Control of FOPDT Process using Sliding Mode Controller with Modified PI-D Sliding Surface

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Abstract: - This paper is about the design of a Sliding Mode Controller (SMC), with modified PI-D sliding surface, for the control of First Order Plus Dead Time (FOPDT) process. In the modified PI-D sliding surface, error is connected to the proportional and integral elements of the controller, and the derivative of the system output is connected to derivative element of the controller. The usage of PI-D sliding surface eliminates the discontinuous switching in SMC. In this work, the integro-differential equation used for representing the sliding surface of SMC is replaced with PI-D sliding surface. The controller designed is used to obtain the desired closed loop response of the FOPDT system considered, in simulation. The closed loop performance of SMC with integro-differential equation and SMC with PI-D sliding surface are compared.

Keywords: Sliding mode controller, PID controller, sliding surface, PI-D sliding surface, integro-differential equation, FOPDT process

1 Introduction

Sliding mode control is based on Variable Structure Control (VSC) theory. It is a robust control technique which uses discontinuous control law to control linear and non-linear processes. SMC has several advantages such as fast response, good transient performance, robustness, computational simplicity, easy implementation, insensitivity to system parameter variations and external disturbances [1-3]. It has the disadvantages of chattering and equivalent dynamic formulation [4].

The first step in the design of SMC is to construct the sliding surface, whose derivative is equal to zero, along which the process variable can move smoothly to its desired value. The design of control law is the second step in the design of SMC, that will drive the process variable to the sliding surface. When sliding occurs the controller structure is intentionally altered according to a prescribed control law [5].

A SMC applied for controlling the level of two coupled tanks, by maintaining the stability of closed loop system, with reduced chattering is developed in [6]. An adaptive SMC with PID controller for a class of uncertain systems is proposed in [3]. Elimination of overshoot in the closed loop response obtained using conventional PID controller is achieved by cascading a SMC in the outer loop [7]. Better step response and minimization of steady state error is achieved when PID controller is combined with variable structure controller [8]. Improved closed loop performance is obtained when VSC is combined with PI controller for level control process [9]. The chattering problem found in SMC is eliminated when SMC is combined with PID controller [10]. The above literature indicates the advantages of combining SMC with PID controller, which was the motivation for this work.

By proper choice of system gain, time constant and dead time, dynamics of many industrial processes can in practice be sufficiently modeled by the stable FOPDT transfer function [11]. This is the reason for selecting the model of a FOPDT process for this work.

The paper is organized as follows. Section two presents the review of controllers. Section three is about tuning the controllers considered in this work. Simulation results are given in Section four. Conclusions are drawn in Section five.

2 Review of Controllers

2.1 Review of P, PI and PID controllers

A PID controller is a simple three-term controller. The letters P, I and D stand for proportional, integral and derivative terms respectively. In proportional control action [12], the controller output is proportional to the error signal, and is given by,

$$u(t) = K_P\, e(t) \tag{1}$$

In proportional plus integral controller, the controller output is proportional to the error and to the integral of the error, and is given by,

$$u(t) = K_P \left(e(t) + \frac{1}{T_i} \int e(t)\, dt \right) \tag{2}$$

In proportional plus integral plus derivative controller, the controller output is proportional to the error, integral of the error, and to the derivative of the error, and is given by,

$$u(t) = K_P \left(e(t) + \frac{1}{T_i} \int e(t)\, dt + T_d \frac{d\, e(t)}{dt} \right) \tag{3}$$

Where, K_p is proportional gain,
$K_i = K_p / T_i$ is the integral gain,
$K_d = K_p\, T_d$ is the derivative gain
T_i and T_d are integral time and derivative time respectively.

2.2 Review of PI-D controller

In PI-D controller structure [12], the error signal will be transferred through PI controller, with the derivative element connected to the system output. The controller output of PI-D controller structure is given as,

$$u(t) = K_P \left(e(t) + \frac{1}{T_i} \int e(t)\, dt - T_d \frac{d\, y(t)}{dt} \right) \tag{4}$$

2.3 Review of sliding mode controller

The FOPDT process can be represented by the following continuous domain transfer function,

$$G(s) = \frac{x(s)}{u(s)} = \frac{K\, e^{-\tau_d s}}{\tau\, s + 1} \tag{5}$$

The sliding surface *s(t)* using integro-differential equation, which acts on the tracking error is [6],

$$s(t) = \left(\frac{d}{dt} + \lambda \right)^n \int_0^t e(t)\, dt \tag{6}$$

Where, *e(t)* is tracking error, λ is the tuning parameter and *n* is the order of the system. The main objective is to force the error to zero, so that the derivative of switching surface is zero.

