Suboptimal dual controller for stochastic systems with variable design parameter

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Abstract: This work concerns a refinement of a suboptimal dual controller for discrete time systems with stochastic parameters. The dual property means that the control signal is chosen so that estimation of the model parameters and regulation of the output signals are optimally balanced. The control signal is computed in such a way so as to minimize the variance of output around a reference value one step further, with the addition of the term referring to a future estimate two steps ahead . An algorithm is used for the adaptive adjustment of the adjustable parameter lambda, for each step of the way. The actual performance of the proposed controller is evaluated through a Monte Carlo simulations method.

Key–Words: Dual control, Stochastic control, Suboptimal control, Time varying systems

1 Introduction

The treatment of control system with unknown parameters and time-varying is an interesting challenge. Starting from Feldbaum's unprecedented work [1], several surveys were stimulated with the purpose of investigating and clarifying the dual control problem [2–4]. The optimal controller for this problem has two objectives that must balance two conflicting purposes: control and estimation action.

Due to the computational and numerical intractability of the optimal solution to the dual control problem, many researchers resorted to suboptimal solutions in which the optimization problem is redesigned, to include cautious and probing features in simple controllers. Wittenmark [5] proposed an Active Suboptimal Dual (ASOD) controller, which uses the objective function one step ahead, including the covariance matrix of the estimated parameters. Next, Witternamrk and Elevitch [6] presented an analytical solution by making a series expansion of the loss function, with respect to the control signal. Hughes and Jacobs [7] proposed, appending an additional excitation to the control signal through a defined minimum value. The variance of the innovation process, through unknown but constant coefficients, is used in Milito et al. [8] to optimize the system performance. Ishihara et al. [9] extended the innovation process, to suit models with input delays and stochastic coefficients containing white noise components.

Maitelli and Yonemama [10] proposed a novel suboptimal dual controller, using optimal predictions of the output. This controller minimizes the output deviations during M steps ahead in time. Lee JM. and Lee JH. [11] applied an approximate dynamic programming (ADP) based strategy to the dual adaptive control problem. This ADP approach could derive a superior control policy, which actively reduces the parameter uncertainty, leading to a significant performance improvement. Flidr and Simandl [12] shows a new implicit dual control method based on Bellman optimization. The stochastic integration rule is employed to determine the control. Kral and Simandl [13] proposed and discussed a predictive dual control for a non linear system with functional uncertainty based on the bicriterial approach.

An important modelling technique in stochastic processes is queueing model. The dual suboptimal controllers must be studied in queueing network to optimize the service rates of the servers. Li Xia et al. [14] derived the performance difference equations and developed a policy iteration algorithm for customer-average performance in state-dependent closed Jackson networks. They also developed a sample path based algorithm for estimating the realization factors. Li Xia et al. [15] extended the MaxMin optimality of service rate control in closed Jackson networks to a much more general form of cost functions. This result is derived based on the difference equation for Markov systems. Due to system features the suboptimal dual controllers may optimize control and estimation data and improve the system performance.

While several suboptimal dual control solutions were proposed and simulated in recent decades, the application in industrial processes is still restricted. Bar-Shalom and Wall [16] presented a method for the quantitative assessment of the effects of uncertainty in macroeconomic policy optimization problems. Ismail et al. [17] successfully implemented a suboptimal dual controller in the paper-coating industry. The article shows the advantages of including probing in the control signal, and the improvement in the quality of the process. The dual controller yielded substantial quality improvement and was able to control the process throughout the entire blade life.

In this work, a modification in the Active Suboptimal Dual Controller is presented. Originally, there is a fixed design parameter λ that is associated with the function $f(P(k+2))$, which is intended to ensure a good parameter estimation. Therefore, it is proposed to use an adaptive adjustment of parameter λ at every step, in order to improve the performance and robustness of the controller.

2 Problem Formulation

The dynamic system to be controlled is assumed to be a time-varying single-input/single-output ARX (AutoRegressive with eXogenous input) model with stochastic parameters described by

$$\begin{aligned} y(k) + a_1(k)y(k-1) + \cdots + a_m(k)y(k-m) = \\ b_1(k)u(k-1) + \ldots + b_n(k)u(k-n) + e(k) \end{aligned} \quad (1)$$

where $y(k)$ is the output, $u(k)$ is the input, and $e(k)$ is a sequence of independent, identically distributed Gaussian variables, with zero mean and variance σ^2. It is also supposed that $e(k)$ is independent of $y(j), u(j), a_i(j), b_i(j), jk$ and $b_1(k) \neq 0$ for each instant k.

The time-varying parameters

$$x(k) = [a_1(k) \cdots a_m(k) | b_1(k) \cdots b_n(k)]^T \quad (2)$$

are modeled by a Gauss-Markov process, which satisfies the stochastic difference equation

$$x(k+1) = \Phi x(k) + v(k) \quad (3)$$

where Φ is a known $(m+n)*(m+n)$ stable matrix and $v(k)$ is a sequence of independent, identically distributed Gaussian random variables, with zero mean and variance matrix R_v. Moreover, it is assumed that $e(k)$ is independent of $v(k)$ and $x(0)$.

