Image Super-Resolution Using Compressed Sensing Based on Learning Sub Dictionary

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Abstract: - Many applications have benefited remarkably from high resolution (HR) imaging models such as, astronomical and biomedical imaging system. In recent years, image enhancement and resolution approach has been embark the great result, and received great deal of attention of the researcher, and many researchers have proposed several methods to achieve the goal of high resolution image. In this paper an efficient method of high resolution image based on the concept of Compressed Sensing (CS) have been introduced, which uses sub dictionary instead of redundant dictionary and traditional orthogonal basis. The new framework is consisted of three phases. Firstly, we designed the sub dictionary that are learned from a range of datasets of high quality patches and then selected adaptively. Secondly, Principal Component Analysis (PCA) has been applied to each data sets of the high quality patches to evaluate the principal component from which the dictionary is constructed. Finally, the HR image is generated by averaging all high resolution patches. In addition, the proposed method has been demonstrated better results on real images in terms of peak to signal and noise ratio (PSNR), structural similarity (SSIM) and root mean square error (RMSE). Furthermore, our method has been evaluated by deriving the modulation transfer function (MTF), the MTF curve showed better reconstruction of HR image and achieved various improvements compared with other methods.

Key-Words: - Compressed Sensing, Compact Dictionary, Image Resolution, Modulation Transfer Function, Principal Component Analysis (PCA)

1 Introduction

High resolution (HR) image is currently a very active area of research and this approach received great deal of attention of the researcher, as it has provided to overcome the limitations of resolution imaging sensors and provide a better resolution display. The HR techniques play a very vital role in biomedical imaging and astronomical observation where analysis from low resolution (LR) images is very difficult.

Meanwhile, the HR image reconstruction is most spotlighted research area, which increases the efficiency of digital image applications and overcome the limitation of imaging system [1]. Digital cameras have limitations, such as cost, integrated circuits and required high resolution. HR images not only provide good brightness and contrast, but also detailed information of the image [2]. Generally the information from low resolution (LR) images can be extracted and used to reconstruct the HR images. Lately several methods for high resolution have been proposed [3-5]. The various factor such, as image blurring, image deformation that causes the decreases of the resolution in the acquisition steps of digital imaging. Based on acquisition step, Yincheng et al. [2] have used compressed sensing based on redundant dictionary to reconstruct the HR image, which the method was simple and provided robustness compared with several other methods, but the training phase was required large amount of calculations and computational time. Normally, Fourier transform has been used to reconstruct the high resolution image which also required large computational time [3]. Besides, maximum a posterior (MAP) is very fast and robust method which enhanced the quality of reconstructed images, but required large amount of calculation which makes algorithm more complex [4-5]. Based on prior knowledge of the images, the HR task is cast as the problem of recovering the original high resolution image by fusing the LR images [1]. The HR image reconstruction is generally creating an ill-posed problem due to insufficient number of low resolution images.

The reconstruction of HR is to recover image after applying the same model that should reproduce from the LR image [1]. When the number of available input images is small this will degrades the performance of HR reconstruction algorithm [6]. Several methods have been proposed to reconstruct the HR image based upon interpolation likewise, Bicubic interpolation which have tendency to generate smooth images with block artefacts [7-9]. Dai et al. [8] presented the local image patches using background combine technique of soft edge smoothness and alpha matting technique for HR image. Dong et al. [10] introduced the robust and fast method to obtain HR images by adaptive sparse domain and regularization approaches which have been extensively used for deblurring of images but require complex algorithm. Lately, many researchers have proposed various ways to achieve the HR reconstruction by using the multiple fames of LR images of the same scene to generate the one full frame of HR image which directly affect image enhancement quality because of LR frames of the scene [11-13]. Several methods in which the valuable detailed information of high frequency have lost in LR images by applying the prior input information, which have been learned from the training phases [1], [14-16].

In this paper, a new method has been proposed based on the concept of CS to overcome the problems of HR image reconstruction, the new algorithm includes designing and learning sub-dictionary which are learned and processed by principal component analysis (PCA) and then selected adaptively in order to tackle all problems. Firstly, the sub-dictionary is learned to structure the image shape and achieved the pixel brightness by construct a datasets of high resolution image patches. Secondly, the PCA is applied to improve the algorithm efficiency and, to reduce efficiently the large amount of calculation from which the dictionary is constructed and then selected adaptively. Finally, the high resolution (HR) image is constructed by averaging all reconstructed patches.

The problem formulation and modeling are discussed in section 2, the complete analysis of learning dictionary and selected adaptively were discussed in section 3, based on the dictionary image reconstruction model is discussed in section 4, the simulation results was introduced in section 5, and finally the work was concluded in section 6.

