Design of observer for one-sided Lipschitz nonlinear systems with interval time-varying delay

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Abstract: - This paper deals with state observer design problem for a class of nonlinear dynamical systems with interval time-varying delay that satisfies the one-sided Lipschitz condition. The systems under consideration include the well-studied Lipschitz system as a special case and possess inherent advantages with respect to conservativeness. By constructing a novel Lyapunov-Krasovskii functional and utilizing free-weighting matrices approach, novel sufficient conditions that guarantee the observer error converge asymptotically to zero are obtained for a class of nonlinear dynamical systems with interval time-varying delay in terms of linear matrix inequalities. The computing method for observer gain matrix is given. Finally, illustrative examples are given to demonstrate the effectiveness of the proposed method.

Key-Words: - Nonlinear observers; One-sided Lipschitz condition; Quadratic inner-boundedness; Asymptotic stability; Interval time-varying delay

1 Introduction

Over the last decades, a large amount of research activities have focused on observer design for nonlinear dynamical systems as can be shown through the vast literature in this field. This topic was inspired by the fact that state estimation can be used for control, diagnosis and supervision purposes. Recently, other applications, such as synchronization and input recovery in communication systems, have become ones of the emerging and interesting research areas [1-4].

It is well known that the existence of time delay in a system may cause instability or bad system performance in closed feedback systems. The time-delay phenomenon may be encountered in many practical systems, such as the AIDS epidemic, aircraft stabilization, chemical engineering systems, inferred grinding model, manual control, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines. Hence, recent years have witnessed investigation of stability analysis and observer design for time-delay systems [4-7].

The conventional Lipschitz condition is commonly used in the existing nonlinear observer design. However, a major limitation in the existing results for Lipschitz nonlinear systems is that most of them work only for adequately small values of the Lipschitz constant [8]. In order to overcome this limitation, the so-called one-sided Lipschitz condition was first introduced in [9] for nonlinear state estimation. Several interesting works on state observers and stabilization for this type of systems were developed in [10-13]. In [12], the problem of observer design for one-sided Lipschitz non-linear systems was investigated by using the linear matrix inequality (LMI) approach. Sufficient conditions that ensure the existence of observers for one-sided Lipschitz non-linear systems were established and expressed in terms of linear matrix inequalities (LMIs). State observer design for a general class of nonlinear discrete-time systems that satisfies the one-sided Lipschitz condition was considered in [13]. The problem of observer-based stabilization for linear systems with parameter inequality was considered in [14]. A design methodology was proposed ascribed to a judicious use of the famous Young relation.

Some new delay dependent sufficient conditions ensuring the stability for uncertain neutral system with discrete and distributed delays were obtained in terms of linear matrix inequalities (LMIs) in [15]. Usually, the range of delays considered in most of the existing references is from zero to an upper bound [15]. In practice, however, the delay may vary in a range for which the lower bound is not restricted to being zero [16]. However, to the best of our knowledge, few results have been given on the study of observers for one-sided Lipschitz nonlinear systems with interval time-varying delay and some special nonlinearities. This motivates our present research.

In this paper, we consider the nonlinear observer design for one-sided Lipschitz systems with interval time-varying delay and nonlinearities. By using Lyapunov-Krasovskii functional and free-weighting matrices approach, we establish novel sufficient conditions for a class of nonlinear dynamical systems with interval time-varying delay in terms of linear matrix inequalities, which guarantee the observer error converges asymptotically to zero. The computing method for observer gain matrix is proposed.

This paper is organized as follows. In Section 2, the system description, some assumptions and lemmas are given, and the corresponding observer is introduced. In Section 3, some sufficient conditions that guarantee the observer error converges asymptotically to zero for a class of nonlinear dynamical systems with interval time-varying delay are presented. In Section 4, simulation examples are given to show the performances of our method. Finally, conclusion is drawn and remarks are made in Section 5.

Notations. R^n denotes the n-dimensional real Euclidean space. $R^{m\times n}$ represents the set of all $m\times n$ real matrices. $\langle\cdot,\cdot\rangle$ is the inner product in R^n, that is given $x, y\in R^n$, then $\langle x,y\rangle = x^T y$, where x^T is the transpose of x. $\|\cdot\|$ denotes the Euclidean norm in R^n. For a square matrix S, $S>0$ ($S<0$) means that the matrix is positive definite (negative definite). In symmetric block matrices, we use an asterisk '$*$' to represent a term induced by symmetry. I represents an identity matrix of appropriate dimension.

2 Problem statement and preliminaries

Consider the following uncertain nonlinear systems with interval time-varying delay

$$\begin{aligned}\dot{x}(t) &= Ax(t)+Bx(t-h(t))+\Phi(x(t),u)\\ &\quad + f(x(t),x(t-h(t)),u), \qquad (1)\\ y(t) &= Cx(t)+Dx(t-h(t)),\end{aligned}$$

where $x\in R^n$ is the state vector, $u\in R^m$ is the control input, $y\in R^p$ is the measured output, $A,B\in R^{n\times n}$ and $C,D\in R^{p\times n}$ are known matrices, the continuous functions $f(x(t),x(t-h(t)),u)$ and $\Phi(x(t),u)$ are unknown and denote the nonlinear terms with respect to x and u. The delay $h(t)$ is time varying and satisfies

$$0\le h_m \le h(t)\le h_M,\quad \dot{h}(t)\le h_d <1,\quad \forall t\ge 0, \qquad (2)$$

where h_m and h_M are constants representing respectively the lower and upper bounds of the delay, h_d is a constant.

Our objective is to derive sufficient conditions that ensure the stability of observer error dynamics. The following assumptions are made throughout this paper.

Assumption 1 $\Phi(x(t),u)$ and $f(x(t),x(t-h(t)),u)$ are one-sided Lipschitz with respect to x and u, i.e.

$$\langle \Phi(x(t),u)-\Phi(\hat{x}(\hat{t}),u), x(t)-x(t)\rangle \le \rho_1\|x(t)-x(t)\|^2, \qquad (3)$$

$$\begin{aligned}&\langle f(x(t),x(t-h(t)),u)-f(\hat{x}(t),x(t-h(t)),u),\\ &x(t)-\hat{x}(t)\rangle \le \rho_2\|x(t)-x(t)\|^2 \qquad (4)\\ &\qquad\qquad +\rho_3\|x(t-h(t))-\hat{x}(t-h(t))\|^2,\end{aligned}$$

for any $x,\hat{x}\in R^n$, $y\in R^p$, $u\in R^m$, where ρ_1, ρ_2, ρ_3 are the one-sided Lipschitz constants which can be positive or negative.