Defining the row vector

$$\begin{aligned} \theta(k) = [-y(k-1)\ -y(k-2)\ \cdots\ -y(k-m)\ | \\ u(k-1)\ u(k-2)\ \cdots\ u(k-n)] \end{aligned} \quad (4)$$

equation (1) can be written as

$$y(k) = \theta(k)x(k) + e(k) \quad (5)$$

Thus, the model is defined in the compact form by (3) and (5).

The purpose of the control action is to keep the system output as close as possible around a reference sequence value, $y_r(k+1)$, during H steps of control. The deviation is measured by the criterion

$$V_H(u) = E\{\sum_{i=1}^{H}(y(k+i) - y_r)^2\} \quad (6)$$

where $u = (u_{k+H-1}, \cdots, u_k)^T$ represents the actual and future control signals and the expected value refers to all underlying random variables. The controller design problem is formulated by selecting the input sequence $u(i), i = k, \cdots, k+H-1$, that minimizes the cost function (6), subject to (3),(5) and by a causality constraint requiring that, at each instant k, the control signal $u(k)$can only depend on the initial information, and on past inputs and outputs; i.e.,

$$\begin{aligned} \Upsilon_k = \{y(k), y(k-1), \cdots, y(0), u(k-1), \\ u(k-2), \cdots, u(0)\} \end{aligned} \quad (7)$$

The parameter estimation $x(k)$ of the system described by (1), standard Kalman-filter equations can be applied to (3) and (5). It can be seen that the conditional distribution of $x(k+1)$, given Υ_k, is Gaussian with mean $\hat{x}(k+1)$ and covariance $P(k+1)$, where $\hat{x}$ and P satisfy the difference equations

$$\hat{x}(k+1) = \Phi\hat{x}(k) + K(k)[y(k) - \theta(k)\hat{x}(k)] \quad (8)$$

$$K(k) = \Phi P(k)\theta^T(k)[\theta(k)P(k)\theta^T(k) + \sigma^2]^{-1} \quad (9)$$

$$P(k+1) = [\Phi - K(k)\theta(k)]P(k)\Phi^T + R_v \quad (10)$$

with initial conditions $P(0)$ and $\hat{x}(0)$.

3 ASOD Controller

In order to acquire a good system performance (5), Wittenmark [5] proposed an Active Suboptimal Dual Controller (ASOD),adding a term to the loss function (6), which reflects the quality of the estimates. By attaching simple terms, this will prevent the cautious controller from turning off the control. It is important to add simple terms to make it easy to numerically be able to find the resulting controller.

$$V_{ASOD}(u_k) = E\{(y(k+1) - y_r(k+1))^2 + \lambda f(P(k+2)|\Upsilon_k\} \quad (11)$$

The minimization of (11) is done with respect to $u(k)$. $P(k+2)$ is the variance of the errors of the parameter estimates and is the first time at which the covariance is influenced by $u(k)$. The function $f(.)$ is assumed to be positive, increasing monotonically, and twice continuously differentiable. Using (11) the control signal must guarantee a good trade-off between control and estimation. The second term in (11) can be regarded as a constraint which is added to the loss function.

In general, it is not possible to minimize (11) analytically, since $P(k+2)$ is a non-linear function of $u(k)$. The solution must be obtained through an iterative procedure. Alternatively, Witternmark and Elevitch [6] proposed a further approximation by making a series expansion of (11) around a nominal control, and keeping first and second-order terms to obtain an analytical expression for the control signal.

The second term in equation (11) also has a parameter design λ which needs to be defined in advance, and remains fixed throughout the steps.

4 ASOD Controller with variable design parameter

In this section, the change in the Active Suboptimal Dual Controller proposed by Wittenmark [5] will be presented. The intended solution is to obtain a weight variable factor lambda (λ) that minimizes the loss function (11).

The loss function (11) is changed to

$$V^{GV}_{ASOD}(u_k) = E\{(y(k+1) - y_r(k+1))^2 + \lambda(k) f(P(k+2)|\Upsilon_k\} \quad (12)$$

where $\lambda(k)$ represents how much the estimated parameter accuracy is important compared to the control action at each step. An appropriate choice of a fixed λ value can involve extensive system simulations. A weight variable factor is necessary,owing to stochastic variables in the system. Once the parameters and the noise interferes randomly at each step in the system, the weight factor $\lambda(k)$ must also keep up with these changes, to conduct better control and estimation.

