A constructive design of state observer for two-dimensional systems

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Abstract : This paper is concerned with the design of 2-D state observer for a class of two-dimensional systems described by the Fornasini-Marchesini local state-space second model. The paper provides a constructive design method for the design of 2-D asymptotic state observer using a 2-D Lyapunov theory and a linear matrix inequalities (LMI) technique. An illustrative example is provided to demonstrate the feasibility of the proposed method.

Key–Words : 2-D systems, Fornasini-Marchesini second model, stability, state observers, Linear Matrix Inequalities (LMI)

1 Introduction

This paper concentrates on Two-dimensional systems (2-D), the study of these systems has received much attention in past decades, because these 2-D models play important roles in image data processing and transformation [10], water stream heating, thermal processes, modeling of partial differential equations [21], gas absorption, and others areas of digital signal processing [18]. A great number of stability and control results related to 2-D systems have been reported in the literature : for example, using 2-D Lyapunov equations, some stability results have already been obtained in [11, 12, 22, 24]. In the recent years a great deal of works have been devoted to the analysis and design techniques for these systems [1, 2, 8, 14]. The problem of observing the state vector of both 1-D and 2-D linear discrete-time system has been the object of numerous studies [7, 9], ever since the original work of Luenberger [20], first appeared. but not fully investigated for 2-D systems, and still not completely solved.

we must point out that although it is known that the observation problem of linear 2-D systems can be reduced to checking a polynomial matrix of 2-D systems [4], it is seems difficult to apply this results in practice for the observer synthesis problem. Very recently, to solve this problem, many authors have proposed to transform the initial 2-D system to an equivalent one (by using some regular transformations) to design an observer controller for this system [16], [17]. In contrast, the aim of this paper is to design an asymptotic state observer for 2-D linear discrete-time system for 2-D systems described by the Fornasini-Marchesini local state-space second model, a new constructive method is them proposed for the design of asymptotic state observers using a Lyapanov equation and a linear matrix inequalities (LMI) techniques, ensures that the estimated states converge to the real value (i.,e. the observation error vanishes asymptotically to zero), followed by an illustrative example. The remainder of the paper is organized as follows : In section 2 the problem under study is formulated and some preliminary results are given. Section 3 is dedicated to the design of a state observer for two-dimensional systems described by the Fornasini-Marchesini local state-space second model. Section 4 uses a numerical example to illustrate the effectiveness of the proposed method . Finally, some conclusions are presented.

Notation : Throughout this paper, for symmetric matrices X and Y, X $>$ Y means that the matrix X-Y is positive definite I is the identity matrix of appropriate dimensions. We use an asterisk (*) to represent symmetric blocks that can be deduced by symmetry. $\| \, . \, \|$ means the euclidean norm. We also denote by A^T the transpose of matrix A, $[X]^+$ the generalized inverse of matrix X and by X^{-1} the inverse of matrix X.

2 Problem formulation and Preliminaries

Consider the linear time-invariant 2-D system (Σ) described by the following Fornasini-Marchesini state-space second model [10] :

$$(\Sigma): \begin{cases} x_{i+1,j+1} &= A_1 x_{i,j+1} + A_2 x_{i+1,j} \\ &+ B_1 u_{i,j+1} + B_2 u_{i+1,j} \\ y_{i,j} &= C x_{i,j} \end{cases} \quad (1)$$

with the following boundary conditions [10] for system (1) are given by

$$\begin{cases} x_{i,0} = x_0(i) & \forall\, i \in N \\ x_{0,j} = x_0(j) & \forall\, j \in N \end{cases} \quad (2)$$

satisfying

$$\begin{cases} sup_i ||x_{i,0}|| < \infty \\ sup_j ||x_{0,j}|| < \infty \end{cases} \quad (3)$$

where $x_{i,j} \in \mathbb{R}^n$ is the state vector, $u_{i,j} \in \mathbb{R}^m$ is the input vector, $y_{i,j} \in \mathbb{R}^l$ is the output vector.

