

# Chaotic Prediction of Fluctuant Voltage Time Series in EAF Power Supply Grid

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*Abstract:* - In order to solve the problem of voltage flicker produced by electric arc furnace (EAF) operation, it is a prerequisite to analyze the characteristics of voltage time series acquired from AC EAF power supply grid and to conduct voltage prediction based on chaos theory. In this paper, the voltage time series is firstly reconstructed in phase space to determine optimal delay time and embedding dimension and the maximum Lyapunov exponent (MLE) is computed to testify the chaotic characteristics of voltage fluctuation, which indicate the chaotic approach is feasible for voltage prediction. Then, the voltage is respectively predicted by MLE method and adding-weight one-rank local method. The comparison of the prediction result shows that the two methods both lead to good prediction effect, but the latter one is better in precision. Therefore, the adding-weight one-rank local method is more suitable to predict the fluctuant voltage of AC EAF power supply grid.

*Key-Words:* - EAF, voltage fluctuation, chaotic prediction, MLE, adding-weight

## 1 Introduction

As a kind of typical impact load in power system, electric arc furnaces (EAF) have significant impact on power quality of the grid, especially voltage fluctuation which is most severe and difficult to overcome. A lot of research has been carried out on the voltage fluctuation existed in EAF power supply grid, including EAF modelling [1-3] and voltage fluctuation mitigation [4-6]. The former regards EAF as a stochastic load and establishes differential equations to simulate voltage fluctuation based on actual physical mechanism of EAF. However, due to high nonlinear and strong stochastic characteristics of the arc, it is very difficult to obtain a precise and practical model, which is also reflected in the current studies. The latter focuses on the installation of reactive power compensation devices such as static Var compensators (SVC) in the grid-side connected to EAF to decrease the level of voltage fluctuation. But the ability of SVC depends on their speed, which is limited by delays in reactive power measurement and thyristor ignition [7]. Therefore, it is difficult to implement dynamic and real-time compensation because of the time difference between actual compensation and

pre-compensation, leading to almost impossible further improvement of SVC capacity.

Indeed, the EAF system is an extremely complicated system, of which the factors that influence voltage fluctuation vary, so it is impractical to establish a precise mathematical model of EAF. Luckily, chaos theory can analyze and simulate those phenomena which are apparently stochastic and irregular and has been applied to analysis of respective complex objects [8-11]. Although randomness is demonstrated in arc variation, it not necessarily exists in the grid-side because the grid has other loads which also affect the voltage. Accordingly, the application of the chaos theory in analysis and prediction of EAF grid voltage can efficiently improve the effect of voltage fluctuation mitigation combining the SVCs. Currently, there are few documents that study EAF parameters using chaos theory and the existing researches are confined to parameters in the terminal EAF rather than the grid-side parameters which are greatly concerned by electric power researchers.

In this paper, voltage time series is acquired from a steel manufacturer. Based on the principles of

chaos theory for analysis and prediction, the time series is reconstructed in phase space under which the maximum Lyapunov exponent (MLE) is computed to testify chaotic characteristics of the voltage time series. Then, the voltage is predicted in ultra-short term respectively by MLE method and adding-weight one-rank local method. The prediction results show that the chaotic approach can predict ultra-short term variation trend of the voltage, and the adding-weight one-rank local method has wider range of precise prediction compared to MLE method. Through chaotic prediction, mastering the voltage variation in ultra-short term ahead of time is significant to ensure safety and stability of power system by combining SVC for effective voltage fluctuation mitigation.

## 2 The Measurement of EAF Power Supply Grid Voltage

The voltage time series is acquired from an AC EAF system which is energized by 110/33kV transformer, as is shown in Fig.1.  $T_1$  is the distribution transformer,  $T_2$  is the furnace transformer. The voltage is measured at 33kV bus by a voltage sensor installed on the secondary side of  $T_1$ . The time of voltage measurement is chosen at melting period of EAF, during which the voltage fluctuates the most drastically. Because response time of SVC has been shorten to milliseconds currently, around 20ms in ideal state, the sampling time voltage acquisition is set as  $\Delta t=0.02s$ . Fig.2 shows the voltage root mean square (RMS) values amounting to 3000 in one minute.

