The DBSCAN Clustering Algorithm by a P System with Active Membranes

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Abstract: - The great characteristic of the P system with active membranes is that not only the objects evolve but also the membrane structure. Using the possibility to change membrane structure, it can be used in a parallel computation for solving clustering problems. In this paper a P system with active membranes for solving DBSCAN clustering problems is proposed. This new model of P system can reduce the time complexity of computing without increasing the complexity of the DBSCAN clustering algorithm. Firstly it specifies the procedure of the DBSCAN clustering algorithm. Then a P system with a sequence of new rules is designed to realize DBSCAN clustering algorithm. For a given dataset, it can be clustered in a non-deterministic way. Through example verification, this new model of P system is proved to be feasible and effective to solve DBSCAN clustering problems. This is a great improvement in applications of membrane computing.

Key-Words: - DBSCAN; Clustering Algorithm; Membrane Computing; P System; Active Membranes

1 Introduction

Clustering analysis, as an important part of data mining, has been widely applied in many areas, including statistics, biology and machine learning. DBSCAN clustering method is an important method of clustering, which can discover clusters with arbitrary shape and filter out noise (outliers) [1]. The key idea is that for each point of a cluster the neighborhood of a given radius has to contain at least a minimum number of points [2].

Inspired by the architecture and functioning of living cells, membrane computing has emerged in recent years as a powerful modeling tool. It provides a new non–deterministic model of computation which starts from the assumption that the processes taking place in the compartmental structure of a living cell can be interpreted as computations [3]. The devices of this model are called P systems. A variety of applications have been reported, such as biological modeling, combinatorial problems and NPC problems [4].

A particularly interesting class of P systems is the P systems with active membranes. In a P system with active membranes not only the objects evolve but also the membrane structure. Simply speaking, for a given set of N objects, the core objects can be determined by comparing the ε-neighborhood of each object *Min*. In this process, a membrane system is designed and the membrane labeled i represents objects i. Then, the two membranes are merged into a single membrane if the two objects are core objects. The membrane mergence is repeated until no change in each membrane. In the end, different membranes represent different clusters. This new method has great significance to the application of membrane computing in clustering problems.

2 Prerequisites

2.1 The DBSCAN Clustering Algorithm

DBSCAN (Density-Based Spatial Clustering of Application with Noise) is a density-based clustering algorithm. The algorithm grows regions with sufficiently high density into clusters and discovers clusters of arbitrary shape in spatial databases with noise. Firstly, a number of new definitions of the density-based clustering are presented in this paper [1, 5].

Definition1. ε-neighborhood

The neighborhood within a radius ε of a given object is called the ε-neighborhood of the object. For a given data set *D*, the ε-neighborhood of an object p is denoted by $N_\varepsilon(p)$, and $N_\varepsilon(p) = \{q \in D|\text{dist}(p, q \leq \varepsilon)\}$.

The ε-neighborhood of the object is obtained by calculating and comparing the distances between the object and other objects. So the distance between the objects can be showed in matrix form [6]:

$$D'_{nn} = \begin{pmatrix} w'_{11} & w'_{12} & \cdots & w'_{1n} \\ w'_{21} & w'_{22} & \cdots & w'_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w'_{n1} & w'_{n2} & \cdots & w'_{nn} \end{pmatrix} \quad (1)$$

Where w'_{ij} is the distance between objects a_i and a_j [7]. For the convenience of calculation, the matrix element w'_{ij} is rounded to integer w_{ij} which results in a new matrix D_{nn}.

$$D_{nn} = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{pmatrix} \quad (2)$$

Definition2. Core object

If the ε-neighborhood of an object contains at least a minimum number, *Min*, of objects, and then the object is called a core object. So an object p is a core object if $|N_\varepsilon(p)| \geq Min$.

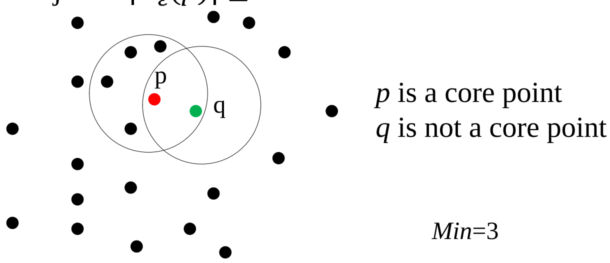

Fig.1 core object

Definition3. Directly density-reachable

Given a set of objects, *D*, an object q is directly density-reachable from an object p if q is within the ε-neighborhood of p, and p is a core object. That is, p and q satisfy the following conditions.

1) $p \in N_\varepsilon(q)$
2) $|N_\varepsilon(q)| \geq Min$

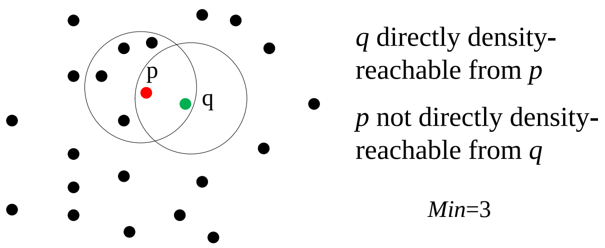

Fig.2 directly density-reachable

Definition4. Density-reachable

An object p is density-reachable from an object q with respect to ε and *Min* in a set of objects, *D*, if there is a chain of points $p_1, \dots, p_1$, where $p_1 = q$ and $p_n = p$ such that p_{i+1} is directly density-reachable from p_i with respect to ε and *Min*, for $1 \leq i \leq n, p_i \in D$.

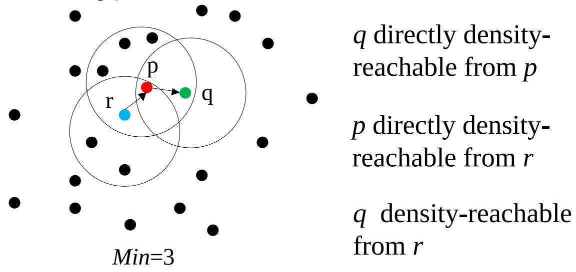

Fig.3 density-reachable

Definition5. Density-connected

An object p is density-connected to an object q with respect to ε and *Min* in a set of objects, *D*, if there is an object $o \in D$ such that both p and q are density-reachable from o with respect to ε and *Min*.

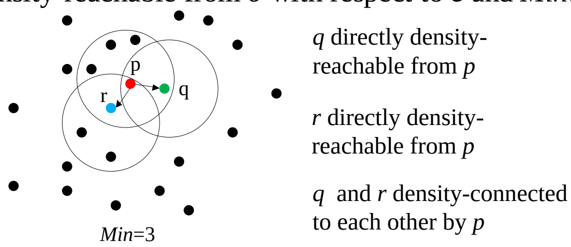

Fig.4 density-connected

Definition6. Cluster

Given a set of objects, *D*, a cluster *C* with respect to ε and *Min* is a non-empty subset of *D* satisfying the following conditions.