2 Problem Formulation and Modeling

As illustrated in the introduction, the several major problems associated with HR image can be divided in two phases. The first one is the inverse problem associated by the training phase is required large amount of calculation and is caused algorithm more complex. The second one is the noise in the training part which affects the redundant dictionary that leads to be unsatisfied result in reconstruction part

Fig.1 shows the block diagram of the proposed method. Our objective is to generate the sub dictionary which will be used to reconstruct the high resolution image (HR). By using bicubic interpolation HR (x_i) and LR (y_i) images are developed. If we directly extract LR image we can lost the LR patches and features information, to inverse this effect we select the different method by using the blurred and down sampled method. Firstly the HR (x_i) images is firstly blurred and down sampled by integer value, and then used bicubic interpolation to obtain the corresponding LR (y_i) images. By applying the blurred and down sampled operator to HR images, the corresponding HR images must contains less features. To overcome this problem, and to achieve the high resolution patches, we used the bilateral filter operator R_i to extract the high resolution patches. After extraction of patch we need to generate the sub dictionary from HR image patches and due to localizes the image structures and forming shape. Learning a sub dictionary is very important part obtained from constructing the datasets, once the dictionary are learned then principal component analysis (PCA) method is applied to reduce the complexity of the algorithm, and minimizes the blurriness of the patch, then select the dictionary adaptively of each quality image patches which leads to satisfied result at the reconstruction part, and finally we reconstruct the HR image by averaging all reconstructed patches.

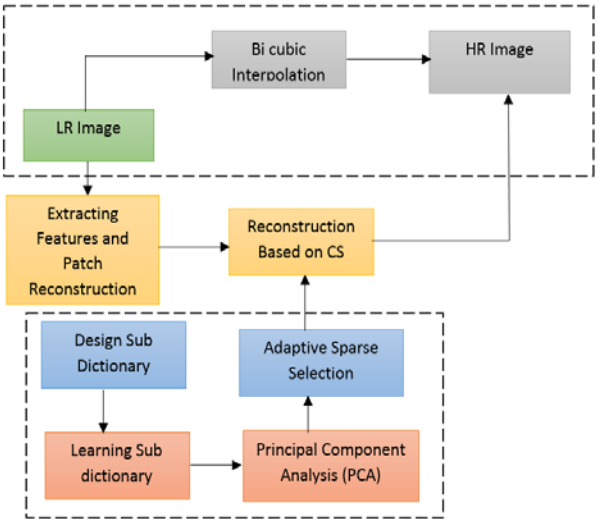


Fig. 1: The problem formulation block diagram.

Normally, the HR images is a generally an ill-posed problem, due to less number of low resolution images. In order to achieve the high resolution image the sub dictionary is introduced by datasets of high quality image patches instead of redundant dictionary. And the same time, in order to achieve the maximum resolution of image at reconstruction part, the amount of calculation in training phase [2] needs to be reduced by applying PCA method to each subset, from which the dictionary is constructed and selected adaptively to increases the efficiency of the algorithm, in addition the noise directly affect the quality of redundant dictionary, which leads to be unsatisfied result in reconstruction part.

2.1 Theory of Compressive Sensing

In the CS theory, a signal is defined as sparse in a certain basis or transform can be acquired a rate below than Nyquist rate[17-18], and the number of sparse signals recovered from their projections into the small number of vectors [19-20], the general process of compressed sensing is define as below.

$$x = \sum_{i=1}^{N} s_i \emptyset_i \text{ or } x = s\emptyset \tag{1}$$

where $\emptyset = \emptyset_1, \emptyset_2, \dots \emptyset_N$ is an N×N orthogonal basis matrix and *s* is weighting coefficients with column size N×1, and the signal *x* is sparse in $\emptyset$ domain [2]. The main objective of CS is to recover a sparse signal from limited number of measurements expressed as below,

$$y = \varphi x + v = \varphi s\emptyset + v = \Theta s + v \tag{2}$$

where the size of basis φ is M, and matrix $\Theta = \varphi\emptyset$ size is M×N and M<N.

The appropriate target of CS and our objective is to reconstruct the original sparsity signal with defined measurements. Because of certain limitation of M<N, the optimum solution is very difficult to be achieved, several methods have proposed to achieve an optimum solution and inversion the limitation, likewise orthogonal matching pursuit (OMP), basis pursuit (BP). These methods are provide great solutions but still have some limitation, because it is complex in nature and require more processing time. Though the BP method is faster, and it provides robustness of reconstructing a sparse signal compared with OMP, but this method also is complex and require large amount of processing time in real application of image processing.

2.2 High Resolution Image Modeling

HR image can be acquired from LR input images, so when it will be converted from LR to HR, the problem such as image blurriness, contrast and brightness are occurred. Assume x_i is a HR image, $x_i \in z^N$ where N is the number of pixel in each frame of HR image vector x_i. The HR image is corrupted with blurriness and Gaussian noise, so it is turned to be LR image vector, which is treated as input image and contain low resolution. So,

$$y_i = [y_{i1}, y_{i2}, \dots\dots, y_{ik}] \in z^N \tag{3}$$

$$y_i = Bx_i + v \tag{4}$$

where v is assumed Gaussian noise with defined value (0, σ^2) and B is the blur operator of HR images.

