Two new algorithms of signal processing in the communication system with Information Feedback

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Abstract: - The signal processing in the receivers of both channels is carried out with elementary signals. The direct channel receiver has memory. The threshold control in the feedback channel receiver is the main goal of the present article. The first algorithm on the criterion of minimum error probability without any restriction is investigated. The result is the set of thresholds changed in the repetitions. The second algorithm is investigated when the consumption of energy is introduced. This problem is solved on the method of the conditional optimization. Once again, the set of thresholds is obtained. The effectiveness of the mentioned two new algorithms is compared with the simple algorithm, when thresholds do not change with the number of repetition.

Key-Words: - Communication system with information feedback, error probability.

1 Introduction

The first wave of the interest to the communication systems with feedback channel is related to the 60's of the twenty century. One of the main applications of these systems is the satellite communications. Since of the channel communication is quite susceptible to interference, noises, etc., the investigations of such kind of communication systems is focused on the reliable transmission and retransmission of information. There are two very important systems within these communication systems. The system with Automatic Retransmission reQuest (ARQ) and the communication systems with information feedback (CSIF). See for instance [1] – [4]. Now the quantity of publications devoted to such type of systems is increased. We mention here some publications related to the CSIF. There are some books [5] - [7], some patents [8] - [15] and some articles [16] - [21] devoted to different aspects of theory and practice of CSIF. In the present paper we use practically the same methodology [2], [3], [19] - [21].

It is necessary to mention one important fact: the signal processing algorithms in the direct and feedback channel receivers do not involve memory operations. It means the information about previous signals coming in umbral devices is giving up. In the present paper we investigate another case when the mentioned information is used. But it is necessary to take into account the different situation in functioning of receivers: during the repetitions the same signals are coming in the entrance of the receiver of the direct channel; in the entrance of the receiver of the feedback channel different signals can be received, when there is an error in the receiver of the direct channel. This is the reason that the positive effect of signal processing with memory can be only existed in the receiver of the direct channel. Below we investigate the efficiency of CSIF with the memory in the receiver of the direct channel. The present paper deals with the investigation of a simple variant of CSIF when elementary binary signals translate in the both channels.

2 Description of the system

The elementary information symbol $a_k\left(k=1,2\right)$ modulates the deterministic signal $s_{dk}\left(t\right)$ (here the below index d means the direct channel), r is the maxima number of a possible cycle). In the direct channel this signal is mixed with the white noise $n_d\left(t\right)$ and the sum

$$\xi_d^{(s)}\left(t\right)=\theta s_{d1}\left(t\right)+\left(1-\theta\right)s_{d2}\left(t\right)+n_d\left(t\right) \quad (1)$$

is coming in the entrance of the receiver of the direct channel. The random binary parameter θ must be determined in the receiver. If $\theta=1$, the receiver accepts the *preliminary* decision that the signal $s_{d1}\left(t\right)$ (or the symbol a_1) is received. If $\theta=0$, the receiver accepts the *preliminary* decision that the signal $s_{d2}(t)$ (or the symbol a_2) is received. In the both cases the corresponding signal $s_{fi}(t)(i=1;2)$ must be sent in the feedback channel. For the sake of simplicity we suppose that a priory probabilities of symbols a_1 and a_2 are equals. Also, we suppose that the signals $s_{d1}(t)$ and $s_{d2}(t)$ are antipodal. In this case the probability of error is determined by the formula:

$$q_d = 1-\Phi\left(z_d\right) \tag{2}$$

where

$$\Phi\left(z_d\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z_d}\exp\left(-\frac{x^2}{2}\right)dx \tag{3}$$

is the probability integral,

$$z_d = \sqrt{\frac{2E_d}{N_{0d}}} \tag{4}$$

is the relation of the signal to noise in the entrance of the umbral device of the receiver, E_d is the energy of the signal $s_{dk}(t)(k=1;2)$, $0.5N_{0d}$ is the spectral density $[-\infty \le \omega \le +\infty]$ of white noise $n_d(t)$. The formulas (2) - (4) do not depend on the number of cycle (s). They are valid for the zero umbral and for the case of the system *without the memory* in the receiver of the direct channel. It means that the random value in the input of the umbral devise is eliminated after the *preliminary* decision.

