Robust H_∞ filtering for uncertain differential linear repetitive processes via LMIs and polynomial matrices

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Abstract: This paper is concerned with the full-order $H\infty$ filtering problem for uncertain differential linear repetitive processes (LRPs). uncertain differential linear repetitive process (LRP) assumed to be stable along the pass, our attention is focused on the design of a full-order filter, which guarantees the filtering error process to be stable along the pass, and minimizes an upper bound for the $H\infty$ norm of its transfer function. approach are proposed to solve the full-order $H\infty$ filtering problem.

Key–Words: Differential repetitive processes, $H\infty$ filtering, Convex polytopic uncertainty.

1 Introduction

As is well known, many practical systems can be modeled as two-dimensional 2D systems [1, 2], such as those in image data processing and transmission, thermal processes, gas absorption and water stream heating. During the last few decades, the investigation of 2D systems in the control and signal processing fields has attracted considerable attention and many important results have been reported to the literature. Among these results, the H_∞ filtering problem for two-dimensional 2D linear systems described by Roesser and FornasiniMarchesini (FM) models in [3, 5, 4, 30, 6, 7, 8, 13, 12, 16, 14, 28, 17, 18, 20], for 2D linear parameter-varying systems, the related work can be found in [28], stability and stabilization of 2D systems in [23, 26, 22, 31], H_∞ control for 2-D nonlinear systems with delays and the nonfragile H_∞ and $l_2 - l_1$ problem for Roesser-type 2D systems in [19]. However, because there is no systematic and general approach to analyze linear repetitive processes systems, many problems still remain.

On the other hand, Many physical systems complete the same finite duration operation over and over again. Repetitive processes have this characteristic where a series of sweeps or passes are made through dynamics defined over a finite duration known as the pass length. Once each pass is complete, the process resets to the original location and the next one begins. The output on each pass is termed the pass profile and the notation for scalar or vector valued variables is $yk(t)$, $0 \leq t \leq \alpha < \infty$, $k \geq 0$, where y is the scalar or vector valued variable, the integer k is the pass number and α is the pass length. Also the previous pass profile contributes to dynamics of the next one and the result can be oscillations in the pass profile sequence $\{y\}_k$ that increase in amplitude from pass-to-pass (k) and cannot be studied by standard systems theory.

The problem becomes more complex when uncertainties affect the model and a robust filter is needed. and can make important effects on the properties of dynamic systems.

This work deals with the problem of robust H_∞ filter design for uncertain differential linear repetitive processes system with time-invariant parameters belonging to a polytope. The main contribution of this paper is to provide new parameter-dependent LMIs for H_∞ robust filtering of uncertain differential linear repetitive processes system. by means of Finsler's lemma, the bounded real lemma for the H_∞ norm is lifted to a larger parameter space. In this new parameter space, extra degrees of freedom provided by slack variables can be used to reduce the conservativeness when deriving LMI conditions for filter design. By imposing a structure to the decision variables. LMI relaxations based on homogeneously polynomially parameter-dependent matrices of arbitrary degree are derived for the robust filter design. As illustrated by several benchmark examples borrowed from the literature, the proposed conditions provide less conservative than other existing method.

Notation : we use standard notation throughout this paper. The notation $P > 0(< 0)$ is used for positive (negative) definite matrices. $*$ stands for the symmetric term of the diagonal elements of square symmetric matrix. I denotes the identity matrix with appropriate dimension. the superscript "T" represents the transpose. $sym(A)$ indicates $A^T + A$, $diag(...)$ stands for a block-diagonal matrix.The Euclidean vector norm is denoted by $\| \ . \ \|$. The l_2 norm of a 2-D signal $f_k(t)$ is given by

$$\|f_k(t)\|_2 = \sqrt{\sum_{k=0}^{\infty} \int_0^{\infty} f_k^T(t) f_k(t) dt}$$

where $f_k(t)$ is said to be in the space $l_2[0, \infty), [0, \infty)$ or l_2, for simplicity, if $\| \ f_k(t) \ \|_2 < \infty$.

2 Preliminaries

Consider the uncertain differential linear repetitive processes described by the following state-space model over $0 \leq t \leq \beta, k \geq 0$:

$$\begin{array}{rl} \dot{x}_{k+1}(t) = & A(\alpha)x_{k+1}(t) + B_0(\alpha)y_k(t) + B(\alpha)w_{k+1}(t) \\ y_{k+1}(t) = & C(\alpha)x_{k+1}(t) + D_0(\alpha)y_k(t) + D(\alpha)w_{k+1}(t) \\ z_{k+1}(t) = & E(\alpha)x_{k+1}(t) + F_0(\alpha)y_k(t) + F(\alpha)w_{k+1}(t) \end{array} \quad (1)$$

where on pass k, $x_k(t) \in \mathbb{R}^n$ is the state vector and $y_k(t) \in \mathbb{R}^m$ is the pass profile vector, and $w_k(t) \in \mathbb{R}^l$ disturbance (or noise) vector which belongs to L_2, $z_{k+1}(t) \in \mathbb{R}^r$ is the measured output, $v_{k+1}(t) \in \mathbb{R}^p$ is the signal to be estimated. respectively; $A(\alpha)$, $B_0(\alpha)$, $B(\alpha)$, $C(\alpha)$, $D_0(\alpha)$, $D(\alpha)$, $E(\alpha)$, $F_0(\alpha)$ and $F(\alpha)$ are time-invariant but unknown real matrices with appropriate dimensions.
the state initial vector on each pass and the initial pass profile (on pass 0). The form of these considered here is

$$\begin{array}{l} x_{k+1}(0) = d_{k+1}, k \geq 0 \\ y_0(t) = f(t) \end{array} \quad (2)$$

where $d_{k+1} \in \mathbb{R}^n$ has known constant entries and $f(t) \in \mathbb{R}^m$ are known functions of t. Throughout the paper, the following assumptions are further made for system (1)