$$\frac{d\, s(t)}{dt} = 0 \tag{7}$$

This means the controlled variable has reached the desired value, during which *s(t)* attains a constant value and *e(t)* is zero. In SMC control law, *u(t)* consists of continuous and discontinuous parts, with the controller output given by,

$$u(t) = u_c(t) + u_d(t) \tag{8}$$

The continuous part of SMC is the function of controlled variable and error signal,

$$u_c(t) = f\,(x(t), e(t)) \tag{9}$$

The dead time term found in the FOPDT process represented by Eqn. 5 is approximated using Taylor's series and is given by,

$$e^{-\tau_d s} \cong \frac{1}{\tau_d\, s + 1} \tag{10}$$

Using Eqn. 10 in Eqn. 5 implies,

$$\frac{x(s)}{u(s)} = \frac{K}{\tau\, s + 1} . \frac{1}{\tau_d\, s + 1} \tag{11}$$

Representing Eqn. 11 in time domain, *u(t)* is given by,

$$u(t) = \left(\frac{\tau_d\, \tau}{K} \right) \frac{d^2\, x(t)}{dt^2} + \left(\frac{\tau_d + \tau}{K} \right) \frac{d\, x(t)}{dt} + \frac{x(t)}{K} \tag{12}$$

Eqn. 12, represents a system of order two, hence the sliding surface given in Eqn. 6 is,

$$s(t) = \frac{d}{dt} e(t) + 2\, \lambda\, e(t) + \lambda^2 \int_0^t e(t)\, dt \tag{13}$$

Differentiating the above sliding surface, gives

$$\frac{d\,s(t)}{dt} = \frac{d^2e(t)}{dt^2} + 2\lambda\frac{d\,e(t)}{dt} + \lambda^2 e(t) = 0 \qquad (14)$$

The error signal *e(t)* is the difference between the reference value *r(t)* and the measured variable *x(t)* and it is represented by,

$$e(t) = r(t) - x(t) \qquad (15)$$

Using Eqn. 15 in Eqn. 14, with a constant reference value and process variable that varies with the time.

$$\frac{d^2\left(r(t)-x(t)\right)}{dt^2} + 2\lambda\frac{d\left(r(t)-x(t)\right)}{dt} + \lambda^2 e(t) = 0 \qquad (16)$$

$$-\frac{d^2\,x(t)}{dt^2} - 2\lambda\frac{d\,x(t)}{dt} + \lambda^2 e(t) = 0 \qquad (17)$$

$$\frac{d^2\,x(t)}{dt^2} = -2\lambda\frac{d\,x(t)}{dt} + \lambda^2 e(t) \qquad (18)$$

Substituting Eqn. 18, into Eqn. 12, gives the continuous controller part,

$$u_C(t) = \frac{\tau_d\,\tau}{K}\left\{\left[\frac{\tau_d + \tau}{\tau_d\,\tau} - 2\lambda\right]\frac{d\,x(t)}{dt} + \lambda^2 e(t) + \frac{x(t)}{\tau_d\,\tau}\right\} \qquad (19)$$

If the derivative of controlled variable *x(t)* is assumed as zero, then from the above equation

$$\frac{\tau_d + \tau}{\tau_d\,\tau} = 2\lambda \qquad (20)$$

Also the above assumption, simplifies the continuous controller equation,

$$u_C(t) = \frac{\tau_d\,\tau}{K}\left\{\lambda^2 e(t) + \frac{x(t)}{\tau_d\,\tau}\right\} \qquad (21)$$

The discontinuous part of SMC is given by [13],

$$u_d(t) = S_D\frac{s(t)}{|s(t)| + \delta} \qquad (22)$$

Where, S_D is the tuning parameter which depends on reaching mode and δ is a tuning parameter.

Now, the continuous and discontinuous part of the controller equations are added to obtain the complete controller equation,

$$u(t) = \frac{\tau_d\,\tau}{K}\left\{\lambda^2 e(t) + \frac{x(t)}{\tau_d\,\tau}\right\} + S_D\frac{s(t)}{|s(t)| + \delta} \qquad (23)$$

The sliding surface equation of SMC [14] using integro-differential equation for a second order system is,

$$s(t) = sign(K)\left[-\frac{d\,x(t)}{dt} + 2\lambda\,e(t) + \lambda^2\int_0^t e(t)\,dt\right] \qquad (24)$$

3. CONTROLLER TUNING

3.1 Tuning of PID controller

The FOPDT process *G(s)* considered in this work is taken from [15],

$$G(s) = \frac{1.68\,e^{-0.26s}}{1.2158\,s + 1} \qquad (25)$$

The PID controller is tuned using Ziegler Nichols (ZN) tuning technique and the parameters obtained are given in Table I. These parameters are used for obtaining the closed loop response of the system in simulation.