Considering previous revealed study, now it is used the theory of CS to overcome high resolution reconstruction of image problem, but when using training phases and reconstruction phases it is required large amount of calculation, more processing time and increasing the complexity of algorithm. Assuming the LR image patch size L and HR patch Q, and matrix K sparse should have the less value, which inverse the condition of using orthogonal basis to convert the HR patches into column vector which has less atoms.

$$K \le \frac{L}{\log\left(\frac{Q}{L}\right)} \tag{5}$$

where *K* is the sparse signal has less values therefore, it is difficult to find orthogonal basis which can transform the high resolution patches. The parameter Q and L are the patches number of the high and low resolution image respectively.

Let x_i be the vector of high resolution image, and y_i be the vector of low resolution image. Two vectors are satisfied relation as $x_i = \sum_i (y_i R_i)$, where index i is the number of patches run from 1 to N, R_i is extraction matrix from low resolution patch to high resolution image patch. For patch x_i it is supposed that sub dictionary $\emptyset_k$ is selected for it, then x_i can be written as below,

$$x_i = \sum_{k=1}^{N} \emptyset_k \, \mu_k \tag{6}$$

where $\|\mu_k\|_1 \leq T$, where T is the operator that controlling the spares signal values, and $\|.\|_1$is the $l_1$norm.

The proposed model based on the concept of compressed sensing is to achieve the high resolution image by using the low resolution input image. The traditional methods of orthogonal basis, measurement matrix and redundant dictionary that are replaced by new proposed method based on learning series of dictionary.

$$\mu_k = \arg\min_{\mu_k}\{\|y_i - B\,\emptyset_k\,\mu_k\|_2^2 + \lambda\|\mu_k\|_1\} \tag{7}$$

$$\text{s.t}\ \|x_i - B\,\mu_k\|_2^2 < v$$

where μ_k termed as sparsing coefficient vectors which contained the original input image information, where λ is the representation of coefficient matrix, the above Eq. 7 represent the optimizing problem of μ_k [10], this problem can be solved by the proposed method based on learning dictionary and selected adaptively and, then reconstruct the HR image by averaging all reconstructed patches, which provides the robustness and fastest convergence rate of proposed method.

3 Dictionary Learning Model

Our objective is to learn the dictionary which can be accurately represent the original signal [21]. Learning a sub dictionaries, let's assume there is M image patches are selected our objective to learn k sub dictionaries $Ø_k$ from dataset S, so that we can find the most sub dictionary.

$$S = [s_1, s_2, \dots\dots, s_M] \tag{8}$$

After creating S dataset, now we are clustering S datasets in to the K cluster$[C_1, C_2, \dots, C_K]$ and start learning the sub dictionary from the each of the K cluster then we passed through the each patch of M patches pass through the high pass filter to form an structure of the image patches and edges. $S_h = [s_1^h, s_2^h, s_3^h, \dots\dots, s_M^h]$ is the dataset of M image patches passed through high pass filter, we need to find the K cluster from S_h the centroid of the cluster C_K , as we need to learn the K sub dictionaries. We assume $\psi_k = [C_1, C_2, \dots, C_K]$, now partitioned the dataset S then we cluster in to k subsets which is S_k, where k = 1,2,…,K with matrix dimension $n \times m_k$ where m_k denotes the number of samples in S_k.Once we clustered S datasets then we need to learn the sub dictionary $Ø_k$ from the cluster S_k.

$$(\emptyset_k, \mu_k) = \arg\min_{\emptyset_k, \mu_k} \{\, \|S_k - \emptyset_k\|_F^2 + \rho\|\mu_k\|_1 \,\} \tag{9}$$

Eq. 9 is an expression to learn the sub dictionary, which is constructed by using the group of high quality image patches, where μ_k is the sparsing coefficient with matrix information of S_k datasets under the dictionary $\emptyset_k$, and$\|.\|_F$ is the Forbenius norm. Also, Eq. 9 is expressed as a joint optimization problem of both $\emptyset_k$ and μ_k, it can be solved by the KSVD algorithm [22].

3.1 Principal Component Analysis (PCA)

Principal component analysis (PCA) is a classical de-correlations and dimensionally reduction technique, which is widely used in pattern recognition and statistical signal processing [23]. Since PCA technique can remove the noise, it has been successfully used in adaptive image denoising by computing the local PCA transform of each image patch [24]. The key objective of PCA is to solve the large amount of calculation and complexity of algorithm [2], by applying PCA to each datasets and evaluating the PCA with constructed dictionary $\emptyset_k$. Given the covariance ω_k matrix of the datasets S_k . To evaluate the s eigenvector in q_k which are used to construct the dictionary $\emptyset_s$ i.e. $\emptyset_s = [q_1, q_2, \dots., q_s]$, the Eq. 9 can be rewritten as follow,

$$s = \arg\min_{s}\{\, \|S_k - \emptyset_s\|_F^2 + \rho\|\mu_k\|_1 \,\} \tag{10}$$

In Eq. 10 we have achieved the sub dictionary s after successfully applied PCA method. The sub dictionary we have learned earlier $\emptyset_k$ from the datasets S_k is $\emptyset_k = [q_1, q_2, \ldots, q_s]$, by applying the above process we can get the K sub dictionaries.