Assumption 1:
$A(\alpha), B_0(\alpha), B(\alpha), C(\alpha), D_0(\alpha), D(\alpha), E(\alpha), F_0(\alpha)$ and $F(\alpha)$ are assumed to satisfy $\Omega(\alpha)$ =($A(\alpha)$,$B_0(\alpha)$,$B(\alpha)$,$C(\alpha)$,$D_0(\alpha)$,$D(\alpha)$,$E(\alpha)$, $F_0(\alpha)$,$F(\alpha)$) $\in \mathfrak{R}$, where $\mathfrak{R}$ is a given convex bounded polyhedral domain defined as

$$\mathfrak{R} = \{\Omega(\alpha) \setminus \Omega(\alpha) = \sum_{i=1}^{N} \alpha_i \Omega_i; \alpha = \sum_{i=1}^{N} \alpha_i, \alpha_i \geq 0\}$$

with $\Omega_i = (A_i$,B_{0i},B_i,C_i,D_{0i},D_i,E_i, F_{0i},$F_i)$ denoting vertices.
Assumption 2: System (1) is stable along the pass for all $\Omega(\alpha) \in \mathfrak{R}$
In the paper, our interest is to estimate the signal $v_{k+1}(t)$ by the using the following differential linear repetitive processes reduced-order filter

$$\begin{array}{rl} \dot{\varphi}_{k+1}(t) = & A_f\varphi_{k+1}(t) + B_{0f}\phi_k(t) + B_f z_{k+1}(t) \\ \phi_{k+1}(t) = & C_f\varphi_{k+1}(t) + D_{0f}\phi_k(t) + D_f z_{k+1}(t) \\ \hat{v}_{k+1}(t) = & G_f\varphi_{k+1}(t) + H_{0f}\phi_k(t) + H_f z_{k+1}(t) \\ \varphi_{k+1}(0) = & 0, k \geq 0, \phi_0(t) = 0, 0 \leq t \leq \beta \end{array} \quad (3)$$

where on pass k, $\varphi_k(t) \in \mathbb{R}^n$ is the state vector of the filter, $\phi_k(t) \in \mathbb{R}^m$ is the pass profile vector and $\hat{v}_{k+1}(t) \in \mathbb{R}^p$ is the estimator of $v_{k+1}(t)$.

Defining the augmented system:

$$\begin{array}{rl} \dot{\xi}_{k+1}(t) = & \tilde{A}(\alpha)\xi_{k+1}(t) + \tilde{B}_0(\alpha)\zeta_k(t) \\ & +\tilde{B}(\alpha)w_{k+1}(t) \\ \zeta_{k+1}(t) = & \tilde{C}(\alpha)\xi_{k+1}(t) + \tilde{D}_0(\alpha)\zeta_k(t) \\ & +\tilde{D}(\alpha)w_{k+1}(t) \\ e_{k+1}(t) = & \tilde{G}(\alpha)\xi_{k+1}(t) + \tilde{H}_0(\alpha)\zeta_k(t) \\ & +\tilde{H}(\alpha)w_{k+1}(t) \\ \xi_{k+1}(0) = & 0, k \geq 0, \zeta_0(t) = 0, 0 \leq t \leq \beta \end{array} \quad (4)$$

where
$\xi_{k+1}(t) = [x_{k+1}^T(t) \quad \varphi_{k+1}^T(t)]^T$
$\zeta_k(t) = [y_k^T(t) \quad \phi_k^T(t)]^T$, $e_{k+1}(t) = v_{k+1}(t) - \hat{v}_{k+1}(t)$
and

$$\begin{array}{l} \tilde{A}(\alpha) = \begin{bmatrix} A(\alpha) & 0 \\ B_f E(\alpha) & A_f \end{bmatrix} \\ \tilde{B}_0(\alpha) = \begin{bmatrix} B_0(\alpha) & 0 \\ B_f F_0(\alpha) & B_{0f} \end{bmatrix} \\ \tilde{B}(\alpha) = \begin{bmatrix} B(\alpha) \\ B_f F(\alpha) \end{bmatrix} \\ \tilde{C}(\alpha) = \begin{bmatrix} C(\alpha) & 0 \\ D_f E(\alpha) & C_f \end{bmatrix} \\ \tilde{D}_0(\alpha) = \begin{bmatrix} D_0(\alpha) & 0 \\ D_f F_0(\alpha) & D_{0f} \end{bmatrix} \\ \tilde{D}(\alpha) = \begin{bmatrix} D(\alpha) \\ D_f F(\alpha) \end{bmatrix} \\ \tilde{G}(\alpha) = \begin{bmatrix} G - H_f E(\alpha) & -G_f \end{bmatrix} \\ \tilde{H}_0(\alpha) = \begin{bmatrix} H_0 - H_f F_0(\alpha) & -H_{0f} \end{bmatrix} \\ \tilde{H}(\alpha) = \begin{bmatrix} -H_f F \end{bmatrix} \end{array} \quad (5)$$