Assumption 2 $\Phi(x(t),u)$ and $f(x(t),x(t-h(t)),u)$ are quadratic inner-boundedness with respect to x and u, i.e.

$$\begin{aligned}&(\Phi(x(t),u)-\Phi(\hat{x}(t),u))^T(\Phi(x(t),u)-\Phi(x(t),u))\\ &\quad\le \gamma_1\langle x(t)-\hat{x}(t),\Phi(x(t),u)-\Phi(x(t),u)\rangle \qquad (5)\\ &\qquad+\beta_1\|x(t)-\hat{x}(t)\|^2,\end{aligned}$$

$$\begin{aligned}&(f(x(t),x(t-h(t)),u)-f(\hat{x}(t),x(t-h(t)),u))^T\\ &\times(f(x(t),x(t-h(t)),u)-f(\hat{x}(t),x(t-h(t)),u))\\ &\quad\le \beta_2\|x(t)-\hat{x}(t)\|^2+\beta_3\|x(t-h(t))-x(t-h(t))\|^2\\ &\qquad+\gamma_2\langle x(t)-\hat{x}(t), f(x(t),x(t-h(t)),u)\\ &\qquad-f(\hat{x}(t),x(t-h(t)),u)\rangle\\ &\qquad+\gamma_3\langle x(t-h(t))-\hat{x}(t-h(t)),\\ &\qquad f(x(t),x(t-h(t)),u)-f(\hat{x}(t),x(t-h(t)),u)\rangle, \qquad (6)\end{aligned}$$

where $\beta_1,\beta_2,\beta_3,\gamma_1,\gamma_2$ and γ_3 are real scalars.

Lemma 1 [17] For any constant matrix $M\in R^{n\times n}$, $M=M^T>0$, scalar $\eta>0$, vector function $\omega:[0,\eta]\to R^n$ such that the integrations concerned are well defined, then

$$\left[\int_0^\eta \omega(s)ds\right]^T M\left[\int_0^\eta \omega(s)ds\right] \le \eta\int_0^\eta \omega^T(s)M\omega(s)ds. \qquad (7)$$

Lemma 2 (Schur Complement [18]). Given constant matrices Σ_1, Σ_2 and Σ_3 with appropriate dimensions, where $\Sigma_1^T = \Sigma_1$ and $\Sigma_2^T = \Sigma_2$, then

$$\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$$

if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0. \quad (8)$$

Remark 1 Unlike the well-known Lipschitz condition, the constants ρ_1, ρ_2, ρ_3, β_1, β_2, β_3, γ_1, γ_2, γ_3 can be positive, negative or zero. In addition, if the functions $\Phi(x(t),u)$ and $f(x(t),x(t-h(t)),u)$ are Lipschitz, then they are also both one-sided Lipschitz and quadratically inner-bounded, but the converse is not true. The one-sided Lipschitz condition provides a less conservative condition than the classical Lipschitz one. The concept of quadratic inner-boundedness is very useful to provide tractable LMI stability conditions.

The observer of system (1) is defined by the following form

$$\begin{aligned} \dot{\hat{x}}(t) &= Ax(t) + Bx(t-h(t)) + \Phi(x(t),u) \\ &\quad + f(\hat{x}(t), x(t-h(t)), u) \\ &\quad + L(y - C\hat{x}(t) - Dx(t-h(t))), \end{aligned} \quad (9)$$

denote $e(t) = x(t) - \hat{x}(t)$ then the observer error dynamics is governed by

$$\begin{aligned} \dot{e}(t) &= (A-LC)e(t) + (B-LD)e(t-h(t)) \\ &\quad + \tilde{\Phi} + \tilde{f}, \end{aligned} \quad (10)$$

where

$$\tilde{\Phi} \triangleq \Phi(x(t),u) - \Phi(\hat{x}(t),u),$$

and

$$\tilde{f} \triangleq f(x(t),x(t-h(t)),u) - f(\hat{x}(t),x(t-h(t)),u).$$

3 Main Results

In this section, we present LMI conditions for the observer synthesis problems of one-sided Lipschitz nonlinear systems with interval time-varying delay.

Theorem 1 Assume that assumption 1 and 2 are satisfied. For given scalars $0 \le h_m \le h_M$, h_d and observer gain matrix L, the observer error dynamics (10) is asymptotically stable, if there exist scalars $\varepsilon_i \ge 0$ $(i = 1,2,3,4)$ and matrices $P > 0$, $X \ge 0$, $Q_i > 0$ $(i = 1,2,\ldots,7)$, $N^T = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix}$, $M^T = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T \end{bmatrix}$, $Y \ge 0$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix} < 0, \quad (11)$$

$$\begin{bmatrix} X & M \\ M^T & Q_4 \end{bmatrix} \ge 0, \qquad \begin{bmatrix} Y & N \\ N^T & Q_5 \end{bmatrix} \ge 0, \quad (12)$$

where

$$\Pi_1 = \begin{bmatrix} \phi + h_m X + h_M Y & -M & -N & \sqrt{h_m} S^T Q_4 \\ * & -Q_1 & 0 & 0 \\ * & * & -Q_3 & 0 \\ * & * & * & -Q_4 \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} \sqrt{h_M} S^T Q_5 & \sqrt{\frac{h_m^2}{2}} S^T Q_6 & \sqrt{\frac{h_M^2}{2}} S^T Q_7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Pi_3 = \begin{bmatrix} -Q_5 & 0 & 0 \\ 0 & -Q_6 & 0 \\ 0 & 0 & -Q_7 \end{bmatrix},$$

$$S = \begin{bmatrix} A-LC & B-LD & I & I \end{bmatrix},$$

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ * & \phi_{22} & 0 & \frac{\varepsilon_4 \gamma_3}{2} I \\ * & * & -\varepsilon_3 I & 0 \\ * & * & * & -\varepsilon_4 I \end{bmatrix}, \quad (13)$$

$$\begin{aligned} \phi_{11} &= A^T P + PA - C^T L^T P - PLC + Q_1 \\ &\quad + Q_2 + Q_3 + M_1 + M_1^T + N_1 + N_1^T \\ &\quad + (\varepsilon_1 \rho_1 + \varepsilon_2 \rho_2 + \varepsilon_3 \beta_1 + \varepsilon_4 \beta_2) I, \end{aligned}$$

$$\phi_{12} = PB - PLD + M_2^T + N_2^T,$$

$$\phi_{13} = P + M_3^T + N_3^T + \frac{\varepsilon_3 \gamma_1 - \varepsilon_1}{2} I,$$

$$\phi_{14} = P + M_4^T + N_4^T + \frac{\varepsilon_4 \gamma_2 - \varepsilon_2}{2} I,$$