1) $\forall p, q \in D$: if $p \in C$ and q is density-reachable from p with respect to ε and *Min*, then $q \in C$.

2) $\forall p, q \in C$: p is density-connected to q with respect to ε and *Min*.

Definition7. Noise

Given a set of objects, *D*, let $C_1, \cdots, C_k$ be the clusters of the database *D* with respect to ε and *Min*, $i = 1, \cdots, k$. Then the noise is defined as the set of points in the database *D* not belonging to any cluster C_i. That is, noise is a set $N = \{p \in D | \forall\, i: p \notin C_i, i = 1, \cdots, k\}$.

Given the parameters ε and *Min*, a cluster is discovered in a two-step approach. First, select an arbitrary point from the database satisfying the core point condition as a seed. Second, retrieve all points that are density-reachable from the seed and group them as a cluster.

According to the definitions of the DBSCAN clustering algorithm above and the characteristics of the P system with active membranes, this paper specifies the procedure of the DBSCAN clustering algorithm. And it is as follows.

1. Compare the ε-neighborhood of each object in the dataset with *Min*;
2. Determine if the object is core object. If it is, then go to step *3*; if not, it is defined to be a noise;
3. Create new clusters with core objects;
4. Collect directly density-reachable objects from one core object and determine if the directly density-reachable object is core object. If it is, combine the two clusters into one;
5. Repeat step *4* until there is no change in the clusters.

2.2 P Systems with Active Membranes

P systems are distributed and parallel models processing multisets of objects using evolution, communication and other types of rules encapsulated into regions delimited by membranes. From a biological point of view, a possible weakness of P system mostly used is the fact that the membrane structure is static and does not evolve during the computation. For this reason, the P system with active membranes was introduced in [4, 8], having rules which directly involve the membranes where the objects evolve and also making the membranes themselves evolve. Using the possibility to change membrane structure, it can create an exponential working space in linear time, which can then be used in a parallel computation for solving computationally hard problems.

In the literature, this new model of P systems has been successfully used to design solutions to some well-known NP–complete problems, such as SAT [9], Subset Sum [10], Knapsack [11], Bin Packing [12], Partition [13], and the Common Algorithmic Problem[14]. Recently, it has also been proposed to solve clustering problems, as its characteristics can improve the efficiency of clustering process.

In this paper, we work with a variant of P systems with active membranes that does not use polarizations but use promoters, which in fact means that we simply may forget the polarizations of membranes and introduce the promoters of rules.

Firstly, we define the model which we work with: P systems with active membranes. A P system with active membranes (without polarizations) is a construct:

$$\Pi = (O, T, H, \mu, w_1, \dots, w_n, R) \quad (3)$$

Where:

1. $n \geq 1$ is the initial degree of the system;
2. O is the alphabet of objects;
3. T is the output of the P system;
4. H is a finite set of labels for membranes, $H = \{1, 2, \cdots, n\}$;
5. μ is a membrane structure, consisting of n membranes, labelled (not necessarily in a one-to-one manner) with elements of H;
6. w_i describes the multisets of objects in membrane *i*;
7. R_i describes the rules in membrane *i*;

Each rule in the P system is in the form of $u \to v$ or $(u \to v)^c$. u is a string representing multisets of objects in O, and v is a string in the form of $v = v'$ or $v = v'\delta$. v' is a string over $O \times \{here, out, in\}$ and δ is a symbol indicating membrane dissolving after executing the rule. c is a promoter or inhibitor of the rules. This paper only introduces the promoters of the rules. We call $p, p \in O$, a promoter for rule r, and we denote this by $(r)^p$, if rule r is active only in the presence of p. Furthermore, the rules in active membranes are developmental rules with the following forms:

1) $[a]_h^p \to [v]_h$, where $h \in H, a, p \in O, v \in O^*$

Object evolution rules. An object a is evolved into v in a membrane h with the presence of promoter p, and the membranes and the promoters are neither taking part in the application of these rules nor are they modified by them.

2) $a[\ ]_h^p \to [b]_h$, where $h \in H, a, b, p \in O$

Send-in communication rules. An object a is introduced into the membrane h with the presence of promoters p, possibly modified to b during the process; but the membranes and the promoters are not evolved during the process.

3) $[a]_h^p \to [\ ]_h b$, where $h \in H, a, b, p \in O$

Send-out communication rules. An object a is sent out of the membrane h with the presence of promoters p, possibly modified to b during the process; but the membranes and the promoters are not evolved during the process.

4) $[a]_h^p \to b$, where $h \in H, a, b, p \in O$

Dissolving rules. An object a is modified into b with the presence of promoter p, at the same time the surrounding membrane is dissolved. And the remaining objects the former membranes are left free in the region immediately above it.

5) $[a]_{h_1}^p \to [b]_{h_2}[c]_{h_3}$,
where $h_1, h_2, h_3 \in H, a, b, c, p \in O$

Division rules. In reaction with the promoter p, the membrane is divided into two membranes with, maybe, different labels; the object a specified in the rule is replaced in the two new membranes by possibly new objects b, c; and the remaining objects are duplicated in the process.

6) $[\ ]_{h_1}^{p_1}[\ ]_{h_2}^{p_2} \rightarrow [\ ]_{h_3}$,

where $h_1, h_2, h_3 \in H, p_1, p_2 \in O$

Merging rules. When there are promoters p_1 in the membrane h_1 and promoters p_2 in the membrane h_2, the two membranes are merged into a single membrane h_3; the objects of the former membranes are put together in the new membrane.

7) $[a]_{h_1}^{p_1}[b]_{h_2}^{p_2} \rightarrow [c]_{h_3}$,

where $h_1, h_2, h_3 \in H, a, b, c, p_1, p_2 \in O$

Fusion rule. With the promoters p_1 in membrane h_1 and promoters p_2 in membrane h_2, the two membranes are merged into a single membrane h_3; the object a, b respectively specified in membrane h_1, h_2 are replaced in the new membrane h_3 by possibly a new object c; and the remaining objects of the former membranes are put together in the new membrane

The rules are applied in the non-deterministic maximally parallel mode. The computation stops when there is no rule which can be applied to objects and membranes in the last configuration. The result of the computation is the collection of objects expelled from the output membrane during the whole computation. Only halting computations give a result, non-halting computations give no output.

3 A P System with Active Membranes for DBSCAN Clustering Algorithm

3.1 Designing a P System for DBSCAN Clustering Algorithm

In this part, a P System with active membranes for DBSCAN clustering algorithm is proposed. The initial and final structures of this P system are shown in Fig.5 and Fig.6.