3.2 Adaptively Selection

In Previous section, we have learned dictionary $\emptyset_k$ earlier from a datasets of high quality image patches, we assign adaptively a sub dictionary to each patch of S_k, and we need to compute the centroid value of each cluster C_K of subset S_k. In past, the initial estimation of signal solved with iterative thresholding algorithm (ITA) [25].We assign adaptively a sub dictionary to each high quality image patch, we denoted the patch P_h and have centroid of each cluster ω_{ph}. The patch P_h is a high resolution patch and has been constructed by high pass filter, so we selects the best dictionary by adaptive way.

$$k_h = \arg\min_{k_h} \|P_h - \omega_{Ph}\|_2 \tag{11}$$

Because the initial estimation of patch P_h is noisy, the distance between P_h and ω_{Ph} can be calculated, which may not be a robust one, then it is evaluated in the subspace $\emptyset_h$.

$$k_h = \arg\min_{k_h} \|\emptyset_h P_h - \emptyset_h \omega_{Ph}\|_2 \tag{12}$$

Eq. 12 increases the robustness of adaptively selected dictionary, so the k_h th dictionary $\emptyset_h$ will be selected and assigned to the patch P_h.

4 Image Reconstruction Model

The basic principal and process flow are described in Fig.2.

1. The low resolution (LR) image $\sqrt{m} \times \sqrt{m}$,
2. Preprocessing of HR images is divided in to $\sqrt{n} \times \sqrt{n}$ patches, by using the bilateral filter which are used to extract the feature of LR images and then learning dictionary.
3. Learned dictionary passed through PCA, and selected adaptively to obtain the HR patches of the images.
4. Final image is constructed by averaging all high resolution patches.

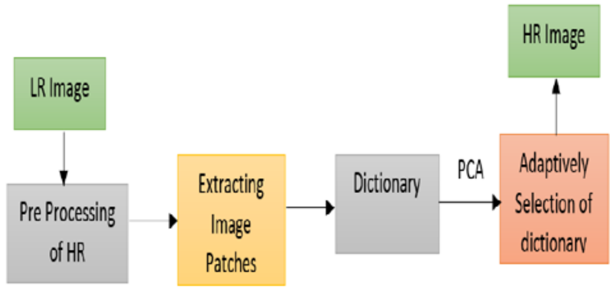


Fig. 2: Reconstruction process of HR image.

In image reconstruction part of the high resolution images we need to magnify the input image which is assumed to be generated the high resolution image x_i from the sub dictionary. We assumed the low resolution input image y_i is used to scale up by using the bicubic method to achieve the corresponding high resolution image x_i and have the same size as x_{i1}. Let assume x_i is the high resolution image vector, and y_i is the low resolution image vector, then feature of image y can be defined as, $y = R_{ki} y_i$ where i is the number of patches, $i = 1,2,3 \ldots N$, and R_{ki} is the matrix extraction feature of patch y from y_i.

$$P_k = \sum_{i=1}^{N} R_{ki}\, y_i \tag{13}$$

$$P_h = \sum_{k=1}^{K} P_k \tag{14}$$

$$P_h = \sum_{k=1}^{K} \sum_{i=1}^{N} R_{ki}\, y_i \tag{15}$$

In Eq. 15 the term $R_{ki}\, y_i$, which builds the high resolution patches.

The optimization problem of sparse representation can also be achieved by an *l*1-homotopy algorithm [2] to approximate the desired solution so it can be written as follow,

$$L = \arg\min_{\mu_k} \|P_h - B\, \emptyset_k\, \mu_k\|_2^2 + \lambda \|\mu_k\|_1 \tag{16}$$

The high resolution image is developed from the patch P_h by solving the above joint optimization problem. We used the adaptive way to learn the sub dictionary and applied PCA, so it is very unlikely to loss the detailed information of the reconstructed patches, and we also consider the noise effect in the patches. To recover the sparse representation we introduces the homotopy selection of path $(1-\omega)$ in Eq. 17.

$$L = \arg\min_{\mu_k} \|P_h - B\,\emptyset_k\,\mu_k\|_2^2 + \lambda\|\mu_k\|_1 + (1-\omega)q^T\mu_k \quad (17)$$

changing parameter ω from 0 to 1, so q is formulate as below,

$$q_i = -\lambda w_i - A^T(A\mu_k - P_h) \ \text{s.t} \quad A = B\,\emptyset_k \quad (18)$$

where i is the iterations, and w_i is the vector direction of the homotopy path, which contain the sign value μ_k.