$$\phi_{22} = -(1-h_d) Q_2 + (\varepsilon_2 \rho_3 + \varepsilon_4 \beta_3) I.$$

Proof. Let us choose the Lyapunov-Krasovskii functional candidate as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$

where

$$
\begin{aligned}
V_1(t) &= e^T(t)Pe(t),\\
V_2(t) &= \int_{t-h_m}^{t} e^T(s)Q_1e(s)ds + \int_{t-h(t)}^{t} e^T(s)Q_2e(s)ds\\
&\quad + \int_{t-h_M}^{t} e^T(s)Q_3e(s)ds,\\
V_3(t) &= \int_{-h_m}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Q_4\dot{e}(s)dsd\theta\\
&\quad + \int_{-h_M}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Q_5\dot{e}(s)dsd\theta\\
&\quad + \int_{-h_m}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{e}^T(s)Q_6\dot{e}(s)dsd\lambda d\theta\\
&\quad + \int_{-h_M}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{e}^T(s)Q_7\dot{e}(s)dsd\lambda d\theta,
\end{aligned}
$$

then

$$
\begin{aligned}
\dot{V}_1(t) &= e^T(t)\left[(A-LC)^TP + P(A-LC)\right]e(t)\\
&\quad + 2e^T(t)P(B-LD)e(t-h(t))\\
&\quad + 2e^T(t)P\tilde{\Phi} + 2e^T(t)P\tilde{f},\\
\dot{V}_2(t) &= e^T(t)Q_1e(t) - e^T(t-h_m)Q_1e(t-h_m)\\
&\quad + e^T(t)Q_2e(t) + e^T(t)Q_3e(t)\\
&\quad - (1-\dot{h}(t))e^T(t-h(t))Q_2e(t-h(t))\\
&\quad - e^T(t-h_M)Q_3e(t-h_M)\\
&\le e^T(t)\left[Q_1+Q_2+Q_3\right]e(t)\\
&\quad - e^T(t-h_m)Q_1e(t-h_m)\\
&\quad - (1-h_d)e^T(t-h(t))Q_2e(t-h(t))\\
&\quad - e^T(t-h_M)Q_3e(t-h_M),
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\dot{V}_3(t) &= h_m\dot{e}^T(t)Q_4\dot{e}(t) + h_M\dot{e}^T(t)Q_5\dot{e}(t)\\
&\quad - \int_{t-h_m}^{t}\dot{e}^T(s)Q_4\dot{e}(s)ds - \int_{t-h_M}^{t}\dot{e}^T(s)Q_5\dot{e}(s)ds\\
&\quad + \frac{h_m^2}{2}\dot{e}^T(t)Q_6\dot{e}(t) - \int_{-h_m}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Q_6\dot{e}(s)dsd\theta\\
&\quad + \frac{h_M^2}{2}\dot{e}^T(t)Q_7\dot{e}(t) - \int_{-h_M}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Q_7\dot{e}(s)dsd\theta.
\end{aligned}
\tag{15}
$$

Define $\xi_1^T(t) = \left[e^T(t) \quad e^T(t-h(t)) \quad \tilde{\Phi}^T \quad \tilde{f}^T\right]$. Then, the following equations hold for any matrices M and N with appropriate dimensions:

$$
\begin{aligned}
&2\xi_1^T(t)M\left[e(t) - e(t-h_m) - \int_{t-h_m}^{t}\dot{e}(s)ds\right] = 0,\\
&2\xi_1^T(t)N\left[e(t) - e(t-h_M) - \int_{t-h_M}^{t}\dot{e}(s)ds\right] = 0.
\end{aligned}
\tag{16}
$$

Moreover, for matrices X and Y with appropriate dimensions, we have

$$
\begin{aligned}
&h_m\xi_1^T(t)X\xi_1(t) - \int_{t-h_m}^{t}\xi_1^T(t)X\xi_1(t)ds = 0,\\
&h_M\xi_1^T(t)Y\xi_1(t) - \int_{t-h_M}^{t}\xi_1^T(t)Y\xi_1(t)ds = 0.
\end{aligned}
\tag{17}
$$

On the other hand, for any scalars $\varepsilon_i \ge 0\ (i = 1,2,3,4)$, if follows from (3)-(6) that

$$
\begin{aligned}
&\varepsilon_1\left[\rho_1e^T(t)e(t) - e^T(t)\tilde{\Phi}\right] \ge 0,\\
&\varepsilon_2\left[\rho_2e^T(t)e(t) + \rho_3e^T(t-h(t))e(t-h(t))\right.\\
&\qquad \left. - e^T(t)\tilde{f}\right] \ge 0,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\varepsilon_3\left[\beta_1e^T(t)e(t) + \gamma_1e^T(t)\tilde{\Phi} - \tilde{\Phi}^T\tilde{\Phi}\right] \ge 0,\\
&\varepsilon_4\left[\beta_2e^T(t)e(t) + \beta_3e^T(t-h(t))e(t-h(t))\right.\\
&\qquad \left. + \gamma_2e^T(t)\tilde{f} + \gamma_3e^T(t-h(t))\tilde{f} - \tilde{f}^T\tilde{f}\right] \ge 0.
\end{aligned}
\tag{19}
$$

From (14)-(15), we have

$$
\begin{aligned}
\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t)\\
&\le e^T(t)\left[(A-LC)^TP + P(A-LC)\right]e(t)\\
&\quad + 2e^T(t)P(B-LD)e(t-h(t)) + 2e^T(t)P\tilde{\Phi}\\
&\quad + 2e^T(t)P\tilde{f} + e^T(t)\left[Q_1+Q_2+Q_3\right]e(t)\\
&\quad - e^T(t-h_m)Q_1e(t-h_m) - e^T(t-h_M)Q_3e(t-h_M)\\
&\quad - (1-h_d)e^T(t-h(t))Q_2e(t-h(t))\\
&\quad + h_m\dot{e}^T(t)Q_4\dot{e}(t) + h_M\dot{e}^T(t)Q_5\dot{e}(t)\\
&\quad + \frac{h_m^2}{2}\dot{e}^T(t)Q_6\dot{e}(t) + \frac{h_M^2}{2}\dot{e}^T(t)Q_7\dot{e}(t)\\
&\quad - \int_{t-h_m}^{t}\dot{e}^T(s)Q_4\dot{e}(s)ds - \int_{t-h_M}^{t}\dot{e}^T(s)Q_5\dot{e}(s)ds\\
&\quad - \int_{-h_m}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Q_6\dot{e}(s)dsd\theta\\
&\quad - \int_{-h_M}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Q_7\dot{e}(s)dsd\theta.
\end{aligned}
\tag{20}
$$

