Trapezoidal Fuzzy AHP and Fuzzy Comprehensive Evaluation Approaches for Evaluating Academic Library Service

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Abstract: -Academic libraries contribute to educational processes; therefore, the evaluation of library services plays an important role in enhancing a university's quality. In this paper, we propose a novel framework for academic library service evaluation based on the combination of fuzzy analytical hierarchy process (FAHP) and fuzzy comprehensive evaluation method. Specifically, the evaluation hierarchical structure is established and then the criterion and attribute weights are determined by the trapezoidal FAHP method. Employing the FAHP in group decision-making facilitates a consensus of decision-makers, and reduces uncertainty in decision-making. The evaluation of the academic library service can then be conducted by the use of the comprehensive evaluation method. A case application is also used to illustrate the proposed framework. The application of this framework can make the evaluation results more scientific, accurate, and objective. It is expected that this work may serve as a tool for managers of higher education institutions in improving the educational quality level.

Key Words: - Academic library service; Fuzzy analytical hierarchy process; Decision making process; Fuzzy sets

1 Introduction

Due to the trends of internationalization and globalization, universities face increased competition from many other higher education institutions. Good services can enhance the satisfaction level of students and graduates, and can attract more prospective students. According to Weber [1], a university can only provide the best services to the community if it commits itself to continuous quality improvement. Many universities have been committed to ongoing improvement, and thus must evaluate the activities and services they provide. In every university, the library has an important role in improving the research quality and motivating the students' study. Academic libraries have become resource centers for permanent learning and research, focusing their efforts on the access to and supply of information, the advanced retrieval of online resources, and the provision of new information services [2]. Hence, the traditional manner of assessing value and activities in academic libraries is being questioned. As a consequence, the assessment activity of academic library service is becoming a more important topic [3, 4].

In recent years, several researchers have focused on the evaluation of library services by employing mathematical models. Huang *et al.* [5] established a fuzzy evaluation model of service quality based on statistical findings. Hua [6] applied a fuzzy comprehensive evaluation method for digital library evaluation, using the M(1,2,3) algorithm to calculate the membership degree transformation. Cao [7] proposed an approach to evaluate academic library service quality that used fuzzy linguistic variables to express opinions on the satisfaction of users. The above mentioned studies are among the main studies on library service evaluation that can be found in the literature. These studies have provided useful tools for library service evaluation in universities, and are useful applications of mathematical models in assessing service quality.

Our study concentrates on the establishment of an evaluation index system with reasonable and objective attribute weights. Based on the evaluation index system, the academic library service is evaluated by the fuzzy comprehensive evaluation method. In order to acquire an objective, accurate and effective assessment, the decisive factor is how to distribute the importance of the attributes in the evaluation system. Determining the importance of the factor is related to multiple criteria decision-making problems. Decision-makers usually feel more confident to give linguistic variables, rather than expressing their judgments in the form of numeric values. Hence, the fuzzy set theory is a useful tool in dealing with imprecise and uncertain data. AHP, proposed by Satty in the mid-1970s, is a practical decision-making method. AHP is an effective method to solve multi-target and multi-layer decision-making problems. The method can deal with the importance of many factors and alternatives. Being an extension of AHP, fuzzy AHP is able to solve hierarchical fuzzy decision-making problems. The fuzzy AHP method has been widely used by various researchers to solve different decision making problems. Mikhailov and Tsvetinov [8] used fuzzy AHP to deal with the uncertainty and imprecision of the service evaluation process. Chan and Kumar [9] presented a fuzzy extended AHP approach to select the best supplier considering risk factors. Huang *et al.* [10] used fuzzy AHP for government-sponsored R&D project selection. Gungor *et al.* [11] proposed a personnel selection system based on fuzzy AHP, and the system evaluated the most suitable personnel dealing with the rating of both qualitative and quantitative criteria. Chou *et al.* [12] employed fuzzy AHP to evaluate the importance of each criterion in human resources for science and technology. Do and Chen [13] proposed a framework for teaching performance evaluation based on fuzzy AHP and fuzzy comprehensive evaluation.

Apart from the above mentioned applications, there are still many studies that use fuzzy AHP for solving different managerial problems. These studies revealed the high applicability of fuzzy AHP for practical purposes. Therefore, fuzzy AHP is appropriate for determining the weights in the evaluation index system. In this study, the extension of the Saaty's AHP method with trapezoidal fuzzy numbers [14, 15] is employed to obtain the criteria weights of the library quality service evaluation index system. On the basis of the index system, the evaluation is carried out.

The application of fuzzy AHP for determining the weights in the evaluation index system can be briefly described as follows. First, a hierarchical structure is developed. A group of decision-makers is then formed and invited to evaluate the criteria and attributes. The comparison of the importance of one criterion over another can be done with the consensus of all the group members that is in the form of a linguistic assessment. The linguistic assessment of the group is converted to trapezoidal fuzzy numbers. After that, these trapezoidal fuzzy numbers are used to build the comparison matrices of decision-makers based on a pair-wise comparison technique. Once accepted by checking consistency ratios, the matrices are used to calculate the weights of criteria and attributes by the fuzzy AHP method.

The remainder of this paper is organized as follows. Section 2 includes fuzzy AHP and some related concepts. Section 3 presents the framework for designing the evaluation index system. The hierarchy for library service quality evaluation based on LibQUAL+^{TM} dimensions is mentioned in section 4. Section 5 deals with establishing the evaluation index system and determining the criterion and attribute weights. Section 6 presents an application of the proposed evaluation index system based on the comprehensive evaluation method. Finally, conclusions are then given in Section 7.