When the receiver of the direct channel has the memory, then all parameters in the formulas (2) – (4) depend on the number of cycle (s):

$$q_d^{(s)} = 1-\Phi\left(z_d^{(s)}\right) \tag{5}$$

where

$$z_d^{(s)} = \sqrt{\frac{2sE_d}{N_{0d}}} = z_d\sqrt{s} \tag{6}$$

The information about the received symbol $a_k(k=1;2)$ is kept up in a special memory and does not give to a recipient. This received symbol is sent by another signal $s_{fk}(t)(k=1;2)$ in the feedback channel. There is a white noise $n_f(t)$ with its spectral density $0.5N_{0f}$. In the entrance of the receiver of the feedback channel is coming the sum

$$\xi_f(t) = \theta s_{f1}(t)+(1-\theta)s_{f2}(t)+n_f(t) \tag{7}$$

In (5) all designations change index "d" with "f". Here it is necessary to emphasize one fact: the umbral $\beta^{(s)}$ in the receiver of the feedback channel places a grand role in the effectiveness of the system under consideration (see for instance [13, 19]. Generally, one can write the error probability in the feedback channel in the form:

$$q_f^{(s)} = 1-\Phi\left(z_f \pm \beta^{(s)}\right) \tag{8}$$

where

$$z_f = \sqrt{\frac{2E_f}{N_{0f}}} \tag{9}$$

The sign of the umbral $\beta^{(s)}$ is determined by the symbol $a_k(k=1;2)$ which is sent in the direct channel. The value of this umbral cannot be chosen on the base of the problem of the error minimization in the feedback channel exclusively. This umbral must be determined in the problem of global optimization of a functionary of the complete system.

The transmission part involves the special comparison scheme. If the symbol $a_k(k=1;2)$ has been sent in the direct channel and the same symbol has been received in the receiver of the feedback channel then the comparison scheme accepts the decision that the symbol a_k has been correctly received in the receiver part. Then the symbol kept in the memory must be given to the recipient and the transmission part begins to send the next symbol in the direct channel. If two different symbols came in inputs of the comparison scheme then this scheme elaborates a special pilot signal which informs the receiver part about incorrect reception of the signal $s_{dk}^{(s)}(t)(k=1;2)$. The symbol kept in the memory does not send to the recipient and the transmission part begins to send the same symbol in the next cycle $(s+1)$. The system can have a limited number r of the repetitions or an infinite one $(r=\infty)$. In the case $s=r$ it is not necessary to send the signal in the feedback channel and the umbral $\beta^{(r)} = \pm\infty$ correspondently to the symbol a_k sent in the direct channel.

3 General formulas

The system under consideration can be descripted by two main statistic characteristics: the error probability of the symbol transmission q; the average normalized energetic consumptions for the symbol transmission z^2. Let us describe the cycle(s):

The probability of the incorrect reception of the symbol (or the error probability) is:

$$q^{(s)} = q_d^{(s)} q_f^{(s)} = \left[1-\Phi\left(z_d^{(s)}\right)\right]\left[1-\Phi\left(z_f^{(s)} \pm \beta^{(s)}\right)\right] \quad (10)$$

The probability of the correct reception of the symbol is:

$$p^{(s)} = p_d^{(s)} p_f^{(s)} = \left[\Phi\left(z_d^{(s)}\right)\right]\left[\Phi\left(z_f^{(s)} \mp \beta^{(s)}\right)\right] \quad (11)$$