The weight factor $\lambda(k)$ definition is performed using an algorithm that takes into account the following information: the estimated vector $\hat{x}(k+1)$, the Kalman gain vector $K(k+1)$, the noise parameters vector $v(k)$, and the certainty equivalence control $u_{CE}(k)$

$$\lambda(k) \Longleftrightarrow \begin{cases} \hat{x}(k+1) \\ K(k+1) \\ R_v(k) \\ u_{CE}(k) \end{cases}$$

where

$$K(k+1) = \Phi P(k+1)\theta^T(k+1)[\theta(k+1) P(k+1)\theta^T(k+1) + \sigma^2]^{-1} \quad (13)$$

$$u_{CE}(k) = \frac{y_r(k+1) - \tilde{\theta}(k)\hat{x}(k+1)}{\hat{b_1}(k+1)} \quad (14)$$

and

$$\tilde{\theta}(k) = [-y(k-1) \cdots -y(k-m)| \; 0\, u(k-2) \cdots u(k-n)] \quad (15)$$
$$\hat{b_1}(k) \neq 0$$

The algorithm 1 presents the sequence of actions made to set the $u(k)$ value at each step. The most important points in the algorithm are the ways to find out which parameter $\lambda_i(k)$ provide the best control signal $u_i(k)$ at each step. The control signal selection is detailed below.

Defining the weight factor vector we have

$$\varphi_\lambda(k) = [\lambda_1(k)\, \lambda_2(k) \cdots \lambda_i(k)] \quad (16)$$

Using (16), an input vector U_λ is calculated. The calculating procedure is performed as described in Witternamrk and Elevitch [6].

$$U_\lambda = [u_1(k)\, u_2(k) \cdots u_i(k)] \quad (17)$$

The choice of $u_i(k)$ influences directly the actual loss function (12), as well as the future cost, since $u_i(k)$ influences $\theta(k+1)$, which also influences $\hat{x}(k+2)$, $P(k+2)$ and $\tilde{\theta}(k+2)$ Åström and Wittenmark [18].

The $u_i(k)$ selection starts by examining $\hat{x}(k+1)$. From the stochastic process (5) we know that the conditional distribution of $y(k)|\Upsilon_{(k-1)}$ is normal, with mean value given by

$$E[y(k)|\Upsilon_{(k-1)}] = \theta(k-1)\hat{x}(k) \quad (18)$$

and variance

$$\sigma^2_y(k) = \theta(k-1)P(k)\theta^T(k-1) + \sigma^2 \quad (19)$$

Moreover, we know that the conditional distribution of $\hat{x}(k+1)|\Upsilon_{(k-1)}$ is normal, with mean value given by

$$E[\hat{x}(k+1)|\Upsilon_{(k-1)}] = \Phi\hat{x}(k) \quad (20)$$

and covariance

$$cov \ [\hat{x}(k+1), \hat{x}(k+1)|\Upsilon_{(k-1)}] = K(k)[\sigma^2 + \theta(k)P(k)\theta^T(k)]K^T(k) \quad (21)$$

Algorithm 1: $\lambda(k)$ definition

Data: A weight factor vector $\varphi_\lambda = [\lambda_1(k) \quad \cdots \quad \lambda_i(k)]$
Result: The parameter $\lambda_i(k)$

repeat
  forall the *elements of* φ_λ **do**
    calculate $u_i(k)$ that minimizes V_{ASOD}^{GV} ; /* shown in (12) */
  end
  if $|\hat{x}(k+1)| \leq \frac{R_v}{2}$ **then**
    foreach $u_i(k)$ *calculated* **do**
      evaluate $K_i(k+1)$;
    end
    Function *select max element(*$K_i(k+1)$*) : array*;
    begin
      return $u_i(k)$ associated a maximum K_i;
    end
  else
    Calculate $u_{CE}(k)$;
    Function *select min element(*r_{min}*) : array*;
    begin
      compute vector $U_\lambda(k) - u_{CE}(k)$; /* shown in (28) */
      return $u_i(k)$ associated a minimum value in vector;
    end
  end
until *simulation stops*;

Given by Wittenmark [18] that the minimum value for the quadratic criterion in time-varying systems with known parameters is

$$min \sum_{i=1}^{H} [y(k+i) - y_r]^2 = k\sigma^2 \quad (22)$$

it is predicted that the estimated vector $\hat{x}(k+1)$ cannot be sensitive to changes in parameters when

$$-\frac{\sigma_v^2}{2} \leq \hat{x}(k+1) \leq \frac{\sigma_v^2}{2} \quad (23)$$

Thus, the selection mode of $u_i(k)$ is directly linked to the condition above. If the $\hat{x}(k+1)$ value is between the condition limits (23), the suitable $u_i(k)$ selection is done based on the estimated parameter gain (8). If the $\hat{x}(k+1)$ value is not between the condition limits (23), the appropriate $u_i(k)$ selection is performed, taking into account the certainty equivalence control signal (14).