2. Fuzzy Analytic Hierarchy Process (FAHP)

2.1. Fuzzy sets and fuzzy numbers

The fuzzy set theory was first introduced by Zadeh [16] to deal with the uncertainty due to imprecision or vagueness. A fuzzy set, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$, is a set of ordered pairs, and X is a subset of the real numbers, R, in which $\mu_{\tilde{A}}(x)$ is called the membership function that assigns to each object, x, a grade of membership ranging from zero to one. Since its introduction, the fuzzy set theory has been widely applied to address real-world problems in which decision makers need to analyze and process information that is imprecise. A fuzzy number is a special case of a convex normalized fuzzy set [17]. It is possible to use different fuzzy numbers in various particular situations. Triangular and trapezoidal fuzzy numbers are usually adopted to deal with the vagueness of decisions related to the performance levels of alternative choices with respect to each criterion. When the two most promising values of a trapezoidal fuzzy number are the same number, it becomes a triangular fuzzy number (TFN). This means that a TFN is a special case of a trapezoidal fuzzy number. Therefore, a trapezoidal fuzzy number can deal with more general situations [18]. In this study, the opinions of decision-makers are described by linguistic variables that have been expressed in trapezoidal fuzzy numbers.

A trapezoidal fuzzy number, denoted as $\tilde{A} = (l, m, n, s)$, has the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l} & l \le x \le m \\ 1 & m \le x \le n \\ \frac{s-x}{s-n} & n \le x \le s \end{cases} \quad (1)$$

where [m, n] is called a mode interval of $\tilde{A}$, and parameters l and s are the lower and upper bound of $\tilde{A}$, which limit the field of possible evaluations (Fig. 1).

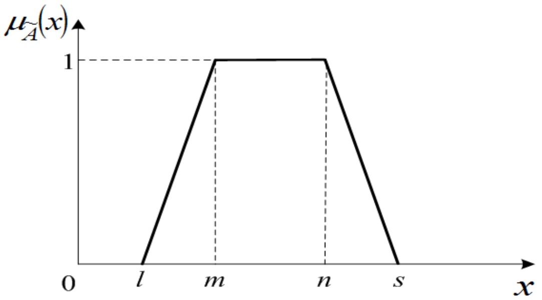

Fig. 1: A trapezoidal fuzzy number.

Consider two trapezoidal fuzzy numbers, $\tilde{A}_1$ and $\tilde{A}_2$, $\tilde{A}_1$ = *(l₁, m₁, n₁, s₁)* and $\tilde{A}_2$ =*(l₂, m₂, n₂, s₂)* The main operational laws for these two trapezoidal fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ are as follows:

$$\tilde{A}_1 + \tilde{A}_2 = (l_1 + l_2, m_1+m_2, n_1+n_2, s_1+s_2) \quad (2)$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = (l_1 l_2, m_1 m_2, n_1 n_2, s_1 s_2), \text{ for } l_i > 0, m_i > 0, n_i > 0, s_i > 0, i = 1, 2 \quad (3)$$

$$\lambda \otimes \tilde{A}_1 = (\lambda l_1, \lambda m_1, \lambda n_1, \lambda s_1), \text{ for } \lambda > 0, \lambda \in R, l_1 > 0, m_1 > 0, n_1 > 0, s_1 > 0 \quad (4)$$

$$\tilde{A}_1^{-1} = \left(\frac{1}{s_1}, \frac{1}{n_1}, \frac{1}{m_1}, \frac{1}{l_1}\right), \text{ for } l_1 > 0, m_1 > 0, n_1 > 0, s_1 > 0 \quad (5)$$

2.2. The trapezoidal fuzzy AHP method

The AHP method [19], the decision-making process, uses pairwise comparison judgments and matrix algebra to identify and estimate the relative importance of criteria and alternatives. It is a powerful method to solve complex decision problems. However, the pure AHP method has some shortcomings. AHP is ineffective when applied to deal with the ambiguity problem. The fuzzy AHP, an extension of the AHP model, has been applied to fuzzy decision-making problems. In the fuzzy AHP, by using fuzzy arithmetic operation laws, the weights of evaluative elements are determined. There are several fuzzy AHP methods reported in the literature [20]. In this study, without loss of generality, the extension of Saaty's AHP method with trapezoidal fuzzy numbers [14, 15] is utilized to obtain the attribute weights.

Let $\tilde{A} = (\tilde{a}_{ij})_{nn}$ be a fuzzy pair-wise comparison matrix, where $\tilde{a}_{ij} = (l_{ij}, m_{ij}, n_{ij}, s_{ij})$. The weights can be calculated as follows:

$$\alpha_j = \left[\prod_{j=1}^{n} l_{ij}\right]^{1/n} \quad (6)$$

$$\beta_j = \left[\prod_{j=1}^{n} m_{ij}\right]^{1/n} \quad (7)$$

$$\gamma_j = \left[\prod_{j=1}^{n} n_{ij}\right]^{1/n} \quad (8)$$

$$\delta_j = \left[\prod_{j=1}^{n} s_{ij}\right]^{1/n} \quad (9)$$

and

$$\alpha = \sum_{j=1}^{n} \alpha_j \quad (10)$$

$$\beta = \sum_{j=1}^{n} \beta_j \quad (11)$$

$$\gamma = \sum_{j=1}^{n} \gamma_j \quad (12)$$

$$\delta = \sum_{j=1}^{n} \delta_j \quad (13)$$

Then the fuzzy weights are defined as follows:

$$\tilde{w}_j = \left(\alpha_j \delta^{-1}, \beta_j \gamma^{-1}, \gamma_j \beta^{-1}, \delta_j \alpha^{-1}\right), \text{ for } j=1,\ldots,n \quad (14)$$

The fuzzy weight vector $\tilde{W}$ can be obtained.