The patches P_h detailed information are achieved by the inner product of learning sub dictionary $\emptyset_k$ and sparsing coefficient μ_k, as stated above. The final high resolution image can be developed by,

$$x_i = x_{i1} + P_h\ R_i^T\,R_i \quad (19)$$

The HR image can be constructed by averaging all high resolution patches P_h in overlap region and adding x_{i1} to achieve the final image, where $R_i^T\,R_i$ is a diagonal matrix which weights the each pixels of high resolution image.

5 Simulation Results and Discussion

In this section several experiment are conducted to analysis the performance of the proposed method, we have chosen the 60 frames of HR images with variant sizes, the selection of HR images is optimal, The image contents varies from image to image because every image varies due to their brightness, intensity and noise. We patches the total amount of 130 000 HR with patch size of (7×7), as it is clustering based method, we partition the training data sets into 200 clusters, to evaluate the patch size on images, then learned the sub dictionary and selected adaptively to test images, if we used the smaller patch sizes such as (3×3, 5×5), it would generates the artefacts and have less brightness in the reconstruction part.

a (5×5) b (7×7)

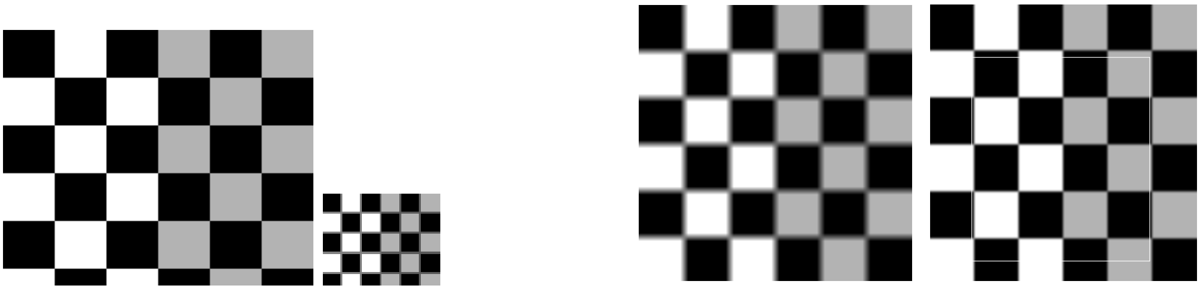
Fig. 3: Visual analysis of deblurred image.

Fig.3 shows the visual comparison and detailed information of two deblurred images with different patches. Yincheng et al. [2] reconstructed the image by using the image patch size (5×5), which was blur with less contrast and brightness, and achieved less PNSR, (see fig.7). The proposed method utilizes the image patch size of (7×7), results show a better resolution with good contrast. Indeed, the selection of patches will leads to be good PNSR (see fig.7) and less RMSE (see fig.6).

5.1 Simulation Results on Ideal Image

We performed a simulation experiments with different images, in order to evaluate the information contained in the dictionary. We used block image with size of (36×36) which consists of white and black blocks respectively, and each size of block is (6×6). Fig.4 represents, the original HR image in 4(a), 4(b) is the degraded LR image and, 4(c) is the bicubic method, the images is blurred with less contrast and low resolution. Fig. 4(d) shows the result with the proposed method, which achieves a good quality image. When the pixel values are in between (0,255), the RMSE is significantly reduced and achieves a better result compared with different methods.

The original HR mage can be reconstructed with any of the method, but resolution and contrast will remain the critical issues. Fig 4 (c) and 4 (d) respectively shows the reconstructed HR image with the bicubic method and the proposed method. The proposed method achieves an enhanced HR image with good contrast and better resolution compared with bicubic method.



(a) (b) (c) (d)

Fig. 4: (a) Original HR image, (b) LR degraded image, (c) Bicubic Interpolation and (d) proposed method.

