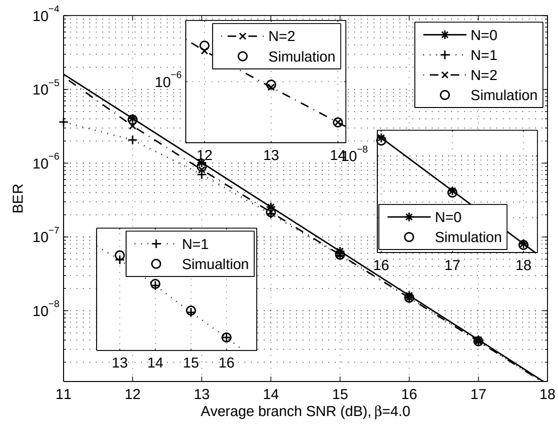

Figure 3: Approximate and simulated BERs of coherent BPSK with triple-branch SC over generalized correlated Weibull fading channels ($[\lambda_1, \lambda_2, \lambda_3] = [0.6, 0.4, 0.9]$) when $\beta = 4.0$.

The error-rate expression for SC can achieve high accuracy by adding a few extra terms in series expression at the convergence region in terms of average branch SNR, which is shown in Figure 3. It can been seen from Figure 3 that as the number of terms in series error-rate expression increases, error-rate performance converges at a lower average branch SNR. For example, the error-rate curve with $N = 0$ begins to agree with the simulation result at $\overline{\gamma} = 18$ dB, while the error-rate curves with $N = 1$ and $N = 2$ begin to agree with their simulated counterparts at 16 dB and 14 dB, respectively. Therefore, using more terms in the series based error-rate expression one can extend the SNR region in which the developed error-rate expression converges.

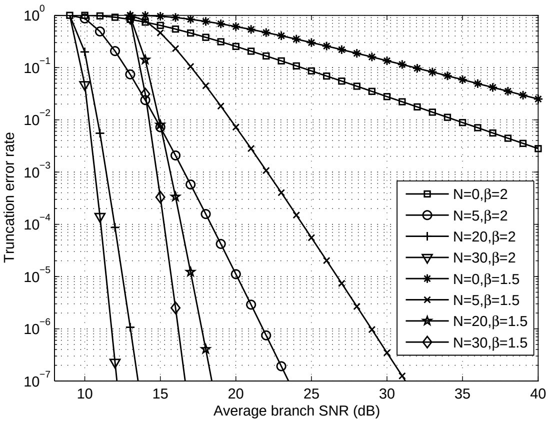

Figure 4: Truncation error rate for coherent modulations using different number of terms of the infinite series error-rate expression of SC over generalized correlated Weibull fading channels ($[\lambda_1, \lambda_2, \lambda_3] = [0.6, 0.4, 0.9]$).

Figure 4 plots the relative truncation error rate of series based error-rate expression of SC versus average branch SNR for various values of fading parameter β. As expected, the truncation error tends to diminish as average branch SNR increases. Let N be the number of terms used in the series error-rate expression. For a given value of N, the truncation error decreases as the value of β increases. One can also predict the accuracy of truncation error using Figure 4. For instance, to guarantee a truncation error less than 1%, it requires $N = 0$ at 36 dB, and $N = 5$ at 15.2 dB. Although the estimated error-rate is more accurate if more terms are used in evaluating the error-rate expression, the computational complexity will also increase.

Without loss of generality, we consider a correlation model with equal average branch SNRs and $\beta_1 = \cdots = \beta_L = 1.5$ in the multivariate Weibull fading channels. The bit-error probability versus SNR is plotted in Figure 5 for dual-branch and four-branch SC with uncorrelated and correlated cases ($\varrho = 0.13$). It is observed that the correlative case, as expected, degrades the error-rate performance. Nevertheless,

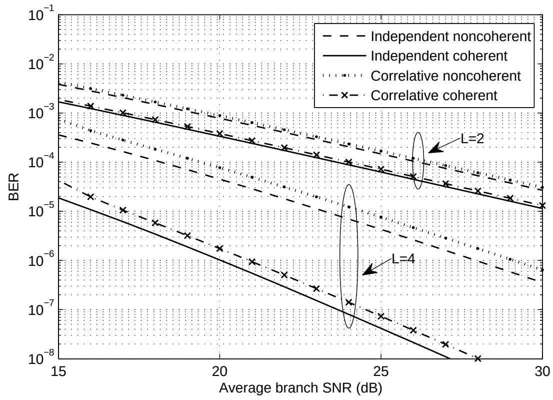

Figure 5: The error-rate against SNR in the case of $L = 2$ and $L = 4$ with uncorrelated and correlated ($\varrho = 0.13$) diversity branches for MRC when $\beta = 1.5$.

the error-rate difference for coherent modulation and noncoherent modulation between the the correlated branches and uncorrelated branches tends to be larger as the number of branches increases under the same condition.