A_1, A_2, B_1, B_2 and C are known constant real matrices of appropriate dimensions.
Before presenting the state observer design for 2-D systems, we will give the following results that can be used in the sequel of this paper.

Definition 1. *[18] The 2-D linear discrete-time system (Σ) is said to be asymptotically stable if*

$$\lim_{k\to\infty} \parallel X(k) \parallel = 0$$

under $\sup_j \parallel X(0) \parallel < \infty$ *where* $X(k) = \{x(i,j) : i + j = k\}$ *and* $\parallel X(k) \parallel = \sup_{x\in X(k)} \parallel x \parallel$.

The following lemma gives a sufficient condition for the asymptotic stability of 2-D linear discrete-time system (Σ) in terms of an LMI.

Lemma 1. *[12]. The 2-D linear discrete-time system (Σ) is asymptotically stable if there exist matrices $P > 0$ and $Q > 0$ such that the following LMI holds*

$$A^T P A - \Pi < 0 \quad (4)$$

where $\Pi = \begin{bmatrix} Q & 0 \\ 0 & P - Q \end{bmatrix}$ *and* $A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}$.

Remark 1. *The LMI (4) is useful for stability analysis but not for synthesis problem, because it involves two variables P and Q that render the linearization of the problem a difficult task.*

The following Lemma will be used for the synthesis problem of 2-D state observer.

Lemma 2. *[15].The 2-D linear discrete-time system (Σ) is asymptotically stable if there exist matrices $P > 0$, $Q > 0$, $R = R^T \geq 0$, $\Pi > 0$, $F \in R^{2n\times n}$ and $G \in R^{n\times n}$ such that :*

$$\begin{bmatrix} -\Pi + FA + A^T F^T & -F + A^T G^T \\ * & P - G - G^T \end{bmatrix} < 0. \quad (5)$$

where $\Pi = \begin{bmatrix} P - Q - 2R & R \\ R^T & Q \end{bmatrix}$, $F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$ *and* $A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}$.

3 Design of 2-D observers

In this section, we will present a constructive method for the 2-D state observer design for system (Σ).

The following Theorem [3] gives a necessary and sufficient condition for the existence of the 2-D state observers for system (Σ).

Theorem 1. *[3] The following facts are equivalent :*
i) (Σ) is observable (detectable).
ii)

$$rank \begin{bmatrix} C \\ I_n - z_1 A_1 - z_2 A_2 \end{bmatrix} = n, \forall (z_1, z_2) \in \bar{U}^2 \quad (6)$$

where : $\bar{U}^2 = (z_1, z_2) \in \mathbb{C}^2 : |z_1| \leq 1, |z_2| \leq 1$

Remark 2. *- Theorem 1 presents a necessary and sufficient condition for the existence of a 2-D state observer based on the coefficient matrices of a given system (Σ). This is an important step towards the design of 2-D asymptotic state observer .*
- For the synthesis of the 2-D asymptotic state observer, this is a very complicated formulation resulting to a very hard problem to solve, since we have a linear constraint mixed with the very highly nonlinear infinite dimensional constraint. The significant contribution of this paper lies in its simplicity and completeness. all the provided main results are easily checkable, it will demonstrate, in the sequel, that we can still completely solve the synthesis of asymptotic 2-D state observer problem in term of Linear Matrix Inequalities which presents unnecessary computational overload.

Remark 3. *It often occurs that all state variables of a system are not accessible to measurement, or that the output is not measurable of lack of technology.*

Let us consider the following 2-D state observer system for 2-D system (Σ).

$$(\hat{\Sigma}) : \begin{cases} w_{i+1,j+1} &= N_1 w_{i,j+1} + N_2 w_{i+1,j} \\ &+ L_1 y_{i,j+1} + L_2 y_{i+1,j} \\ &+ M_1 u_{i,j+1} + M_2 u_{i+1,j} \\ \hat{x}_{i,j} &= w_{i,j} + E y_{i,j} \end{cases} \quad (7)$$

Where $w_{i,j} \in \mathbb{R}^n$ is the state vector of the observer and $\hat{x}_{i,j} \in \mathbb{R}^n$ is the estimate of the state vector $x_{i,j}$.
N_1, N_2, L_1, L_2 M_1 and M_2 are matrices to be determineted to have an observation error which asymptotically vanishes at zero.