From (16)-(20), it follows that

$$
\begin{aligned}
\dot{V}(t) &\le e^T(t)\left[(A-LC)^TP + P(A-LC)\right]e(t)\\
&\quad + 2e^T(t)P(B-LD)e(t-h(t)) + 2e^T(t)P\tilde{\Phi}\\
&\quad + 2e^T(t)P\tilde{f} + e^T(t)\left[Q_1+Q_2+Q_3\right]e(t)\\
&\quad - e^T(t-h_m)Q_1e(t-h_m)\\
&\quad - (1-h_d)e^T(t-h(t))Q_2e(t-h(t))\\
&\quad - e^T(t-h_M)Q_3e(t-h_M)\\
&\quad + h_m\dot{e}^T(t)Q_4\dot{e}(t) + h_M\dot{e}^T(t)Q_5\dot{e}(t)\\
&\quad + \frac{h_m^2}{2}\dot{e}^T(t)Q_6\dot{e}(t) + \frac{h_M^2}{2}\dot{e}^T(t)Q_7\dot{e}(t)\\
&\quad - \int_{t-h_m}^{t}\dot{e}^T(s)Q_4\dot{e}(s)ds - \int_{t-h_M}^{t}\dot{e}^T(s)Q_5\dot{e}(s)ds\\
&\quad - \int_{-h_m}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Q_6\dot{e}(s)dsd\theta\\
&\quad - \int_{-h_M}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Q_7\dot{e}(s)dsd\theta
\end{aligned}
$$

$$
\begin{aligned}
&+2\xi_1^T(t)M\left[e(t)-e(t-h_m)-\int_{t-h_m}^{t}\dot{e}(s)ds\right]\\
&+2\xi_1^T(t)N\left[e(t)-e(t-h_M)-\int_{t-h_M}^{t}\dot{e}(s)ds\right]\\
&+h_m\xi_1^T(t)X\xi_1(t)-\int_{t-h_m}^{t}\xi_1^T(t)X\xi_1(t)ds\\
&+h_M\xi_1^T(t)Y\xi_1(t)-\int_{t-h_M}^{t}\xi_1^T(t)Y\xi_1(t)ds\\
&+\varepsilon_1\left[\rho_1 e^T(t)e(t)-e^T(t)\tilde{\Phi}\right]\\
&+\varepsilon_2\left[\rho_2 e^T(t)e(t)-e^T(t)\tilde{f}\right.\\
&\left.+\rho_3 e^T(t-h(t))e(t-h(t))\right]\\
&+\varepsilon_3\left[\beta_1 e^T(t)e(t)+\gamma_1 e^T(t)\tilde{\Phi}-\tilde{\Phi}^T\tilde{\Phi}\right]\\
&+\varepsilon_4[\beta_2 e^T(t)e(t)+\beta_3 e^T(t-h(t))e(t-h(t))\\
&+\gamma_2 e^T(t)\tilde{f}+\gamma_3 e^T(t-h(t))\tilde{f}-\tilde{f}^T\tilde{f}].
\end{aligned}
\tag{21}
$$

From (21), we get

$$
\begin{aligned}
\dot{V}(t)\le e^T(t)&\left[(A-LC)^T P+P(A-LC)\right.\\
&\left.+Q_1+Q_2+Q_3\right]e(t)+2e^T(t)P\tilde{\Phi}\\
&+2e^T(t)P(B-LD)e(t-h(t))\\
&+2e^T(t)P\tilde{f}-e^T(t-h_m)Q_1 e(t-h_m)\\
&-e^T(t-h_M)Q_3 e(t-h_M)\\
&-(1-h_d)e^T(t-h(t))Q_2 e(t-h(t))\\
&+h_m\dot{e}^T(t)Q_4\dot{e}(t)+h_M\dot{e}^T(t)Q_5\dot{e}(t)\\
&+\frac{h_m^2}{2}\dot{e}^T(t)Q_6\dot{e}(t)+\frac{h_M^2}{2}\dot{e}^T(t)Q_7\dot{e}(t)\\
&-\int_{t-h_m}^{t}\dot{e}^T(s)Q_4\dot{e}(s)ds-\int_{t-h_M}^{t}\dot{e}^T(s)Q_5\dot{e}(s)ds\\
&+2\xi_1^T(t)M\left[e(t)-e(t-h_m)-\int_{t-h_m}^{t}\dot{e}(s)ds\right]\\
&+2\xi_1^T(t)N\left[e(t)-e(t-h_M)-\int_{t-h_M}^{t}\dot{e}(s)ds\right]\\
&+h_m\xi_1^T(t)X\xi_1(t)-\int_{t-h_m}^{t}\xi_1^T(t)X\xi_1(t)ds\\
&+h_M\xi_1^T(t)Y\xi_1(t)-\int_{t-h_M}^{t}\xi_1^T(t)Y\xi_1(t)ds\\
&+\varepsilon_1\left[\rho_1 e^T(t)e(t)-e^T(t)\tilde{\Phi}\right]\\
&+\varepsilon_2\left[\rho_2 e^T(t)e(t)-e^T(t)\tilde{f}\right.\\
&\left.+\rho_3 e^T(t-h(t))e(t-h(t))\right]\\
&+\varepsilon_3\left[\beta_1 e^T(t)e(t)+\gamma_1 e^T(t)\tilde{\Phi}-\tilde{\Phi}^T\tilde{\Phi}\right]\\
&+\varepsilon_4\left[\beta_2 e^T(t)e(t)+\beta_3 e^T(t-h(t))e(t-h(t))\right.\\
&\left.+\gamma_2 e^T(t)\tilde{f}+\gamma_3 e^T(t-h(t))\tilde{f}-\tilde{f}^T\tilde{f}\right]
\end{aligned}
$$

$$
\begin{aligned}
&=\xi_3^T(t)\left[\Theta+h_m\bar{S}^T Q_4\bar{S}+h_M\bar{S}^T Q_5\bar{S}\right.\\
&\left.+\frac{h_m^2}{2}\bar{S}^T Q_6\bar{S}+\frac{h_M^2}{2}\bar{S}^T Q_7\bar{S}\right]\xi_3(t)\\
&-\int_{t-h_m}^{t}\xi_2^T(t,s)\begin{bmatrix}X & M\\ M^T & Q_4\end{bmatrix}\xi_2(t,s)ds\\
&-\int_{t-h_M}^{t}\xi_2^T(t,s)\begin{bmatrix}Y & N\\ N^T & Q_5\end{bmatrix}\xi_2(t,s)ds,
\end{aligned}
\tag{22}
$$

where

$$
\xi_3^T(t)=\left[\xi_1^T(t)\quad e^T(t-h_m)\quad e^T(t-h_M)\right],
$$

$$
\xi_2^T(t,s)=\left[\xi_1^T(t)\quad \dot{e}^T(s)\right],
$$

$$
\bar{S}=\left[S\quad 0\quad 0\right],
$$

$$
\Theta=\begin{bmatrix}\phi+h_m X+h_M Y & -M & -N\\ * & -Q_1 & 0\\ * & * & -Q_3\end{bmatrix}.
$$

From (11), (12) and Lemma 2, we get that $\dot{V}(t)<0$ for all $e(t)\ne 0$. This completes the proof.