In order to study the power correlation effect on diversity reception over correlated fading channels, we compare SC with MRC and EGC in terms of error-rate. Following the method of changing the CDF of γ_{SC} into infinite series form in Section 3, we can rewrite the joint pdf of $\mathbf{W}$ as

$$f_{\mathbf{W}}(w_1, ..., w_L) = \prod_{k=1}^{L} \frac{\beta_k}{\Omega_k^2} \sum_{i=0}^{\infty} \frac{\Gamma(1+i)}{(1+\Lambda)^{1+i}} \sum_{j_1+\cdots j_L=i} g_{\mathbf{W}} \quad (33)$$

where

$$g_{\mathbf{W}} = \prod_{k=1}^{L} \frac{\sigma_k^{2j_k} \lambda_k^{2j_k}}{\Gamma^2(j_k+1)\,\Omega_k^{4j_k}} w_k^{\beta_k j_k + \beta_k - 1} \exp\left(-\frac{w_k^{\beta_k}}{\Omega_k^2}\right).$$

Since the MGF of γ_{MRC} and the MGF of w_{EGC} for EGC are defined as

$$M_{\gamma_{\text{MRC}}}(s) = \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{L-fold} \exp\left(-s \sum\nolimits_{k=1}^{L} w_k^2\right) \times f_{\mathbf{W}}(w_1, \ldots, w_L)\, \mathrm{d}w_1 ... \mathrm{d}w_L, \quad (34)$$

and

$$M_{w_{\text{EGC}}}(s) = \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{L-fold} \exp\left(\frac{s}{\sqrt{L}} \sum\nolimits_{k=1}^{L} w_k\right) \times f_{\mathbf{W}}(w_1, \ldots, w_L)\, \mathrm{d}w_1 ... \mathrm{d}w_L, \quad (35)$$

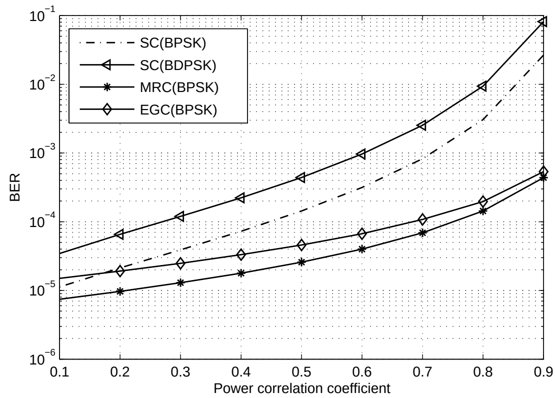

Figure 6: Exact error-rate for diversity receptions versus power correlation parameter with triple branches when $\beta = 2.0$, and $\overline{\gamma} = 15$ dB.

resectively, averaging over the pdf of $\mathbf{W}$ given by (33), we can obtain $M_{\gamma_{\mathrm{MRC}}}(s)$ and $M_{w_{\mathrm{EGC}}}(s)$ which can be used to eveluate the error-rate performance for MRC and EGC. It is assumed that the branches are equally correlated with power correlation coefficient ϱ defined in (3), and the fading parameter $\beta = 2.0$. Figure 6 indicates that the power correlation parameter affects the error-rate performance of different diversity receptions for a given value of SNR. Clearly, Fig. 6 shows that error-rate increases as ϱ increases, and MRC outperforms EGC and SC in terms of error-rate performance over the entire region of power correlation coefficient. It is also important to point out that the error-rate performance of SC with coherent or noncoherent modulation is more sensitive to the value of ϱ than that of MRC and EGC, especially when the value of ϱ is high.

6 Conclusion

We studied the error probability of SC over generalized correlated Weibull fading channels with arbitrary number of diversity branches. We derived an exact infinite series based error-rate expressions for SC with coherent and noncoherent modulations. Presented numerical examples showed that taking the first several terms of the series error-rate expression results in improved error-rate estimation. We also developed asymptotic error probability in terms of closed-form for SC. These error-rate expressions can be used to estimate error-rate readily without resorting to the time-consuming Monte Carlo simulations.

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References:

[1] N. S. Adawi, H. L. Bertoni, J. R. Child, W. A. Daniel, J. E. Dettra, R. P. Eckert, E. H. Flath Jr., and R. T. Forrest, “Coverage prediction for mobile radio systems operating in the 800/900 MHz frequency range,” *IEEE Trans. Veh. Technol.*, vol. 37, no. 1, Feb. 1988, pp. 3–72.

[2] M. H. Ismail and M. M. Matalgah, “Exact and approximate error-rate analysis of BPSK in Weibull fading with cochannel interference,” *IET. Commun.*, vol. 1, no. 2, Apr. 2007, pp. 203–208.