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n) \quad (15)$$

3. The proposed framework for designing a performance evaluation index system based on fuzzy AHP

In order to search for a consensus, it is necessary to establish a representative and democratic decision-making process when designing the evaluation index system. The proposed framework is composed of the following steps:

3.1. Developing the hierarchical structure of the evaluation index system

The hierarchical structure is constructed by combining all of the criteria and attributes specific to the research problem. Based on the identified criteria and attributes, the hierarchical structure for evaluation is obtained. In the system, the objective is in the first level and criteria and attributes are in successive levels. This step also dissects the problem into elements according to their common characteristics.

3.2. Selecting decision-makers

A group of decision-makers is formed. The members of the group are experts who have experience in the research field. The decision-makers are required to provide the relative importance of each criterion and attribute.

3.3. Determining the linguistic variables and fuzzy conversion scale

The decision-makers make pair-wise comparisons of the importance or preference between each pair of criteria. Consider a problem at a level with n elements. Each set of pair-wise comparisons for a level requires $n(n-1)/2$ judgments, which are further used to construct a positive fuzzy reciprocal comparison matrix. The comparison of one criterion over another can be done with the help of questionnaires, which are in the form of linguistic variables. A linguistic variable is a variable whose values are words or sentences in a natural or artificial language [21]. In this study, trapezoidal fuzzy numbers are used to represent subjective pairwise comparisons of decision-makers, namely, “Equally important”, “Weakly important”, “Essentially important”, “Very strongly important”, and “Absolutely more important”. The trapezoidal fuzzy number and linguistic variable, which are proposed by Mou [22], are used to convert such linguistic values into fuzzy numbers and is demonstrated in Table 1.

Table 1: Linguistic scales and fuzzy scales of importance.

Linguistic variable	Trapezoidal fuzzy number
Equally important	$(1,1,1,1)$
Weakly important	$\left(2,\frac{5}{2},\frac{7}{2},4\right)$
Essentially important	$\left(4,\frac{9}{2},\frac{11}{2},6\right)$
Very strongly important	$\left(6,\frac{13}{2},\frac{15}{2},8\right)$
Absolutely more important	$\left(8,\frac{17}{2},9,9\right)$

3.4. Establishing comparison matrices

Consider a problem at one level with n criteria, where the relative importance of criterion i to j is represented by trapezoidal fuzzy numbers $\tilde{a}_{ij}=(l_{ij}, m_{ij}, n_{ij}, s_{ij})$). If criterion i is very strongly important in comparison with the criterion j; $\tilde{a}_{ij}$ is (6, 13/2, 15/2, 8). If criterion j is thought to be very strongly more important than criterion i, the pairwise comparison between i and j could be presented by $\tilde{a}_{ij}$ = (1/8, 2/15, 2/13, 1/6).

As in the traditional AHP, the comparison matrix $\tilde{A}=\{\tilde{a}_{ij}\}$ can be constructed as

$$\tilde{A}=\begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & \dots & 1 \end{bmatrix} \quad (16)$$

3.5. Calculating the consistency index and consistency ratio of comparison matrix

To assure a certain quality level of a decision, the consistency of an evaluation has to be analyzed. Saaty [19] proposed an index to measure consistency. This index can be used to indicate the consistency of the pairwise comparison matrices. To investigate the consistency, the fuzzy comparison matrices need to be converted into crisp matrices [23]. If $\tilde{A}=[\tilde{a}_{ij}]$ is a fuzzy positive reciprocal matrix, then $A=[a_{ij}]$ is a positive reciprocal crisp matrix. When $A=[a_{ij}]$ is consistent, $\tilde{A}=[\tilde{a}_{ij}]$ is also consistent. The operation of converting fuzzy numbers into crisp numbers is called defuzzification. There are several defuzzification methods [24]. In our work, a trapezoidal fuzzy number denoted as $\tilde{a}_{ij}=(l_{ij},m_{ij},n_{ij},s_{ij})$ is defuzzified to a crisp number as follows [24, 25].

$$a_{ij}=\frac{l_{ij}+2m_{ij}+2n_{ij}+s_{ij}}{6}, \quad (17)$$

where a_{ij} is the defuzzified crisp value.

After all, the elements in the comparison matrix are converted from trapezoidal fuzzy numbers to crisp numbers, and the comparison matrix is now expressed as follows:

$$A=\begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{21} & 1 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & 1 \end{bmatrix} \quad (18)$$

The consistency index, CI, for a comparison matrix can be computed with the use of the following equation:

$$CI = \frac{\lambda_{\max} - n}{n-1}, \quad (19)$$

where $\lambda_{\max}$ is the largest eigenvalue of the comparison matrix, n is the dimension of the matrix.

The consistency ratio (CR) [19] is defined as a ratio between the consistency of a given evaluation matrix and consistency of a random matrix:

$$CR = \frac{CI}{RI(n)} \quad (20)$$

where $RI(n)$ is a random index [26] that depends on n, as shown in Table 2.

Table 2: Random index (RI) of random matrices.

n	3	4	5	6	7	8	9
RI(n)	0.58	0.9	1.12	1.24	1.32	1.41	1.45

If the consistency ratio of a comparison matrix is equal to or less than 0.1, it can be acceptable. When the CR is unacceptable, the group is encouraged to repeat the pair-wise comparisons. In this step, the MATLAB package is employed to calculate the eigenvalues of all comparison matrices.

3.6. Calculating the weights of criteria and attributes

The extension of the Saaty's AHP method with trapezoidal fuzzy number proposed by Buckley is then employed to identify the weights of criteria and attributes.

3.7. Calculating the global weights for the attributes

Global attribute weights are computed by multiplying the local weight of the attribute with the local weight of the criterion to which it belongs.

