A Common Structure for H-Infinity Complementary Sensitivity Design of DTCT-PID Controller Parameters

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***Abstract*—**This paper presents a successful graphical design methodology for Discrete-Time or Continuous-Time Proportional Integral Derivative (DTCT-PID) controller parameters that satisfy H-infinity complementary sensitivity constraint for a single input single output (SISO) system. These problems can be solved by finding all achievable DTCT-PID parameters that simultaneously stabilize the closed-loop characteristic polynomial and satisfy constraints defined by a set of related complex polynomials. The bilinear transformation is used to describe the Discrete-Time Proportional Integral Derivative (DTPID) controller parameters since this methodology is based on frequency response and sampled data collection of the system. This approach allows the user to apply the same procedures for discrete-time or continuous-time H-infinity complementary sensitivity design of PID controller parameters. This methodology has been applied to a DC motor data to demonstrate the application of this methodology.

Key-Words: - H-infinity complementary sensitivity, PID controller, time delay, discrete- time, continuous time

1 Introduction

The design of Proportional Integral Derivative (PID) controllers have been researched in the literatures for many decades. PID controllers have been implemented in many industrial and manufactory applications. Design control algorithms that can handle communication delays and robustness for unknown disturbance environment are particularly important in process control systems. Recently, a design method that works directly in the digital domain is more prevalent in an autonomous system. H-infinity complementary sensitivity design is particularly the target of this paper since in many applications there is no information of the uncertain disturbances.

Most of the early work in designing PID controllers required finding all PID controllers that stabilized the nominal plant model, where the rational transfer function model was known [1] and [2]. The method introduced by Tan in [3] broke the numerator and denominator of plant transfer function into even and odd parts. Ho introduced a generalization of the Hermite-Biehler theorem for H_∞ PID controller design [4]. Bhattacharyya and Keel looked at a similar problem for first-order controllers [5]. In [6] , Ho and Lin looked at PID controller design for robust performance of a rational plant transfer function. In [7], Keel and Bhattacharyya allowed the time-delays in the nominal model when they investigated the weighted sensitivity and robust stability problems. In [8], Emami and Watkins developed a method for robust performance design of PID controller parameters. In [9], Saeki introduced a method for determining the number of unstable poles in each region of PI and PD planes. In [10], Žáková introduced constrained pole assignment to design PD controllers for a double integrator plant model with time delays or time constant.

A particular application in this area is in an autonomous vessel design. Ker-Wei, Yu and Hsu introduced particle swarm optimization method for a ship coordinate system [11]. Xia, Shi, Fu, Wang, and Bian applied hybrid PID controllers with a neural network to make an adaptive control for a ship [12].

Pettersen and Fossen added an integral action in the feedback path to control the orientation and position of a ship with experimental results [13].

Tzamtzi, Koumboulis, and Skarpetis looked at the parameters of PID controller using a metaheuristic algorithm method [14]. They developed the metaheuristic algorithm to adjust the PID parameters and meet the performance requirement for a pouring task. Chang and Chen applied a fractional PID controller to meet the performance requirement for an active magnetic bearing system [15]. In [15], an adaptive genetic algorithm applied to determine the PID controller parameters that optimized a multi-objective cost function.

Unfortunately, most of the work in this area has concentrated on the design of continuous-time PID controllers [1-10]. However, in a networking autonomous system the controllers should be implemented as digital compensators. In [16], Emami and Watkins applied discrete time H-infinity complementary sensitivity PID controller design to a DC motor based on the frequency data measurement. They used delta operator to describe the PID controllers. In [17] and [18], Keel, Bhattacharyya, Datta, and Rego developed backward differences to design discrete time PID controllers that stabilized the Tchebysheve representation of a discrete time system. In [19] and [20], Emami, Watkins, and Lee applied the delta operator to obtain a unified approach for finding stability boundaries of PID controllers for arbitrary order transfer functions with time delay in the frequency domain. The determination of all achievable PID controllers in the parameter space was introduced in [9], [19], and [20].

The current paper is the extension of our previous work in [16], [21], and [22]. The extension is to introduce a unified approach for H-infinity Discrete-Time or Continuous-Time Proportional Integral Derivative (DTCT-PID) controller parameters for both discrete or continues time systems under a common frame work. In addition, an experimental data of SRV-02 DC motor from Quanser Innovate Educate in Figure 1 is used to demonstrate the application of the current paper.