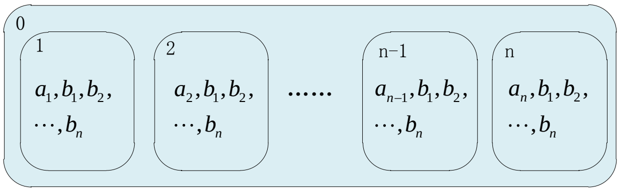

Fig.5 the initial structure

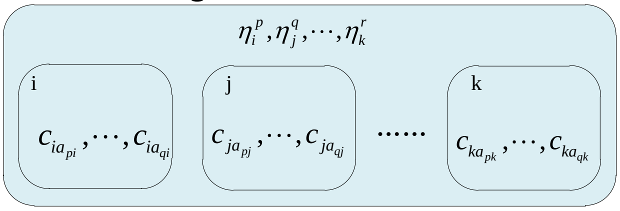

Fig.6 the final structure

In this P system, there are *n+1* membranes. Membrane *0* is the skin membrane and Membranes *1* to *n* respectively represents *n* objects in the initial. In the process of clustering, not only the objects evolve but also the membrane structure. Through the rules of active membranes, the k membranes obtained finally indicate k clusters of the results. Significantly, it uses the matrix D_{nn} to compare the distances between the n objects and we define each individual w_{ij} as nonnegative integer variables in this P system.

The P system with active membranes for DBSCAN clustering is defined as follows:

$\Pi = (O, T, \mu, w_0, w_1, \dots, w_n, R_0, R_1, \dots, R_n, \rho)$ (4)

Where:

1. Working alphabet:

$O = \{a_i, b_j, d_{i,j}, \eta_i, \gamma_i, \sigma, B_j, c_{pa_i}, N_i\}$

2. Output:

$T = \{\eta_i^k\},\ 1 \le i, k \le n$

3. Membrane structure:

$\mu = [_0[_1]_1[_2]_2 \cdots [_{n-1}]_{n-1}[_n]_n]_0$

4. Initial multisets:

$\omega_0 = \{\lambda\}$; ω_0 represents the initial set of objects in skin membrane;

$\omega_i = \{a_i, b_1, b_2, \cdots, b_n\text{: } 1 \le i \le n\}$;

5. The set R of evolution rules consists of the following rules:

Rules in the membrane labelled i $\{1 \le i \le n\}$:

$r_1 = \{a_i b_1 b_2 \cdots b_n \rightarrow d_{i,1}^{w_{i1}} d_{i,2}^{w_{i2}} \cdots d_{i,n}^{w_{in}} | 1 \le i, j \le n\}$

$r_2 = \{d_{i,j}^t \rightarrow \eta_i B_j | 1 \le i, j \le n, 0 \le t \le \varepsilon\} \cup \{d_{i,j}^t \rightarrow \sigma | 1 \le i, j \le n, t > \varepsilon\}$

$r_3 = \{(\eta_i^k \rightarrow \gamma_i^k \eta_i^k | Min \le k \le \mathrm{n}) \cup (B_j \gamma_i \rightarrow c_{i,a_j} | 1 \le i, j \le n\ )\}$

$\cup \{B_j \eta_i^k \rightarrow N_i \eta_i^1\ | 1 \le i, j \le n, 0 < k < Min\}$

$r_4 = \{\sigma \rightarrow \lambda\} \cup \{B_j \rightarrow \lambda\}$

Rules in the skin membrane labelled *0*:

$r_5 = \{[c_{ja_p}\eta_j]_j^{c_{ja_i}}[\eta_i^k]_i^{c_{ia_j}} \rightarrow [c_{ia_p}]_j[\eta_i^{k+1}]_i | 1 \le i < j \le n,\ 1 \le p,k \le n,\ i \ne p\}$

$r_6 = \{([\,]_j^{c_{ja_i}}[\,]_i^{c_{ia_j}} \rightarrow [\,]_i) \cup ([c_{qa_j}]_q \rightarrow [c_{qa_i}]_q) | 1 \le i < j < q \le n\}$

$r_7 = \{([c_{ia_p}c_{ia_p}\eta_i^k]_i \rightarrow [c_{ia_p}\eta_i^{k-1}]_i) \cup ([c_{ja_i}\eta_j]_i \rightarrow [\lambda]_i) | 1 \le i,j,p,k \le n\}$

$r_8 = \{[c_{ia_p}]_i^{c_{ia_p}}[N_p\eta_p^k]_p \rightarrow [c_{ia_p}]_i | 1 \le i,p,k \le n\}$

$r_9 = \{[\eta_i^k]_i \rightarrow \eta_i^k[\,]_i | 1 \le i,k \le n\}$

6. The priority relations ρ over R: $\rho = \{r_1 > r_2 > r_3 > r_4\} \cup \{r_5 > r_6 > r_7 > r_8 > r_9\}$

3.2 The Computations in P System

At the beginning of a computation, the membrane labelled $i\ \{1 \le i \le n\}$ contains objects $a_i, b_1, \cdots, b_n$. The object a_i represents the *i*-th object of the dataset and $b_1, \cdots, b_n$ represents all objects in dataset. Initially in the P system the only rules that can be applied is r_1 [15] in membrane labelled i. It produces the object $D_{i,j}^{w_{ij}}$ $(1 \le i,j \le n)$ which is used to compare the distances between a_i and other objects in dataset. Then η_i and σ is obtained according to the distance $D_{i,j}^{w_{ij}}$. The object η_i indicates that the distance between a_i and b_j is less than ε. The object σ indicates that the distance between a_i and b_j is more than ε. If the number of η_i is more than minimum number *Min*, we call a_i a core point and mark its directly density-reachable members as c_{ia_j}. Or this point is defined to be a noise and it is marked as N_i. At the same time, the multiplicity of the object η_i represents the number of density–reachable objects for the core object a_i.

After determining if each object is core object, the rules in skin membrane are triggered. The object evolution rule r_5 with promoters is executed extremely to find all directly density-reachable objects of the core object. When there is promoter c_{ja_i} in the membrane j and promoters c_{ia_j} in the membrane i, the rule r_5 is activated to transform c_{ja_p} into c_{ia_p} and increase the multiplicity of the object η_i and decrease the multiplicity of the object η_j. Then the two membranes i and j are merged into a single membrane i, and the objects of the former membranes are put together in the membrane i. Meanwhile, the rule r_7 remove the same object in the new membrane and decrease the multiplicity of the object η_i. This indicates the combination of the similar object, which is the process of DBSCAN clustering. And next, the object c_{qa_j} in membrane q is transformed by c_{qa_i} to make the core object a_i continue to collect density-reachable objects. After the core objects collecting all the density-reachable objects, the rule r_6 is executed to remove redundant object N_p, as the object a_p is a density-reachable object from a core object. Finally, the *send-out* communication rule r_9 is carried out to send the object η_i to the skin membrane. Every index i indicates one cluster and the multiplicity of the object η_i indicate the number of members in each cluster. Particularly, when the multiplicity of the object η_p is 1, it means a_p is a noise in this dataset.