4. Establishing the hierarchy for library service quality evaluation

In this study, we adopt LibQUAL+™ dimensions. LibQUAL+™ is a result of the collaboration of the Association of Research Libraries (ARL) with Texas A&M University for benchmarking perceptions of library service quality [27]. The three dimensions determined by LibQUAL+™ are "Service affect", "Information control" and "Library as place". The "Service affect", "Information Control" and "Library as Place" are decomposed into nine, eight and five components, respectively. Based on LibQUAL+™ dimensions, the problem taken here has three levels of hierarchy. The overall objective is library service. The criteria are denoted by C_i (where, $i=1$-3), attributes by A_j (where, $j=1$-22). The hierarchy of library service can be seen in Fig. 2.

5. Application of the proposed framework to determine the evaluation index system

In developing a performance evaluation index system, the importance of each criterion must be significantly considered. Our study is related to library service quality evaluation in higher education institutions in Vietnam. In order to acquire the weights of criteria and attributes, a group of 10 decision-makers including institution managements, experienced lecturers and librarians was formed. The questionnaires were provided to receive their viewpoints. After reaching a consensus, the comparison matrix was determined from their judgments of the relative importance of one criterion over another. The comparison matrix of the group, when making pair-wise comparisons of the criteria, is shown in Table 3.

Table 3: Comparison matrix of the criteria.

	C_1	C_2	C_3
C_1	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(2,2.5,3.5,4)
C_2	(4,4.5,5.5,6)	(1,1,1,1)	(8,8.5,9,9)
C_3	(0.25,0.286,0.4,0.5)	(0.111,0.111,0.118,0.125)	(1,1,1,1)

The next step is to calculate the consistency ratio of the comparison matrix. By employing Eq. 17, all the trapezoidal fuzzy numbers in the matrix were defuzzified to crisp numbers. The CR value of the comparison matrix was then calculated by using Eqs. (14) and (15). This value is 0.0503. Hence, it is acceptable.

When making comparisons with all attributes at the corresponding level with respect to the upper level criteria, the matrices were then obtained and are shown in Tables 4-6. Then, the CR values for all matrices were determined by making the same calculations, as in Table 3. From the results of the consistency test of the comparison matrices, it was found that they are all less than 10%. Therefore, the consistency in each matrix is acceptable.

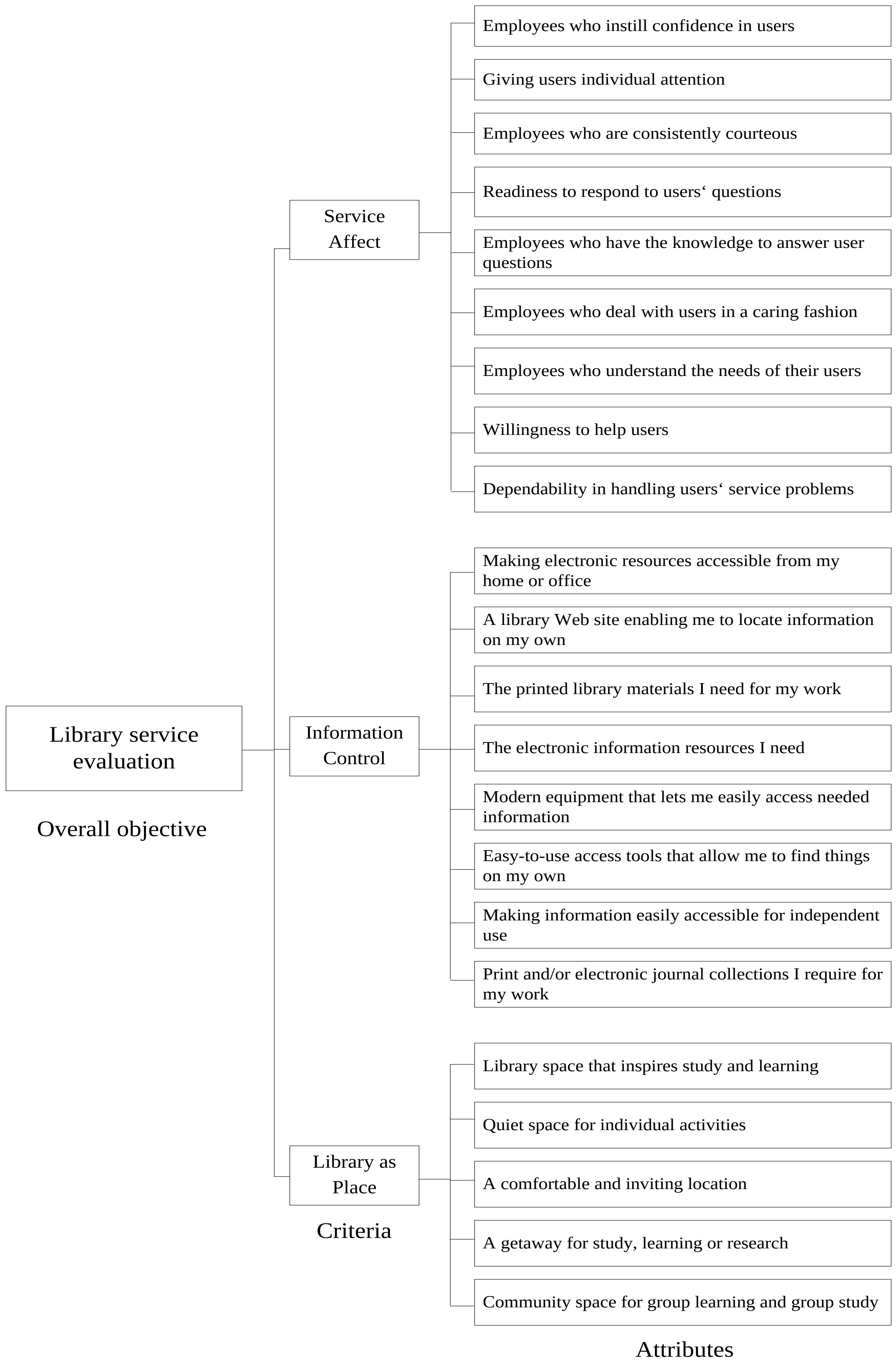

Fig. 2: Hierarchy for library service.