The parameter uncertainties considered in this paper are assumed to be of polytopic type, entering into all the vertices of the system model. The polytopic uncertainty has be widely used in the problems of robust control and filtering for uncertain systems (see, for

instance, [25] and the references therein), and many practical systems possess parameter uncertainties which can be either exactly modeled or overbounded by the polytope $\mathfrak{R}$.
Then, the differential linear repetitive processes $H\infty$ filtering problem to be addressed in this paper can be expressed as follows: given the differential linear repetitive processes system (1), design a suitable a full-order filter (3) such that the following tow requirements are satisfied.

1. The resulting error process (4) with $w(i,j) \equiv 0$ is stable along the pass for all $\Omega(\alpha) \in \mathfrak{R}$.
2. Under zero boundary condition, and for all non-zero $w_{k+1}(t) \in L_2$, we require that

$$\|e_{k+1}(t)\|_2 < \gamma \|w_{k+1}(t)\|_2 \qquad (6)$$

Lemma 2.1 *Let $\xi \in \mathbb{R}^n$, $Q \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{m\times n}$ with $rank(B) < n$ and $B^\perp$ such that $BB^\perp = 0$. Then, the following conditions are equivalent:*
(i) $\xi^T Q \xi < 0, \forall \xi \neq 0 : B\xi = 0$
(ii) ${B^\perp}^T Q B^\perp < 0$
(iii) $\exists \mu \in \Re : Q - \mu B^T B < 0$
(iv) $\exists \chi \in \Re^{n\times m} : Q + \chi B + B^T \chi^T < 0$

3 Robust H_∞ filtering analysis

Using theorem 1 in ([11]) with $P_1(\alpha) > 0$ and $P_2(\alpha) > 0$ are parameter-dependent symmetric matrices. we have the following result.

Lemma 3.1 *The filtering error process (4) is stable along the pass with prescribed H_∞ performance level $\gamma > 0$ if there exist matrices $P_1 > 0$ and $P_2 > 0$ such that the following LMIs hold for all $\Omega(\alpha) \in \mathfrak{R}$:*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & P_1(\alpha)\tilde{B}(\alpha) & \tilde{G}^T(\alpha) & \tilde{C}^T(\alpha)P_2(\alpha) \\ * & -P_2(\alpha) & 0 & {\tilde{H}_0}^T(\alpha) & {\tilde{D}_0}^T(\alpha)P_2(\alpha) \\ * & * & -\gamma^2 I & \tilde{H}^T(\alpha) & \tilde{D}^T(\alpha)P_2(\alpha) \\ * & * & * & -I & 0 \\ * & * & * & * & -P_2(\alpha) \end{bmatrix} < 0 \qquad (7)$$

where
$\Gamma_{11} = P_1(\alpha)\tilde{A}(\alpha) + \tilde{A}^T(\alpha)P_1(\alpha)$
$\Gamma_{12} = P_1(\alpha)\tilde{B}_0(\alpha)$

Theorem 1 *The filtering error process (4) is stable along the pass with prescribed H_∞ performance level $\gamma > 0$ if there exist a parameter-dependent symmetric positive definite matrices $P_1(\alpha)$, $P_2(\alpha)$ and parameter-dependent matrices $L(\alpha)$, $K(\alpha)$, $M(\alpha)$ and $N(\alpha)$ such that the following LMIs (8) hold for all $\Omega(\alpha) \in \mathfrak{R}$:*

$$\begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ * & \psi_{22} & \psi_{23} & \psi_{24} \\ * & * & -I & -N(\alpha) \\ * & * & * & \psi_{44} \end{bmatrix} < 0 \qquad (8)$$

where
$\psi_{11} = -\bar{P}_2(\alpha) + L(\alpha)\bar{A}(\alpha) + \bar{A}^T(\alpha)L^T(\alpha)$
$\psi_{12} = L(\alpha)\bar{B}(\alpha) + \bar{A}^T(\alpha)M^T(\alpha)$
$\psi_{13} = \bar{G}^T(\alpha) + \bar{A}^T(\alpha)N^T(\alpha)$
$\psi_{14} = -L(\alpha) + \bar{A}^T(\alpha)K^T(\alpha) + \bar{P}_1(\alpha)$
$\psi_{22} = M(\alpha)\bar{B}(\alpha) + \bar{B}^T(\alpha)M^T(\alpha) - \gamma^2 I$
$\psi_{23} = \tilde{H}(\alpha) + \bar{B}^T(\alpha)N^T(\alpha)$
$\psi_{24} = -M(\alpha) + \bar{B}^T(\alpha)K^T(\alpha)$
$\psi_{44} = -K(\alpha) - K^T(\alpha) + \bar{P}_2(\alpha)$
and
$\bar{A}(\alpha) = \begin{bmatrix} \tilde{A}(\alpha) & \tilde{B}_0(\alpha) \\ \tilde{C}(\alpha) & \tilde{D}_0(\alpha) \end{bmatrix}$
$\bar{B}(\alpha) = \begin{bmatrix} \tilde{B}(\alpha) \\ \tilde{D}(\alpha) \end{bmatrix}$
$\bar{G}(\alpha) = [\tilde{G}(\alpha) \quad \tilde{H}_0(\alpha)]$
$\bar{P}_2(\alpha) = diag(0, P_2(\alpha))$
$\bar{P}_1(\alpha) = diag(P_1(\alpha), 0)$
Proof: The LMIs (8) is obtained by considering
$\chi = \begin{bmatrix} L(\alpha) \\ M(\alpha) \\ N(\alpha) \\ K(\alpha) \end{bmatrix}$
$B = \begin{bmatrix} \bar{A}(\alpha) & \bar{B}(\alpha) & 0 & -I \end{bmatrix}$