These rules are used maximum parallel in each membrane when calculating. This P system will halt if no more rules can be executed and no more objects η_i can be obtained [16]. And at this moment, the configuration of each membrane is achieved to be stable. As a consequence, each membrane in the skin membrane represents one cluster, and the objects in each membrane represent the members in each cluster. The DBSCAN clustering algorithm is achieved in this P system with active membranes successfully.

4 Test and Analysis

In order to verify the feasibility and effectiveness of this P system with active membranes for DBSCAN, we cluster an example of dataset to obtain the final results. As an example, the 20 integral points are considered to be clustered. (0,0), (1,4), (1,5), (2,6), (3,6), (5,6), (6,5), (6,4), (7,3), (7,2), (7,1), (9,1), (10,1), (11,1), (11,5), (13,5), (15,5), (14,6), (12,6), (13,7). And this example suppose that $\varepsilon = 3$, *Min*=2. The initial structure of this P system is shown in Fig.7.

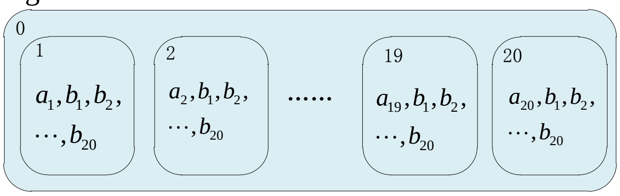

Fig.7 the initial structure of this P system

The distance matrix $D_{20,20}$ of these integral *20* points can be obtained as follows. (The choice of a distance function for two points is Manhattan distance in this paper for convenience.)

$$D_{20,20}=\begin{pmatrix} 0&5&6&8&9&11&11&10&10&9&8&10&11&12&16&18&20&20&18&20\\ 5&0&1&3&4&6&6&5&7&8&9&11&12&13&11&13&15&15&13&15\\ 6&1&0&2&3&5&5&6&8&9&10&12&13&14&10&12&14&14&12&14\\ 8&3&2&0&1&3&5&6&8&9&10&12&13&14&10&12&14&12&10&12\\ 9&4&3&1&0&2&4&5&7&8&9&11&12&13&9&11&13&11&9&11\\ 11&6&5&3&2&0&2&3&5&6&7&9&10&11&7&9&11&9&7&9\\ 11&6&5&5&4&2&0&1&3&4&5&7&8&9&5&7&9&9&7&9\\ 10&5&6&6&5&3&1&0&2&3&4&6&7&8&6&8&10&10&8&10\\ 10&7&8&8&7&5&3&2&0&1&2&4&5&6&6&8&10&10&8&10\\ 9&8&9&9&8&6&4&3&1&0&1&3&4&5&7&9&11&11&9&11\\ 8&9&10&10&9&7&5&4&2&1&0&2&3&4&8&10&12&12&10&12\\ 10&11&12&12&11&9&7&6&4&3&2&0&1&2&6&8&10&10&8&10\\ 11&12&13&13&12&10&8&7&5&4&3&1&0&1&5&7&9&9&7&9\\ 12&13&14&14&13&11&9&8&6&5&4&2&1&0&4&6&8&8&6&8\\ 16&11&10&10&9&7&5&6&6&7&8&6&5&4&0&2&4&4&2&4\\ 18&13&12&12&11&9&7&8&8&9&10&8&7&6&2&0&2&2&2&2\\ 20&15&14&14&13&11&9&10&10&11&12&10&9&8&4&2&0&2&4&4\\ 20&15&14&12&11&9&9&10&10&11&12&10&9&8&4&2&2&0&2&2\\ 18&13&12&10&9&7&7&8&8&9&10&8&7&6&2&2&4&2&0&2\\ 20&15&14&12&11&9&9&10&10&11&12&10&9&8&4&2&4&2&2&0 \end{pmatrix} \quad (5)$$

Operating in P system with DBSCAN clustering algorithm, we can obtain the computational process of P system. It is shown in Table 1- 4. Particularly, some repeated steps in the process are omitted.