Table 4: Comparison matrix of the attributes within “Service affect” criterion.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	(1,1,1,1)	(1,1,1,1)	(0.25,0.286,0.4,0.5)	(0.125,0.133,0.154,0.167)	(0.167,0.182,0.222,0.25)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(0.125,0.133,0.154,0.167)	(1,1,1,1)
A_2	(1,1,1,1)	(1,1,1,1)	(0.25,0.286,0.4,0.5)	(0.125,0.133,0.154,0.167)	(0.167,0.182,0.222,0.25)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(0.125,0.133,0.154,0.167)	(2,2.5,3.5,4)
A_3	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(0.125,0.133,0.154,0.167)	(4,4.5,5.5,6)
A_4	(6,6.5,7.5,8)	(6,6.5,7.5,8)	(4,4.5,5.5,6)	(1,1,1,1)	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(0.25,0.286,0.4,0.5)	(8,8.5,9,9)
A_5	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(2,2.5,3.5,4)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(6,6.5,7.5,8)
A_6	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(0.25,0.286,0.4,0.5)	(2,2.5,3.5,4)	(1,1,1,1)	(2,2.5,3.5,4)	(0.25,0.286,0.4,0.5)	(6,6.5,7.5,8)
A_7	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(4,4.5,5.5,6)
A_8	(6,6.5,7.5,8)	(6,6.5,7.5,8)	(6,6.5,7.5,8)	(2,2.5,3.5,4)	(4,4.5,5.5,6)	(2,2.5,3.5,4)	(4,4.5,5.5,6)	(1,1,1,1)	(8,8.5,9,9)
A_9	(1,1,1,1)	(0.25,0.286,0.4,0.5)	(0.167,0.182,0.222,0.25)	(0.111,0.111,0.118,0.125)	(0.125,0.133,0.154,0.167)	(0.125,0.133,0.154,0.167)	(0.167,0.182,0.222,0.25)	(0.111,0.111,0.118,0.125)	(1,1,1,1)

Table 5: Comparison matrix of the attributes within “Information control” criterion.

	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}
A_{10}	(1,1,1,1)	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(2,2.5,3.5,4)	(4,4.5,5.5,6)	(0.167,0.182,0.222,0.25)
A_{11}	(1,1,1,1)	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(2,2.5,3.5,4)	(4,4.5,5.5,6)	(0.167,0.182,0.222,0.25)
A_{12}	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(1,1,1,1)	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(6,6.5,7.5,8)	(6,6.5,7.5,8)	(1,1,1,1)
A_{13}	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(2,2.5,3.5,4)	(6,6.5,7.5,8)	(6,6.5,7.5,8)	(0.25,0.286,0.4,0.5)
A_{14}	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(0.25,0.286,0.4,0.5)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(0.25,0.286,0.4,0.5)
A_{15}	(0.25,0.286,0.4,0.5)	(0.25,0.286,0.4,0.5)	(0.125,0.133,0.154,0.167)	(0.125,0.133,0.154,0.167)	(0.167,0.182,0.222,0.25)	(1,1,1,1)	(2,2.5,3.5,4)	(0.125,0.133,0.154,0.167)
A_{16}	(0.167,0.182,0.222,0.25)	(0.167,0.182,0.222,0.25)	(0.125,0.133,0.154,0.167)	(0.125,0.133,0.154,0.167)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(1,1,1,1)	(0.125,0.133,0.154,0.167)
A_{17}	(4,4.5,5.5,6)	(4,4.5,5.5,6)	(1,1,1,1)	(2,2.5,3.5,4)	(2,2.5,3.5,4)	(6,6.5,7.5,8)	(6,6.5,7.5,8)	(1,1,1,1)

Table 6: Comparison matrix of the attributes within “Library as place” criterion.

	A_{18}	A_{19}	A_{20}	A_{21}	A_{22}
A_{18}	(1,1,1,1)	(4,4.5,5.5,6)	(6,6.5,7.5,8)	(2,2.5,3.5,4)	(4,4.5,5.5,6)
A_{19}	(0.167,0.182,0.222,0.25)	(1,1,1,1)	(4,4.5,5.5,6)	(0.25,0.286,0.4,0.5)	(2,2.5,3.5,4)
A_{20}	(0.125,0.133,0.154,0.167)	(0.167,0.182,0.222,0.25)	(1,1,1,1)	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)
A_{21}	(0.25,0.286,0.4,0.5)	(2,2.5,3.5,4)	(4,4.5,5.5,6)	(1,1,1,1)	(2,2.5,3.5,4)
A_{22}	(0.167,0.182,0.222,0.25)	(0.25,0.286,0.4,0.5)	(2,2.5,3.5,4)	(0.25,0.286,0.4,0.5)	(1,1,1,1)

Table 7: Computed global weights for attributes.