The probability of the repetition of the symbol is:

$$\begin{aligned} u^{(s)} &= p_d^{(s)} q_f^{(s)} + q_d^{(s)} p_f^{(s)} = \\ &= \left[\Phi\left(z_d^{(s)}\right)\right]\left[1-\Phi\left(z_f^{(s)} \pm \beta^{(s)}\right)\right] + \\ &+ \left[1-\Phi\left(z_d^{(s)}\right)\right]\left[\Phi\left(z_f^{(s)} \mp \beta^{(s)}\right)\right] \end{aligned} \quad (12)$$

When $s = r$ the error probability is

$$q^{(r)} = q_d^{(r)} = \left[1-\Phi\left(z_d^{(r)}\right)\right] \quad (13)$$

Now let us determine the principal characteristics of CSIF. The probability of the error is:

$$q = \sum_{s=1}^{r} q^{(s)} \prod_{i=0}^{s-1} u^{(i)} \quad (14)$$

The average energetic consumptions for the symbol transmission is:

$$E = \sum_{s=1}^{r} E_d \prod_{i=0}^{s-1} u^{(i)} + \sum_{s=1}^{r-1} E_f \prod_{i=0}^{s-1} u^{(i)} \quad (15)$$

where $u^{(0)} = 1$.

Let us assume that white noises in the both channels are equals, i.e. $n_d(t) = n_f(t) = n(t)$ with the same spectral density $N_0/2$. This is not a restriction, because below one can change the parameters normalized z_d and z_f arbitrary. We divide (15) by $N_0/2$, then

$$z^2 = \frac{2E}{N_0} = z_d^2 \sum_{s=1}^{r} \prod_{i=0}^{s-1} u^{(i)} + z_f^2 \sum_{s=1}^{r-1} \prod_{i=0}^{s-1} u^{(i)} \quad (16)$$

The effectiveness of SCIF is described by the error probability q and the energetic normalized consumptions z^2. In the other words we have to use couples of formulas (14), (16).

4 The first algorithm of the optimization problem

From (14) and (16), one can see the both principal characteristics q and z depend on some parameters, namely: $z_d^2, z_f^2, \beta^{(1)}, \beta^{(2)}, ..., \beta^{(r-1)}$. If we vary these parameters, one can change a regime of system functioning. The set of parameters which determines minimum of the error probability q is called optimal. The optimization problem is connected with a calculation of the extremum of the expression (14). For the sake of simplicity we suppose that parameters z_d^2 and z_f^2 are fixed and known.

We equal to zero the partial derivative with $\beta^{(s)}$, i.e.

$$\frac{\partial q}{\partial \beta^{(s)}} = 0 \quad (17)$$

The calculation of (17) is following.

One can write:

$$\begin{aligned} &\frac{\partial q^{(s)}}{\partial \beta^{(s)}} + q^{(s+1)} \frac{\partial u^{(s)}}{\partial \beta^{(s)}} + q^{(s+2)} \frac{\partial u^{(s)}}{\partial \beta^{(s)}} u^{(s+1)} + ... + \\ &+ q^{(r)} \frac{\partial u^{(s)}}{\partial \beta^{(s)}} u^{(s+1)} ... u^{(r-1)} = 0 \end{aligned} \quad (18)$$

Let us write (18) in another form:

$$-\frac{\partial q^{(s)} \big/ \partial \beta^{(s)}}{\partial u^{(s)} \big/ \partial \beta^{(s)}} = q^{(s+1)} + q^{(s+2)} u^{(s+1)} + ... + q^{(r)} u^{(s+1)} ... u^{(r-1)} =$$

$$q^{(s+1)} + u^{(s+1)} \left[q^{(s+2)} + q^{(s+3)} u^{(s+2)} + ... + q^{(r)} u^{(s+2)} ... u^{(r-1)} \right] \quad (19)$$