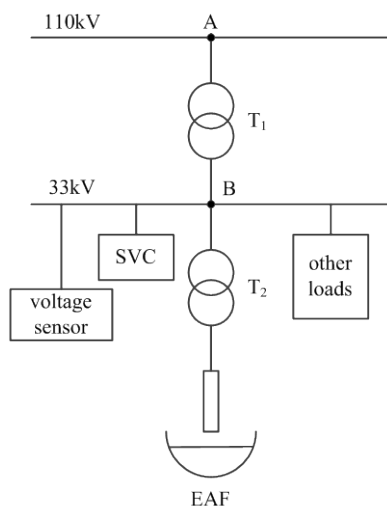


Fig. 1 Power supply system of AC EAF

As can be seen in Fig.2, the voltage RMS values fluctuate near the baseline value of 18.95kV, and the amplitude variation is within 100V. The voltages between each point have no obvious rule. Compared

to the baseline value, the amplitude is not very great because the voltage has been smoothed by SVC in steel manufacturer, but this 100V can also harm other loads sensitive to voltage fluctuation in the same power supply grid. Therefore, it is essential to further reduce the level of voltage fluctuation.

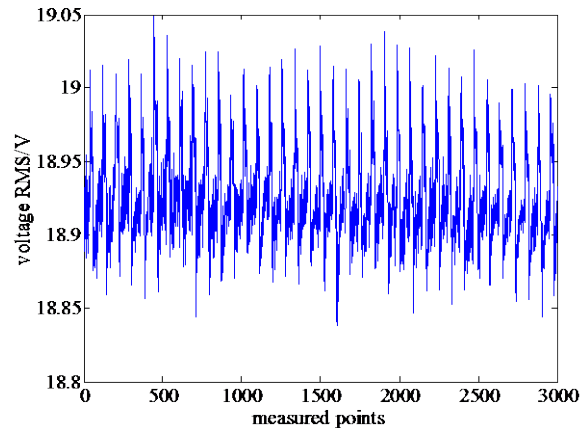


Fig. 2 Measured data of voltage effective value

## 3 Analysis of Chaotic Characteristics of Voltage Time Series

### 3.1 Determination of phase space reconstruction parameters

Phase space reconstruction is the basis of chaotic time series analysis and prediction. By reconstruction, a time series seemingly complex and irregular can be extended from one-dimension to multi-dimension, restoring a chaotic attractor from which underlying rules and rich information can be extracted.

Arrange the voltage RMS values in Fig.2 as a time series, named  $u(t_i)$ ,  $i=1,2,\dots,N$  ( $N$  is length of the series), and the series can be embedded in a  $m$ -dimension phase space by defining the delay vectors

$$U_1 = [u(t_1), u(t_1 + \tau), \dots, u(t_1 + (m-1)\tau)]$$

$$U_2 = [u(t_2), u(t_2 + \tau), \dots, u(t_2 + (m-1)\tau)]$$

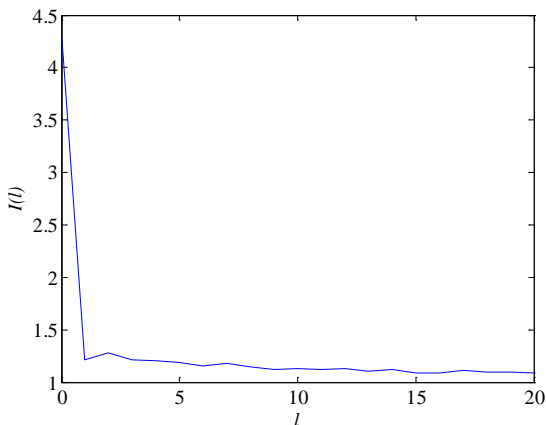
⋮

$$U_M = [u(t_M), u(t_M + \tau), \dots, u(t_M + (m-1)\tau)]$$

where  $\tau=l\Delta t$  ( $l=1, 2, \dots$ ) is the delay time,  $m$  is the embedding dimension, and  $M=N-(m-1)l$  is the amount of delay vectors.

The key of phase space reconstruction lies in the determination of the delay time and the embedding dimension, both of which must be chosen within appropriate ranges [12]. The delay time can be determined by mutual information method, which calculates the parameter from probability perspective and takes into account the linear and

nonlinear relation between points of the series. The first local minimum of the mutual information function is considered as the optimal delay time [13]. In actual calculation, the mutual information is the function of  $l$  which is defined as delay quantity because the delay time  $\tau$  is relevant to  $l$ . In this paper, the mutual information method is adopted to determine the delay time of the voltage time series. Fig.3 shows the relationship between mutual information  $I(l)$  and delay quantity  $l$  based on the voltage time series in Fig.2.