Criteria and local weight	Attribute	Local weight	Global weight
Service affect (C_1) (0.181)	Employees who instill confidence in users (A_1)	0.026	0.0047
	Giving users individual attention (A_2)	0.030	0.0054
	Employees who are consistently courteous (A_3)	0.050	0.009
	Readiness to respond to users' questions (A_4)	0.218	0.0395
	Employees who have the knowledge to answer user questions (A_5)	0.097	0.0175
	Employees who deal with users in a caring fashion (A_6)	0.157	0.0283
	Employees who understand the needs of their users (A_7)	0.083	0.0151
	Willingness to help users (A_8)	0.321	0.0581
	Dependability in handling users' service problems (A_9)	0.018	0.0033
Information Control (C_2) (0.746)	Making electronic resources accessible from my home or office (A_{10})	0.057	0.0428
	A library Web site enabling me to locate information on my own (A_{11})	0.057	0.0428
	The printed library materials I need for my work (A_{12})	0.270	0.2015
	The electronic information resources I need (A_{13})	0.182	0.136
	Modern equipment that lets me easily access needed information (A_{14})	0.113	0.0846
	Easy-to-use access tools that allow me to find things on my own (A_{15})	0.030	0.0222
	Making information easily accessible for independent use (A_{16})	0.020	0.0148
	Print and/or electronic journal collections I require for my work (A_{17})	0.270	0.2015
Library as Place (C_3) (0.073)	Library space that inspires study and learning (A_{18})	0.489	0.0357
	Quiet space for individual activities (A_{19})	0.142	0.0104
	A comfortable and inviting location (A_{20})	0.041	0.003
	A getaway for study, learning or research (A_{21})	0.244	0.0178
	Community space for group learning and group study (A_{22})	0.084	0.0061

Table 8: The results of library service quality rating.

Criteria	Attribute	Very good	Good	Medium	Poor	Very poor
Service affect (C_1)	Employees who instill confidence in users (A_1)	124	23	1	4	0
	Giving users individual attention (A_2)	34	113	3	2	0
	Employees who are consistently courteous (A_3)	15	123	5	9	0
	Readiness to respond to users‘ questions (A_4)	24	112	4	12	0
	Employees who have the knowledge to answer user questions (A_5)	12	134	5	1	0
	Employees who deal with users in a caring fashion (A_6)	16	127	4	5	0
	Employees who understand the needs of their users (A_7)	3	134	8	7	0
	Willingness to help users (A_8)	37	81	23	11	0
	Dependability in handling users‘ service problems (A_9)	78	69	2	3	0
Information Control (C_2)	Making electronic resources accessible from my home or office (A_{10})	0	0	5	29	118
	A library Web site enabling me to locate information on my own (A_{11})	0	0	4	45	103
	The printed library materials I need for my work (A_{12})	125	22	5	0	0
	The electronic information resources I need (A_{13})	0	47	82	23	0
	Modern equipment that lets me easily access needed information (A_{14})	112	23	12	5	0
	Easy-to-use access tools that allow me to find things on my own (A_{15})	34	59	40	19	0
	Making information easily accessible for independent use (A_{16})	2	67	67	16	0
	Print and/or electronic journal collections I require for my work (A_{17})	8	75	48	21	0
Library as Place (C_3)	Library space that inspires study and learning (A_{18})	123	19	10	0	0
	Quiet space for individual activities (A_{19})	98	32	22	0	0
	A comfortable and inviting location (A_{20})	0	0	0	107	45
	A getaway for study, learning or research (A_{21})	42	65	45	0	0
	Community space for group learning and group study (A_{22})	34	55	58	5	0

The trapezoidal fuzzy AHP method was then employed to identify the weights of criteria and attributes. Taking pairwise comparison matrix of the criteria in Table 4 as an example, the weights of the criteria were acquired as follows:

Using Eqs. (6)-(15), we obtained the fuzzy weight vector as follows:

$\tilde{W} = \left(\tilde{W}_{C1}, \tilde{W}_{C2}, \tilde{W}_{C3}\right)^T$ = ((0.134, 0.155, 0.206, 0.24),(0.613, 0.68, 0.824, 0.906),(0.059,0.064,0.081,0.095))^T

The above fuzzy weight vector was defuzzified by Eq. (17).

$W= (0.183, 0.755, 0.074)^T$

We then normalized the weight vector and obtained the relative weights of the three criteria.

$W= (0.181, 0.746, 0.073)^T$

The calculation results show that the weight of "Information Control" is largest. Hence, this factor plays the most important part in library service quality, followed by "Service Affect".

Following a similar calculation, the weight vectors of attributes at the successive level were determined. They are as shown below:

The weight vector from Table 4 was calculated as

$W_{C1} = (W_{A1}, W_{A2}, W_{A3}, W_{A4}, W_{A5}, W_{A6}, W_{A7}, W_{A8}, W_{A9})^T$
$= (0.026, 0.03, 0.05, 0.218, 0.097, 0.157, 0.083, 0.321, 0.018)^T$

The weight vector from Table 5 was calculated as

$W_{C2}= (W_{A10}, W_{A11}, W_{A12}, W_{A13}, W_{A14}, W_{A15}, W_{A16}, W_{A17})^T$
$= (0.057, 0.057, 0.27, 0.182, 0.113, 0.03,0.02, 0.27)^T$

The weight vector from Table 6 was calculated as

$W_{C3}=(W_{A18}, W_{A19}, W_{A20}, W_{A21}, W_{A22})^T= (0.489, 0.142, 0.041, 0.244, 0.084)^T$

Global weights for attributes are then calculated. Global weights for the attributes are computed by multiplying the local weight of the attribute with the local weight of the criteria in which it belongs. Hence, the global weights can be derived as shown in Table 7. According to the global attribute weights, the three most important attributes that can affect overall academic library service quality are "The printed library materials I need for my work (A_{12})", "Print and/or electronic journal collections I require for my work (A_{17})", and "The electronic information resources I need (A_{13})".

6. Evaluation of academic library service

From the above evaluation index system and acquired criterion and attribute weights, the fuzzy comprehensive evaluation method is introduced to assess the academic library service. In order to illustrate the method, we took a case application as an example.