Remark 2 Theorem 1 provides the interval time-varying delay dependent stability criteria for given gain matrix L, which can be illustrated by Example 1 in the next section. However, our design goal is to find an observer gain matrix L such that the error dynamics (10) is asymptotically stable. We can derive the following theorem.

Theorem 2 Assume that assumption 1 and 2 are satisfied. For given scalars $h_m>0, h_M>0$ and h_d, the observer error dynamics (10) is asymptotically stable, if there exist scalars $\varepsilon_i\ge 0\quad (i=1,2,3,4)$ and matrices $P>0$, $Q_i>0(i=1,2,\cdots,5)$ of appropriate dimensions such that the following LMI holds:

$$
\Omega=\begin{bmatrix}\bar{\Omega} & 0\\ 0 & \hat{\Omega}\end{bmatrix}<0,
\tag{23}
$$

where

$$
\bar{\Omega}=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14}\\ * & \Omega_{22} & 0 & \frac{\varepsilon_4\gamma_3}{2}I\\ * & * & -\varepsilon_3 I & 0\\ * & * & * & -\varepsilon_4 I\end{bmatrix},
$$

$$
\begin{aligned}
\Omega_{11}=&A^T P+PA-R^T C-C^T R+Q_1\\
&+Q_2+Q_3+h_m Q_4+h_M Q_5\\
&+(\varepsilon_1\rho_1+\varepsilon_2\rho_2+\varepsilon_3\beta_1+\varepsilon_4\beta_2)I,
\end{aligned}
$$

$$
\Omega_{12}=PB-R^T D,\quad \Omega_{13}=P+\frac{\varepsilon_3\gamma_1-\varepsilon_1}{2}I,
$$

$$\Omega_{14} = P + \frac{\varepsilon_4\gamma_2 - \varepsilon_2}{2} I,$$

$$\Omega_{22} = -(1-h_d)Q_2 + (\varepsilon_2\rho_3 + \varepsilon_4\beta_3)I,$$

$$\hat{\Omega} = \begin{bmatrix} -Q_1 & 0 & 0 & 0 \\ 0 & -Q_3 & 0 & 0 \\ 0 & 0 & -\frac{1}{h_m}Q_4 & 0 \\ 0 & 0 & 0 & -\frac{1}{h_M}Q_5 \end{bmatrix}.$$

If (23) holds a feasible solution, then the gain matrix L is given by $L = P^{-1}R^T$.

Proof. Choose the Lyapunov-Krasovskii functional candidate as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$

where

$$V_1(t) = e^T(t)Pe(t),$$

$$V_2(t) = \int_{t-h_m}^{t} e^T(s)Q_1e(s)ds + \int_{t-h(t)}^{t} e^T(s)Q_2e(s)ds + \int_{t-h_M}^{t} e^T(s)Q_3e(s)ds,$$

$$V_3(t) = \int_{-h_m}^{0}\int_{t+\theta}^{t} e^T(s)Q_4e(s)dsd\theta + \int_{-h_M}^{0}\int_{t+\theta}^{t} e^T(s)Q_5e(s)dsd\theta.$$

We have

$$\begin{aligned}\dot{V}_1(t) &= e^T(t)\left[(A-LC)^T P + P(A-LC)\right]e(t) \\ &\quad + 2e^T(t)P(B-LD)e(t-h(t)) \\ &\quad + 2e^T(t)P\tilde{\Phi} + 2e^T(t)P\tilde{f},\end{aligned}$$

$$\begin{aligned}\dot{V}_2(t) &= e^T(t)Q_1e(t) - e^T(t-h_m)Q_1e(t-h_m) \\ &\quad + e^T(t)Q_2e(t) + e^T(t)Q_3e(t) \\ &\quad - (1-\dot{h}(t))e^T(t-h(t))Q_2e(t-h(t)) \\ &\quad - e^T(t-h_M)Q_3e(t-h_M) \\ &\le e^T(t)\left[Q_1+Q_2+Q_3\right]e(t) \\ &\quad - e^T(t-h_m)Q_1e(t-h_m) \\ &\quad - (1-h_d)e^T(t-h(t))Q_2e(t-h(t)) \\ &\quad - e^T(t-h_M)Q_3e(t-h_M),\end{aligned}$$

$$\begin{aligned}\dot{V}_3(t) &= h_m e^T(t)Q_4e(t) + h_M e^T(t)Q_5e(t) \\ &\quad - \int_{t-h_m}^{t} e^T(s)Q_4e(s)ds - \int_{t-h_M}^{t} e^T(s)Q_5e(s)ds.\end{aligned}$$

By Lemma 1, we have

$$-\int_{t-h_m}^{t} e^T(s)Q_4e(s)ds \le -\frac{1}{h_m}\left[\int_{t-h_m}^{t} e(s)ds\right]^T Q_4\left[\int_{t-h_m}^{t} e(s)ds\right],$$

and

$$-\int_{t-h_M}^{t} e^T(s)Q_5e(s)ds \le -\frac{1}{h_M}\left[\int_{t-h_M}^{t} e(s)ds\right]^T Q_5\left[\int_{t-h_M}^{t} e(s)ds\right],$$

then

$$\begin{aligned}\dot{V}_3(t) &\le h_m e^T(t)Q_4e(t) + h_M e^T(t)Q_5e(t) \\ &\quad - \frac{1}{h_m}\left[\int_{t-h_m}^{t} e(s)ds\right]^T Q_4\left[\int_{t-h_m}^{t} e(s)ds\right] \\ &\quad - \frac{1}{h_M}\left[\int_{t-h_M}^{t} e(s)ds\right]^T Q_5\left[\int_{t-h_M}^{t} e(s)ds\right].\end{aligned}$$