But as far as

$$q^{(s+2)} + q^{(s+3)} u^{(s+2)} + ... + q^{(r)} u^{(s+2)} ... u^{(r-1)} = = -\frac{\partial q^{(s+1)} \big/ \partial \beta^{(s+1)}}{\partial u^{(s+1)} \big/ \partial \beta^{(s+1)}}$$

so instead (19) we write

$$-\frac{\partial q^{(s)} \big/ \partial \beta^{(s)}}{\partial u^{(s)} \big/ \partial \beta^{(s)}} = q^{(s+1)} - u^{(s+1)} \frac{\partial q^{(s+1)} \big/ \partial \beta^{(s+1)}}{\partial u^{(s+1)} \big/ \partial \beta^{(s+1)}} \quad (20)$$

Following (10) and (12) we have

$$\begin{aligned} &\frac{\partial q^{(s)}}{\partial \beta^{(s)}} = -q_d^{(s)} \varphi\left(z_f + \beta^{(s)}\right) \\ &\frac{\partial u^{(s)}}{\partial \beta^{(s)}} = q_d^{(s)} \varphi\left(z_f + \beta^{(s)}\right) + p_d^{(s)} \varphi\left(z_f - \beta^{(s)}\right) \end{aligned} \quad (21)$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \quad (22)$$

Taking into account (21) and (22) we write (20) in the form

$$\frac{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)}{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)+p_d^{(s)}\varphi\left(z_f-\beta^{(s)}\right)}=q^{(s+1)}+ \\ +\frac{u^{(s+1)}q_d^{(s+1)}\varphi\left(z_f+\beta^{(s+1)}\right)}{q_d^{(s+1)}\varphi\left(z_f+\beta^{(s+1)}\right)+p_d^{(s+1)}\varphi\left(z_f-\beta^{(s+1)}\right)} \quad (23)$$

We calculate

$$\frac{\varphi\left(z_f-\beta^{(s)}\right)}{\varphi\left(z_f+\beta^{(s)}\right)}=\exp\left(2z_f\beta^{(s)}\right)$$

Then the left part of (23) can be written in the form:

$$\frac{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)}{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)+p_d^{(s)}\varphi\left(z_f-\beta^{(s)}\right)}= \\ =\frac{q_d^{(s)}}{q_d^{(s)}+p_d^{(s)}\exp\left(2z_f\beta^{(s)}\right)} \quad (24)$$

Substituting (24) into (23) we obtain

$$\frac{q_d^{(s)}}{q_d^{(s)}+p_d^{(s)}\exp\left(2z_f\beta^{(s)}\right)}= \\ =q^{(s+1)}+\frac{u^{(s+1)}q_d^{(s+1)}}{q_d^{(s+1)}+p_d^{(s+1)}\exp\left(2z_f\beta^{(s+1)}\right)} \quad (25)$$

From (25) one can find the final expression:

$$\beta^{(s)}=\frac{1}{2z_f}ln\frac{q_d^{(s)}}{p_d^{(s)}}\times \\ \times\left\{\frac{q_d^{(s+1)}+p_d^{(s+1)}\exp\left(2z_f\beta^{(s+1)}\right)}{q^{(s+1)}\left(q_d^{(s+1)}+p_d^{(s+1)}\exp\left(2z_f\beta^{(s+1)}\right)+u^{(s+1)}q_d^{(s+1)}\right)}-1\right\} \quad (26)$$

We remember: when $a_k=a_1$ the threshold $\beta^{(r)}=-\infty$. Then using (26) one can determine all values $\beta^{(r-1)},\beta^{(r-2)},...,\beta^{(1)}$. For instance we have

$$\beta^{(r-1)}=\frac{1}{2z_f}\ln\frac{q_d^{(r-1)}\left(1-q_d^{(r)}\right)}{p_d^{(r-1)}q_d^{(r)}} \quad (27)$$

If we put $\beta^{(r-1)}=-\infty$ then $\beta^{(s)}=-\infty$ for all s.This means that the CSIF transforms into one direction system.