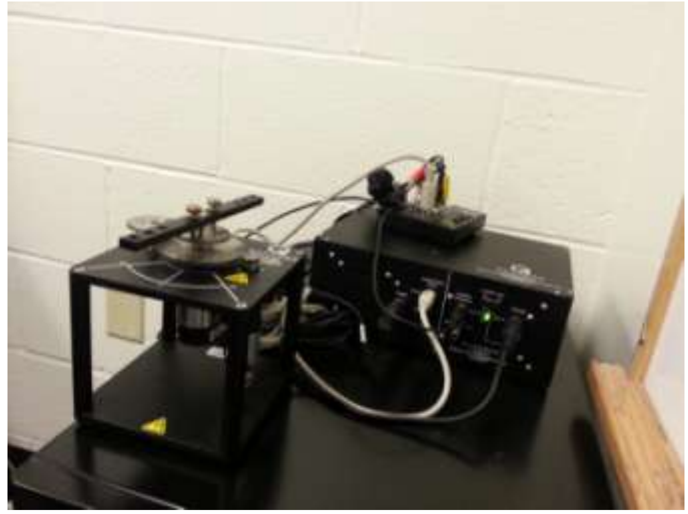

Fig 1. SRV02 DC motor

The goal of current paper is to define all achievable DTCT-PID controller parameters that simultaneously stabilize the closed-loop discrete time or continuous time system and satisfy an H_∞ complementary sensitivity constraints. Complementary sensitivity constraint is particularly the target here since the assumption is that there is no information of disturbance environment. Designing DTCT-PID controller for complementary sensitivity constraint allows the system to perform reasonably more robust. The bilinear transformation is used to describe the Discrete-Time Proportional Integral Derivative (DTPID) controller parameters for systems with the modeling in digital and sampled-data environment [22], [23], [24], and [25]. In addition, this methodology is based on frequency response analysis. This work builds upon the pervious development in [8], [16], [21], [22], and [24].

The remainder of this paper is organized as follows. The design statement is introduced in Section 2. This design methodology is performed by using SRV-02 DC motor data from Quanser Innovate Educate [26] in Section3. The conclusion is presented in Section 4. Finally, the acknowledgement of this paper is in Section 5.

2 Design Statement

Consider a Single Input Single Output (SISO) System in Figure 2, where the nominal continuous-time plant transfer function is $G_o(s)$ with a time delay of τ such as:

$$G_p(s) = G_0(s)e^{-\tau s}. \tag{1}$$

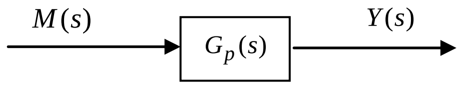

Fig. 2. Open loop system

The bilinear discrete time modeling of system is defined as:

$$G_p(z) = \Gamma\left(G_p(s), T_s\right), \tag{2}$$

where, Γ is the bilinear transformation and sampling period is T_s. The relationship between the bilinear transformation and Laplace transformation is $s = \frac{2}{T_s}\frac{z-1}{z+1}$. Using the bilinear transformation for discrete time modeling allows the designer to consider the fractional time delay in the model for the discrete time analysis in frequency response.

The unity feedback closed-loop system shown in Figure 3, where G_p can be selected either discrete time or continuous time model of the system. G_c is either the discrete or continuous time PID controller. The either sampled or analog reference input and output signals are R and Y, respectively.

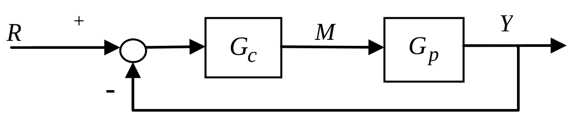

Fig. 3. Block diagram of the closed loop system

The discrete time bilinear frequency domain transformation is used as:

$$\beta = \frac{2}{T_s}\frac{e^{j\omega T_s}-1}{e^{j\omega T_s}+1} =: \frac{2\left(\cos(\omega T_s)-1+j\sin(\omega T_s)\right)}{T_s\left(\cos(\omega T_s)+1+j\sin(\omega T_s)\right)}, \tag{3}$$

where ω is the frequency-scale warping range between $0 \le \omega < \omega_s$. ω_s is the Nyquist frequency.