$$Q = \begin{bmatrix} -\bar{P}_2(\alpha) & 0 & \bar{G}^T(\alpha) & \bar{P}_1(\alpha) \\ * & -\gamma^2 I & \tilde{H}(\alpha) & 0 \\ * & * & -I & 0 \\ * & * & * & \bar{P}_2(\alpha) \end{bmatrix} \qquad (9)$$

in condition (iv) of lemma II.1 with
$B^\perp = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ \bar{A}(\alpha) & \bar{B}(\alpha) & 0 \end{bmatrix}$
and then by calculation and schur complement, using condition (ii) of lemma II.1, we can obtain the equivalence between $B^{\perp T} Q B^\perp < 0$ and LMIs (7), (7) is equivalent to (8). Thus Theorem 1 is equivalent to lemma III.1.□

Remark 1: When the matrices ($\tilde{A}(\alpha)$, $\tilde{B}_0(\alpha)$, $\tilde{B}(\alpha)$, $\tilde{C}(\alpha)$, $\tilde{D}_0(\alpha)$, $\tilde{D}(\alpha)$, $\tilde{G}(\alpha)$, $\tilde{H}_0(\alpha)$, $\tilde{H}(\alpha)$) are known. However, if the matrices are from an uncertain polytope, (8) would render a less-conservative evaluation of the upper-bound of the H_∞ norm of the system (4) due to the freedom given by slack variables

$L(\alpha)$, $M(\alpha)$, $N(\alpha)$ and $K(\alpha)$ and the fact that $P_1(\alpha)$ and $P_2(\alpha)$ are allowed to vertex-dependent in (8). This additional matrix variable will enable use to derive a less conservative robust full-order filtering design

4 Robust H_∞ filter design

The above result is useful for H_∞ analysis when the full-order filter is given. In order to facilitate the robust full-order H_∞ filter design, we need to consider a special case of the above theorem. To this end, we specify the matrices $L(\alpha)$, $M(\alpha)$, $N(\alpha)$ and $K(\alpha)$ as follows:

$$L(\alpha) = diag\left\{\begin{bmatrix} T_{11}(\alpha) & T_{12} \\ T_{21}(\alpha) & T_{12}\end{bmatrix}, \begin{bmatrix} F_{11}(\alpha) & F_{12} \\ F_{21}(\alpha) & F_{12}\end{bmatrix}\right\} \quad (10)$$

$$K(\alpha) = diag\left\{\begin{bmatrix} G_{11}(\alpha) & \lambda_1 T_{12} \\ G_{21}(\alpha) & \lambda_2 T_{12}\end{bmatrix}, \begin{bmatrix} L_{11}(\alpha) & \lambda_3 F_{12} \\ L_{21}(\alpha) & \lambda_4 F_{12}\end{bmatrix}\right\}$$

$M(\alpha) = [T_1(\alpha) \quad 0 \quad T_2(\alpha) \quad 0]$
and
$N(\alpha) = [T_3(\alpha) \quad 0 \quad T_4(\alpha) \quad 0]$

where

and T_{12}, F_{12}, λ_1, λ_2, λ_3, λ_4 are scalar variables to be determined. For convenience, matrices $P_1(\alpha)$ and $P_2(\alpha)$ is also partitioned as following

$$P_1(\alpha) = \begin{bmatrix} P_{11}(\alpha) & P_{12}(\alpha) \\ P_{12}^T(\alpha) & P_{22}(\alpha)\end{bmatrix}$$

$$P_2(\alpha) = \begin{bmatrix} Q_{11}(\alpha) & Q_{12}(\alpha) \\ Q_{12}^T(\alpha) & Q_{22}(\alpha)\end{bmatrix} \quad (11)$$

and the following change of variables is adopted $\bar{A}_f = T_{12}A_f$, $\bar{B}_f = T_{12}B_f$, $\bar{B}_{0f} = T_{12}B_{0f}$, $\bar{C}_f = F_{12}C_f$, $\bar{D}_f = F_{12}D_f$, and $\bar{D}_{0f} = F_{12}D_{0f}$. With this particular choice for the decision variables, the sufficient condition in lemma III.1 for the existence of a robust $H\infty$ full-order filter, as presented below.
A sufficient parameter-dependent condition