Under definitions (8) and (9), it is seen that if the estimated vector meets the condition (23), then the chosen control signal $u_i(k)$ will be the one to apply a higher gain in the future estimated vector (step forward); i.e.:

Introducing the gain vector

$$K_\lambda(k+1) = [K_1(k+1) \ K_2(k+1) \ \cdots \ K_i(k+1)] \quad (24)$$

where

$$K_i(k+1) = \frac{\Phi P(k+1)\theta^T(k+1)}{\sigma_y^2(k+1)} \quad (25)$$

and

$$\sigma_y^2(k+1) = \theta(k+1)P(k+1)\theta^T(k+1) + \sigma^2 \quad (26)$$

It selects the largest absolute value of the gain vector (24),

$$K_{max}(k+1) = max|K_\lambda(k+1)| \quad (27)$$

Therefore, the $u_i(k) \in U_\lambda(k)$ selected matches $u_i(k)$,associated with the maximum $K_i(k+1)$ value.

If the requirement (23) is false, the chosen control signal $u_i(k)$ will be that which is, in absolute terms, the closest to $u_{CE}(k)$;

Considering the values of input vector (17) subtracted from (14) we have

$$U_\lambda(k) - u_{CE}(k) = [(u_1(k) - u_{CE}(k)), (u_2(k) - u_{CE}(k), \cdots, (u_i(k) - u_{CE}(k))] \quad (28)$$

Consequently, it selects the lowest absolute value of equation (27).

5 Simulation Results

The performance of the suboptimal dual controller with variable parameter λ proposed is illustrated through three examples. Comparisons will be made with the innovation dual controller presented in Milito et al. [8], as well as the ASOD controller presented in Witternmark and Elevitch [6]. The evaluation of the controller performance is made from NS simulations, where each one consists of NP steps. Thus, the average loss per step, which evaluates the control quality, is given by

$$\overline{V} = \frac{1}{NS}\left\{\sum_{j=1}^{NS}\frac{1}{NP}\left[\sum_{i=1}^{NP}(y(k+i)- y_r(k+1))^2\right]\right\} \quad (29)$$

and the standard deviation is given by

$$\sigma_V = \sqrt{\frac{1}{NS}\left\{\sum_{j=1}^{NS}\left(\frac{1}{NP}\left[\sum_{i=1}^{NP}a^2\right]-\overline{V}\right)^2\right\}} \quad (30)$$

where

$$a = y(k+i) - y_r(k+1) \quad (31)$$

The estimation quality can be measured by the mean value of p_{bb} the parameter variance, which is the main parameter of interest.

$$\overline{P}_{bb} = \frac{1}{NS}\left\{\sum_{j=1}^{NS}\frac{1}{NP}\left[\sum_{i=1}^{NP}p_{bb}(i)\right]\right\} \quad (32)$$

The changes between different realizations can be very large, and it can be difficult to know which suboptimal solution is best for each example. So, pairwise comparisons of the average loss per step for each simulation were made, and the statistical hypothesis test was used to decide the controller ranking. The following three examples were simulated:

Example 1: Consider the first-order system with the reference signal as $y_r = 1$. This system has often been used in to evaluate the performance of different adaptive suboptimal controllers Witternamrk and Elevitch [6].

$$y(k) + a(k)y(k-1) = b(k)u(k-1) + e(k) \quad (33)$$

where

$$\begin{aligned} a(k) &= -0.9 \\ b(k+1) &= 0.9b(k) + v(k) \end{aligned}$$

Example 2: Consider the first-order system with the reference signal as $y_r = 0$

$$y(k) + a(k)y(k-1) = b(k)u(k-1) + e(k) \quad (34)$$

where

$$\begin{aligned} a(k+1) &= -1 \\ b(k+1) &= 0.8b(k) + v(k) \end{aligned}$$

Example 3: Consider an $ARX(2,2)$ system with the reference signal as $y_r = 0$.

$$\begin{aligned} &y(k) + a_1(k)y(k-1) + a_2(k)y(k-2) = \\ &b_1(k)u(k-1) + b_2(k)u(k-2) + e(k) \end{aligned} \quad (35)$$

where

$$\begin{aligned} a_1(k+1) &= -1.2 \\ a_2(k+1) &= 0.7 \\ b_1(k+1) &= 0.9b(k) + v(k) \\ b_2(k+1) &= 1 \end{aligned}$$

For each system, 200 Monte Carlo simulations ($NS = 200$) were done, where each simulation consisted of 400 time steps ($NP = 400$). In all simulations, the noise $e(k)$ and $v(k)$ had a mean value zero and standard deviation of 0.5 and 1.0, respectively. The initial time of the simulation ($k < 20$) is disregarded to avoid the influence of initial effects. The weighting factor λ in the ASOD controller and in the innovation controller was chosen to be 0.5 in all examples, since this value has been shown to give a good exchange between control action and estimation (see [8] and [6]).

The simulation results are presented in Tables 1, 2, 3, 4, 5 and 6. Figures 1, 4 and 9 summarize the performance of the parameter λ during the simulation and Figures 2, 3, 5, 6, 7 and 8 show the performance of the proposed controller for one realization of each system.