The unified frequency transformation for both continuous time and discrete time data are defined as:

$$\beta = \begin{pmatrix} j\omega, & T_s = 0, & \omega \in [0,\infty) \\ \frac{2}{T_s}\frac{e^{j\omega T_s}-1}{e^{j\omega T_s}+1}, & T_s \neq 0, & \omega \in [0,\omega_s) \end{pmatrix}. \tag{4}$$

The unified transfer function of the system in (1) or (2) in frequency domain can be defined in terms of their real and imaginary parts as

$$G_p(\beta) = R_p(\beta) + j\,I_p(\beta). \tag{5}$$

The unified DTCT-PID controller is defined in frequency domain as:

$$G_c(\beta) = K_p + \frac{K_i}{\beta} + K_d\,\beta, \tag{6}$$

where K_p, K_i, and K_d are the proportional, integral, and derivative gains, respectively.

The deterministic values of K_p, K_i, and K_d for which the closed-loop characteristic polynomial is Hurwitz stable was defined in [9], [19], and [20]. In the current paper, the problem is to find all achievable DTCT-PID controllers that stabilize the system and satisfy the H_∞ complementary sensitivity constraint such as:

$$\|T(\beta)\|_\infty \le \gamma_0, \tag{7}$$

where $T(\beta) = \frac{G_p(\beta)G_c(\beta)}{1+G_p(\beta)G_c(\beta)}$ is the complementary sensitivity function and γ_0 is a real positive scalar. The complex function in (7) for a SISO system for each value of ω can be written in terms of its magnitude and phase angles as:

$$\left\| T(\beta) \right| e^{j\angle T(\beta)} \right| \le \gamma_0 \qquad \forall\omega. \tag{8}$$

If (8) holds, then for each value of ω:

$$T(\beta)e^{j\theta_T} \le \gamma_0 \qquad \forall\omega, \tag{9}$$

must be true for some $\theta_T \in [0, 2\pi)$, where $\theta_T = -\angle T(\beta)$. Consequently, all PID controllers that satisfy (7) must lie at the intersection of all controllers that satisfy (9) for all $\theta_T \in [0, 2\pi)$.

To solve this problem for each value of $\theta_T \in [0, 2\pi)$ all DTCT-PID controller parameters on the boundary of (9) will be defined. It is easy to show from (9), that all the PID controllers on the boundary must satisfy the following characteristic equation:

$$P(\omega, \theta_T, \gamma_0, T_s) = 0, \qquad (10)$$

where,

$$P(\omega, \theta_T, \gamma_0, T_s) = 1 + G_p(\beta)G_c(\beta) - \frac{1}{\gamma_0} G_p(\beta)G_c(\beta)\, e^{j\theta_T}.$$

Note that (10) reduces to the frequency response of the standard closed-loop characteristic polynomial as $\gamma_0 \to \infty$. Substituting (5), (6), and $e^{j\theta_T} = \cos\theta_T + j\sin\theta_T$ into (10) and solving for the real and imaginary parts yields:

$$X_{Rp}K_p + X_{Ri}K_i + X_{Rd}K_d = Y_R, \qquad (11)$$

and

$$X_{Ip}K_p + X_{Ii}K_i + X_{Id}K_d = Y_I, \qquad (12)$$

where

$$X_{Rp} = \omega R_p(\beta)\left(1 - \frac{1}{\gamma_0}\cos\theta_T\right) + \omega I_p(\beta)\left(\frac{1}{\gamma_0}\sin\theta_T\right),$$

$$X_{Ri} = \frac{\left(1 + \cos(\omega T_s)\right)}{2\mathrm{sinc}(\omega T_s)} \left(\left(\frac{1}{\gamma_0}\sin\theta_T\right)R_p(\beta) + \left(\frac{1}{\gamma_0}\cos\theta_T - 1\right)I_p(\beta)\right),$$

$$X_{Rd} = \frac{2\omega^2 \mathrm{sinc}(\omega T_s)}{\left(1 - \cos(\omega T_s)\right)} \left(\left(\frac{1}{\gamma_0}\sin\theta_T\right)R_p(\beta) + \left(\frac{1}{\gamma_0}\cos\theta_T - 1\right)I_p(\beta)\right),$$

$$Y_R = -\omega,$$

$$X_{Ip} = \omega R_p(\beta)\left(\frac{1}{\gamma_0}\sin\theta_T\right) + \omega I_p(\beta)\left(\frac{1}{\gamma_0}\cos\theta_T - 1\right),$$

$$X_{Ii} = \frac{\left(1 + \cos(\omega T_s)\right)}{2\mathrm{sinc}(\omega T_s)} \left(\left(1 - \frac{1}{\gamma_0}\cos\theta_T\right)R_p(\beta) + \left(\frac{1}{\gamma_0}\sin\theta_T\right)I_p(\beta)\right),$$

$$X_{Id} = \frac{2\omega^2 \mathrm{sinc}(\omega T_s)}{\left(1 - \cos(\omega T_s)\right)} \left(\left(1 - \frac{1}{\gamma_0}\cos\theta_T\right)R_p(\beta) + \left(\frac{1}{\gamma_0}\sin\theta_T\right)I_p(\beta)\right),$$

$$Y_I = 0.$$

This is a three-dimensional system in terms of the controller parameters K_p, K_i, and K_d, that corresponds to three degrees of freedom. The solution of this problem is defined in three main methods.