Table 1 the clustering process in P system

	Step0(initial state)	*Step1*	*Step2*
Mem0	λ	λ	λ
Mem1	$a_1, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{1,1}^{0}, d_{1,2}^{5}, \cdots, d_{1,19}^{18}, d_{1,20}^{20}(r_1)$	$B_1, \eta_1^1, \sigma^{19}(r_2)$
Mem2	$a_2, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{2,1}^{5}, d_{2,2}^{0}, \cdots, d_{2,19}^{13}, d_{2,20}^{15}(r_1)$	$B_2, B_3, B_4, \eta_2^3, \sigma^{17}(r_2)$
Mem3	$a_3, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{3,1}^{6}, d_{3,2}^{1}, \cdots, d_{3,19}^{12}, d_{3,20}^{14}(r_1)$	$B_2, B_3, B_4, B_5, \eta_3^4, \sigma^{16}(r_2)$
Mem4	$a_4, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{4,1}^{8}, d_{4,2}^{3}, \cdots, d_{4,19}^{10}, d_{4,20}^{12}(r_1)$	$B_2, B_3, B_4, B_5, B_6, \eta_4^5, \sigma^{15}(r_2)$
Mem5	$a_5, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{5,1}^{9}, d_{5,2}^{4}, \cdots, d_{5,19}^{9}, d_{5,20}^{11}(r_1)$	$B_3, B_4, B_5, B_6, \eta_5^4, \sigma^{16}(r_2)$
Mem6	$a_6, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{6,1}^{11}, d_{6,2}^{6}, \cdots, d_{6,19}^{7}, d_{6,20}^{9}(r_1)$	$B_4, B_5, B_6, B_7, B_8, \eta_6^5, \sigma^{15}(r_2)$
Mem7	$a_7, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{7,1}^{11}, d_{7,2}^{6}, \cdots, d_{7,19}^{7}, d_{7,20}^{9}(r_1)$	$B_6, B_7, B_8, B_9, \eta_7^4, \sigma^{16}(r_2)$
Mem8	$a_8, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{8,1}^{10}, d_{8,2}^{5}, \cdots, d_{8,19}^{8}, d_{8,20}^{10}(r_1)$	$B_6, B_7, B_8, B_9, B_{10}, \eta_8^5, \sigma^{15}(r_2)$
Mem9	$a_9, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{9,1}^{10}, d_{9,2}^{7}, \cdots, d_{9,19}^{8}, d_{9,20}^{10}(r_1)$	$B_7, B_8, B_9, B_{10}, B_{11}, \eta_9^5, \sigma^{15}(r_2)$
Mem10	$a_{10}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{10,1}^{9}, d_{10,2}^{8}, \cdots, d_{10,19}^{9}, d_{10,20}^{11}(r_1)$	$B_8, B_9, B_{10}, B_{11}, B_{12}, \eta_{10}^5, \sigma^{15}(r_2)$
Mem11	$a_{11}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{11,1}^{8}, d_{11,2}^{9}, \cdots, d_{11,19}^{10}, d_{11,20}^{12}(r_1)$	$B_9, B_{10}, B_{11}, B_{12}, \eta_{11}^4, \sigma^{16}(r_2)$
Mem12	$a_{12}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{12,1}^{10}, d_{12,2}^{11}, \cdots, d_{12,19}^{8}, d1_{2,20}^{10}(r_1)$	$B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, \eta_{12}^5, \sigma^{15}(r_2)$
Mem13	$a_{13}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{13,1}^{11}, d_{13,2}^{12}, \cdots, d_{13,19}^{7}, d_{13,20}^{9}(r_1)$	$B_{11}, B_{12}, B_{13}, B_{14}, \eta_{13}^4, \sigma^{16}(r_2)$
Mem14	$a_{14}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{14,1}^{12}, d_{14,2}^{13}, \cdots, d_{14,19}^{6}, d_{14,20}^{8}(r_1)$	$B_{12}, B_{13}, B_{14}, \eta_{14}^3, \sigma^{17}(r_2)$
Mem15	$a_{15}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{15,1}^{16}, d_{15,2}^{11}, \cdots, d_{15,19}^{2}, d_{15,20}^{4}(r_1)$	$B_{15}, B_{16}, B_{19}, \eta_{15}^3, \sigma^{17}(r_2)$
Mem16	$a_{16}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{16,1}^{18}, d_{16,2}^{13}, \cdots, d_{16,19}^{2}, d_{16,20}^{2}(r_1)$	$B_{15}, B_{16}, B_{17}, B_{18}, B_{19}, B_{20}, \eta_{16}^6, \sigma^{14}(r_2)$
Mem17	$a_{17}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{17,1}^{20}, d_{17,2}^{15}, \cdots, d_{17,19}^{4}, d_{17,20}^{4}(r_1)$	$B_{16}, B_{17}, B_{18}, \eta_{17}^3, \sigma^{17}(r_2)$
Mem18	$a_{18}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{18,1}^{20}, d_{18,2}^{15}, \cdots, d_{18,19}^{2}, d_{18,20}^{2}(r_1)$	$B_{16}, B_{17}, B_{18}, B_{19}, B_{20}, \eta_{18}^5, \sigma^{15}(r_2)$
Mem19	$a_{19}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{19,1}^{18}, d_{19,2}^{13}, \cdots, d_{19,19}^{0}, d_{19,20}^{2}(r_1)$	$B_{15}, B_{16}, B_{18}, B_{19}, B_{20}, \eta_{19}^5, \sigma^{15}(r_2)$
Mem20	$a_{20}, b_1, b_2, \cdots, b_{19}, b_{20}$	$d_{20,1}^{20}, d_{20,2}^{15}, \cdots, d_{20,19}^{2}, d_{20,20}^{0}(r_1)$	$B_{16}, B_{18}, B_{19}, B_{20}, \eta_{20}^4, \sigma^{16}(r_2)$

Table 1(continued) the clustering process in P system

	Step3	*Step4*
Mem0	λ	λ
Mem1	$N_1, \eta_1^1, \sigma^{19}(r_3)$	$N_1, \eta_1^1, \sigma^{19}(r_3)$
Mem2	$B_2, B_3, B_4, \gamma_2^3, \eta_2^3, \sigma^{17}(r_3)$	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^3, \sigma^{17}(r_3)$
Mem3	$B_2, B_3, B_4, B_5, \gamma_3^4, \eta_3^4, \sigma^{16}(r_3)$	$c_{3a_2}, c_{3a_3}, c_{3a_4}, c_{3a_5}, \eta_3^4, \sigma^{16}(r_3)$
Mem4	$B_2, B_3, B_4, B_5, B_6, \gamma_4^5, \eta_4^5, \sigma^{15}(r_3)$	$c_{4a_2}, c_{4a_3}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^5, \sigma^{15}(r_3)$
Mem5	$B_3, B_4, B_5, B_6, \gamma_5^4, \eta_5^4, \sigma^{16}(r_3)$	$c_{5a_3}, c_{5a_4}, c_{5a_5}, c_{5a_6}, \eta_5^4, \sigma^{16}(r_3)$
Mem6	$B_4, B_5, B_6, B_7, B_8, \gamma_6^5, \eta_6^5, \sigma^{15}(r_3)$	$c_{6a_4}, c_{6a_5}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^5, \sigma^{15}(r_3)$
Mem7	$B_6, B_7, B_8, B_9, \gamma_7^4, \eta_7^4, \sigma^{16}(r_3)$	$c_{7a_6}, c_{7a_7}, c_{7a_8}, c_{7a_9}, \eta_7^4, \sigma^{16}(r_3)$
Mem8	$B_6, B_7, B_8, B_9, B_{10}, \gamma_8^5, \eta_8^5, \sigma^{15}(r_3)$	$c_{8a_6}, c_{8a_7}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^5, \sigma^{15}(r_3)$
Mem9	$B_7, B_8, B_9, B_{10}, B_{11}, \gamma_9^5, \eta_9^5, \sigma^{15}(r_3)$	$c_{9a_7}, c_{9a_8}, c_{9a_9}, c_{9a_{10}}, c_{9a_{11}}, \eta_9^5, \sigma^{15}(r_3)$
Mem10	$B_8, B_9, B_{10}, B_{11}, B_{12}, \gamma_{10}^5, \eta_{10}^5, \sigma^{15}(r_3)$	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^5, \sigma^{15}(r_3)$
Mem11	$B_9, B_{10}, B_{11}, B_{12}, \gamma_{11}^4, \eta_{11}^4, \sigma^{16}(r_3)$	$c_{11a_9}, c_{11a_{10}}, c_{11a_{11}}, c_{11a_{12}}, \eta_{11}^4, \sigma^{16}(r_3)$
Mem12	$B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, \gamma_{12}^5, \eta_{12}^5, \sigma^{15}(r_3)$	$c_{12a_{10}}, c_{12a_{11}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^5, \sigma^{15}(r_3)$
Mem13	$B_{11}, B_{12}, B_{13}, B_{14}, \gamma_{13}^4, \eta_{13}^4, \sigma^{16}(r_3)$	$c_{13a_{11}}, c_{13a_{12}}, c_{13a_{13}}, c_{13a_{14}}, \eta_{13}^4, \sigma^{16}(r_3)$
Mem14	$B_{12}, B_{13}, B_{14}, \gamma_{14}^3, \eta_{14}^3, \sigma^{17}(r_3)$	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3, \sigma^{17}(r_3)$
Mem15	$B_{15}, B_{16}, B_{19}, \gamma_{15}^3, \eta_{15}^3, \sigma^{17}(r_3)$	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^3, \sigma^{17}(r_3)$
Mem16	$B_{15}, B_{16}, B_{17}, B_{18}, B_{19}, B_{20}, \gamma_{16}^6, \eta_{16}^6, \sigma^{14}(r_3)$	$c_{16a_{15}}, c_{16a_{16}}, c_{16a_{17}}, c_{16a_{18}}, c_{16a_{19}}, c_{16a_{20}}, \eta_{16}^6, \sigma^{14}(r_3)$
Mem17	$B_{16}, B_{17}, B_{18}, \gamma_{17}^3, \eta_{17}^3, \sigma^{17}(r_3)$	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^3, \sigma^{17}(r_3)$
Mem18	$B_{16}, B_{17}, B_{18}, B_{19}, B_{20}, \gamma_{18}^5, \eta_{18}^5, \sigma^{15}(r_3)$	$c_{18a_{16}}, c_{18a_{17}}, c_{18a_{18}}, c_{18a_{19}}, c_{18a_{20}}, \eta_{18}^5, \sigma^{15}(r_3)$
Mem19	$B_{15}, B_{16}, B_{18}, B_{19}, B_{20}, \gamma_{19}^5, \eta_{19}^5, \sigma^{15}(r_3)$	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^5, \sigma^{15}(r_3)$
Mem20	$B_{16}, B_{18}, B_{19}, B_{20}, \gamma_{20}^4, \eta_{20}^4, \sigma^{16}(r_3)$	$c_{20a_{16}}, c_{20a_{18}}, c_{20a_{19}}, c_{20a_{20}}, \eta_{20}^4, \sigma^{16}(r_3)$