[3] A. M. Magableh and M. M. Matalgah, “Error probability performance and ergodic capacity of l-branch switched and examine combining in weibull fading channels,” *IET. Commun.*, vol. 5, no. 9, Jul. 2011, pp. 1173–1181.

[4] B. S. Bithas, G. K. Karagiannidis, N. C. Sagias, P. T. Mathiopoulos, S. A. Kotsopoulos, and G. E. Corazza, “Performance analysis of a class of GSC receivers over nonidentical Weibull fading channels,” *IEEE Trans. Veh. Technol*, vol. 54, no. 6, Oct. 2006, pp. 1963–1970.

[5] P. R. Sahu and A. K. Chaturvedi, “Performance analysis of predetection EGC receiver in Weibull fading channel,” *Electron. Lett.*, vol. 41, no. 2, Jan. 2005, pp. 47–48.

[6] S. Liu, X. Song, J. Cheng, and N. C. Beaulieu. “Asymptotic analysis of multi-branch EGC and SC over equally correlated Rician channels,” *Wireless Communications and Mobile Computing*, vol. 15, no. 5, May 2015, pp. 812–822.

[7] K. T. Hemachandra and N. C. Beaulieu, “New representations for the Gaussian class multivariate Weibull distribution with constant correlation and applications,” *IEEE Trans. Commun.*, vol. 59, no. 10, Oct. 2011, pp. 2648–2653.

[8] M. H. Ismail and M. M. Matalgah, “Bit error rate analysis of diversity M-phase-shift keying receivers in Weibull fading with cochannel interference,” *IET Commun.*, vol. 4, no. 1, Jan. 2010, pp. 13–25.

[9] N. C. Sagias, G. K. Karagiannidis, D. A. Zogas, P. T. Mathiopoulos, and G. S. Tombras, “Performance analysis of dual selection diversity in

correlated Weibull fading channels," *IEEE Trans. Commu.*, vol. 52, no. 7, Jul. 2004, pp. 1063–1067.

[10] H. Xiao, Z. Nie, and S. Yang, "Channel capacity and digital modulation schemes in correlated Weibull fading channels with nonidentical statistics," *J. of Syst. Eng. and Electron.*, vol. 18, no. 2, Jul. 2004, pp. 205–209.

[11] M. Č. Stefanović, D. L. Drača, A. S. Panajotović, and N. M. Sekulović, "Performance analysis of system with L-branch selection combining over correlated Weibull fading channels in the presence of cochannel interference," *International Journal of Communication Systems*, vol. 23, no. 2, Feb. 2010, pp. 139–150.

[12] Z. Papadimitriou, P. Mathiopoulos, and N. C. Sagias, "The trivariate and quadrivariate Weibull fading distributions with arbitrary correlation and their applications to diversity reception ," *IEEE Trans. Commun.*, vol. 57, no. 11, Nov. 2009, pp. 3230–3234.

[13] N. C. Sagias and G. Karagiannidis, "Gaussian class multivariate weibull distributions: theory and applications in fading channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, Oct. 2005, pp. 3608–3619.

[14] N. C. Beaulieu and K. T. Hemachandra, "Novel simple representations for Gaussian class multivariate distributions with generalized correlation," *IEEE Trans. Inf. Theory*, vol. 57, no. 12, Dec. 2011, pp. 8072–8083.

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Seires and Products*, 6th ed. New York: Academic, 2000.

[16] J. Schlenker, J. Cheng, and R. Schober, " Improving and bounding asymptotic approximations for diversity combiners in correlated generalized Rician fading," *IEEE Trans. Wirel. Commun.*, vol. 13, no. 2, Feb. 2014, pp. 736–748.

[17] X. G. Song, J. Cheng, and N. C. Beaulieu, "Asymptotic analysis of different multibranch diversity receivers with arbitrarily correlated Rician channels," *IEEE Trans. Wirel. Commun.*, vol. 13, no. 10, Oct. 2014, pp. 5676–5689.

[18] Z. Wang and G. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vo. 58, no. 5, Aug. 2003, pp. 1351–1355.

[19] X. Li and J. Cheng, "Asymptotic error rate analysis of selection combining on generalized correlated Nakagami-*m* channels," *IEEE Trans. Commun.*, vol. 60, Jul. 2012, pp. 1765–1771.

[20] S. Liu, J. Cheng, and N. C. Beaulieu, "Asymptotic error analysis of diversity schemes on arbitrarily correlated Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 59, no. 10, Oct. 2011, pp. 2648–2653.

[21] S. Andras, A. Baricz, and Y. Sun, "The generalized Marcum *Q*-function: an orthogonal polynomial approach," *Acta Universitatis Sapientiae Mathematica*, vol. 3, pp. 60-76, Feb. 2011.

[22] M. Abramowitz and I. A. Stegun, Eds., Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 10th ed. New York: Dover Publications, 1972.