The first method obtains all PID controller parameters that satisfy the H-infinity complementary sensitivity constraint and stability boundaries in the $(K_p,\ K_i)$ plane. The boundary of characteristic equation in (10) achieves in this plane for a fixed value of derivative gain. After setting K_d to the fixed value of $\tilde{K}_d$, (11) and (12) can be rewritten as:

$$\begin{bmatrix} X_{Rp} & X_{Ri} \\ X_{Ip} & X_{Ii} \end{bmatrix} \begin{bmatrix} K_p \\ K_i \end{bmatrix} = \begin{bmatrix} Y_R - X_{Rd}\tilde{K}_d \\ Y_I - X_{Id}\tilde{K}_d \end{bmatrix}. \qquad (13)$$

Solving (13), for $0 < \omega < \omega_s$ and $0 < \theta_T < 2\pi$, gives the following equations for the discrete time proportional and integral parameters:

$$K_p(\omega, \theta_T, \gamma_0, T_s) = \frac{\left(R_p(\beta)\left(\frac{1}{\gamma_0}\cos\theta_T - 1\right) + I_p(\beta)\left(-\frac{1}{\gamma_0}\sin\theta_T\right)\right)}{\left|G_p(\beta)\right|^2 \left(1 - \frac{2}{\gamma_0}\cos\theta_T + \frac{1}{\gamma_0{}^2}\right)}, \qquad (14)$$

$$K_i\left(\omega,\theta_T,\gamma_0,T_s\right)=\omega^2\tilde{K}_d\,\frac{4\mathrm{sinc}^2\left(\omega T_s\right)}{\left(\cos\left(\omega T_s\right)+1\right)^2}+\frac{2\omega\mathrm{sinc}\left(\omega T_s\right)\begin{pmatrix}R_p(\beta)\left(\dfrac{1}{\gamma_0}\sin\theta_T\right)+\\ I_p(\beta)\left(\dfrac{1}{\gamma_0}\cos\theta_T-1\right)\end{pmatrix}}{\left|G_p(\beta)\right|^2\left(\cos\left(\omega T_s\right)+1\right)\left(1-\dfrac{2}{\gamma_0}\cos\theta_T+\dfrac{1}{\gamma_0{}^2}\right)}. \tag{15}$$

As $T_s\to 0$, for $0<\omega<\infty$ and $0<\theta_T<2\pi$ this result corresponds to the continuous time systems. In continuous time the proportional parameters is the same expression as (14) and the following equation is for integral gains.

$$K_i(\omega,\theta_T,\gamma_0)=\omega^2\tilde{K}_d+\frac{\omega\left(R_p(\beta)\left(\dfrac{1}{\gamma_0}\sin\theta_T\right)+I_p(\beta)\left(\dfrac{1}{\gamma_0}\cos\theta_T-1\right)\right)}{\left|G_p(\beta)\right|^2\left(1-\dfrac{2}{\gamma_0}\cos\theta_T+\dfrac{1}{\gamma_0{}^2}\right)}. \tag{16}$$

Note, if $\gamma_0=1,$ then $\theta_T=0$ should be avoided as the denominators of (14), (15), and (16) go to zero. Setting $\omega=0$ in (13) obtains:

$$\begin{bmatrix}0 & X_{Ri}(0)\\ 0 & X_{Ii}(0)\end{bmatrix}\begin{bmatrix}K_p\\ K_i\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}, \tag{17}$$

and conclude that $K_p\left(0,\theta_T,\gamma_0,T_s\right)$ is arbitrary and $K_i\left(0,\theta_T,\gamma_0,T_s\right)=0$, unless $I_p(0)=R_p(0)=0$, which holds only when the plant has a zero at the origin.