The φ_λ vector used in simulations is given by

$$\varphi_\lambda(k) = [0.1 \;\; 0.3 \;\; 0.5 \;\; 0.7 \;\; 0.9] \quad (36)$$

Table 1: Simulation results - Example 1.

Control law	$\overline{V}$	σ_V	$\overline{P}_{bb}$
ASOD $\lambda = 0.5$	0,7990	0,1701	2,0296
Innovation	0,8705	0,1992	2,9907
ASOD λ variable	0,7466	0,1683	2,1163

In all examples, the modified ASOD controller achieved results that indicate a better performance compared to the fixed λ ASOD controller, as well as

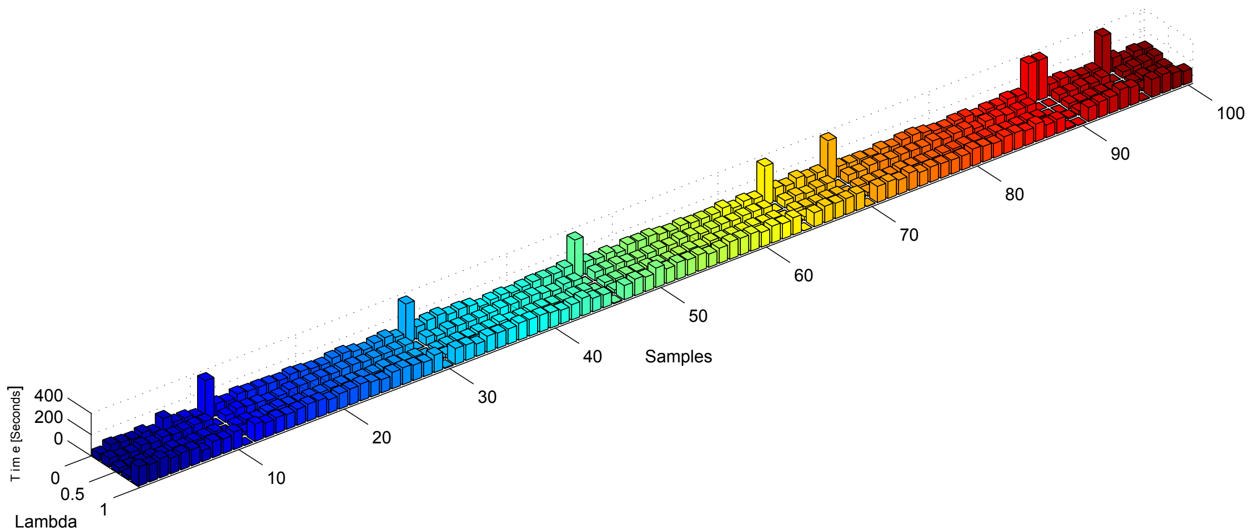

Figure 1: 3D lambda vector map over 100 simulations in Example 1.

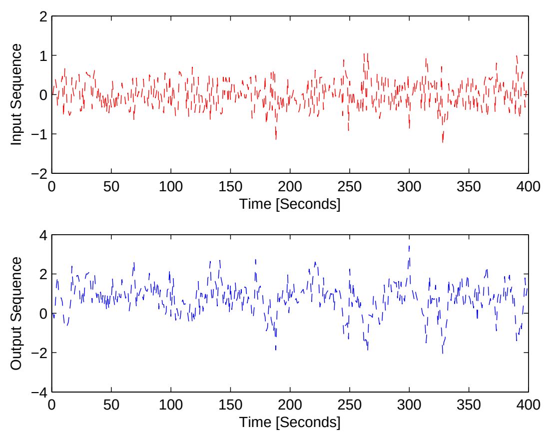

Figure 2: Simulation of Example 1 with graphics: output y and control signal.

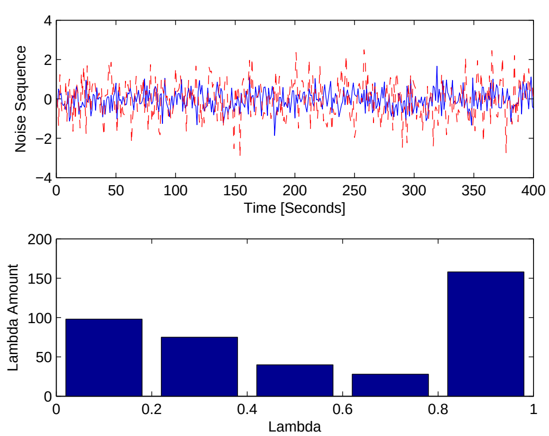

Figure 3: Simulation of Example 1 with graphics: noises and φ_λ vector.

Table 2: Pairwise comparison between controllers - Example 1.