Theorem 2 *The full-order filtering error process (3) is stable along the pass with prescribed H_∞ performance level $\gamma > 0$ if there exist a parameter-dependent symmetric positive definite matrices $P_1(\alpha)$, $P_2(\alpha)$ as in (11) and parameter-dependent matrices $L(\alpha)$, $M(\alpha)$, $N(\alpha)$, $K(\alpha)$ as in (10) and matrices $\bar{A}_f$, $\bar{B}_{0f}$, $\bar{D}_f$, $\bar{D}_{0f}$, $\bar{B}_f$, $\bar{C}_f$, $\bar{G}_f$, $\bar{H}_{0f}$, $\bar{H}_f$ and scalars λ_1, λ_2, λ_3, λ_4 such that the following LMIs hold for all $\alpha \in \Lambda_N$:*

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ * & * & \Lambda_{33} & \Lambda_{34} \\ * & * & * & -I \end{bmatrix} < 0 \quad (12)$$

where

$$\Lambda_{11} = \begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} & \eta_{14} \\ * & sym(\bar{A}_f) & \eta_{23} & \eta_{24} \\ * & * & \eta_{33} & \eta_{34} \\ * & * & * & \eta_{44} \end{bmatrix}$$

with
$\eta_{11} = sym(T_{11}(\alpha)A(\alpha)) + sym(\bar{B}_f E(\alpha))$
$\eta_{12} = \bar{A}_f + A^T(\alpha)T_{21}^T(\alpha) + E^T(\alpha)\bar{B}_f^T$
$\eta_{13} = T_{11}(\alpha)B_0(\alpha) + C^T(\alpha)F_{11}^T(\alpha) + \bar{B}_f F_0(\alpha) + E^T(\alpha)\bar{D}_f^T$
$\eta_{14} = \bar{B}_{0f} + C^T(\alpha)F_{21}^T(\alpha) + E^T(\alpha)\bar{D}_f^T$
$\eta_{23} = T_{21}(\alpha)B_0(\alpha) + \bar{B}_f F_0(\alpha) + \bar{C}_f^T$
$\eta_{24} = \bar{B}_{0f} + \bar{C}_f^T$
$\eta_{33} = sym(F_{11}(\alpha)D_0(\alpha)) + sym(\bar{D}_f F_0(\alpha)) - Q_{11}(\alpha)$
$\eta_{34} = \bar{D}_{0f} + D_0^T(\alpha)F_{21}^T(\alpha) + F_0^T(\alpha)\bar{D}_f^T - Q_{12}(\alpha)$
$\eta_{34} = sym(\bar{D}_{0f}) - Q_{22}(\alpha)$

$$,\Lambda_{12} = \begin{bmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \\ \phi_{31} \end{bmatrix}$$

with
$\phi_{11} = T_{11}(\alpha)B(\alpha) + \bar{B}_f F(\alpha) + A^T(\alpha)T_1^T(\alpha) + C^T(\alpha)T_2^T(\alpha)$
$\phi_{21} = T_{21}(\alpha)B(\alpha) + \bar{B}_f F(\alpha)$
$\phi_{31} = F_{11}(\alpha)D(\alpha) + \bar{D}_f F(\alpha) + B_0^T(\alpha)T_1^T(\alpha) + D_0^T(\alpha)T_2^T(\alpha)$
$\phi_{31} = F_{21}(\alpha)D(\alpha) + \bar{D}_f F(\alpha)$

$$,\Lambda_{13} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ \varphi_{21} & \varphi_{21} & \varphi_{23} & \lambda_4\bar{C}_f^T \\ \varphi_{31} & \varphi_{32} & \varphi_{33} & \varphi_{34} \\ \varphi_{41} & \lambda_2\bar{B}_{0f}^T & \varphi_{43} & \varphi_{44} \end{bmatrix}$$

with
$\varphi_{11} = A^T(\alpha)G_{11}^T(\alpha) + P_{11}(\alpha) + \lambda_1 E^T(\alpha)\bar{B}_f^T - T_{11}(\alpha)$
$\varphi_{12} = A^T(\alpha)G_{21}^T(\alpha) + P_{12}(\alpha) + \lambda_2 E^T(\alpha)\bar{B}_f^T - T_{12}$
$\varphi_{13} = C^T(\alpha)L_{11}^T(\alpha) + \lambda_3 E^T(\alpha)\bar{D}_f^T$
$\varphi_{14} = C^T(\alpha)L_{21}^T(\alpha) + \lambda_4 E^T(\alpha)\bar{D}_f^T$
$\varphi_{21} = \lambda_1\bar{A}_f^T + P_{12}^T(\alpha) - T_{21}(\alpha)$
$\varphi_{22} = \lambda_2\bar{A}_f^T + P_{22}(\alpha) - T_{12}$
$\varphi_{23} = \lambda_3\bar{C}_f^T$