Adding the terms on the left sides of (18)-(19) to the time derivative of $V(t)$, yields

$$\begin{aligned}\dot{V}(t) &\le e^T(t)\left[(A-LC)^T P + P(A-LC)\right]e(t) \\ &+ 2e^T(t)P(B-LD)e(t-h(t)) \\ &+ 2e^T(t)P\tilde{\Phi} + 2e^T(t)P\tilde{f} \\ &+ e^T(t)\left[Q_1+Q_2+Q_3\right]e(t) \\ &- e^T(t-h_m)Q_1e(t-h_m) \\ &- (1-h_d)e^T(t-h(t))Q_2e(t-h(t)) \\ &- e^T(t-h_M)Q_3e(t-h_M) \\ &+ h_m e^T(t)Q_4e(t) + h_M e^T(t)Q_5e(t) \\ &- \frac{1}{h_m}\left[\int_{t-h_m}^{t} e(s)ds\right]^T Q_4\left[\int_{t-h_m}^{t} e(s)ds\right] \\ &- \frac{1}{h_M}\left[\int_{t-h_M}^{t} e(s)ds\right]^T Q_5\left[\int_{t-h_M}^{t} e(s)ds\right] \\ &+ \varepsilon_1\left[\rho_1 e^T(t)e(t) - e^T(t)\tilde{\Phi}\right] \\ &+ \varepsilon_2\left[\rho_2 e^T(t)e(t) + \rho_3 e^T(t-h(t))e(t-h(t))\right. \\ &\left. - e^T(t)\tilde{f}\right] + \varepsilon_3\left[\beta_1 e^T(t)e(t) + \gamma_1 e^T(t)\tilde{\Phi} - \tilde{\Phi}^T\tilde{\Phi}\right] \\ &+ \varepsilon_4\left[\beta_2 e^T(t)e(t) + \beta_3 e^T(t-h(t))e(t-h(t))\right. \\ &\left. + \gamma_2 e^T(t)\tilde{f} + \gamma_3 e^T(t-h(t))\tilde{f} - \tilde{f}^T\tilde{f}\right] \\ &= \zeta^T(t)\Omega\zeta(t),\end{aligned} \tag{24}$$

where

$$\zeta^T(t) = \left[e^T(t) \quad e^T(t-h(t)) \quad \tilde{\Phi}^T \quad \tilde{f}^T \quad e^T(t-h_m) \quad e^T(t-h_M) \quad \int_{t-h_m}^{t} e^T(s)ds \quad \int_{t-h_M}^{t} e^T(s)ds\right].$$

Let $L = P^{-1}R^T$. Then it follows from (24) that $\dot{V}(t) < 0$ for all $e(t) \ne 0$ provided that the LMI (23) holds a feasible solution. The proof is completed.

Remark 3 In some existing literature, the range of varying delay considered in many existing references is from zero to an upper bound. When $h_m = 0,$ i.e., the time-varying delay satisfies

$$0 \le h(t) \le h_M, \quad \dot{h}(t) \le h_d,$$

we can obtain the following result.

Corollary 1 Assume that assumption 1 and 2 are satisfied. Consider system (1) with $h_m = 0.$ For given scalars $h_M \ge 0,$ h_d and observer gain matrix L, the observer error dynamics (10) is asymptotically stable, if there exist scalars $\varepsilon_i \ge 0$ $(i = 1,2,3,4)$ and matrices $P > 0,$ $N^T = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T \end{bmatrix},$ $Y \ge 0,$ $Q_i > 0\,(i = 1,2,3,4)$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \phi + h_M Y & -N & \sqrt{h_m} S^T Q_3 & \sqrt{\dfrac{h_m^2}{2}} S^T Q_4 \\ * & -Q_2 & 0 & 0 \\ * & * & -Q_3 & 0 \\ * & * & * & -Q_4 \end{bmatrix} < 0,$$

$$\begin{bmatrix} Y & N \\ N^T & Q_3 \end{bmatrix} \ge 0, \tag{25}$$

where

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ * & \phi_{22} & 0 & \dfrac{\varepsilon_4 \gamma_3}{2} I \\ * & * & -\varepsilon_3 I & 0 \\ * & * & * & -\varepsilon_4 I \end{bmatrix},$$

$$S = \begin{bmatrix} A - LC & B - LD & I & I \end{bmatrix},$$

$$\begin{aligned} \phi_{11} &= A^T P + PA - C^T L^T P - PLC \\ &\quad + Q_1 + Q_2 + N_1 + N_1^T \\ &\quad + (\varepsilon_1 \rho_1 + \varepsilon_2 \rho_2 + \varepsilon_3 \beta_1 + \varepsilon_4 \beta_2) I, \\ \phi_{12} &= PB - PLD + N_2^T, \\ \phi_{13} &= P + N_3^T + \frac{\varepsilon_3 \gamma_1 - \varepsilon_1}{2} I, \\ \phi_{14} &= P + N_4^T + \frac{\varepsilon_4 \gamma_2 - \varepsilon_2}{2} I, \\ \phi_{22} &= -(1 - h_d) Q_1 + (\varepsilon_2 \rho_3 + \varepsilon_4 \beta_3) I. \end{aligned}$$

Proof. Let us choose the Lyapunov-Krasovskii functional candidate as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$

where

$$V_1(t) = e^T(t) P e(t),$$

$$V_2(t) = \int_{t-h(t)}^{t} e^T(s) Q_1 e(s) ds + \int_{t-h_M}^{t} e^T(s) Q_2 e(s) ds,$$

$$\begin{aligned} V_3(t) &= \int_{-h_M}^{0} \int_{t+\theta}^{t} \dot{e}^T(s) Q_3 \dot{e}(s) ds d\theta \\ &\quad + \int_{-h_M}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{e}^T(s) Q_4 \dot{e}(s) ds d\lambda d\theta. \end{aligned}$$

Using the similar method shown in the proof of Theorem 1, the inequality (25) can be obtained and the detailed proof is omitted.

Corollary 2 Assume that assumption 1 and 2 are satisfied. Consider system (1) with $h_m = 0.$ For given scalars $h_M \ge 0,$ and h_d, the observer error dynamics (10) is asymptotically stable, if there exist scalars $\varepsilon_i \ge 0$ $(i = 1,2,3,4)$ and matrices $P > 0,$ $O_i > 0$ $(i = 1,2,3)$ of appropriate dimensions such that the following LMI holds:

$$\begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & 0 \\ * & \Omega_{22} & 0 & \dfrac{\varepsilon_4 \gamma_3}{2} I & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_4 I & 0 & 0 \\ * & * & * & * & -O_2 & 0 \\ * & * & * & * & * & -\dfrac{1}{h_M} O_3 \end{pmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \Omega_{11} &= A^T P + PA - R^T C - C^T R + O_1 + O_2 \\ &\quad + h_M O_3 + (\varepsilon_1 \rho_1 + \varepsilon_2 \rho_2 + \varepsilon_3 \beta_1 + \varepsilon_4 \beta_2) I, \\ \Omega_{12} &= PB - R^T D, \qquad \Omega_{13} = P + \frac{\varepsilon_3 \gamma_1 - \varepsilon_1}{2} I, \\ \Omega_{14} &= P + \frac{\varepsilon_4 \gamma_2 - \varepsilon_2}{2} I, \\ \Omega_{22} &= -(1 - h_d) O_1 + (\varepsilon_2 \rho_3 + \varepsilon_4 \beta_3) I. \end{aligned}$$

If (26) holds a feasible solution, then the gain matrix L is given by $L = P^{-1} R^T.$

Proof. Similar to the proof of Theorem 2, the condition (26) in Corollary 2 can be obtained and the detailed proof is omitted.