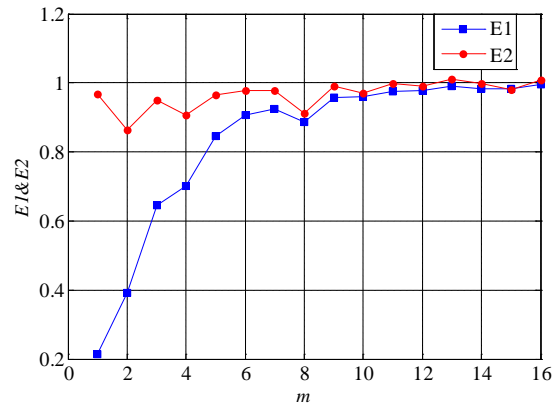
**Fig. 3 The relationship between mutual information and delay quantity**

As can be seen from Fig.3, the mutual information curve is not smooth. The mutual information quantity  $I(l)$  reaches the first local minimum when  $l=1$ , so the optimal delay time  $\tau$  can be determined as 0.02s considering the sampling time.

Cao method is often used to determine the embedding dimension  $m$ . The Cao method overcomes the shortage of false nearest neighbors (FNN) method in selecting threshold value, and defines two variables, namely  $E_1(m)$  and  $E_2(m)$ , whose detailed implication can be seen from [14]. For stochastic time series, of which the data variation have no predictability,  $E_2(m)$  equals to “1” all the time. For chaotic time series,  $E_2(m)$  is related to  $m$  and not constant, so there must be some  $m$  making  $E_2(m) \neq 1$ . Therefore,  $E_2(m)$  can be used to judge whether a time series is chaotic or not. The procedure of calculating the embedding dimension is as follows. The delay quantity  $l$  is firstly given by the mutual information method. When  $E_1(m)$  reaches saturation with the increase of  $m$ , the minimum embedding dimension is  $m+1$ . The curve of embedding dimension variation is shown in Fig.4.

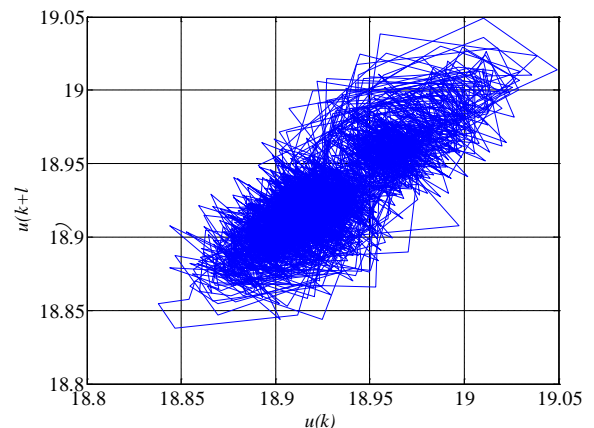
As can be seen in Fig.4,  $E_2(m)$  demonstrates obvious fluctuation around “1”, indicating that the voltage time series of EAF power supply grid is

deterministic to an extent rather than purely stochastic, which means that the voltage time series is a compound containing both deterministic and stochastic mechanism. This kind of mechanism is codetermined by the operational characteristics of EAF and other loads connected to the same grid. When  $m=6$ ,  $E_1$  reaches saturation and slightly fluctuates around “1”, so the minimum embedding dimension is  $m=7$ .



**Fig. 4 The curve of embedding dimension variation**

Use the obtained delay time and embedding dimension above to reconstruct the phase space, as is depicted in Fig.5. The two-dimensional phase space has similar trajectories and demonstrates a double-scroll structure, which is similar to the Lorenz attractor. So it can be concluded qualitatively that the voltage time series of EAF has chaotic characteristics. The reason why the phase space trajectories are not as smooth and clear as the Lorenz attractor is that the original voltage time series acquired from the spot contains noise which has damaged the relativity between vectors in the phase space and made the evolution of these vectors dislocated.