5 Example

Let us choose $r=3, z_d=3$ and calculate the thresholds $\beta^{(s)}$ for some values z_f following (26). The results of such calculation are given in the Table 1:

Table 1

z_f	1	2	3	4
$\beta^{(2)}$	2.35	1.17	0.78	0.51
$\beta^{(1)}$	4.8	4.7	2.3	2.06
z	5.3	4.92	4.7	5.06
q	$9.55 \cdot 10^{-8}$	$2.34 \cdot 10^{-8}$	$6.1 \cdot 10^{-10}$	$2.2 \cdot 10^{-12}$
$log\ q$	-7.02	-7.63	-9.21	-11.66

It is quite possible to calculate the effectiveness of CSIF with another algorithm of functioning. In this variant the thresholds do not change their values with the one exception when $s=r$. The calculation must be carried out following (10), (12), (14) and (16). Fixing the parameters z_d and z_f it is necessary to change the value $\beta^{(s)}=\beta$ in the wide limits $-\infty\le\beta\le\infty$. The obtained curves will have minimum with correspondent values z. Once again, we choose $r=3$, $z_d=3$. After this we will find for some parameters z_f the minimal error probability and the average consumption z. The calculation results are given in the Table 2.

Table 2

z_f	1	2	3	4
z	5.3	4.92	4.7	5.06
log q	-6.99	-7.2	-8.3	-10.8

The comparison of the error probability of the both variants under consideration demonstrates (see the lower lines of Tables 1 and Table 2) the following facts: 1) The effectiveness of CSIF with the optimal change of thresholds is better with respect of the system with the constant thresholds; 2) The difference of the values of the error probability is rather small. This means that one can recommend using in practice the second variant which can be realized by simple schemes.

6 The second algorithm of the optimization problem

In the comparison of the problem in section 4 and 5, now we will investigate the algorithm of the error probability optimization with the condition that the energy consumption is restricted. Formally, we need to find the minimum of the error probability

$$q=f_1\left(z_d,z_f,\beta^{(1)},\beta^{(2)},\dots,\beta^{(r-1)}\right)$$

when the energy parameter

$$z^2=f_2\left(z_d,z_f,\beta^{(1)},\beta^{(2)},\dots,\beta^{(r-1)}\right)$$

is fixed. Once again, we suppose that the values z_d and z_f are known.

We use the method of Lagrange's multipliers. We write the expression

$$q+\lambda z^2 \tag{28}$$

where λ is the Lagrange's multiplier and values q and z^2 are determined by (14) and (21).

Let us calculate the partial derivative with $\beta^{(s)}$. In order to obtain the set of optimal thresholds $\beta^{(s)}$, it is necessary to solve the equation

$$\frac{\partial}{\partial\beta^{(s)}}\left(q+\lambda z^2\right)=0 \tag{29}$$

Let is insert (14) and (16) into (29), then we cancel out the term $\prod_{i=0}^{s-1}u^{(i)}$. After this, the equation (29) can be written

$$\begin{aligned}&\frac{\partial q^{(s)}}{\partial\beta^{(s)}}+q^{(s+1)}\frac{\partial u^{(s)}}{\partial\beta^{(s)}}+q^{(s+2)}\frac{\partial u^{(s)}}{\partial\beta^{(s)}}u^{(s+1)}+\cdots\\&+q^{(r)}\frac{\partial u^{(s)}}{\partial\beta^{(s)}}u^{(s+1)}\cdots u^{(r-1)}+\lambda\Bigg[z_d^2\left(\frac{\partial u^{(s)}}{\partial\beta^{(s)}}+\frac{\partial u^{(s)}}{\partial\beta^{(s)}}u^{(s+1)}+\right.\\&\left.+\cdots+\frac{\partial u^{(s)}}{\partial\beta^{(s)}}u^{(s+1)}\cdots u^{(r-1)}\right)+z_f^2\left(\frac{\partial u^{(s)}}{\partial\beta^{(s)}}+\frac{\partial u^{(s)}}{\partial\beta^{(s)}}u^{(s+1)}+\right.\\&\left.+\cdots+\frac{\partial u^{(s)}}{\partial\beta^{(s)}}u^{(s+1)}\cdots u^{(r-2)}\right)\Bigg]=0\end{aligned} \tag{30}$$