The second method obtains all PID controller parameters that satisfy the H-infinity complementary sensitivity constraint and stability boundaries in the $(K_p,\,K_d)$ plane. The boundary of characteristic equation in (10) achieves in this plane for a fixed value of integral gain. After setting K_i to the fixed value $\tilde{K}_i$, (11) and (12) can be rewritten as:

$$\begin{bmatrix}X_{Rp} & X_{Rd}\\ X_{Ip} & X_{Id}\end{bmatrix}\begin{bmatrix}K_p\\ K_d\end{bmatrix}=\begin{bmatrix}Y_R-X_{Ri}\tilde{K}_i\\ Y_I-X_{Ii}\tilde{K}_i\end{bmatrix}. \tag{18}$$

Solving (18) for $0<\omega<\omega_s$ and $0<\theta_T<2\pi$, gives the same expression as (14) for the discrete time or continuous time proportional parameters and it gives the following equation for the discrete time derivative parameters:

$$K_d\left(\omega,\theta_T,\gamma_0,T_s\right)=\tilde{K}_i\,\frac{\left(\cos\left(\omega T_s\right)+1\right)^2}{4\omega^2\mathrm{sinc}^2\left(\omega T_s\right)}+\frac{\begin{pmatrix}R_p(\beta)\left(-\dfrac{1}{\gamma_0}\sin\theta_T\right)+\\ I_p(\beta)\left(-\dfrac{1}{\gamma_0}\cos\theta_T+1\right)\end{pmatrix}\left(\cos\left(\omega T_s\right)+1\right)}{2\omega\left|G_p(\beta)\right|^2\left(1-\dfrac{2}{\gamma_0}\cos\theta_T+\dfrac{1}{\gamma_0{}^2}\right)\mathrm{sinc}\left(\omega T_s\right)}. \tag{19}$$

As $T_s\to 0$, for $0<\omega<\infty$ and $0<\theta_T<2\pi$, this result corresponds to the following expression for the continuous time derivative parameters.

$$K_d(\omega,\theta_T,\gamma_0)=\frac{\tilde{K}_i}{\omega^2}+\frac{\left(R_p(\beta)\left(-\dfrac{1}{\gamma}\sin\theta_T\right)+I_p(\beta)\left(-\dfrac{1}{\gamma}\cos\theta_T+1\right)\right)}{\omega\left|G_p(\beta)\right|^2\left(1-\dfrac{2}{\gamma_0}\cos\theta_T+\dfrac{1}{\gamma_0{}^2}\right)}. \tag{20}$$

Note, if $\omega=0$, $\tilde{K}_i$ must be equal to zero for a solution to exist. Furthermore, as $I_p(0)=0$ for all real plants, $K_d\left(0,\theta_T,\gamma_0,T_s\right)$ is arbitrary and:

$$K_p\left(0,\theta_T,\gamma_0,T_s\right)=\frac{-1}{R_p(0)\left(1-\dfrac{1}{\gamma_0}\cos\theta_T\right)}. \tag{21}$$

If $\gamma_0=1,$ then $\theta_T=0$ should be avoided as the denominators of (14), (19), (20), and (21) go to zero.

The third method applies all achievable PID controller parameters that satisfy the H-infinity complementary sensitivity constraint and stability boundaries in the in the (K_i, K_d) plane. The boundary of characteristic equation in (10) obtains for a fixed value of proportional gain. After setting K_p to the fixed value $\tilde{K}_p$, (11) and (12) correspond the following:

$$\begin{bmatrix} X_{Ri} & X_{Rd} \\ X_{Ii} & X_{Id} \end{bmatrix} \begin{bmatrix} K_i \\ K_d \end{bmatrix} = \begin{bmatrix} Y_R - X_{Rp}\tilde{K}_p \\ Y_I - X_{Ip}\tilde{K}_p \end{bmatrix}. \tag{22}$$

Despite the fact that the coefficients matrix are singular, a solution occurs at any frequency of ω_i. These frequencies can be found graphically by plotting $K_p(\omega, \theta_T, \gamma_0, T_s)$ from (14) verses the frequency range of $0 < \omega < \omega_s$, and $0 < \theta_T < 2\pi$ for the discrete time PID controllers. Next step is to find interception of the fixed value of proportional gain, i.e., $\tilde{K}_p$ and the $K_p(\omega, \theta_T, \gamma_0, T_s)$ curves. These interceptions correspond the frequencies of ω_i's. At these frequencies, $K_d(\omega_i, \theta_T, \gamma_0, T_s)$ and $K_i(\omega_i, \theta_T, \gamma_0, T_s)$ satisfy the straight lines equation such as:

$$\begin{gathered} K_d(\omega_i, \theta_T, \gamma_0, T_s) = \\ K_i(\omega_i, \theta_T, \gamma_0, T_s)\frac{\left(\cos(\omega_i T_s)+1\right)^2}{4\omega_i^2 \operatorname{sinc}^2(\omega_i T_s)} + \\ \frac{\begin{pmatrix} R_p(\beta_i)\left(-\frac{1}{\gamma_0}\sin\theta_T\right) + \\ I_p(\beta_i)\left(-\frac{1}{\gamma_0}\cos\theta_T + 1\right) \end{pmatrix}\left(\cos(\omega_i T_s)+1\right)}{2\omega_i \left|G_p(\beta_i)\right|^2 \left(1 - \frac{2}{\gamma_0}\cos\theta_T + \frac{1}{\gamma_0^2}\right)\operatorname{sinc}(\omega_i T_s)}. \end{gathered} \tag{23}$$

This result corresponds to the continuous-time cases as $T_s \to 0$ for $0 < \omega < \infty$ and $0 < \theta_T < 2\pi$ that gives the continuous time derivative and integral PID parameters such as:

$$K_d(\omega_i, \theta_T, \gamma_0) = \frac{K_i(\omega_i, \theta_T, \gamma_0)}{\omega_i^2} + \frac{\left(R_p(\beta_i)\left(-\frac{1}{\gamma}\sin\theta_T\right) + I_p(\beta_i)\left(-\frac{1}{\gamma}\cos\theta_T + 1\right)\right)}{\omega_i \left|G_p(\beta_i)\right|^2 \left(1 - \frac{2}{\gamma_0}\cos\theta_T + \frac{1}{\gamma_0^2}\right)}. \tag{24}$$

Note that at $\omega = 0$, $K_d(0, \theta_T, \gamma_0, T_s)$ is arbitrary and $K_i(0, \theta_T, \gamma_0, T_s) = 0$, unless $I_p(0) = R_p(0) = 0$, which holds only when the plant has a zero at the origin. In such case, a PID compensator should be avoided as the PID pole cancels the zero at the origin and the system becomes internally unstable. Note that if $\gamma_0 = 1$, $\theta_T = 0$ should be avoided as the denominators of (23) and (24) go to zero.

3 Application Example

In this section, a discrete-time PID controller is designed to regulate the shaft position of a SRV-02 DC motor from Quanser Innovate Educate [26], in Figure 1. The goal is to find all DTPID controllers that stabilize the system and satisfy the complementary sensitivity constraint in (7), where $\gamma_0 = 1.25$ [27], and the sampling period is $T_s = 0.025$ seconds. A communication delay of $\tau = 0.1$ seconds is defined here in the forward path. The nominal model of SRV-02 in continuous time with the commutation delay has been identified as:

$$G_p(s) = \frac{1.53}{s(0.024s+1)}e^{-\tau s}. \tag{25}$$

The procedures to achieve this goal are:

1) Use (2) to find the discrete time bilinear transformation of the continuous time model in (25):

$$G_p(z) = \frac{0.006299z^2 + 0.0126z + 0.006299}{z^2 - 1.34z + 0.3404}z^{-4}. \tag{26}$$

2) Equations (14) and (15) in method 1 are used in the (K_p, K_i) plane for a fixed value of $\tilde{K}_d = 0.002$. All DTPID controllers that satisfy the complementary sensitivity constraint in (7) are found by setting

$\gamma_0 = 1.25$ and $T_s = 0.025$ seconds then finding the intersection of all regions for frequency of $0 < \omega < \omega_s$ and $\theta_T \in [0, 2\pi)$.

3) The DTPID stability boundary can be found the same as stage 2 by setting $\gamma_0 = \infty$ in (14) and (15). This boundary shows with the red-bold line in Figure 4.

4) The region that satisfies the complementary sensitivity constraint is shown in Figure 4 with blue-solid lines. The intersection of all regions inside the stability boundary of the (K_p, K_i) plane corresponds all DTPID controller parameters that satisfy the complementary sensitivity constraint in (7).

5) The procedure 2-4 can be repeated for the continuous time case as $T_s = 0$.