Table 1(continued) the clustering process in P system

	Step5	*Step6*
Mem0	λ	λ
Mem1	$N_1, \eta_1^1(r_4)$	$N_1, \eta_1^1(r_4)$
Mem2	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^3(r_4)$	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^4(r_5)$
Mem3	$c_{3a_2}, c_{3a_3}, c_{3a_4}, c_{3a_5}, \eta_3^4(r_4)$	$c_{3a_2}, c_{2a_3}, c_{3a_4}, c_{3a_5}, \eta_3^3(r_5)$
Mem4	$c_{4a_2}, c_{4a_3}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^5(r_4)$	$c_{4a_2}, c_{4a_3}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^6(r_5)$
Mem5	$c_{5a_3}, c_{5a_4}, c_{5a_5}, c_{5a_6}, \eta_5^4(r_4)$	$c_{4a_3}, c_{5a_4}, c_{5a_5}, c_{5a_6}, \eta_5^3(r_5)$
Mem6	$c_{6a_4}, c_{6a_5}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^5(r_4)$	$c_{6a_4}, c_{6a_5}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^6(r_5)$
Mem7	$c_{7a_6}, c_{7a_7}, c_{7a_8}, c_{7a_9}, \eta_7^4(r_4)$	$c_{7a_6}, c_{6a_7}, c_{7a_8}, c_{7a_9}, \eta_7^3(r_5)$
Mem8	$c_{8a_6}, c_{8a_7}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^5(r_4)$	$c_{8a_6}, c_{8a_7}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^6(r_5)$
Mem9	$c_{9a_7}, c_{9a_8}, c_{9a_9}, c_{9a_{10}}, c_{9a_{11}}, \eta_9^5(r_4)$	$c_{8a_7}, c_{9a_8}, c_{9a_9}, c_{9a_{10}}, c_{9a_{11}}, \eta_9^4(r_5)$
Mem10	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^5(r_4)$	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^6(r_5)$

Mem11	$c_{11a_9}, c_{11a_{10}}, c_{11a_{11}}, c_{11a_{12}}, \eta_{11}^4(r_4)$	$c_{10a_9}, c_{11a_{10}}, c_{11a_{11}}, c_{11a_{12}}, \eta_{11}^3(r_5)$
Mem12	$c_{12a_{10}}, c_{12a_{11}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^5(r_4)$	$c_{12a_{10}}, c_{12a_{11}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^6(r_5)$
Mem13	$c_{13a_{11}}, c_{13a_{12}}, c_{13a_{13}}, c_{13a_{14}}, \eta_{13}^4(r_4)$	$c_{12a_{11}}, c_{13a_{12}}, c_{13a_{13}}, c_{13a_{14}}, \eta_{13}^3(r_5)$
Mem14	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3(r_4)$	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3(r_5)$
Mem15	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^3(r_4)$	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^4(r_5)$
Mem16	$c_{16a_{15}}, c_{16a_{16}}, c_{16a_{17}}, c_{16a_{18}}, c_{16a_{19}}, c_{16a_{20}}, \eta_{16}^6(r_4)$	$c_{16a_{15}}, c_{15a_{16}}, c_{16a_{17}}, c_{16a_{18}}, c_{16a_{19}}, c_{16a_{20}}, \eta_{16}^5(r_5)$
Mem17	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^3(r_4)$	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^4(r_5)$
Mem18	$c_{18a_{16}}, c_{18a_{17}}, c_{18a_{18}}, c_{18a_{19}}, c_{18a_{20}}, \eta_{18}^5(r_4)$	$c_{17a_{16}}, c_{18a_{17}}, c_{18a_{18}}, c_{18a_{19}}, c_{18a_{20}}, \eta_{18}^4(r_5)$
Mem19	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^5(r_4)$	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^6(r_5)$
Mem20	$c_{20a_{16}}, c_{20a_{18}}, c_{20a_{19}}, c_{20a_{20}}, \eta_{20}^4(r_4)$	$c_{19a_{16}}, c_{20a_{18}}, c_{20a_{19}}, c_{20a_{20}}, \eta_{20}^3(r_5)$