6.1. Fuzzy comprehensive evaluation

Fuzzy comprehensive evaluation is an application of fuzzy mathematics. It uses the principles of fuzzy transformation and the maximum membership degree to evaluate all relevant factors to make a comprehensive evaluation. This is an efficient evaluation method to evaluate objects that are affected by various factors. For objects that are influenced by a few factors, we can use one-level models. If the objects are complicated and the number of the factors is large, we can use models with two or more levels. In this study, we used a fuzzy comprehensive evaluation model with two levels as a tool for library service evaluation. The application steps of fuzzy comprehensive evaluation for one level [28] are as follows:

Step 1: Establishment of the evaluation index system

According to the nature of the characteristics of the evaluation index system, the factor set in the evaluating relationship is determined.

In Section 4, the academic library service evaluation system was established and the weights of criteria and attributes were calculated. The evaluation system has two levels, the first level is $U=\{u_{C1}, u_{C2}, u_{C3}\}$ corresponding to {Service affect, Information Control, Library as Place}, the second level is $u_{C1}=\{u_{A1},u_{A2},\ldots, u_{A9}\}$, $u_{C2}=\{ u_{A10},u_{A1},\ldots, u_{A17}\}$ and $u_{C3}=\{ u_{A18},u_{A19},\ldots, u_{A22}\}$ corresponding to each level one evaluation item.

Step 2: Determining the set of comments

The evaluation comment set is as follows:

$$V=\{v_1, v_2, v_3,\ldots,v_m\}$$

In this study, we used five grades to set up the comment for evaluation: V={very good, good, middle, poor, very poor}.

In order to make the index quantitative, we provide grades for the corresponding comment sheet: $V=(100,85,70,55,40)$

Step 3: Establishing the single-factor evaluation matrix R from U to V

Each factor u_{Ai} should be an evaluated single factor. As there are different types of evaluation levels, the evaluation result of each factor is a fuzzy set of evaluation set V that can be written as the fuzzy vector:

$$R_{Ai}=(r_{Ai1}, r_{Ai2}, r_{Ai3},\ldots,r_{Aim}),\ i=1,2,..n,\ R_{Ai} \in \mu(V)$$

where n is the number of evaluated elements. For example, when we define the evaluation matrix R_{C1}

of the criterion "Service affect" at the second level, we have $n=9$.

The results of these evaluations meet the normalized conditions, and the sum of the weight of the vector is 1; that is, for every i, there is: $r_{Ai1}+r_{Ai2}+r_{Ai3}+\ldots+r_{Aim}=1$

All of the single-factor evaluation constitutes the fuzzy relationship R from U to V: $R=\left(r_{Aij}\right)_{n\times m}$

That is,

$$R=\left(r_{Aij}\right)_{n\times m}=\begin{pmatrix} r_{A11} & r_{A12} & r_{A13} & \cdots & r_{A1m} \\ r_{A21} & r_{A22} & r_{A23} & \cdots & r_{A2m} \\ \cdots & \cdots & \cdots & \cdots & \\ r_{An1} & r_{An2} & r_{An3} & \cdots & r_{Anm} \end{pmatrix} \tag{21}$$

r_{Aij} presents the grade of membership of factor u_{Ai} aiming at the comment v_j.

Step 4: Determining of the weight of factors

Weight means the proportion of each evaluation criteria in the evaluation index system based on relative importance. If a weight is given to an element, the weight distribution set W can be seen as a fuzzy set of set U. How to determine the weight of each factor is the core task of the evaluation system. As discussed in Section 4, we employed fuzzy AHP to determine the weights of criteria and attributes in the evaluation index system.

Step 5: Producing the evaluation results

The results of evaluation can be obtained through multiplying the vector of the factor weight and the matrix R of single-factor evaluation:

$$B=W.R=(b_1, b_2, b_3, \ldots b_m) \tag{22}$$

B is the evaluation result based on all factors in index system U. The k-th element b_k is membership of the evaluation object with regard to k-th element in the comment set. The conclusion of the comprehensive evaluation can be obtained by the max membership principle.

6.2. A case application

The application was carried out in evaluating the academic library at the University of Transport Technology-one of the public universities in Vietnam. According to the evaluation index system and the comment set proposed in the previous sections, we collected opinions of students, lecturers and staff about the academic library service in the second semester of the 2011-2012 academic year. The questionnaires were sent to two hundred users. One hundred and fifty two users completed and returned the questionnaires. Twenty-two items of the standard questionnaire are categorized into three criteria that are "Service Affect", "Information Control", and "Library as Place". The results are represented in Table 8.

As discussed earlier, the fuzzy comprehensive evaluation model for library service evaluation has two levels. The evaluation result is derived by employing the application steps of fuzzy comprehensive evaluation for each level.

Firstly, the evaluation matrices of criteria R_{C1}, R_{C2}, and R_{C3} at the second level were formed. Taking A_1 (Employees who instill confidence in users) as an illustration, when "Employees who instill confidence in users" was concerned, 82% of users rated it "very good", 15% rated it "Good", 1% rated it "Medium", and 3% rated it "Poor", and 0% rated it "Very poor". Hence, its evaluation membership vector is (0.82, 0.15, 0.01, 0.03, 0). In the same way, we can obtain the evaluation matrix of the criterion "Service affect" *(C_1)* as follows:

$$R_{C1}=\begin{bmatrix} 0.82 & 0.15 & 0.01 & 0.03 & 0 \\ 0.22 & 0.74 & 0.02 & 0.01 & 0 \\ 0.1 & 0.81 & 0.03 & 0.06 & 0 \\ 0.16 & 0.74 & 0.03 & 0.08 & 0 \\ 0.08 & 0.88 & 0.03 & 0.01 & 0 \\ 0.11 & 0.84 & 0.03 & 0.03 & 0 \\ 0.02 & 0.88 & 0.05 & 0.05 & 0 \\ 0.24 & 0.53 & 0.15 & 0.07 & 0 \\ 0.51 & 0.45 & 0.01 & 0.02 & 0 \end{bmatrix}$$