The boundary conditions for 2-D state observer system $(\hat{\Sigma})$ are given by

$$\begin{cases} w_{i,0} = w_0(i) & \forall\, i \in N \\ w_{0,j} = w_0(j) & \forall\, j \in N \end{cases} \quad (8)$$

satisfying

$$\begin{cases} sup_i ||w_{i,0}|| < \infty \\ sup_j ||w_{0,j}|| < \infty \end{cases} \quad (9)$$

Based on the results presented in the previous sections, we now give a new constructive method for the design of the 2-D asymptotic state observer so that the estimate state converges to the real value (i. e. the observation error is vanishing asymptotically at zero).
The next Theorem gives sufficient conditions for the design of the state observer for 2-D system (Σ) using LMI approach.

Theorem 2. *The 2-D state observer* (7) *will estimate asymptotically the state vector $x(i,j)$ if the following conditions hold.*

i) There exists $P > 0$, $Q > 0$, $R = R^T \geq 0$,, W_1, , W_2, W_3, X_1, X_2, X_3, X_4, X_5, X_6, $F \in R^{2n\times n}$ and $G \in R^{n\times n}$ such that the following LMI holds

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ * & \Gamma_4 & \Gamma_5 \\ * & * & \Gamma_6 \end{bmatrix} < 0 \quad (10)$$

Where :
$\Gamma_1 = -P + Q + 2R + F_1A_1 + A_1^T F_1^T - W_1CA_1 - A_1^T C^T W_1^T - X_1C - C^T X_1^T$
$\Gamma_2 = -R + F_1A_2 + A_1^T F2^T - W_1CA_2 - A_1^T C^T W_2^T - X_3C - C^T X_4^T$
$\Gamma_3 = -F_1 + A_1^T G^T - A_1^T C^T W_3^T - C^T X_5^T$
$\Gamma_4 = -Q + F_2A_2 + A_2^T F_2^T - W_2CA_2 - A_2^T C^T W_2^T - X_2C - C^T X_2^T$
$\Gamma_5 = -F_2 + A_2^T G^T - A_2^T C^T W_3^T - C^T X_6^T$
$\Gamma_6 = P - G - G^T$

ii) $M_p = (I - EC)B_p$, where $p = 1, 2$

iii) $N_p = A_p - ECA_p - K_pC$, $\quad L_p = K_p + N_pE$ where $p = 1, 2$

Proof : From the equation of the 2-D systems (Σ) and $(\hat{\Sigma})$, the observation error is :

$$e_{i,j} = x_{i,j} - \hat{x}_{i,j}$$

The 2-D observer system $(\hat{\Sigma})$ is state observer for the 2-D system (Σ) if $lim_{i+j\to\infty} e_{i,j} \to 0$, for any boundary conditions $x_{i,0}$, $x_{0,j}$ satisfying (3) and $w_{i,0}$, $w_{0,j}$ satisfying (9), and for every input sequence $u_{i,j}$.
the observation error has the following dynamics

$$e_{i+1,j+1} = \Phi x_{i+1,j+1} - w_{i+1,j+1}$$

where $\Phi = I_n - EC$
Replacing $w_{i,j}$ by $\Phi x_{i,j} - e_{i,j}$, $y_{i,j+1}$ by $Cx_{i,j+1}$ and $y_{i+1,j}$ by $Cx_{i+1,j}$.
Then, the dynamics of this estimation error are given by

$$\begin{aligned} e_{i+1,j+1} &= N_1 e_{i,j+1} + N_2 e_{i+1,j} \\ &+ \left[\Phi A_1 - N_1\Phi - L_1C\right] x_{i,j+1} \\ &+ \left[\Phi A_2 - N_2\Phi - L_2C\right] x_{i+1,j} \\ &+ \left[\Phi B_1 - M_1\right] u_{i,j+1} \\ &+ \left[\Phi B_2 - M_2\right] u_{i+1,j} \end{aligned} \quad (11)$$