$\varphi_{31} = B_0^T(\alpha)G_{11}^T(\alpha) + \lambda_1 F_0^T(\alpha)\bar{B}_f^T$
$\varphi_{32} = B_0^T(\alpha)G_{21}^T(\alpha) + \lambda_2 F_0^T(\alpha)\bar{B}_f^T$
$\varphi_{33} = D_0^T(\alpha)L_{11}^T(\alpha) + \lambda_3 F_0^T(\alpha)\bar{D}_f^T - F_{11}(\alpha)$
$\varphi_{34} = D_0^T(\alpha)L_{21}^T(\alpha) + \lambda_4 F_0^T(\alpha)\bar{D}_f^T - F_{12}$
$\varphi_{41} = \lambda_1 \bar{B}_{0f}^T$
$\varphi_{43} = \lambda_3 \bar{D}_{0f}^T - F_{21}(\alpha)$
$\varphi_{44} = \lambda_4 \bar{D}_{0f}^T - F_{12}$

$$,\Lambda_{14} = \begin{bmatrix} \Lambda_{14}^1 \\ -G_f^T \\ \Lambda_{14}^3 \\ -H_{0f}^T \end{bmatrix}$$

with
$\Lambda_{14}^1 = G^T(\alpha) - E^T(\alpha)H_f^T + A^T(\alpha)T_3^T(\alpha) + C^T(\alpha)T_4^T(\alpha)$
$\Lambda_{14}^3 = H_0^T(\alpha) - F_0^T(\alpha)H_f^T + B_0^T(\alpha)T_3^T(\alpha) + D_0^T(\alpha)T_4^T(\alpha)$

$$,\Lambda_{22} = \begin{bmatrix} sym(T_1(\alpha)B(\alpha)) + sym(T_2(\alpha)D(\alpha)) - \gamma^2 I_l \end{bmatrix}$$

$$,\Lambda_{23} = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} & \chi_{14} \end{bmatrix}$$

with
$\chi_{11} = B^T(\alpha)G_{11}^T(\alpha) + \lambda_1 F^T(\alpha)\bar{B}_f^T - T_1(\alpha)$
$\chi_{12} = B^T(\alpha)G_{21}^T(\alpha) + \lambda_2 F^T(\alpha)\bar{B}_f^T$
$\chi_{13} = D^T(\alpha)L_{11}^T(\alpha) + \lambda_3 F^T(\alpha)\bar{D}_f^T - T_2(\alpha)$
$\chi_{14} = D^T(\alpha)L_{21}^T(\alpha) + \lambda_4 F^T(\alpha)\bar{D}_f^T$

$$,\Lambda_{24} = -F^T(\alpha)H_f^T + B^T(\alpha)T_3^T(\alpha) + D^T(\alpha)T_4^T(\alpha)$$

$$,\Lambda_{33} = \begin{bmatrix} -sym(G_{11}^T(\alpha)) & \psi_{12} & 0 & 0 \\ * & -\lambda_2 sym(T_{12}) & 0 & 0 \\ * & * & \psi_{33} & \psi_{34} \\ * & * & * & \psi_{44} \end{bmatrix}$$

with
$\psi_{12} = -\lambda_1 T_{12} - G_{21}^T(\alpha)$
$\psi_{33} - sym(L_{11}(\alpha)) + Q_{11}(\alpha)$
$\psi_{34} = -\lambda_3 F_{12} - L_{21}^T(\alpha) + Q_{12}(\alpha)$
$\psi_{44} - \lambda_4 sym(F_{12}) + Q_{22}(\alpha)$

$$,\Lambda_{34} = \begin{bmatrix} -T_3^T(\alpha) \\ 0 \\ -T_4^T(\alpha) \\ 0 \end{bmatrix}$$

and

Then there exists a full-order filter of the form of (3) such that the full-order filtering error dynamics are stable along the pass and the prescribed H_∞ performance level γ is achieved. This H_∞ full-order filter can be computed from

$$\begin{bmatrix} A_f & B_{0f} & B_f \\ C_f & D_{0f} & D_f \\ G_f & H_{0f} & H_f \end{bmatrix} = \begin{bmatrix} T_{12}^{-1} & 0 & 0 \\ 0 & F_{12}^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_f & \bar{B}_{0f} & \bar{B}_f \\ \bar{C}_f & \bar{D}_{0f} & \bar{D}_f \\ \bar{G}_f & \bar{H}_{0f} & \bar{H}_f \end{bmatrix}$$

Remark 2: The searching parameters λ_1, λ_2, λ_3 and λ_4 in Theorem 1 can be searched using some optimization program (eg MATLAB fminsearch), to attain better optimization result. When they are set to be fixed constants, (12) is linear in the variables. Thus the following convex optimization problem:
Minimize γ subject to (12)
can be solved easily by using Yalmip ([33]) and SeDumi ([34]).