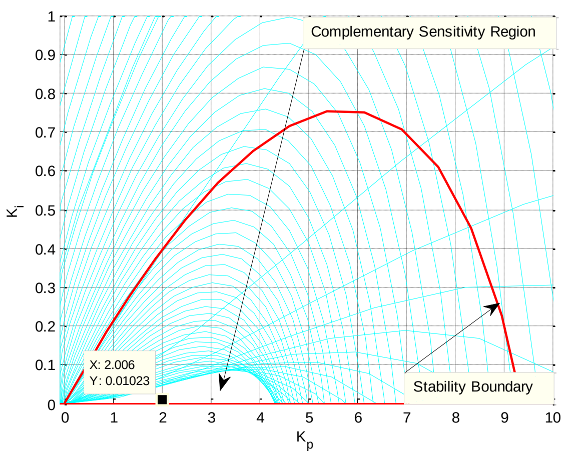

Fig. 4. Discrete-time stability boundary and complementary sensitivity region in the (K_p, K_i) plane

To verify the results, an arbitrary controller from this region is chosen, giving us the DTPID controller from (6) such as:

$$G_c(\beta) = 2.01 + \frac{0.01}{\beta} + 0.002\beta. \tag{27}$$

The Bode response of the discrete-time complementary sensitivity function with the PID controller in (27) is shown in Figure 5. As can be seen, $\|T(\beta)\|_\infty = 1$, which is less than $\gamma_0 = 1.25$. Consequently, the design goal is met. To verify the stability of system the closed loop step response is shown in Figure 6.

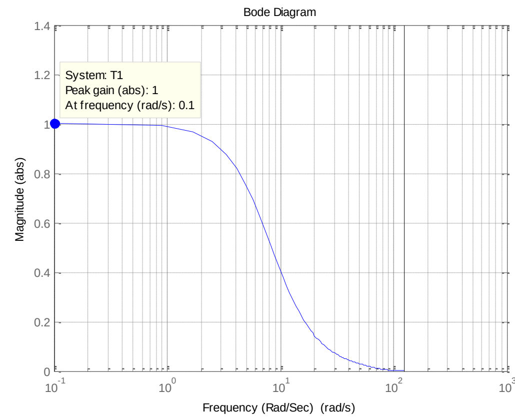

Fig. 5. Magnitude of discrete-time complementary sensitivity frequency response SRV-02 DC motor with DTPID controller in equation(27)

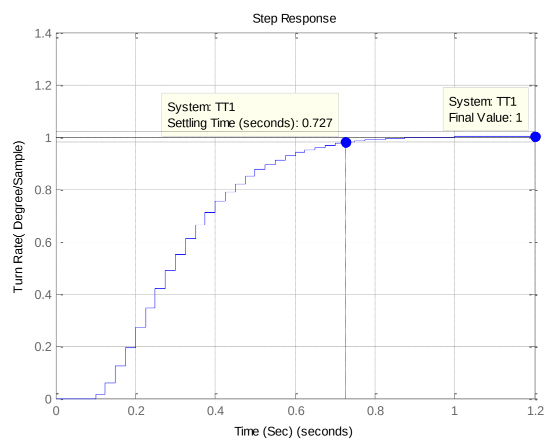

Fig. 6. Discrete time step response of the closed-loop SRV-02 DC motor with DTPID controller

The second method applies equations (14) and (19) in the (K_p, K_d) plane for a fixed value of $\tilde{K}_i = 0.05$. The DTPID controller is designed to satisfy the complementary sensitivity constraint with $\gamma_0 = 1.25$ and $T_s = 0.025$ seconds. The region that satisfies the discrete-time complementary sensitivity constraint and the stability boundary are shown in Figure 7. The intersection of all regions inside the stability boundary of the (K_p, K_d) plane is the discrete-time complementary sensitivity region.

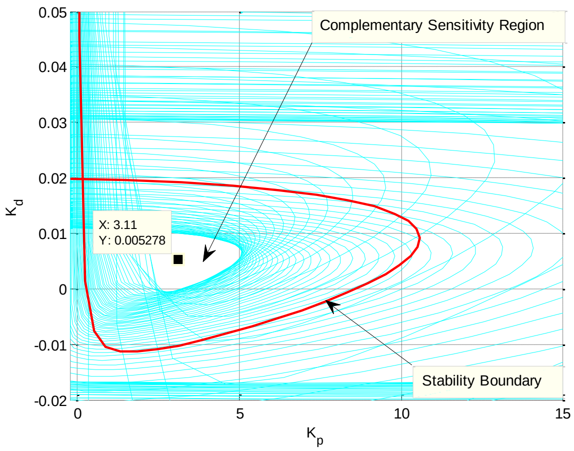

Fig. 7. Discrete-time stability boundary and complementary sensitivity region in the (K_p, K_d) plane

To verify the results, an arbitrary controller from this region is chosen as:

$$G_c(\beta)=3.11+\frac{0.05}{\beta}+0.005\beta. \tag{28}$$

The discrete-time complementary sensitivity constraint with the DTPID controller in (28) gives $\|T(\beta)\|_\infty=1.06$, which is less than $\gamma_0=1.25$. Consequently, the design goal is met.