**Fig. 5 The two-dimensional phase space of voltage**

### 3.2 The computing of the maximum Lyapunov exponent

In the phase space, if the initial distance between two trajectories is extremely small, but the distance increases exponentially over time, leading to a state that the voltage prediction is impossible. This is the extreme sensitivity of chaotic motion to initial conditions. This kind of sensitivity can be quantitatively measured by the Lyapunov exponent. The exponent denotes the mean divergence rate of neighboring trajectories in the chaotic attractor over time. Therefore, the exponent can be used to identify whether the motion of a system is chaotic or not. In actual identification, the maximum Lyapunov exponent (MLE)  $\lambda$  is only needed. If  $\lambda > 0$ , then the time series displays chaotic characteristics, otherwise no chaotic characteristics exist in the time series. In this paper, the small-amount-data method is chosen to compute the MLE, the computing detail can be seen in [15]. On the basis of phase space reconstruction, the MLE of the voltage time series in Fig.2 can be computed, as is shown in Fig.6.

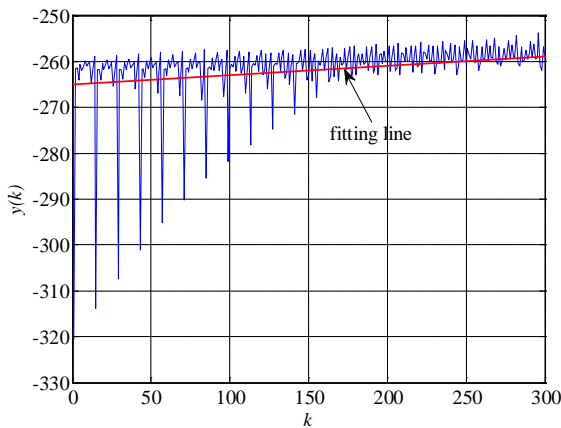


Fig. 6 The  $y(k)$ - $k$  curve for maximum Lyapunov exponent computing

In Fig.6, the red line stands for the mean line obtained by least square method, and its slope is the MLE. The slope of the red line is measured as 0.0221, which is greater than zero, indicating that the voltage time series of EAF has chaotic characteristics and voltage prediction can be implemented by chaotic approach based on the MLE.

### 4 Chaotic Prediction of the Voltage Time Series

For simplicity and convenience, the sampling time is not considered temporarily and the voltage time series is set down as  $u(1), u(2), \dots, u(N)$ . Keep the

reconstruction parameters unchanged, then the reconstructed phase space become as follows:

$$\begin{aligned} U_1 &= [u(1), u(1+l), \dots, u(1+(m-1)l)] \\ U_2 &= [u(2), u(2+l), \dots, u(2+(m-1)l)] \\ &\vdots \\ U_M &= [u(M), u(M+l), \dots, u(M+(m-1)l)] \end{aligned}$$

where  $u(M+(m-1)l)$  is equivalent to  $u(N)$ . The following is the voltage prediction by two different methods.

#### 4.1 Voltage prediction based on MLE

Suppose  $\lambda_1$  is the MLE and  $U_M$  is the central prediction vector in the reconstructed phase space. Search the nearest neighboring vector of  $U_M$ , named as  $U_K$ , and the distance between  $U_M$  and  $U_K$  is set down as  $d_M(0)$  which can be denoted as:

$$d_M(0) = \min_j \|U_M - U_j\| = \|U_M - U_K\|. \quad (1)$$

After a sampling time,  $U_M$  and  $U_K$  respectively evolve into  $U_{M+1}$  and  $U_{K+1}$ . According to the definition of MLE, the following equation of can be obtained:

$$\|U_{M+1} - U_{K+1}\| = d_M(0) \cdot e^{\lambda_1}, \quad (2)$$

where only the last element of  $U_{M+1}$ , namely  $U_{M+1,m}$  which is equivalent to  $u(N+1)$ , remains unknown. It is  $u(N+1)$  that we need to predict, which can be solved from (2) as:

$$u(N+1) = U_{K+1,m} \pm \sqrt{P_1 - P_2}, \quad (3)$$

where  $P_1 = (d_M e^{\lambda_1})^2$ ,  $P_2 = \sum_{i=1}^{m-1} (U_{M+1,i} - U_{K+1,i})^2$ .