The estimation error converges asymptotically to zero if the error system $e_{i+1,j+1} = N_1e_{i,j+1} + N_2e_{i+1,j}$ is asymptotically stable and both $ii)$ and $iii)$ hold. Hence, $e_{i,j} \to 0$, as $i + j \to \infty$. Then the dynamics of this estimation error are given by

$$e_{i+1,j+1} = N_1 e_{i,j+1} + N_2 e_{i+1,j} \quad (12)$$

where N_1 and N_2 are chosen such that the observation error (12) is stable.

we present the determination of matrices procedures N_1, N_2, L_1, L_2, M_1, M_2 and E satisfying the constraints

$$\begin{cases} e_{i+1,j+1} = N_1 e_{i,j+1} + N_2 e_{i+1,j} \;\text{ is stable} \\ \Phi A_1 - N_1\Phi - L_1C = 0 \\ \Phi A_2 - N_2\Phi - L_2C = 0 \\ \Phi B_1 - M_1 = 0 \\ \Phi B_2 - M_2 = 0 \end{cases} \quad (13)$$

The L_p matrices where ($p = 1, 2$) are determined from the equation (13). $L_pC - \Phi A_p + N_p\Phi = 0 \Rightarrow N_p(I_n - EC) + L_pC - \Phi A_p = 0$ then,

$$N_p = A_p - ECA_1 - K_pC \tag{14}$$

With

$$K_p = L_p - N_pE \tag{15}$$

Therefore, the matrices L_p are given by :

$$L_p = K_p + N_pE \tag{16}$$

Determining the matrices L_p requires the determination of the matrices N_p (matrices K_p).

The Matrices L_p are determined so that the observation error system (12) is asymptotically stable (i. e. the observation error converges asymptotically to zero) if there exist matrices $P > 0$, $Q > 0$, $R = R^T \geq 0$, $\Pi > 0$, $F \in R^{2n\times n}$ and $G \in R^{n\times n}$ such that :

$$\begin{bmatrix} -\Pi + FN + N^TF^T & -F + N^TG^T \\ * & P - G - G^T \end{bmatrix} < 0. \tag{17}$$

where $\Pi = \begin{bmatrix} P - Q - 2R & R \\ R^T & Q \end{bmatrix}$ and $N = \begin{bmatrix} N_1 & N_2 \end{bmatrix}$.

Since $N = \begin{bmatrix} N_1 & N_2 \end{bmatrix}$ and $F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$, which allows :

$$\begin{bmatrix} \Upsilon_1 & \Upsilon_2 & \Upsilon_3 \\ * & \Upsilon_4 & \Upsilon_5 \\ * & * & \Gamma_6 \end{bmatrix} < 0. \tag{18}$$

where :
$\Upsilon_1 = -P + Q + 2R + F_1N_1 + N_1^TF_1^T$
$\Upsilon_2 = -R + F_1N_2 + N_1^TF_2^T$
$\Upsilon_3 = -F_1 + N_1^TG^T$
$\Upsilon_4 = -Q + F_2N_2 + N_2^TF_2^T$
$\Upsilon_5 = -F_2 + N_2^TG^T$
From equation (14) we have $N_1 = \Phi A_1 - K_1C$ and $N_2 = \Phi A_2 - K_2C$.

Given that the inequality (18) is bilinear, a resolution method imposes a change of these variables : $W_1 = F_1E$, $W_2 = F_2E$, $W_3 = GE$, $X_1 = F_1K_1$, $X_2 = F_2K_2$, $X_3 = F_1K_2$, $X_4 = F_2K_1$, $X_5 = GK_1$ and $X_6 = GK_2$.