Remark 4 When $f(x(t), x(t - h(t)), u) = 0$ and $\Phi(x, u) = 0,$ the system (1) can be written as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t - h(t)), \\ y(t) = Cx(t) + Dx(t - h(t)). \end{cases} \tag{27}$$

The observer of system (27) is defined by the following form

$$\begin{aligned} \dot{\hat{x}}(\hat{t}) &= Ax(t) + Bx(t - h(t)) \\ &\quad + L(y - C\hat{x}(t) - Dx(t - h(t))). \end{aligned} \tag{28}$$

Denote $e(t) = x(t) - \hat{x}(t),$ then the observer error dynamics is governed by

$$\dot{e}(t) = (A - LC) e(t) + (B - LD) e(t - h(t)). \tag{29}$$

For system (29), we have the following corollary.

Corollary 3 Assume that assumption 1 and 2 are satisfied. For given scalars $0 \le h_m \le h_M$, and h_d, the observer error dynamics (29) is asymptotically stable, if there exist matrices $P > 0$, and $Q_i > 0\ (i = 1, 2, \dots, 5)$ of appropriate dimensions such that the following LMI holds:

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & 0 & 0 & 0 \\ * & \Sigma_{22} & 0 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & -Q_3 & 0 & 0 \\ * & * & * & * & -\frac{1}{h_m}Q_4 & 0 \\ * & * & * & * & * & -\frac{1}{h_M}Q_5 \end{bmatrix} < 0, \tag{30}$$

where

$$\Sigma_{11} = A^T P + PA - R^T C - C^T R + Q_1 + Q_2 + Q_3 + h_m Q_4 + h_M Q_5,$$

$$\Sigma_{12} = PB - R^T D, \quad \Sigma_{22} = -(1 - h_d)Q_2.$$

If (30) holds a feasible solution, then the gain matrix L is given by $L = P^{-1} R^T$.

Proof. Similar to the proof of Theorem 2, the condition (29) in Corollary 3 can be obtained and the detailed proof is omitted.

4 Illustrative examples

In this section, we present two numerical examples in order to show the superiority of our design method.

Example 1. Consider the uncertain nonlinear systems (1) with the following parameters:

$$A = \begin{bmatrix} -2 & -1 \\ -4 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 2 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$\Phi(x(t), u) = \begin{bmatrix} 0 \\ -0.333\sin(x_1) \end{bmatrix} + \begin{bmatrix} 0 \\ 21.6 \end{bmatrix} u(t),$$

$$f = \begin{bmatrix} -x_1(x_1^2 + x_2^2) \\ -x_2(x_1^2 + x_2^2) \end{bmatrix}.$$

$$l = 0.333, \quad h_m = 0.1, \quad h_M = 1.2, \quad h_d = 0.9,$$
$$\gamma_1 = 0.1, \quad \gamma_2 = 0.9, \quad \gamma_3 = 0.7, \quad \rho_1 = 0.7,$$
$$\rho_2 = 0.5, \quad \rho_3 = 0.2, \quad \beta_1 = 0.8, \quad \beta_2 = 1.4,$$
$$\beta_3 = 1.6.$$

Take $L = \begin{pmatrix} -2 & 4 \end{pmatrix}^T$. Theorem 1 can be applied to design an observer, and solving the LMIs (11) and (12) yields

$$P = \begin{bmatrix} 57.95 & 61.01 \\ 61.01 & 144.83 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 484.10 & 86.20 \\ 86.20 & 1096.40 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 203.80 & 76.40 \\ 76.40 & 1420.10 \end{bmatrix}, Q_3 = \begin{bmatrix} 145.04 & 34.04 \\ 34.04 & 905.75 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 964.51 & -7.12 \\ -7.12 & 918.64 \end{bmatrix}, \quad Q_5 = \begin{bmatrix} 8.05 & 0.90 \\ 0.90 & 1.84 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 899.10 & 0 \\ 0 & 899.10 \end{bmatrix}, \quad Q_7 = \begin{bmatrix} 7.79 & 0.84 \\ 0.84 & 1.98 \end{bmatrix},$$

$$R = \begin{bmatrix} -81.70 & 1544.60 \end{bmatrix},$$

$$M = \begin{bmatrix} -411.19 & 78.76 \\ 78.76 & 197.96 \\ 14.37 & 28.93 \\ 28.93 & -55.24 \\ -2.39 & -7.54 \\ -7.54 & -55.17 \\ -11.40 & -6.35 \\ -6.35 & -62.36 \end{bmatrix},$$

$$N = \begin{bmatrix} -46.81 & -2.81 \\ -2.81 & -0.34 \\ 12.48 & 0.83 \\ 0.83 & -0.09 \\ -7.62 & -7.46 \\ -7.46 & -5.06 \\ -12.97 & -7.49 \\ -7.49 & -5.41 \end{bmatrix},$$

$$X = \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 & Y_2 \\ * & Y_3 \end{bmatrix},$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 899.10,$$

where

$$X_1 = \begin{bmatrix} 997.97 & -10.00 & -1.66 & -2.55 \\ -10.00 & 923.82 & -2.55 & -4.58 \\ -1.66 & -2.55 & 899.44 & -0.58 \\ -2.55 & -4.58 & -0.58 & 900.36 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} -0.04 & -7.3 & 2.51 & -1.23 \\ -7.3 & -6.71 & -1.23 & -7.52 \\ -0.17 & -0.34 & -0.40 & -0.62 \\ -0.34 & 1.76 & -0.62 & 2.22 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 898.90 & 0.15 & -0.10 & 0.00 \\ 0.15 & 900.35 & 0.00 & 1.39 \\ -0.10 & 0.00 & 898.92 & -0.12 \\ 0.00 & 1.39 & -0.12 & 900.61 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 1062.40 & 9.80 & -43.46 & -2.82 \\ 9.80 & 899.80 & -2.82 & -0.17 \\ -43.46 & -2.82 & 910.48 & 0.73 \\ -2.82 & -0.17 & 0.73 & 899.06 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} 21.66 & 12.26 & 40.55 & 12.77 \\ 12.26 & 1.54 & 12.77 & 1.56 \\ -5.92 & -3.15 & -10.96 & -3.34 \\ -3.15 & -0.13 & -3.34 & -0.15 \end{bmatrix},$$

$$Y_3 = \begin{bmatrix} 916.86 & 12.74 & 19.74 & 14.27 \\ 12.74 & 908.67 & 14.27 & 10.03 \\ 19.74 & 14.27 & 922.98 & 15.76 \\ 14.27 & 10.03 & 15.76 & 909.61 \end{bmatrix}.$$

According to Theorem 1, the estimation error (10) converges asymptotically towards zero.