Control law A	Control law B	$(A < B) of 200$
ASOD λ variation	ASOD $\lambda = 0.5$	$148(74\%)$
ASOD λ variation	Innovations	$163(81,5\%)$
ASOD $\lambda = 0.5$	Innovations	$136(68\%)$

the innovation controller. The average loss per step obtained was 20% lower and, moreover, as shown in Tables 2, 4 and 6, there is also often an average loss smaller in each simulation.

Figures 1, 4 and 9 show the results associated with each φ_λ vector during each step, for 100 simulations of examples $1-3$, respectively. Comparing Figures 1, 4 and 9, it is possible to distinguish how the lambda λ selection algorithm adapts, according to the system, the estimation and control conditions. In Figure 1, there is a good balance in choosing lambdas, by equalizing the control action and estimate.

Table 3: Simulation results - Example 2.

Control law	$\overline{V}$	σ_V	$\overline{P}_{bb}$
ASOD $\lambda = 0.5$	1,3599	0,4373	1,6808
Innovation	1,6143	0,6203	2,1256
ASOD λ variable	1,2605	0,4086	1,6159

In Figure 1, there is a greater balance in choosing lambdas, by equalizing the control action and estimate. In Figure 4, the lambda selection is more defined at the ends vector emphasizing control or estimation in each step. It is also important to note that, in

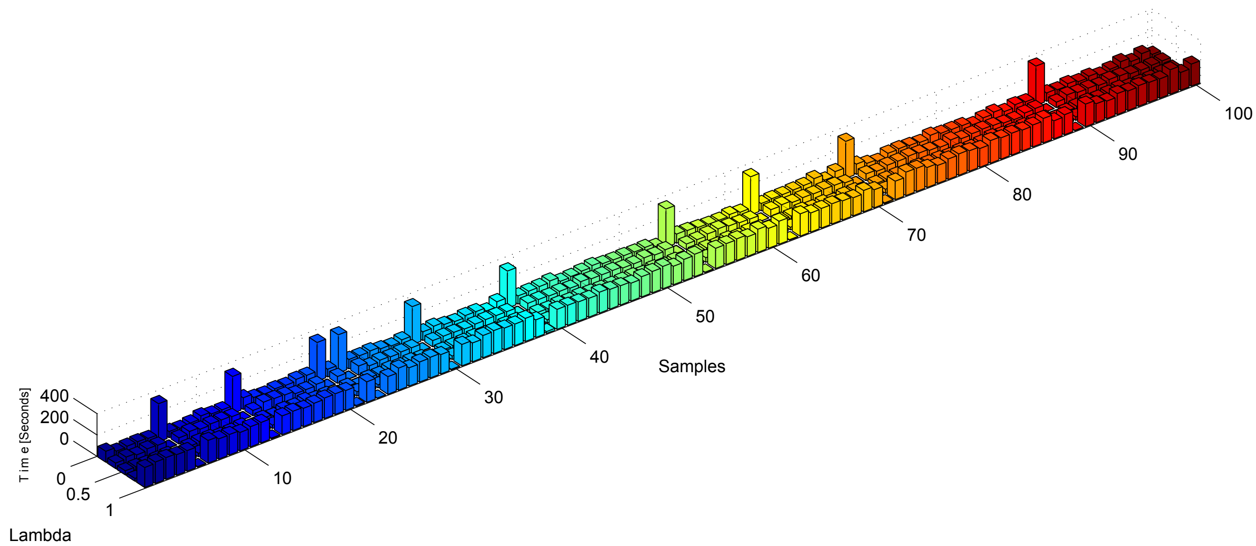

Figure 4: 3D lambda vector map over 100 simulations in Example 2.

Table 4: Pairwise comparison between controllers - Example 2.

Control law A	Control law B	$(A < B) of 200$
ASOD λ variable	ASOD $\lambda = 0.5$	$133(66,5\%)$
ASOD λ variable	Innovations	$144(72\%)$
ASOD $\lambda = 0.5$	Innovations	$164(82\%)$

Example 3, the simulations with the innovation controller presented the turn-off phenomenon, while the other controllers avoided this behaviour of the control signal. Owing to this phenomenon, the control signal remained turned off, and the pairwise comparison between the innovation controller and the others in Table 6 showed unusual results.

The general behaviour of the vector selection $\varphi_\lambda(k)$ will depend on the plant characteristics $\theta(k)$ and $x(k)$, of the associated stochastic variables $e(k)$ and $v(k)$, of the initial plant parameter $P(0)$ and of the known matrix Φ.

Table 5: Simulation result - Example 3.