One can rewrite (30) in the form:

$$\begin{aligned}&-\frac{\partial q^{(s)}/\partial\beta^{(s)}}{\partial u^{(s)}/\partial\beta^{(s)}}=q^{(s+1)}+q^{(s+2)}u^{(s+1)}+\cdots+\hat{q}^{(r)}u^{(s+1)}\cdots u^{(r-1)}+\\&+\lambda\left[z_d^2\left(1+u^{(s+1)}+u^{(s+2)}u^{(s+3)}+\cdots+u^{(s+1)}\cdots u^{(r-1)}\right)+\right.\\&\left.+z_f^2\left(1+u^{(s+1)}+u^{(s+2)}u^{(s+3)}+\cdots+u^{(s+1)}\cdots u^{(r-2)}\right)\right]=\\&=q^{(s+1)}+\lambda\left(z_d^2+z_f^2\right)+u^{(s+1)}\Big\{q^{(s+2)}+q^{(s+3)}u^{(s+2)}+\cdots\\&\cdots+q^{(r)}u^{(s+2)}\cdots u^{(r-1)}+\lambda\Big[z_d^2\Big(1+u^{(s+2)}+u^{(s+2)}u^{(s+3)}+\\&\cdots+u^{(s+2)}u^{(s+3)}\cdots u^{(r-1)}\Big)+z_f^2\Big(1+u^{(s+2)}+u^{(s+2)}u^{(s+3)}+\\&\cdots+u^{(s+2)}u^{(s+3)}\cdots u^{(r-2)}\Big)\Big]\Big\}\end{aligned} \tag{31}$$

Taking into account

$$\begin{aligned}&q^{(s+2)}+q^{(s+3)}u^{(s+2)}+\cdots+q^{(r)}u^{(s+2)}\cdots u^{(r-1)}+\\&+\lambda\left[z_d^2\left(1+u^{(s+2)}+u^{(s+2)}u^{(s+3)}+\cdots+u^{(s+2)}u^{(s+3)}\cdots u^{(r-1)}\right)+\right.\\&\left.+z_f^2\left(1+u^{(s+2)}+u^{(s+2)}u^{(s+3)}+\cdots+u^{(s+2)}u^{(s+3)}\cdots u^{(r-2)}\right)\right]=\\&=-\frac{\partial q^{(s+1)}/\partial\beta^{(s+1)}}{\partial u^{(s+1)}/\partial\beta^{(s+1)}}\end{aligned} \tag{32}$$

We can write (31) in the form

$$\begin{aligned}&-\frac{\partial q^{(s)}/\partial\beta^{(s)}}{\partial u^{(s)}/\partial\beta^{(s)}}=\\&=q^{(s+1)}+\lambda\left(z_d^2+z_f^2\right)-u^{(s+1)}\frac{\partial q^{(s+1)}/\partial\beta^{(s+1)}}{\partial u^{(s+1)}/\partial\beta^{(s+1)}}\end{aligned} \tag{33}$$

Let us calculate

$$\frac{\partial q^{(s)}}{\partial\beta^{(s)}}=-q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right) \tag{34}$$

$$\frac{\partial u^{(s)}}{\partial\beta^{(s)}}=q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)+p_d^{(s)}\varphi\left(z_f-\beta^{(s)}\right) \tag{35}$$

where $\varphi(x)$ is determined by (22)