Table 1(continued) the clustering process in P system

	Step7	*Step8*
Mem0	λ	λ
Mem1	$N_1, \eta_1^1(r_4)$	$N_1, \eta_1^1(r_4)$
Mem2	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^5(r_5)$	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^6(r_5)$
Mem3	$c_{3a_2}, c_{2a_3}, c_{2a_4}, c_{3a_5}, \eta_3^2(r_5)$	$c_{3a_2}, c_{2a_3}, c_{2a_4}, c_{2a_5}, \eta_3^1(r_5)$
Mem4	$c_{4a_2}, c_{4a_3}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^7(r_5)$	$c_{4a_2}, c_{4a_3}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^8(r_5)$
Mem5	$c_{4a_3}, c_{5a_4}, c_{4a_5}, c_{5a_6}, \eta_5^2(r_5)$	$c_{4a_3}, c_{5a_4}, c_{4a_5}, c_{4a_6}, \eta_5^1(r_5)$
Mem6	$c_{6a_4}, c_{6a_5}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^7(r_5)$	$c_{6a_4}, c_{6a_5}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^8(r_5)$
Mem7	$c_{7a_6}, c_{6a_7}, c_{6a_8}, c_{7a_9}, \eta_7^2(r_5)$	$c_{7a_6}, c_{6a_7}, c_{6a_8}, c_{6a_9}, \eta_7^1(r_5)$
Mem8	$c_{8a_6}, c_{8a_7}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^7(r_5)$	$c_{8a_6}, c_{8a_7}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^8(r_5)$
Mem9	$c_{8a_7}, c_{9a_8}, c_{8a_9}, c_{9a_{10}}, c_{9a_{11}}, \eta_9^3(r_5)$	$c_{8a_7}, c_{9a_8}, c_{8a_9}, c_{8a_{10}}, c_{9a_{11}}, \eta_9^2(r_5)$
Mem10	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^7(r_5)$	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^8(r_5)$
Mem11	$c_{10a_9}, c_{11a_{10}}, c_{10a_{11}}, c_{11a_{12}}, \eta_{11}^2(r_5)$	$c_{10a_9}, c_{11a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{11}^1(r_5)$
Mem12	$c_{12a_{10}}, c_{12a_{11}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^7(r_5)$	$c_{12a_{10}}, c_{12a_{11}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^8(r_5)$
Mem13	$c_{12a_{11}}, c_{13a_{12}}, c_{12a_{13}}, c_{13a_{14}}, \eta_{13}^2(r_5)$	$c_{12a_{11}}, c_{13a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{13}^1(r_5)$
Mem14	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3$	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3$
Mem15	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^5(r_5)$	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^6(r_5)$
Mem16	$c_{16a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{16a_{18}}, c_{16a_{19}}, c_{16a_{20}}, \eta_{16}^4(r_5)$	$c_{16a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{15a_{18}}, c_{16a_{19}}, c_{16a_{20}}, \eta_{16}^3(r_5)$
Mem17	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^5(r_5)$	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^6(r_5)$
Mem18	$c_{17a_{16}}, c_{18a_{17}}, c_{17a_{18}}, c_{18a_{19}}, c_{18a_{20}}, \eta_{18}^3(r_5)$	$c_{17a_{16}}, c_{18a_{17}}, c_{17a_{18}}, c_{17a_{19}}, c_{18a_{20}}, \eta_{18}^2(r_5)$
Mem19	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^7(r_5)$	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^8(r_5)$
Mem20	$c_{19a_{16}}, c_{19a_{18}}, c_{20a_{19}}, c_{20a_{20}}, \eta_{20}^2(r_5)$	$c_{19a_{16}}, c_{19a_{18}}, c_{20a_{19}}, c_{19a_{20}}, \eta_{20}^1(r_5)$

Table 2 the clustering process in P system

	Step9	*Step10*
Mem0	λ	λ
Mem1	N_1, η_1^1	N_1, η_1^1
Mem2	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^6(r_5)$	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^6,$
		$c_{3a_2}, c_{2a_3}, c_{2a_4}, c_{2a_5}, \eta_3^1(r_6)$
Mem4	$c_{4a_2}, c_{4a_2}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^8,$	$c_{4a_2}, c_{4a_2}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^8,$
	$c_{4a_3}, c_{5a_4}, c_{4a_5}, c_{4a_6}, \eta_5^1(r_6)$	$c_{4a_2}, c_{5a_4}, c_{4a_5}, c_{4a_6}, \eta_5^1(r_6)$
Mem6	$c_{6a_4}, c_{6a_4}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^8,$	$c_{6a_4}, c_{6a_4}, c_{6a_6}, c_{6a_7}, c_{6a_8}, \eta_6^8,$
	$c_{7a_6}, c_{6a_7}, c_{6a_8}, c_{6a_9}, \eta_7^1(r_6)$	$c_{7a_6}, c_{6a_7}, c_{6a_8}, c_{6a_9}, \eta_7^1(r_6)$
Mem8	$c_{8a_6}, c_{8a_7}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^8(r_5)$	$c_{8a_6}, c_{8a_6}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^9(r_5)$
Mem9	$c_{8a_7}, c_{9a_8}, c_{8a_9}, c_{8a_{10}}, c_{9a_{11}}, \eta_9^2(r_5)$	$c_{8a_6}, c_{9a_8}, c_{8a_9}, c_{8a_{10}}, c_{8a_{11}}, \eta_9^1(r_5)$
Mem10	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^8,$	$c_{10a_8}, c_{10a_9}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^8,$
	$c_{10a_9}, c_{11a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{11}^1(r_6)$	$c_{10a_9}, c_{11a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{11}^1(r_6)$
Mem12	$c_{12a_{10}}, c_{12a_{11}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^8,$	$c_{12a_{10}}, c_{12a_{10}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^8,$
	$c_{12a_{11}}, c_{13a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{13}^1(r_6)$	$c_{12a_{10}}, c_{13a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{13}^1(r_6)$
Mem14	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3$	$c_{14a_{12}}, c_{14a_{13}}, c_{14a_{14}}, \eta_{14}^3$
Mem15	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^6(r_5)$	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^7(r_5)$
Mem16	$c_{16a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{15a_{18}}, c_{16a_{19}}, c_{16a_{20}}, \eta_{16}^3(r_5)$	$c_{16a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{15a_{18}}, c_{15a_{19}}, c_{16a_{20}}, \eta_{16}^2(r_5)$
Mem17	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^6(r_5)$	$c_{17a_{16}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^7(r_5)$
Mem18	$c_{17a_{16}}, c_{18a_{17}}, c_{17a_{18}}, c_{17a_{19}}, c_{18a_{20}}, \eta_{18}^2(r_5)$	$c_{17a_{16}}, c_{18a_{17}}, c_{17a_{18}}, c_{17a_{19}}, c_{17a_{20}}, \eta_{18}^1(r_5)$
Mem19	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^8,$	$c_{19a_{15}}, c_{19a_{16}}, c_{19a_{18}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^8,$
	$c_{19a_{16}}, c_{19a_{18}}, c_{20a_{19}}, c_{19a_{20}}, \eta_{20}^1(r_5)$	$c_{19a_{16}}, c_{19a_{18}}, c_{20a_{19}}, c_{19a_{20}}, \eta_{20}^1(r_6)$