Control law	$\overline{V}$	σ_V	$\overline{P}_{bb}$
ASOD $\lambda = 0.5$	0,9887	0,1755	1,8295
Innovation	0,9140	0,3062	3,4298
ASOD λ variable	0,9759	0,1966	1,9776

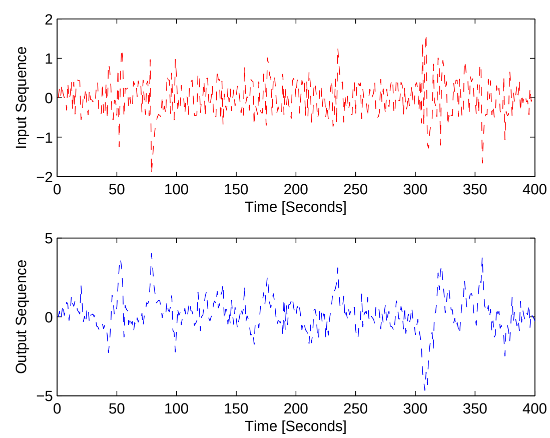

Figure 5: Simulation of Example 2 with graphics: output y and control signal.

Table 6: Pairwise comparison between controllers - Example 3.

Control law A	Control law B	$(A < B) of 200$
ASOD λ variable	ASOD $\lambda = 0.5$	$115(57,5\%)$
ASOD λ variable	Innovations	$55(27,5\%)$
ASOD $\lambda = 0.5$	Innovations	$44(22\%)$

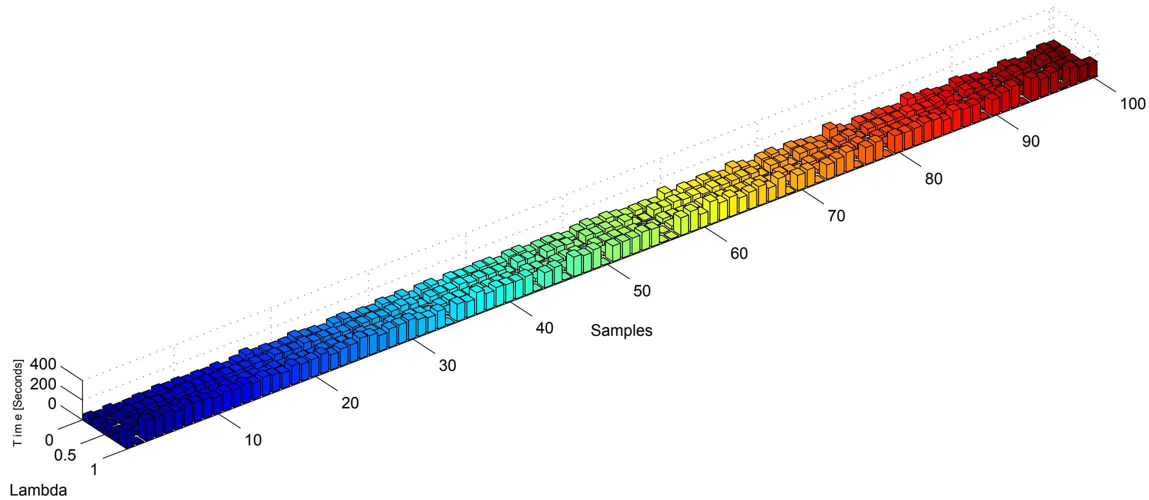

Figure 9: 3D lambda vector map over 100 simulations in Example 3.

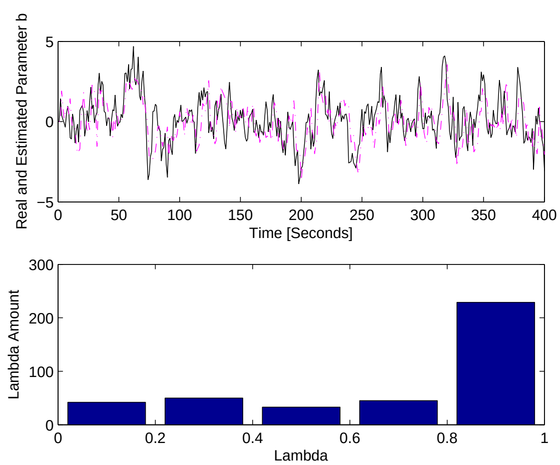

Figure 6: Simulation of Example 2 with graphics: parameter b and φ_λ vector.

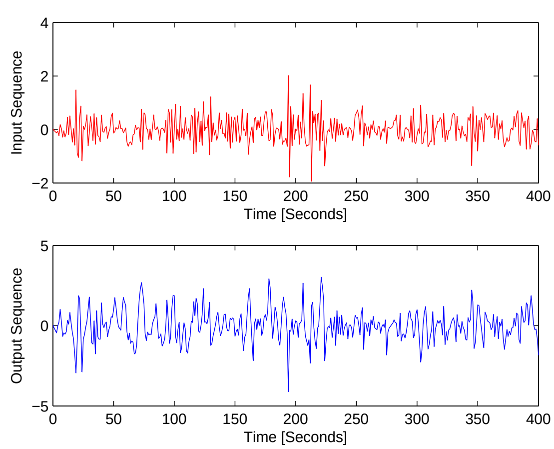

Figure 7: Simulation of Example 3 with graphics: output y and control signal.