On the base (33) – (22) we have:

$$\begin{aligned}&\frac{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)}{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)+p_d^{(s)}\varphi\left(z_f-\beta^{(s)}\right)}=q^{(s+1)}+\lambda\left(z_d^2+z_f^2\right)+\\&+\frac{u^{(s+1)}q_d^{(s+1)}\varphi\left(z_f+\beta^{(s+1)}\right)}{q_d^{(s+1)}\varphi\left(z_f+\beta^{(s+1)}\right)+p_d^{(s+1)}\varphi\left(z_f-\beta^{(s+1)}\right)}\end{aligned} \tag{36}$$

Taking into account

$$\frac{\varphi\left(z_f-\beta^{(s)}\right)}{\varphi\left(z_f+\beta^{(s)}\right)}=e^{2z_f\beta^{(s)}}$$

One can write the left part of (36) in the form:

$$\frac{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)}{q_d^{(s)}\varphi\left(z_f+\beta^{(s)}\right)+p_d^{(s)}\varphi\left(z_f-\beta^{(s)}\right)}=\frac{q_d^{(s)}}{A^{(s)}} \tag{37}$$

where

$$A^{(s)}=q_d^{(s)}+p_d^{(s)}e^{2z_f\beta^{(s)}} \tag{38}$$

Then, we can rewrite (36) in the new form:

$$\frac{q_d^{(s)}}{A^{(s)}}=q^{(s+1)}+\lambda\left(z_d^2+z_f^2\right)+\frac{u^{(s+1)}q_d^{(s+1)}}{A^{(s+1)}} \tag{39}$$

From (39) one can obtain the final recurrent expression for the threshold $\beta^{(s)}$:

$$\beta^{(s)}=\frac{1}{2z_f}\ln\frac{q_d^{(s)}}{p_d^{(s)}}\left\{\frac{A^{(s+1)}}{\left[q^{(s+1)}+\lambda\left(z_d^2+z_f^2\right)\right]\left(A^{(s+1)}+u^{(s+1)}q_d^{(s+1)}\right)}-1\right\} \tag{40}$$

We remember that $\beta^{(r)} = -\infty$, then using (40) one can obtain all values $\beta^{(s)}$ from $\beta^{(r-1)}$ to $\beta^{(1)}$. But there is a difference between (26) and (40), because (40) depends on the value λ. The discussion of this problem is in the next section.

7 Discussion of results

The main idea of the calculation is following: it is necessary to choose z_d and z_f on the base (40) to find the thresholds $\beta^{(r-1)},\ldots,\beta^{(1)}$. After this, one can obtain the principal characteristics of the system q and z (see the expressions (14) and (16)).

Now, we shall determine the limits of the change of the parameter λ.

Let us put $\beta^{(r)} = -\infty$, then

$$\beta^{(r-1)} = \frac{1}{2z_f}\ln\frac{q_d^{(r-1)}}{p_f^{(r-1)}}\left\{\frac{1}{\left[q_d^{(r)} + \lambda\left(z_d^2 + z_f^2\right)\right]} - 1\right\} \qquad (41)$$

The condition $\beta^{(r-1)} = -\infty$ means that $\beta^{(s)} = -\infty$ for all s. In this case, the system with the feedback is transformed into the direct system. The value λ can be found from the expression:

$$\frac{1}{q_d^{(r)} + \lambda\left(z_d^2 + z_f^2\right)} - 1 = 0$$

The solution is

$$\lambda \le \lambda_{\max} = \frac{p_d^{(r)}}{z_d^2 + z_f^2} \qquad (42)$$

If we put $\lambda = 0$, so we have the case studied in the section 6. The minimum value $\lambda_{\min}$ is determined from the expression

$$q_d^{(r)} + \lambda\left(z_d^2 + z_f^2\right) > 0;$$
$$\lambda \ge \lambda_{\min} = -\frac{q_d^{(r)}}{z_d^2 + z_f^2} \qquad (43)$$

So, when λ decreases from $\lambda_{\max}$ to $\lambda = 0$, the error probability q must be decreased from the value $q_0\left(z_0\right)$

$$q_0 = 1 - \varphi\left(z_0\right) \qquad (44)$$

The formula (44) is related to the one channel system $\left(z_0 = z_d\right)$ to the value $q = q_{\min}$ in the system under consideration. The interval $\left[\lambda = 0, \lambda_{\min}\right]$ corresponds to the increase of the error probability q and it is not interesting.