Similarly, the matrix R_{C2}, R_{C3} were obtained. They are as shown below:

$$R_{C2}=\begin{bmatrix} 0 & 0 & 0.03 & 0.19 & 0.78 \\ 0 & 0 & 0.03 & 0.3 & 0.68 \\ 0.82 & 0.14 & 0.03 & 0 & 0 \\ 0 & 0.31 & 0.54 & 0.15 & 0 \\ 0.74 & 0.15 & 0.08 & 0.03 & 0 \\ 0.22 & 0.39 & 0.26 & 0.13 & 0 \\ 0.01 & 0.44 & 0.44 & 0.11 & 0 \\ 0.05 & 0.49 & 0.32 & 0.14 & 0 \end{bmatrix}$$

$$R_{C3}=\begin{bmatrix} 0.81 & 0.13 & 0.07 & 0 & 0 \\ 0.64 & 0.21 & 0.14 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 \\ 0.28 & 0.43 & 0.3 & 0 & 0 \\ 0.22 & 0.36 & 0.38 & 0.03 & 0 \end{bmatrix}$$

Then we can get the evaluation result of B_{C1}

$$B_{C1}=W_{C1}.R_{C1}=(0.026, 0.03, 0.05, 0.218, 0.097, 0.157, 0.083, 0.321, 0.018).$$

$$\begin{bmatrix} 0.82 & 0.15 & 0.01 & 0.03 & 0 \\ 0.22 & 0.74 & 0.02 & 0.01 & 0 \\ 0.1 & 0.81 & 0.03 & 0.06 & 0 \\ 0.16 & 0.74 & 0.03 & 0.08 & 0 \\ 0.08 & 0.88 & 0.03 & 0.01 & 0 \\ 0.11 & 0.84 & 0.03 & 0.03 & 0 \\ 0.02 & 0.88 & 0.05 & 0.05 & 0 \\ 0.24 & 0.53 & 0.15 & 0.07 & 0 \\ 0.51 & 0.45 & 0.01 & 0.02 & 0 \end{bmatrix}$$

$$=(0.18, 0.7, 0.07, 0.05, 0)$$

Similarly, we got the evaluation result of B_{C2}, and B_{C3} through calculations

$$B_{C2}=W_{C2}.R_{C2}=(0.33, 0.27, 0.22, 0.1, 0.08)$$

$$B_{C3}=W_{C3}.R_{C3}=(0.57, 0.23, 0.16, 0.03, 0.01)$$

Then, we established the evaluation matrix R at the first level from the above matrices as follows:

$$R = \begin{bmatrix} B_{C1} \\ B_{C2} \\ B_{C3} \end{bmatrix} = \begin{bmatrix} 0.18 & 0.7 & 0.07 & 0.05 & 0 \\ 0.33 & 0.27 & 0.22 & 0.1 & 0.08 \\ 0.57 & 0.23 & 0.16 & 0.03 & 0.01 \end{bmatrix}$$

The comprehensive evaluation of the library service is

$$B = W.R = (0.181, 0.746, 0.073) \begin{bmatrix} 0.18 & 0.7 & 0.07 & 0.05 & 0 \\ 0.33 & 0.27 & 0.22 & 0.1 & 0.08 \\ 0.57 & 0.23 & 0.16 & 0.03 & 0.01 \end{bmatrix} = (0.32, 0.34, 0.19, 0.09, 0.06)$$

The result shows that the “Good” probability of the library service quality is 0.34; the probability of “Very good”, “Medium”, “Poor” and “Very poor” is 0.32, 0.19, 0.09 and 0.06, respectively. According to the maximum membership degree principle, the comprehensive evaluation result of the library service is “Good”. The evaluation result, which is based on the opinions of users, also provided the managers with suggestion on how to improve the service. We interviewed the general manager of the library and librarians about the evaluation result. They agreed that the result in the proposed evaluation method is transparent and objective. Moreover, the proposed method makes it easier to explain the evaluation result and provides managers with useful information. Based on the results, it can be also concluded that the library service quality is acceptable to users, but improvement is needed to reach the desired level of service. The library service quality can be improved by: (1) maintaining and improving the employees' professional qualifications, (2) making electronic resources accessible from home or office, and (3) providing more comfortable and inviting locations.

7. Conclusions

The academic library plays an important role in the overall quality of a university. Evaluation of academic library service is aimed at clarifying the rate of success in providing users with specific services. Accurate evaluation results can provide the administrators with valuable information to improve the overall quality and offer quality services. This study presents an evaluation index system for an academic library service based on fuzzy AHP, and develops a library service evaluation framework. Application of the framework to evaluate a library can not only reflect the overall service quality, but also reflect the achievement regarding each evaluating attribute. This helps administrators know what improvements are needed to enhance user satisfaction. One contribution of this approach is the introduction of fuzzy AHP to determine the weights. Because fuzzy AHP can capture the vagueness of human judgments, it makes the derived weights of the evaluation index system more objective and reasonable. A case application shows the applicability of this approach to higher education institutions. It is expected that this approach may provide an effective and scientific measurement, not only for assessing the academic library service, but for other services as well. Additionally, this study proposed a systematic procedure of the fuzzy AHP approach in the group decision making environment. Applying this procedure can get an accurate solution with a high degree of consensus. Hence, it may also be used as a reference for management practitioners when solving real world problems. For the future research recommendation, this study will be better if it can collect the opinions from a large number of decision-makers. Regarding employing the fuzzy AHP in group decision-making process, dispersion and homogeneity in individual judgments and its effect on the group decision could be taken into consideration, especially when only one or few decision makers deliver extreme comparison results. A limitation of the current study is that the number of decision makers is not large enough to provide a generalized conclusion.

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