Method 3 applies in the (K_i, K_d) plane for a fixed value of $\tilde{K}_p$. Plot of $K_p(\omega,\theta_T,\gamma_0,T_s)$ and $K_p(\omega,\theta_T,\infty,T_s)$ from (14) for various values of $0<\omega<\omega_s$ and $\theta_T\in[0,2\pi)$ are shown in Figure 8.

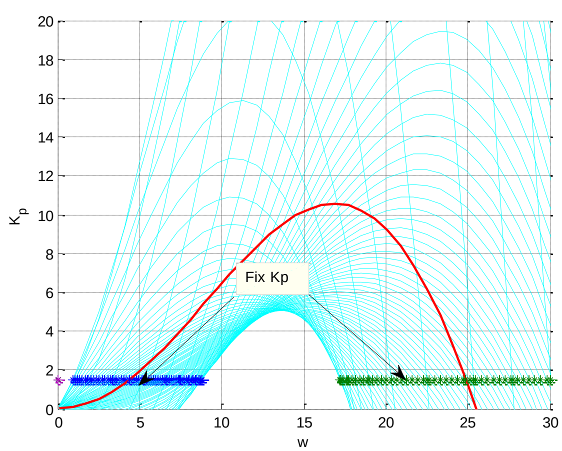

Fig. 8. Plots of $K_p(\omega,\theta_T,\gamma_0,T_0)$ versus ω

For each curve in Figure 8, the ω_i's are the frequencies at which $K_p(\omega,\theta_T,\gamma_0,T_s)=\tilde{K}_p=1.5$. Each ω_i is substituted into (23) to find the required region for choosing PID controller parameters. In addition, the boundary at $K_i(0,\theta_T,\gamma_0,T_s)=0$. The region that satisfies the discrete-time complementary sensitivity constraint and the stability boundary are shown in Figure 9. The intersection of all regions inside the discrete-time stability boundary of the (K_i, K_d) plane is the complementary sensitivity region.

To verify the results, an arbitrary controller from this region is chosen, giving us the DTPID controller as:

$$G_c(\beta)=1.5+\frac{1.04}{\beta}+0.3\beta. \tag{29}$$

The discrete-time complementary sensitivity function with the DTPID controller in (29) result is $\|T(\beta)\|_\infty=1.17$, which is less than $\gamma_0=1.25$. Consequently, the design goal is met.

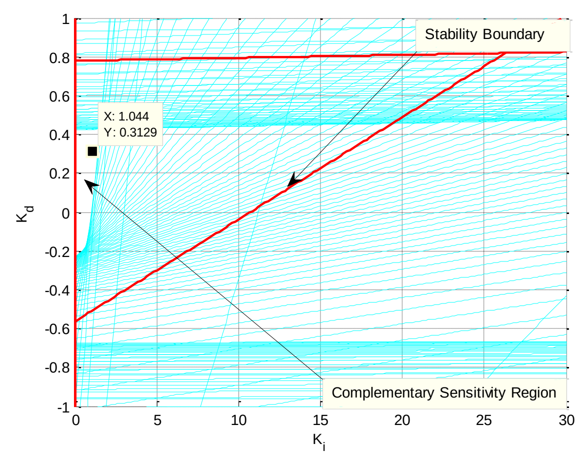

Fig. 9. Discrete-time stability boundary and complementary sensitivity region in the (K_i, K_d) plane

4 Conclusions

All achievable Discrete-Time or Continuous-Time Proportional Integral Derivative (DTCT-PID) controllers that satisfy H_∞ complementary sensitivity constraint of a single input single output (SISO) system with time delay can be found from a graphical technique. The bilinear transformation is used to describe the Discrete-Time Proportional Integral Derivative (DTPID) controller parameters since this methodology is based on frequency response and sample data collection of the system. It is shown that the continuous-time or discrete-time designs can be understood under a common structure through the bilinear transformation. This methodology is simple to understand and easy to implement. A DC motor data with a communication delay in the feedback loop is used to demonstrate the application of this methodology.

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