Table 3 the clustering process in P system

	Step11	*Step12*	⋯
Mem0	λ	λ	⋯
Mem1	N_1, η_1^1	N_1, η_1^1	⋯
Mem2	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^5, c_{2a_4}, c_{2a_5}(r_7)$	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^4, c_{2a_5}(r_7)$	⋯
Mem4	$c_{4a_2}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^7, c_{4a_2}, c_{4a_5}, c_{4a_6}(r_7)$	$c_{4a_2}, c_{4a_4}, c_{4a_5}, c_{4a_6}, \eta_4^6, c_{4a_5}, c_{4a_6}(r_7)$	⋯
Mem6	$c_{6a_4}, c_{6a_4}, c_{6a_6}, c_{6a_8}, \eta_6^7,$	$c_{6a_4}, c_{6a_4}, c_{6a_6}, \eta_6^6,$	⋯
	$c_{6a_7}, c_{6a_8}, c_{6a_9}(r_7)$	$c_{6a_7}, c_{6a_8}, c_{6a_9}(r_7)$	
Mem8	$c_{8a_6}, c_{8a_6}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^9$	$c_{8a_6}, c_{8a_8}, c_{8a_9}, c_{8a_{10}}, \eta_8^8$	⋯
	$c_{8a_6}, c_{8a_9}, c_{8a_{10}}, c_{8a_{11}}(r_6)$	$c_{8a_6}, c_{8a_9}, c_{8a_{10}}, c_{8a_{11}}(r_6)$	
Mem10	$c_{10a_8}, c_{10a_8}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^7,$	$c_{10a_8}, c_{10a_8}, c_{10a_{10}}, c_{10a_{11}}, c_{10a_{12}}, \eta_{10}^6,$	⋯
	$c_{10a_{11}}, c_{10a_{12}}(r_7)$	$c_{10a_{12}}(r_7)$	

Mem12	$c_{12a_{10}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^{6}, c_{12a_{10}}, c_{12a_{14}} (r_7)$	$c_{12a_{10}}, c_{12a_{10}}, c_{12a_{12}}, c_{12a_{13}}, c_{12a_{14}}, \eta_{12}^{5}, c_{12a_{14}} (r_7)$	$\cdots$
Mem14	$c_{14a_{12}}, c_{14a_{12}}, c_{14a_{14}}, \eta_{14}^{3}$	$c_{14a_{12}}, c_{14a_{12}}, c_{14a_{14}}, \eta_{14}^{3}$	$\cdots$
Mem15	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^{8}, c_{16a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{15a_{18}}, c_{15a_{19}}, c_{15a_{20}}, \eta_{16}^{1} (r_6)$	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{19}}, \eta_{15}^{7}, c_{15a_{17}}, c_{15a_{18}}, c_{15a_{19}}, c_{15a_{20}} (r_7)$	$\cdots$
Mem17	$c_{17a_{15}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^{6}, c_{17a_{18}}, c_{17a_{19}}, c_{17a_{20}} (r_7)$	$c_{17a_{15}}, c_{17a_{17}}, c_{17a_{18}}, \eta_{17}^{5}, c_{17a_{19}}, c_{17a_{20}} (r_7)$	$\cdots$
Mem19	$c_{19a_{15}}, c_{19a_{15}}, c_{19a_{17}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^{6}, c_{19a_{20}} (r_7)$	$c_{19a_{15}}, c_{19a_{15}}, c_{19a_{17}}, c_{19a_{19}}, c_{19a_{20}}, \eta_{19}^{5} (r_7)$	$\cdots$

Table 4 the clustering process in P system

	Step42	*Step43*
Mem0	λ	$\eta_1^1, \eta_2^4, \eta_{15}^7 (r_9)$
Mem1	N_1, η_1^1	$N_1 (r_9)$
Mem2	$c_{2a_2}, c_{2a_3}, c_{2a_4}, \eta_2^{13}, c_{2a_5}, c_{2a_7}, c_{2a_8}, c_{2a_9}, c_{2a_{10}}, c_{2a_{12}}, c_{2a_{13}}, c_{2a_{14}} (r_7)$	$c_{2a_2}, c_{2a_3}, c_{2a_4} c_{2a_5}, c_{2a_7}, c_{2a_8}, c_{2a_9}, c_{2a_{10}}, c_{2a_{12}}, c_{2a_{13}}, c_{2a_{14}} (r_9)$
Mem15	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{15a_{18}}, c_{15a_{19}}, c_{15a_{20}}, \eta_{15}^{6} (r_7)$	$c_{15a_{15}}, c_{15a_{16}}, c_{15a_{17}}, c_{15a_{18}}, c_{15a_{19}}, c_{15a_{20}} (r_9)$

In the process, Table 1 shows the rules execution in the membrane labeled i , which is used to determine if the object is core object. Table 2 and Table 3 show the membrane mergence in the skin membrane. Two membranes are merged to produce a new cluster by collecting directly density-reachable objects from one core object. Table 4 shows the final membranes and objects remaining in the skin membrane.

Finally, the clustering of these *20* points in this P system is finished. The membrane structure of this P system is changed by the rules of active membranes. So the final structure of this P system is shown in Fig.8.

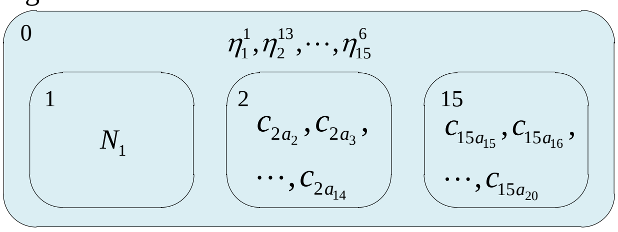

Fig.8 the final structure of this P system

According to the objects $\eta_1^1, \eta_2^4, \eta_{15}^7$ from the skin membrane, which represent three clusters, and the numbers of objects in these three clusters are separately *1*, *13* and *6*. Furthermore there are three members which also indicate three clusters. The objects in each membrane represent the members in each cluster. The members are respectively a_1 and $a_2, a_3, \cdots, a_{14}$ and $a_{15}, a_{16}, \cdots, a_{20}$. The result of the clustering is shown in Fig.9.

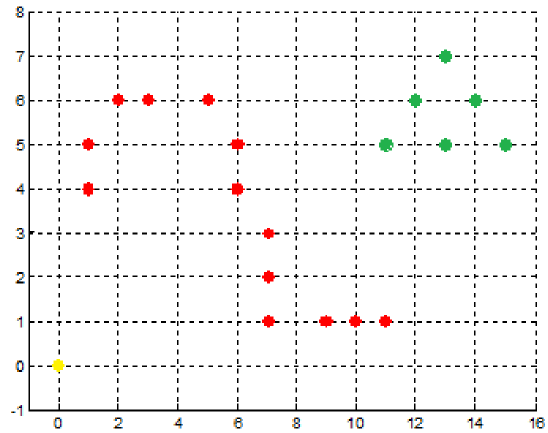

Fig.9 The result of DBSCAN clustering

5 Conclusion

The main feature of the P system with active membranes is that not only the objects evolve but also the membrane structure. Its characteristics of changing membrane structure make it more efficient to solve clustering problems. This paper proposes a P system with active membranes to solve DBSCAN clustering problems. This new model of P system can improve the efficiency of the clustering process. Then the paper provides a general example to verify the feasibility and effectiveness of this P system. The result we obtained can be a strong proof of this P system. In recent years, with the development at full speed of membrane computing, the P system is used in the clustering problems more and more. But it is still in the initial stage and there is much more work for membrane computing to do. In future, we will continue to research how to using membrane computing techniques to realize more clustering algorithms and solve more practical problems with large database.

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