In order to illustrate the obtained formulas, we calculated some curves $q\left(z\right),\left(z_d = 3, r = 3\right)$ depicted in the Fig. 1. The curves 1-4 correspond to the regime with the thresholds (40) and $0 \le \lambda \le \lambda_{\max}$. The initial points are characterized by values:

$$q = q_0\left(z_0 = z_d\right), z = \sqrt{z_d^2 + z_f^2} \qquad (45)$$

The curve 5 is the envelope of the curve family $q\left(z_0, z_f\right)$. This curve describes the optimal functioning regime: for any value z (or q) there are such values $z_d, z_f, \beta^{(r-1)}, \beta^{(r-2)}, \ldots, \beta^{(1)}$ when the value q (or z) will be minimal.

The curves 1'-4' correspond to the system when the thresholds do not change with one exception $\beta^{(r)}$. The thresholds are obtained by the calculation of q and z following (14) and (16) when $-\infty < \beta < +\infty$.

If $\beta = -\infty$, $u^{(s)} = 0$ and (45) are valid. If $\beta = +\infty$, $u^{(s)} = 1, q^{(s)} = 0$ for $s \le r - 1$. Then

$$q = q^{(r)} = 1 - \varphi\left(z_d\sqrt{r}\right), z = \sqrt{r z_d^2 + \left(r-1\right) z_f^2}$$

The curve 5' is the envelope of some particular curves.

The comparison of the curve 5 and 5' demonstrate that the optimal variant with the changed thresholds has a small advantage.

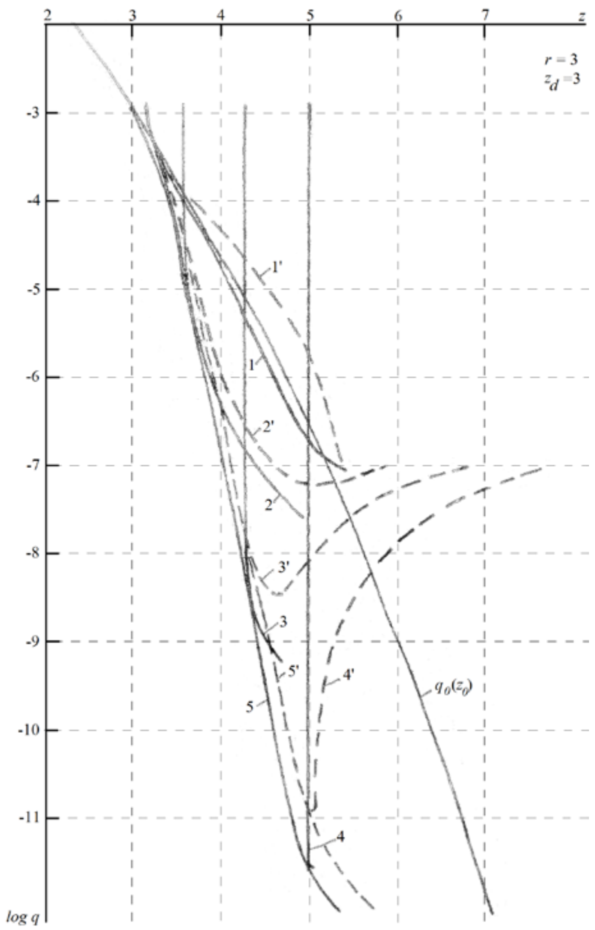

Fig. 1

8 Conclusions

The results if the investigations of two new regimes shows that in practice is quite possible to use the simple algorithm when thresholds do not change in the repetition with one exception $\beta^{(r)}$.

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