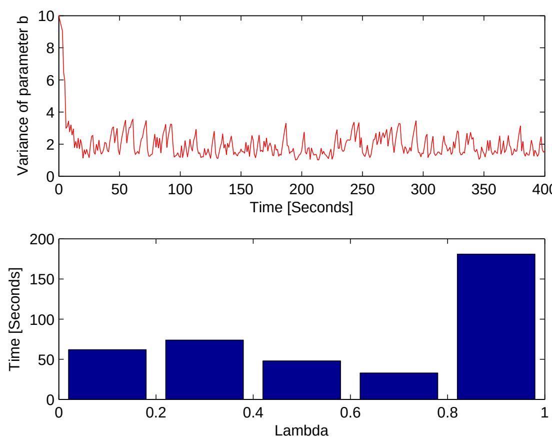

Figure 8: Simulation of Example 3 with graphics: variance and φ_λ vector.

6 Conclusion

The proposed controller achieved good performance without requiring great computational effort. The controller features dual characteristics; i.e. the controller does not perform action controls due only the immediate control, but also consider the future parameters estimation of the system, resulting in a better control.

The controller avoids the necessity for setting the lambda design parameter, which constitutes an advantage since this adjustment can be critical in some cases. An algorithm for the adaptive adjustment of the lambda parameter is employed at each step. Thus, the controller performs a better action control, as well as a better estimation, according to the plant characteristics, and the stochastic variables that affect the system. The proposed controller performed satisfactorily better than the dual innovation controller [8] and the ASOD controller [5]. Once ratified the proposed technique in this paper, the focus will be perform practical tests in stochastic processes such as queueing networks.

References:

[1] Feldbaum, A. A., Dual control theory. I-IV , *Automation Remote Control* vol 21,22, 1960-1961.

[2] Åström, K. J. and Wittenmark, B. *Adaptive Control* Addison-Wesley Longman Publishing Co., Inc. 1994

[3] Filatov, N.M. and Unbehauen, H., Survey of adaptive dual control methods, *Control Theory and Applications, IEE Proceedings* - **147**, 2000, pp. 118-128.

[4] Lindoff, B., Holst, J. and Wittenmark, B., Analysis of approximations of dual control, *Int. J. Adapt. Control Signal Process.* **13**, 1999, pp. 593-620.

[5] Wittenmark, B., An active suboptimal dual controller for systems with stochastic parameters, *Automat.Control Theory Appl.* **3**, 1975, pp. 13–19.

[6] Wittenmark, B. and C. Elevitch, An adaptive control algorithm with dual features, *7th IFAC/IFORS Symp. on Identification and Systems Parameter Estimation*, 1985, pp. 587–592.

[7] Hughes, D.J. and O.L.R. Jacobs, Turn-off, escape and probing in non-linear stochastic control, *Preprint IFAC Symposium on Stochastic Control*, 1974, pp. 343–352.

[8] Milito, R., Padilla, R.A., Padilla, R. and Cadorin, D., Turn-off, An innovations approach to dual control, *Automatic Control, IEEE Transactions on* **27**, 1982, pp. 132–137.

[9] Ishihara, T., Abe, K.I. and Takeda, Extensions of innovations dual control, *Int. J. Systems Sei* **19**, 1988, pp. 653–667.

[10] Maitelli, A. and Yoneyama, T., A multistage suboptimal dual controller using optimal predictors, *Automatic Control, IEEE Transactions on* **44**, 1999, pp. 1002–1008.

[11] Lee JM, Lee JH (2009) An approximate dynamic programming based approach to dual adaptive control. Journal of Process Control 19(5):859 – 864

[12] Flidr M, Simandl M (2013) Implicit dual controller based on stochastic integration rule. In: Control Conference (ECC), 2013 European, pp 896–901

[13] Kral L, Simandl M (2011) Predictive dual control for nonlinear stochastic systems modelled by neural networks. In: Control Automation (MED), 2011 19th Mediterranean Conference on, pp 1277–1282

[14] Li Xia, Xi Chen, Xi-Ren Cao, Policy iteration for customer-average performance optimization of closed queueing systems, *Automatica* **45**, 2009, pp. 1639–1648.

[15] Li Xia; Shihada, B., Max-Min Optimality of Service Rate Control in Closed Queueing Networks, *Automatic Control, IEEE Transactions on* **58**, 2013, pp. 1051–1056.

[16] Bar-Shalom Y, Wall K (1980) Dual adaptive control and uncertainty effects in macroeconomic systems optimization. Automatica 16(2):147 – 156

[17] Ismail, A., Dumont, G.A. and Backstrom, J., Dual adaptive control of paper coating, *Control Systems Technology, IEEE Transactions on* **11**, 2003, pp. 289–309.

[18] Åström, K. J. and B. Wittenmark, Problems of identification and control, *Journal of Mathematical Analysis and Applications* **34**, 1971, pp. 90–113.