The gain matrices are obtained by :
- first solving the LMI (10) with respect to $P > 0$, $Q > 0$, $R \geq 0$, F_1, F_2, G, W_1, W_2, W_3, X_1, X_2, X_3, X_4, X_5 and X_6.
-second, matrices E, K_1 and K_2 are determined by :

$$\begin{cases} E = \begin{bmatrix} F_1 \\ F_2 \\ G \end{bmatrix}^+ \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} \\ K_1 = \begin{bmatrix} F_1 \\ F_2 \\ G \end{bmatrix}^+ \begin{bmatrix} X_1 \\ X_4 \\ X_5 \end{bmatrix} \\ K_2 = \begin{bmatrix} F_2 \\ F_1 \\ G \end{bmatrix}^+ \begin{bmatrix} X_2 \\ X_3 \\ X_6 \end{bmatrix} \end{cases} \tag{19}$$

The matrices N_p where ($p = 1, 2$) are calculated using (14)

$$\begin{cases} N_1 = \Phi A_1 - K_1C \\ N_2 = \Phi A_2 - K_2C \end{cases} \tag{20}$$

where $\Phi = I_n - EC$
The Matrices L_k where ($k = 1, 2$) are obtained by :

$$\begin{cases} L_1 = K_1 + N_1E \\ L_2 = K_2 + N_2E \end{cases} \tag{21}$$

Using the condition (13), M_1 and M_2 are determined by :

$$\begin{cases} M_1 = (I_n - EC)B_1 \\ M_2 = (I_n - EC)B_2 \end{cases} \tag{22}$$

where B_1 and B_2 are the input matrices of system(Σ).

Remark 4.
From Theorem 1, we notice that when condition (7) holds (the 2-D state observer exists), the 2-D asymptotic state observer design problem reduces to finding a feasible LMI (11) as presented in Theorem 2, that provides a new sufficient LMI condition for the design of the state observer for 2-D system (Σ) leading the observation error to vanishing at zero. Note that if system (Σ) reduces to a 1-D system, Theorem 2 can be viewed as an extension of the existing results on the design of asymptotic state observer for 1-D system to the 2-D case.

In the remainder of this paper, an illustrative example is used to show the applicability of the proposed approach.

4 Numerical Example

Consider the 2-D linear discrete-time system described by Fornasini-Marchesini second model, defined by the following system matrices :

$$A_1 = \begin{bmatrix} 0.6940 & 0.0084 & 0.6894 \\ 0.4825 & -0.4581 & 0.2131 \\ -0.1822 & 0.1165 & -0.3406 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.2735 & -0.0045 & -0.1972 \\ -0.5940 & 0.1916 & -0.1894 \\ 0.0319 & 0.2618 & 0.4121 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

It is easy to verify that $rank \begin{bmatrix} C \\ I_n - z_1 A_1 - z_2 A_1 \end{bmatrix} = 3$, which shows that condition existence of Theorem 1 is satisfied.

Now we want to design a 2-D state observer ensures that the estimated states converge asymptotically to the real value (i.,e. the error $e(i,j)$ vanishes asymptotically at zero). Then solving, the LMI conditions (11) by Theorem 2, we obtain the coefficient matrices,P, Q, R, W_1, W_2, W_3, X_1, X_2, X_3, X_4, X_5 and X_6. Finally, from (ii) and (iii) of Theorem 2, we obtain the 2-D state observer matrices :

$$N_1 = \begin{bmatrix} 0.6828 & 0.0155 & -0.1032 \\ 0.2318 & -0.2978 & -0.1622 \\ -0.0143 & 0.0092 & 0.1589 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} -0.2715 & 0.0115 & -0.4588 \\ -0.5501 & 0.5518 & 0.6017 \\ 0.0025 & 0.0206 & 0.5960 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.6134 \\ 0.1528 \\ -0.0510 \end{bmatrix}, L_2 = \begin{bmatrix} -0.1351 \\ -0.3952 \\ -0.0429 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0.0613 \\ 1.3759 \\ 0.0786 \end{bmatrix} \text{ and }$$
$$E = \begin{bmatrix} -0.0613 \\ -1.3759 \\ 0.9214 \end{bmatrix}$$

the corresponding trajectories of the observation error system (starting from random initial boundary conditions) are given in Figures 1, 2 and 3, all of which clearly converge asymptotically to zero when $i + j \to \infty$.