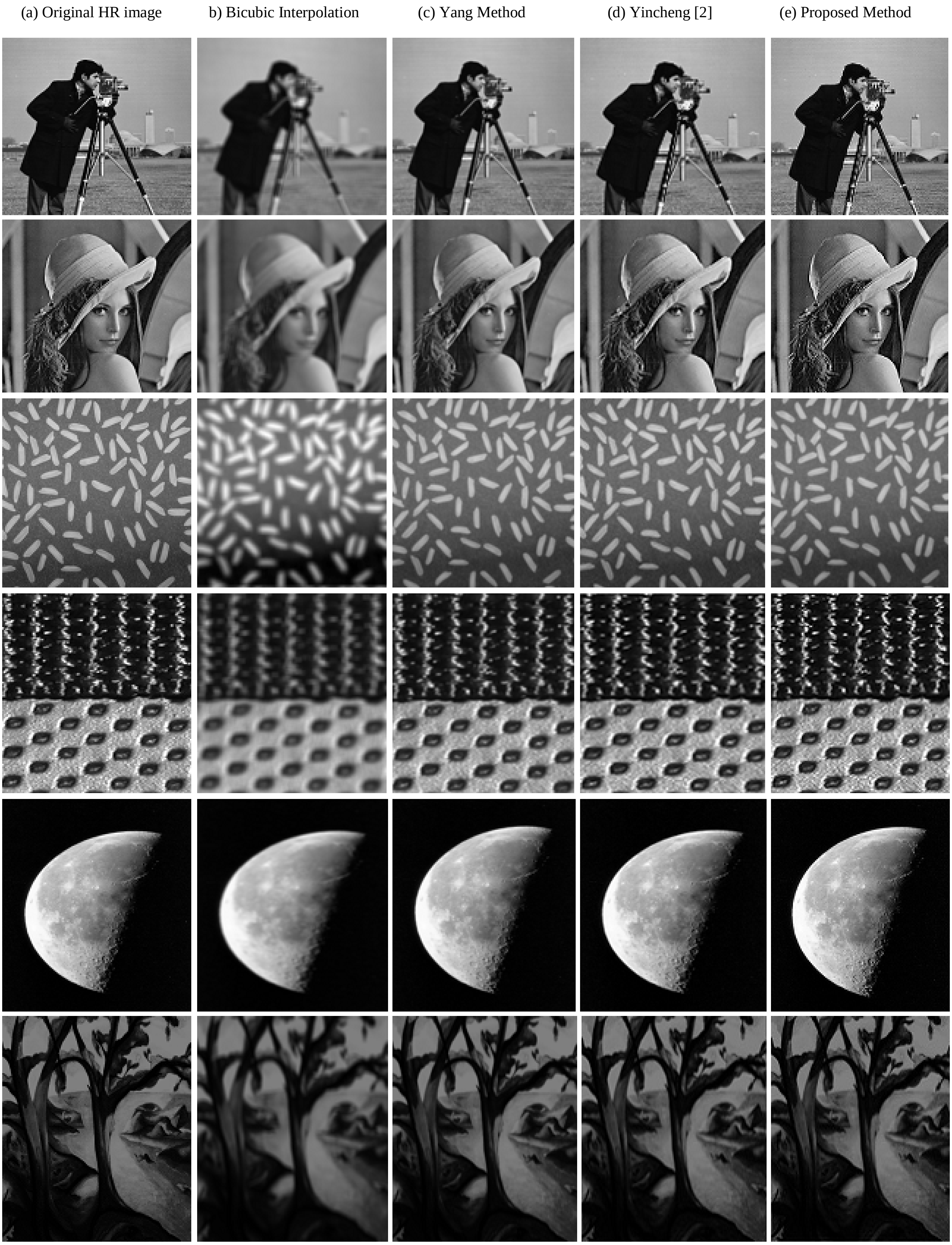


Fig. 5: Visual analysis (Cameraman, Lena, Rice, Bag, Moon and Trees), and comparison of proposed method with others.

5.2 Simulation Results on Different Images

In Reconnection part firstly, we down sampled the original image with reference size of 256×256 to generate the input LR image by using bicubic interpolation. Secondly, extracting the features of patch information. Finally, the HR image is generated by averaging all high resolution patches. To evaluate the performance of the proposed method, several experiments has conducted, we constructed and learn the dictionary by using the datasets of high image patches, and then selected adaptively to achieve the better performance. At last, we introduced the PCA method to suppress the large amount of calculation which is the limitation of [2].

The original image size is 256×256, and input LR image size of 64×64 respectively, we implemented by using six LR images (cameraman, lena, rice, trees, moon, bag), we used the patch size of (7×7) to reconstruct the HR image, the simulation results of proposed method are compared with bicubic interpolation, Yang method and Yincheng [2] method. Fig.5 (a) shows the original HR image, 5(b) is the bicubic interpolation method, and 5(c)-(e) the results of various algorithm Yang method, Yincheng method [2], and our proposed method respectively. The result of bicubic interpolation is blurred image compared with our method, because of the patch sizes and training samples, the image is less clear and we cannot visualizes the image. Yang method is good and better than bicubic, but have less clear and low resolution, there is some blurriness that make the visual quality lower and have smooth artefacts. We learned the sub dictionary, and selected adaptively, which enhanced the visual quality and resolution of proposed method and achieved faster convergence rate, compare with Yang and Yincheng [2].

We used the RMSE PNSR, and SSIM to evaluate and analyse the quality of proposed method, Table 1 is the error comparisons of proposed method with several methods with six different images, and error of bicubic interpolation method is quite high of every image. Yang [1] provides good inversion of the limitation of bicubic method and achieved less error compared with bicubic interpolation. Yincheng [2] have less error than Yang and Bicubic interpolation, it is good scheme but required large amount of calculations to reconstruct the HR image. Our method provides robustness and have less error in all of the test images compared with Yincheng [2], Yang and Bicubic interpolation.