4.1 LMI conditions for robust full-order filtering

Theorem 2 is parameter-dependent sufficient LMI conditions for the existence of a robust H_∞ reduced-order filter. obtained directly from lemma 3 by imposing a particular structure to $L(\alpha)$, $M(\alpha)$, $N(\alpha)$, and $K(\alpha)$ (11). Moreover, they depend on scalar variables λ_1, λ_2, λ_3 and λ_4 that need to be searched. the main difference with respect to lemma 3 is that A_f, B_{0f}, B_f, C_f, D_{0f}, D_f, G_f, H_{0f} and H_f can be readily obtained from a feasible solution by simple change of variables. Another important point is concerned with the scalar parameters λ_1, λ_2, λ_3 and λ_4. the role of the scalar variables in the LMI conditions is to provide extra degrees of freedom and, possibly, to reduce the conservativeness of the LMI tests. In most cases, simple choices such as 0 or 1, for λ_1, λ_2, λ_3 and λ_4. provide less conservative results. This is mainly due to the presence of extra variables in the proposed conditions.
Although only the H_∞ norms have been used in this paper, other performance criteria could be investigated as well for full-order filter design by means of polynomial lyapunov functions.
To solve the parameter-dependent LMI conditions of Theorems 2 the technique proposed in ([24]) to handle parameter-dependent LMIs with parameters in the unit simplex can be applied. To this end, the polynomial matrices (decision variables in the parameter-dependent LMIs, i.e. $P_1(\alpha)$, $P_2(\alpha)$, $T_{11}(\alpha)$, $T_{21}(\alpha)$, $F_{11}(\alpha)$, $F_{21}(\alpha)$, $G_{11}(\alpha)$, $G_{21}(\alpha)$, $L_{11}(\alpha)$, $L_{21}(\alpha)$, $T_1(\alpha)$, $T_2(\alpha)$, $T_3(\alpha)$ and $T_4(\alpha)$) are treated as homogeneous polynomials of arbitrary degree g and sufficient LMI conditions, more and more precise with the increase of g, are expressed only in terms of the vertices of the polytope

5 Illustrative examples

Example 1:
Consider Differential Linear Repetitive Processes system borrowed from [11]:

A=$\begin{bmatrix} -1.45+0.01\delta & 0.64 & -0.40+0.01\delta \\ -0.60 & -1.41 & 0.00 \\ 0.30+0.01\delta & -0.20 & -0.70+0.01\delta \end{bmatrix}$
B_0=$\begin{bmatrix} 1.3+0.01\delta & 0.10+0.01\delta \\ -0.20 & -0.90 \\ 0.20+0.01\delta & -0.40+0.01\delta \end{bmatrix}$
B=$\begin{bmatrix} 0.60+0.01\delta \\ -1.20 \\ 0.20+0.01\delta \end{bmatrix}$
D=$\begin{bmatrix} 1.20+0.01\delta \\ 1.00+0.01\delta \end{bmatrix}$
C=$\begin{bmatrix} 1.3+0.01\delta & -0.60 & -0.10+0.01\delta \\ 0.30+0.01\delta & -0.20 & 0.60+0.01\delta \end{bmatrix}$
D_0=$\begin{bmatrix} -0.60+0.01\delta & 0.10+0.01\delta \\ 0.01\delta & -0.60+0.01\delta \end{bmatrix}$
E=$\begin{bmatrix} -0.80+0.01\delta & 0.40 & 0.20+0.01\delta \end{bmatrix}$
F_0=$\begin{bmatrix} -0.30+0.01\delta & 0.20+0.01\delta \end{bmatrix}$
G=$\begin{bmatrix} -1.00 & 0.60 & 0.30 \end{bmatrix}$ H_0=$\begin{bmatrix} -0.40 & 0.30 \end{bmatrix}$
$F=-0.10+0.01\delta$

Consider the case when δ is non-zero and satisfies $|\delta| \leq 1$. Then in the polytopic uncertainty model for this case the uncertainties in the parameters are represented by a tow-vertex polytope and we take the vertices to be at $\delta = 1$ and -1, respectively.
Table 1 shows the minimum γ obtained with Theorem 1

Table 1
the minimum γ obtained with several arbitrary degree g

	T1(g=0)	T1(g=1)	T1(g=2)	[11]
γ_{min}	0.2750	0.2522	0.2522	0.2750
λ_1	0.1405	0.1935	0.1940	
λ_2	0.3689	0.2259	0.2269	
λ_3	80.9105	34.2888	42.6769	
λ_4	134.0608	22.1290	31.1118	

for this example Theorem 1 with $\lambda_1 = 0.1935$, $\lambda_2 = 0.2259$, $\lambda_3 = 34.2888$, $\lambda_4 = 22.1290$ and g=1, provides a $\gamma_{min} = 0.2522$, while ([11]) yields 0.2750. in this case, the γ_{min} obtained by Theorem 1 with g=2 is smaller than the one provided by ([11]).
Now we Consider the case when δ is non-zero and satisfies $|\delta| \leq 7$. Then in the polytopic uncertainty model for this case the uncertainties in the parameters are represented by a tow-vertex polytope and we take the vertices to be at $\delta = 7$ and -7, respectively.
Table 2 shows the minimum γ obtained with Theorem 1

Table 2
the minimum γ obtained with several arbitrary degree g

	T1(g=0)	T1(g=1)	T1(g=2)	[11]
γ_{min}	12.5651	1.8689	1.6719	12.5686
λ_1	0.1910	0.0632	0.5597	
λ_2	0.1881	1.1746	1.0123	
λ_3	31.8588	10.1105	17.8291	
λ_4	21.3712	23.0610	292.5050	

It is clearly shown that less conservative filter designs are achieved as g grows by applying the structured polynomially parameter-dependent method. illustrating that the proposed method can outperform the other when the system is subject to more uncertainty
Finally, we Consider the case when δ is non-zero and satisfies $|\delta| \leq 7.5$.
Table 3 shows the minimum γ obtained with Theorem 1