TABLE I: PID CONTROLLER PARAMETERS OBTAINED USING ZN METHOD

Controller	*Controller parameters*		
	Proportional Gain(K_P)	*Integral Gain(K_P)*	*Derivative Gain(K_P)*
P	2.78	-	-
PI	2.52	2.84	-
PID	3.34	6.42	0.43

3.2 Tuning of sliding mode controller

Eqn. 23 and Eqn. 24 gives the complete controller expression. The values S_D and δ are determined using the Nelder-Mead tuning algorithm [14, 16],

$$S_D = \frac{0.51}{|K|}\left(\frac{\tau}{\tau_d}\right)^{0.76} \quad (26)$$

$$\delta = 0.68 + 0.12|K|S_D * 2 * \lambda \quad (27)$$

The SMC controller parameters obtained using the above algorithm is given in Table II.

TABLE II. TUNING PARAMETERS OF SMC USING NELDER-MEAD TUNING ALGORITHM

Tuning Parameters	*Values*
S_D	0.98
δ	1.6
λ	2.33

4 SIMULATION RESULTS

The closed loop response of the FOPDT process considered, for an applied unit step change in input is obtained using simulation. The closed responses obtained using PI, PID and PI-D controllers, which are tuned using Ziegler Nichols method are given in Fig.1. It can be seen that, the responses are oscillatory and contains overshoot. In the case of PID controller, the response is more oscillatory and settles faster than the response obtained using PI controller, which is because of the presence of derivative action in PID. It can be observed that closed loop response obtained with PI-D controller is the same as the response obtained with PID controller.

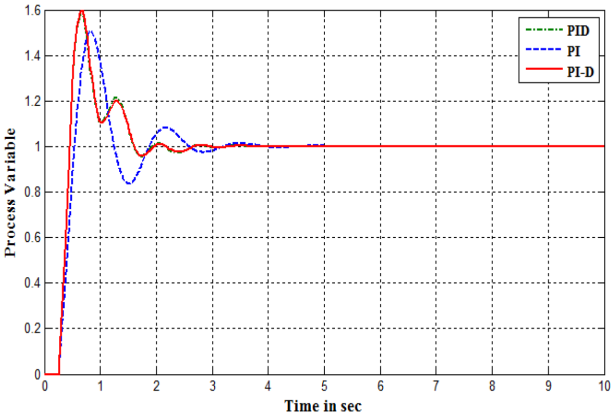

Fig. 1 Step response of PI, PID and PI-D controllers

The closed loop responses obtained using SMC with sliding surface represented by integro-differential equation and SMC with PI-D sliding surface are shown in Fig. 2. The above controllers are tuned using Nelder-Mead algorithm and Ziegler-Nichols algorithm respectively. It is seen that, the responses are less oscillatory, with less peak overshoot when compared to the responses obtained using PID controllers. Also the closed loop responses of SMC with integro-differential sliding surface and SMC with PI-D sliding surface are exactly the same, hence it can be concluded that, the sliding surface in Eqn.23 is equivalent to the PI-D controller equation.

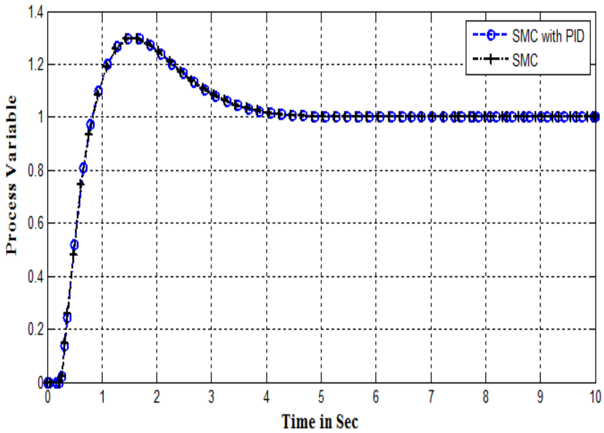

Fig. 2 Comparison of closed loop responses of SMC and SMC with PI-D controllers

In addition to the above comparison, the process gain K, of the FOPDT process considered is 1.6128 and hence the sliding surface given in Eqn. 24 is,

$$s(t) = -\frac{d\,x(t)}{dt} + 2\lambda\, e(t) + \lambda^2 \int_0^t e(t)\,dt \quad (28)$$

Comparing the expression of PI-D controller in Eqn. 4 and the sliding surface represented in Eqn. 28, it is seen that the proportional gain is 2λ, integral gain constant is λ^2 and derivative gain constant is -1. Hence the sliding surface equation is equivalent to PI-D controller structure.

5. CONCLUSIONS

In this work, a FOPDT process is controlled using SMC with PI-D sliding surface. The closed loop response of the above controller is compared with the closed loop response obtained using PI, PID, PI-D and SMC with integro-differential sliding surface. It can be seen that the closed loop performance of the controllers SMC with integro-differential sliding surface and SMC with PI-D sliding surface are identical. Hence it is claimed that SMC with integro-differential sliding surface is equivalent to SMC with PI-D based sliding surface.

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