TABLE 1: RMSE comparisons of proposed method and others

Image	Bicubic Interpolation	Yang Method	Yicheng Method	Proposed Method
Lena	27.1	24.02	22.45	21.8
Trees	13.2	11.9	10.6	9.7
Rice	16.8	16.1	14.21	13.2
Bag	14.8	13.8	12.17	11.3
Moon	12.5	11.6	10.5	9.8
Camera man	15.6	14.2	12.8	11.8

$$\mathrm{RMSE} = \sqrt{\sum_{i}^{N} \frac{(x_{h-}x_i)^2}{N}} \qquad (20)$$

$$\mathrm{PNSR} = 10\log_{10} \frac{N \text{ x } R^2}{\sum(x_{h-}x_i)^2} \qquad (21)$$

where R is the maximum input data type, 2^8= 255, and N is the number of columns in the input image vector. $x_h, x_i \in [0,255]$.

TABLE 2: PNSR comparisons of proposed method and others

Image	Bicubic Interpolation	Yang Method	Yicheng Method	Proposed Method
Lena	19.8	21	22.1	22.9
Trees	25.7	26.5	27.55	28.1
Rice	24.25	25.28	25.9	26.5
Bag	23.2	24.12	24.8	25.5
Moon	26.25	27	28.3	28.6
Camera man	27.2	27.9	28.8	29.3

Table 2 is the PNSR comparison of our proposed method with different methods, using six test images, Bicubic interpolation method has lower PNSR in all of the method because of the in exact detailed and low resolution of image patches, Yang method have achieved better PNSR values than the Bicubic method, but lower than Yincheng method

[2].The proposed method demonstrate the good PNSR in all of the test images compared with several other methods, and achieved clear and good contrast HR image.

TABLE 3: SSIM comparisons of proposed method and others

Image	Bicubic Interpolation	Yang Method	Yicheng Method	Proposed Method
Lena	0.877	0.882	0.891	0.905
Trees	0.765	0.784	0.795	0.802
Rice	0.725	0.734	0.751	0.765
Bag	0.653	0.672	0.698	0.723
Moon	0.762	0.792	0.862	0.872
Camera man	0.832	0.842	0.845	0.85

We are now comparing the behaviour of our proposed method interms of structural similarity (SSIM) with several other method, see (Table 3) which demonstrated that the proposed method achieves better SR reconstruction quality, and SSIM value for every test image. The higher the SSIM value which leads to be better SR reconstruction of quality image.

Fig.6 shows the RMSE comparison of proposed method with different methods, the error was significantly reduced by proposed method due to high quality image patches of 7×7, in addition learning sub dictionary from datasets of high quality image patches and selected adaptively which enhances the quality of image and provides robustness of our method, compared with Yang [1], and Yincheng [2].

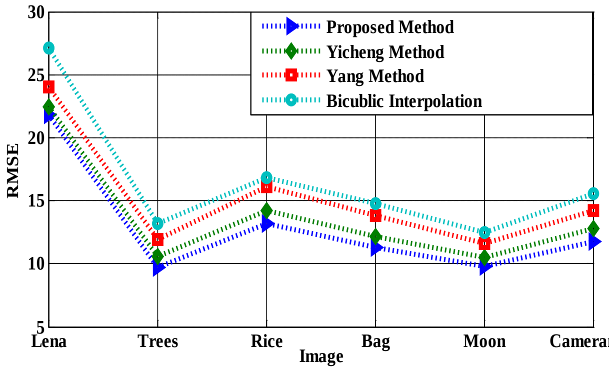


Fig. 6: RMSE comparison of proposed method and other methods.

Fig.7 shows the image reconstruction of HR image results, illustrating the PNSRs of the different methods of six test image were taken as an input LR image, it is found that we achieved good PNSR of our method compared with other methods such as, bicubic interpolation, Yang [1], and Yincheng [2].

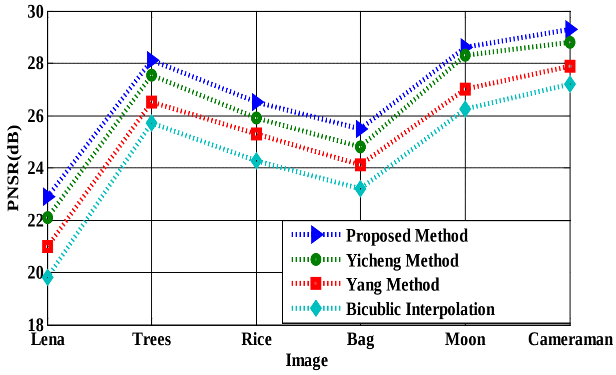


Fig. 7: PNSR comparison of proposed method and other methods.

Fig.8, shows the SSIM response of our proposed method with different approaches by implementing on six test images (Cameraman, Lena, Rice, Bag, Moon, and Trees). The objective assessment in terms of SSIM values demonstrates that the proposed method achieves good reconstruction quality of SR image with better resolution and contrasts, and outperformed several other methods.