As to the selection of “ $\pm$ ” in (3), we can use the intersection angle between two space vectors. The smaller the included angle, the closer the two vectors in space. The detailed rule is explained below:

Suppose the two vectors are:

$$V = (x_1, x_2, \dots, x_m), \quad W = (y_1, y_2, \dots, y_m),$$

and their intersection angle is:

$$\begin{aligned} \theta &= \arccos[(V \cdot W) / (|V| |W|)] \\ &= \arccos \frac{\sum_{i=1}^m x_i y_i}{\left( \sqrt{\sum_{i=1}^m x_i^2} \cdot \sqrt{\sum_{i=1}^m y_i^2} \right)}. \quad (4) \end{aligned}$$

Mark the voltage predicted value as  $u_i^+$  and  $u_i^-$  respectively corresponding to “+” and “-” in (3), and mark  $U^+ = (u_i^+, u_{i-1}^+, \dots, u_{i-n}^+)$ ,  $U^- = (u_i^-, u_{i-1}^-, \dots, u_{i-n}^-)$ ,  $U^j = (u_j^+, u_{j-1}^+, \dots, u_{j-n}^+)$ , then calculate the intersection angle  $\theta^+$  between  $U^+$  and  $U^j$ , and  $\theta^-$  between  $U^-$  and  $U^j$ . If  $\theta^+ < \theta^-$ , choose “+”, otherwise choose “-”.

### 4.2 Voltage prediction considering the influence of neighboring vectors

In this method, a series of neighboring vectors of  $U_M$  are firstly determined. Then, suppose an extremely little positive number  $\varepsilon$  as the distance in phase space. Make the neighboring vectors  $U_j$  of  $U_M$  meet

$$\|U_M - U_j\| < \varepsilon, j = 1, 2, \dots, k, \quad (5)$$

then determine the next vector  $U_{j+1}$  of  $U_j$  according to the evolution of phase space trajectories, and establish a function relationship by

$$U_{j+1} = \Gamma(U_j) = A + BU_j, \quad (6)$$

where  $A$  and  $B$  are fitting parameters which are unknown:

$$A = [a_1, a_2, \dots, a_m]^T, \quad B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mm} \end{bmatrix}.$$

In  $U_{j+1}$ , only the last element  $u_{j+1+(m-1)l}$  is unknown, which is also the quantity needed to predict. It can be described as

$$u_{j+1+(m-1)l} = a_m + b_{m1}u_j + b_{m2}u_{j+1} + \dots + b_{mm}u_{j+(m-1)l} \quad (7)$$

Indeed, different neighboring vectors have different influence on the evolution of the central prediction vector  $U_M$ . The closer these neighboring vectors approach  $U_M$ , the greater the influence can be. For this reason, a weight is introduced, as is described in the following:

$$P_i = \frac{\exp(-(d_i - d_{\min}))}{\sum_{j=1}^k \exp(-(d_j - d_{\min}))} \quad (8)$$

where  $d_i$  is the distance between  $U_i$  and the  $U_M$ , and  $d_{\min}$  is the minimum. Then (6) can be written as:

$$U_{i+1} = A + BU_i \quad (9)$$

Use the least square method to calculate  $A$  and  $B$

by making  $\sum_{i=1}^k P_i (U_{i+1} - A - BU_i)^2$  minimum:

$$\begin{cases} \sum_{i=1}^k P_i (U_{i+1} - A - BU_i) = 0 \\ \sum_{i=1}^k P_i (U_{i+1} - A - BU_i) U_i = 0 \end{cases} \quad (10)$$

The prediction vector is

$$\hat{U}_{M+1} = A + B\hat{U}_M \quad (11)$$

Separate the last element of  $\hat{U}_{M+1}$ , and the prediction value  $u(N+1)$  can be obtained. Because this method establishes a prediction model between the central prediction vector and its neighboring

vectors, and during the prediction, a series of weights are added to the model considering the influence of neighboring vectors on the prediction, therefore this method is called the adding-weight one-rank local method.