Let the input $u(t) = \sin(t)$. For simulation, we set initial conditions as follows:

$$x(0) = (-2 \quad -2 \quad -1.5 \quad -2)^T,$$

$$\hat{x}(0) = (0.5 \quad 1 \quad 1 \quad -1)^T.$$

Figs. 1 and 2 show the simulation results for states $x_1(k)$ and $x_2(k)$, respectively. From the simulation, we know that the effect of state trajectory tracking is satisfactory.

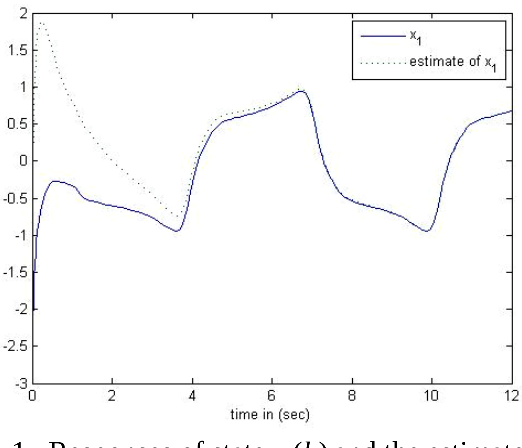

Fig. 1. Responses of state $x_1(k)$ and the estimate of $x_1(k)$ in Example 1

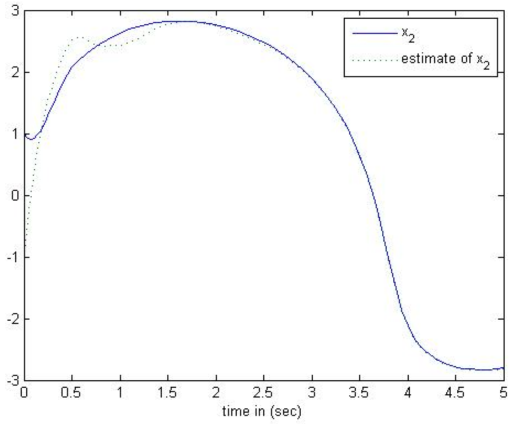

Fig. 2. Responses of state $x_2(k)$ and the estimate of $x_2(k)$ in Example 1

Example 2 Consider the time-varying delay system (27) with the following parameters

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \qquad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.2 & -0.1 \end{bmatrix}, \qquad D = 0.$$

For given

$h_m = 0.1,\quad h_M = 3,\quad h_d = 0.9,\quad \rho_1 = 0.7,$
$\rho_2 = 0.5,\quad \rho_3 = 0.2,\quad \beta_1 = 0.8,\quad \beta_2 = 1.4,$
$\beta_3 = 1.6,\quad \gamma_1 = 0.1,\quad \gamma_2 = 0.9,\quad \gamma_3 = 0.3,$

solving the LMIs (30) yields

$$P = \begin{bmatrix} 59.8841 & 18.0632 \\ 18.0632 & 5.6596 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 55.5489 & 27.3897 \\ 27.3897 & 14.4644 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 509.5623 & 236.0600 \\ 236.0600 & 110.0697 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 55.5489 & 27.3897 \\ 27.3897 & 14.4644 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 11.0080 & 3.1534 \\ 3.1534 & 6.2778 \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} 82.8122 & 41.2942 \\ 41.2942 & 20.8709 \end{bmatrix},$$

$$R = \begin{bmatrix} -2273.2 & -1110.5 \end{bmatrix},$$

then the gain matrix is given by

$$L = P^{-1}R^T = \begin{bmatrix} -692.0397 \\ -2012.5011 \end{bmatrix}.$$

For simulation, we set initial conditions as follows:

$$x(0) = (-2 \quad -2 \quad -1.5 \quad -2)^T,$$

$$\hat{x}(0) = (0.5 \quad 1 \quad 1 \quad -1)^T.$$

Figs. 3 and 4 show the simulation results for states $x_1(k)$ and $x_2(k)$, respectively. From the simulation, we know that the effect of state trajectory tracking is satisfactory.

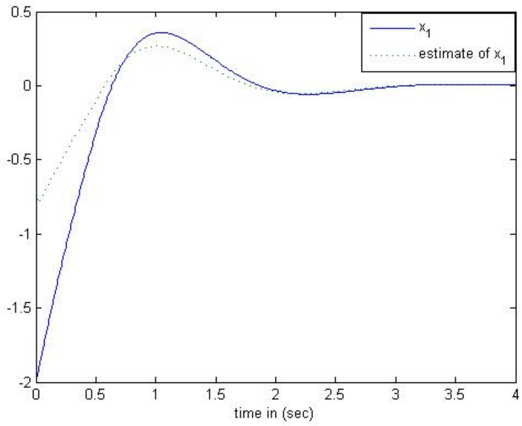

Fig. 3. Responses of state $x_1(k)$ and the estimate of $x_1(k)$ in Example 2

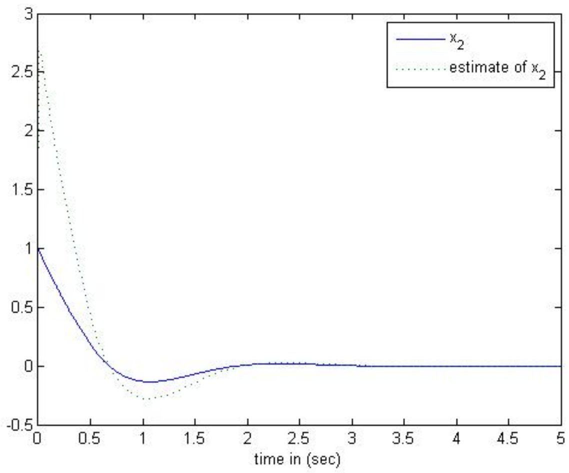

Fig. 4. Responses of state $x_2(k)$ and the estimate of $x_2(k)$ in Example 2

5 Conclusion

In this paper, for a general class of continuous-time nonlinear dynamical system that satisfies the one-sided Lipschitz condition with interval time-varying delay, we propose new observer design approach. Some novel sufficient conditions ensuring the existence of state observer are established for the class of nonlinear dynamical systems with interval time-varying delay in terms of linear matrix inequalities. The computing method for observer gain matrix is presented. Some simulation examples are given to show the efficiency of the proposed approach.

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