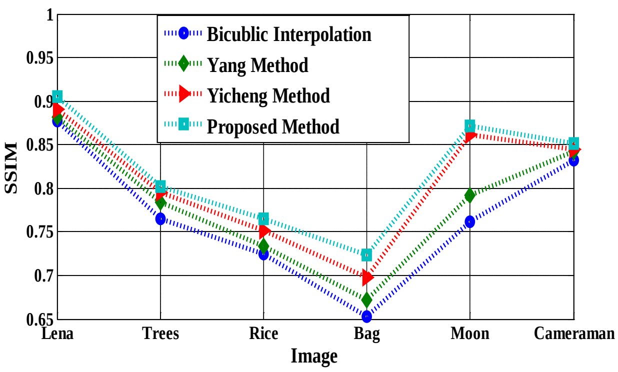


Fig. 8: SSIM comparison of proposed method and other methods.

5.3 Modulation Transfer Function Behavior

In this subsection, we further evaluate our method by using modulation transfer function (MTF) which is used for measuring the spatial resolution of image [26], some of the images that are specified by various kinds of resolution. In digital image spatial resolution and cell resolution play a key role.

$$\mathrm{M} = \mathrm{KN} \tag{22}$$

where K is the kell coefficient of contrast of image ground scene, M is the spatial resolution and N is the cell resolution in an image.

The performance of our reconstructed HR image is analyse by calculating the spatial resolution of reconstructed image and comparing with original HR image. Firstly, we select the edge of an image, let suppose $\mathrm{N}+1$ of reconstructed image $\mathrm{x_i}$ where i =0,1,2,…,N are edges on both sides, secondly derived the discrete linear function $\mathrm{F(x_i)}$ of $\mathrm{x_i}$ by calculating the first order difference of $\mathrm{N}+1$ points, then obtained the DFT of the discrete linear function and, finally obtained the MTF by$\mathrm{Z(k)}$.

$$\mathrm{X(k)} = \sum_{i=1}^{N-1} \mathrm{F(x_i)} e^{-\frac{j2\pi ai}{N}} \tag{23}$$

$$\mathrm{Z(k)} = \frac{|\mathrm{X(k)}|}{\sum_{i=0}^{N-1} \mathrm{F(x_i)}} \tag{24}$$

Fig.9 shows the MTF comparison of proposed method with original HR image, Yincheng [2], Yang [1] and Bicubic interpolation respectively. The MTF curve of Bicubic interpolation, Yang [1] and Yincheng [2] method have different kell coefficient, thus it effect the brightness and contrast of the image, see (Fig.9). The simulation results demonstrated that our proposed method achieves better spatial resolution curve compared with Yincheng [2], Yang [1], and bicubic method and leads to be good resolution HR image.

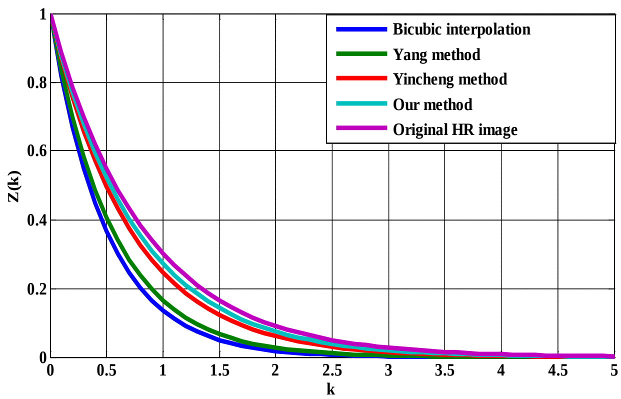


Fig. 9: MTF comparison of proposed method and other methods.

6 Conclusions

In this paper, we have proposed a new method to reconstruct the high resolution (HR) image based on the concept of compressed sensing (CS), and learning sub dictionary that are learned from datasets of high quality image patches which are selected adaptively to reconstruct the HR image. In reconstruction part the algorithm have done by using the compressed sensing based on the learning sub dictionary, and averaging all high resolution patches which are used to reconstruct the HR image. The experiment conducted on six test images using a patch size of (7×7), and simulation results indicate that our approach yielding better reconstruction and achieves good PNSR, and SSIM values compared with other SR method such as, Yincheng method [2] and Yang [1]. In addition to demonstrate the performance of proposed method the RMSE of our method is significantly reduce compared with Yincheng method [2] and Yang [1]. Our presented method is quantitatively and qualitatively robust method compared to other SR scheme, and achieves fastest convergence rate, which is the limitation of several methods. Furthermore we derived the modulation transfer function (MTF) to evaluate the performance of our method in terms of spatial resolution, and experimental results have shown that MTF curve of proposed method is better than the Yincheng [2] and Yang [1], and leads to be good reconstruction quality of high resolution (HR) image.

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