5 Conclusion

In this paper, we have presented a new sufficient LMI conditions to design a 2-D state observer for a class of 2-D systems described by the Fornasini-Marchesini local state-space second model. The proposed approach is novel, in the sense it can be efficiently solved by using standard numerical software. Numerical example is given to demonstrate the applicability of the proposed method.

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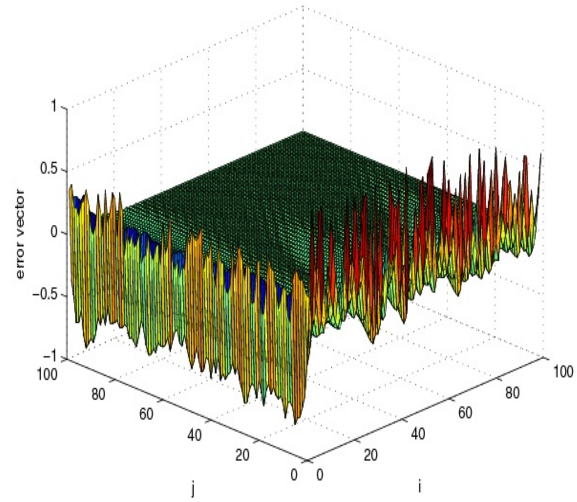

Figure 1. Error $e_1(i, j)$

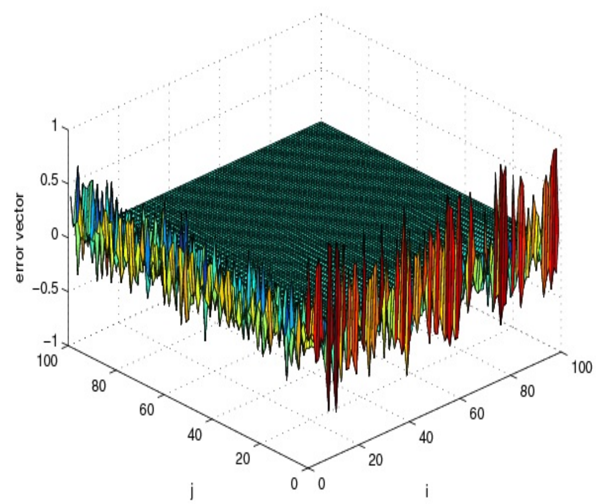

Figure 2. Error $e_2(i, j)$

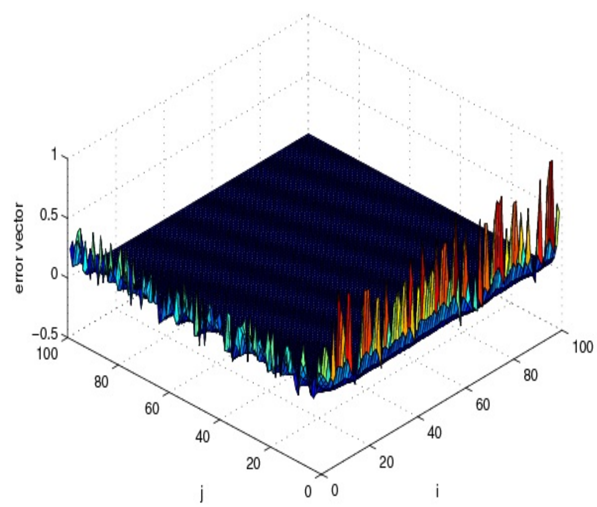

Figure 3. Error $e_3(i, j)$