### 4.3 Prediction results and analysis

The prediction results by using the two methods above are shown in Fig. 7, where 60 points are predicted and they are divided into three groups in time sequence to make a comparison. The mean relative error (MRE) of actual values and predicted values are shown in Table 1. For convenience, we call MLE method as method A, and the add-weight one rank method as method B. As we can see from Fig. 7 and Table 1, both methods can reflect the trend of voltage variation in short term, and the MRE of method B is smaller than method A. The MRE gradually increases with the increase of prediction length, namely the prediction steps, but the MLE increases faster by method A than method B, indicating that method A will go beyond the approved error range ahead of method B. The reason can be set forth as follows. The method A is based on the MLE which is a mean value denoting the divergence rate of the phase space trajectories, so (2) is only an approximation of the actual model, leading to a cumulating error during the prediction. The method B, however, establishes a close relation between neighboring vectors and the central prediction vector by taking into account the influence of different position of neighboring vectors on the prediction, so its prediction error increases slower than method A. To sum up, the method B is more suitable to predict the fluctuant voltage of AC EAF power supply grid.

Table 1 MRE of 60-step prediction by two different methods

Prediction methods	1~20 steps	21~40 steps	41~60 steps
Method A	0.053%	0.062%	0.106%
Method B	0.033%	0.041%	0.065%



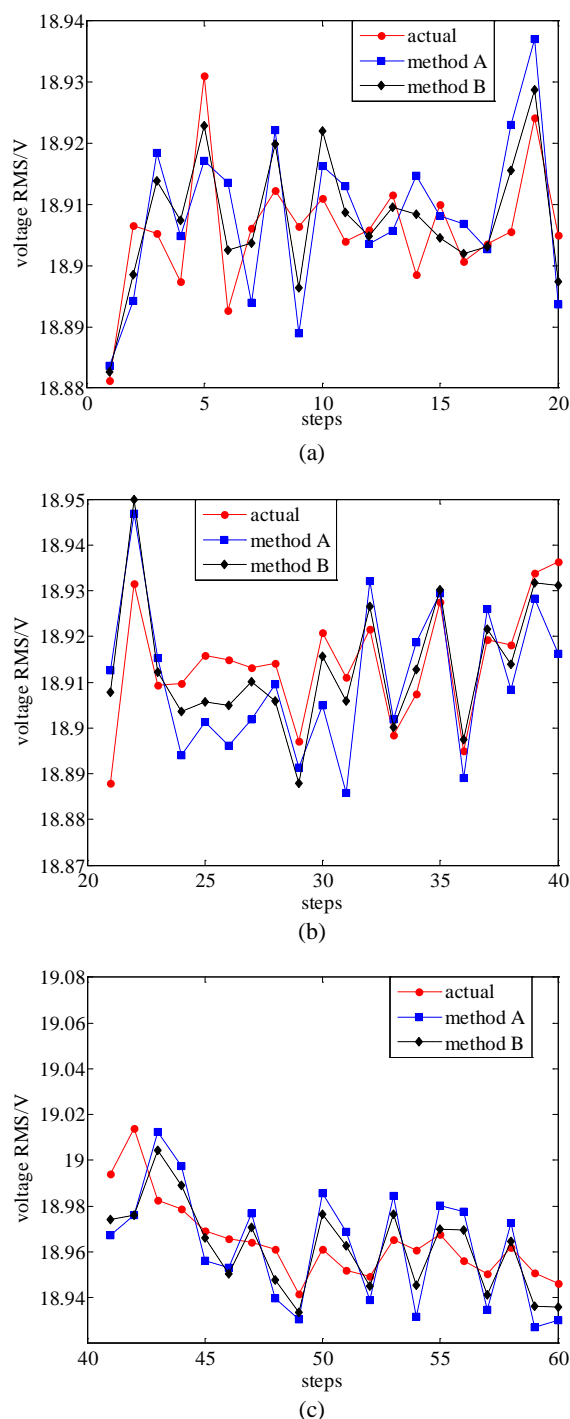


Fig. 7 Voltage prediction result of the two methods

## 5 Conclusion

A chaotic approach is proposed in this paper to analyze and predict the voltage time series acquired from EAF power supply grid. The mutual information method and Cao method are respectively adopted to determine the delay time and the embedding dimension. Through the depiction of a two-dimension phase space and the computing of the MLE, it is proved that the voltage fluctuation in EAF power supply grid has chaotic characteristics and the prediction of voltage is

feasible by chaotic approach. The MLE method and adding-weight one-rank local method are used to predict the voltage in ultra-short term. Results show that chaotic prediction can well reflect the changing trend of the voltage time series. Both methods have high level in precision but compared to the MLE method, the adding-weight one-rank local method has higher precision, which is more suitable to predict the fluctuant voltage of AC EAF power supply grid.

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