Table 3
the minimum γ obtained with several arbitrary degree g

	T1(g=0)	T1(g=1)	T1(g=2)	[11]
γ_{min}	infeasible	2.5318	1.9103	infeasible
λ_1		0.2335	0.4765	
λ_2		0.2142	0.9022	
λ_3		28.1220	16.5565	
λ_4		63.6108	29.3395	

Example 2:
As an example, the metal rolling process is considered . This process is an extremely common industrial process where, in essence, deformation of the workpiece takes place between tow rolls with parallel axes revolving in opposite directions.
Thus the following linear differential equation represent Metal Rolling dynamics: ([27]).

$$\ddot{S}_k(t)+\frac{\alpha}{M}S_k(t) = \frac{\alpha}{\alpha_1}\ddot{S}_{k-1}(t)+\frac{\alpha}{M}S_{k-1}(t)-\frac{\alpha}{M\alpha_2}F_M(t) \quad (13)$$

Where:
$S_k(t)$:current passes through the rolls.
$S_{k-1}(t)$:previous passes through the rolls.
M: is the lumped mass of the roll-gap adjusting mechanism
α_1: the stiffness of the adjustment mechanism spring
α_2: the hardness of the metal strip
$\alpha = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$:the composite stiffness of the metal strip and the roll mechanism
$F_M(t)$:the force developed by the motor.
The linear differential equation (13) can be modelled as a system (1) by imposing :
$x_{k+1}(t)$=$\begin{bmatrix} s_k(t) \\ \dot{s}_k(t) \end{bmatrix}$ and $y_k(t)$=$\begin{bmatrix} s_{k-1}(t) \\ \ddot{s}_{k-1}(t) \end{bmatrix}$
Thus The linear differential equation (13) can be

writhed:

$$\dot{x}_{k+1}(t)=\begin{bmatrix} 0 & 1 \\ -\frac{\alpha}{M} & 0 \end{bmatrix} x_{k+1}(t)+\begin{bmatrix} 0 & 0 \\ \frac{\alpha}{M} & \frac{\alpha}{\alpha_1} \end{bmatrix} y_k(t)+\begin{bmatrix} 0 \\ -\frac{\alpha}{M\alpha_2} \end{bmatrix} F_M(t)$$

$$y_{k+1}(t)=\begin{bmatrix} 1 & 0 \\ -\frac{\alpha}{M} & 0 \end{bmatrix} x_{k+1}(t)+\begin{bmatrix} 0 & 0 \\ \frac{\alpha}{M} & \frac{\alpha}{\alpha_1} \end{bmatrix} y_k(t)+\begin{bmatrix} 0 \\ -\frac{\alpha}{M\alpha_2} \end{bmatrix} F_M(t)$$

In these design studies, the data used are $\alpha_1 = 600$, $\alpha_1 = 2000$ and $M = 100$ This yields $\alpha = 461.54$ and the following matrices in (1):

$$A=\begin{bmatrix} 0 & 1 \\ -7.6035 + 0.1\delta & -3.7722 \end{bmatrix}$$

$$B_0=\begin{bmatrix} 0 & 0 \\ 4.6153846 + 0.1\delta & 0.7692307 + 0.1\delta \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ -0.00230769 + 0.1\delta \end{bmatrix}$$

$$D=\begin{bmatrix} 0 \\ -0.00230769 + 0.1\delta \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 \\ -7.6035 + 0.1\delta & -3.7722 \end{bmatrix}$$

$$D_0=\begin{bmatrix} 0 & 0 \\ 4.6153846 + 0.1\delta & 0.7692307 + 0.1\delta \end{bmatrix}$$

$$E=\begin{bmatrix} -0.80 + 0.1\delta & 0.20 + 0.1\delta \end{bmatrix}$$

$$F_0=\begin{bmatrix} -0.30 + 0.1\delta & 0.20 + 0.1\delta \end{bmatrix}$$

$$G=\begin{bmatrix} -1.00 & 0.30 \end{bmatrix} \; H_0=\begin{bmatrix} -0.40 & 0.3 \end{bmatrix}$$

$$F = -0.10 + 0.01\delta$$

Consider the case when δ is non-zero and satisfies $|\delta| \leq 2.3$. Then in the polytopic uncertainty model for this case the uncertainties in the parameters are represented by a tow-vertex polytope and we take the vertices to be at $\delta = 2.3$ and -2.3, respectively.

Table 1 shows the minimum γ obtained with Theorem 1

Table 1
the minimum γ obtained with several arbitrary degree g

	T1(g=0)	T1(g=1)	T1(g=2)	T1(g=3)	[11]
γ_{min}	0.3423	0.0902	0.0814	0.0798	0.3423
λ_1	0.0001	0.0106	0.0106	0.0037	
λ_2	0.0001	0.0106	0.0106	0.0037	
λ_3	13.0477	33.7882	33.7882	17.9131	
λ_4	12.9183	36.9735	36.9735	17.7059	

6 Conclusions

New parameter-dependent LMI conditions for the design of full order robust and $H\infty$ filters have been proposed, for both uncertain polytopic differential linear repetitive processes with time-invariant parameters. LMI relaxations based on homogeneous polynomials of arbitrary degrees provided less conservative results when compared the other existing technique.

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