Economic Model Predictive Control for Power Plant Process

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Abstract:- Economic model predictive control (EMPC) is a combined control strategy of real time optimization of time-varying process economics and a feedback Model Predictive Controller (MPC) to track the time-varying set-point. In this work, we focus on the two-layer integrated framework of EMPC for non-linear processes. The upper layer consists of an EMPC, carries out economic process optimization to obtain the optimal steady states. A Lyapunov based MPC (LMPC) in the lower layer is forced to operate on these steady states. The LMPC tracks the time varying set point by maintaining the closed loop system states in a predefined stable region. The improved economic performance of the non-processes is demonstrated through some closed-loop simulation for the optimal operation of a power plant.

Keywords:- economic model predictive control, nonlinear systems, process optimization, stability analysis.

1 Introduction

The fundamental objective for a plant operation is to obtain the desired product from the given raw materials by optimizing the economic measure thereby increasing the net profit. The optimal operating performance in chemical process industries is traditionally achieved by a two-layer hierarchical structure. In the upper layer, real time optimization (RTO) is perform to address the time-varying economic considerations such as inconstant energy prices, changing market demand, product transitions, and variable feedstock using steady-state process models to obtain the optimal steady states [1]. The lower layer, generally designed using model predictive control (MPC) methods is bound to maintain the actual process states on these optimal steady states subject to the states and inputs constraints [2]. The popularity of MPC in industries is because of its ability to handle multivariable control problems and allowing the plant operation very close to the boundary of the constraints leading to increase in profitability. The working policy of a MPC is to compute optimal inputs by solving a optimization problem subject to the process dynamics, which forces the process output to operate on the desired set point [3-6] over a finite prediction horizon.

The main drawback of the two-layer paradigm is the process dynamics is not taken into account by the RTO layer to compute the steady states and thus the reachability of the actual process states cannot be certain [7]. To address the disabilities of the two layer approach, many researchers have derived alternatives by replacing the steady state model with a dynamic model, called dynamic real time optimization (D-RTO) strategy[8-9], introducing an intermediate layer to compute steady states reachable by the lower layer, called steady-state target optimization [10-11], computing optimal steady states using nonlinear MPC [12], and also by analysing the rate of unfolding the upper layer [8][13].

In the recent years, another alternative has evolved to overcome the drawbacks in traditional RTO/MPC control scheme where both the layers are integrated to a single layer by substituting the quadratic cost function by a dynamic economic cost function [14]. The resulting combined control scheme is referred as economic model predictive control (EMPC). However, the replacement of the quadratic cost function has lead to the challenge of developing a stable controller which account to the dynamic economic variations. Some of the recent works on EMPC address the closed loop performance by designing the controller with Lyapunov techniques [15]. In [16] two EMPCs are implemented on a catalytic distillation process to showcase the improvement in performance over traditional tracking controllers. In the manufacturing of vinyl acetate, the effect of a Lyapunov based EMPC is discussed for a huge network process. Many adaptive EMPC schemes were also developed and demonstrated through simulation results. Additionally, the optimization problem of EMPC has to be solved faster to control a process in real time. Thus, formulating an EMPC that explicitly accounts to time-varying process economics with provable stability properties is an open topic for research.

The present work focuses on the integrated framework of the two-layer approach. The time-varying economic considerations are optimized by the EMPC and the computed optimal states are fetched to the lower layer to operate on these states. Lyapunov techniques are used to design the lower layer MPC controller to guarantee closed-loop stability. Secondly, the stability of the process is analysed through set theoretic methods. The set theoretic approach provides a constructive way to find a suitable Lyapunov function. A set is constructed which is compatible with the constraints and is positively invariant such that the computed optimal steady states are bounded and reachable by the lower layer process states [17]. A suitable choice for candidate invariant sets - 'ellipsoids' is considered for the construction of the stability region. Lastly, this control scheme is demonstrated for the optimal operation of a fossil fuel power unit (FFPU) through extensive closed-loop simulations.

2 Preliminaries

2.1 Notation

The Euclidean norm of a vector is denoted by $|\cdot|$ and the weighted Euclidean norm of a vector is denoted by the notation $|\cdot|_Q (i.e. |x|_Q = x^T Qx)$, where Q is a positive definite matrix. A continuous function $\alpha: [0, a) \rightarrow [0, \infty)$ is said to be from class K, if it is strictly increasing and $\alpha(0) = 0$. We have used diag(v) to describe a square diagonal matrix whose diagonal elements are equal to vector v while the rest of the elements are equal to zero. The fixed parameter $\Omega_{\rho(x_s)}$ is used to denote the level set of a lyapunov function $V(x, x_S)$, where $x_S \in I \subset \mathbb{R}^n$(i.e., $\Omega_{\rho(x_s)} = \{x \in \mathbb{R}^n \mid V(x, x_S) \leq \rho(x_S)\}$).

2.2 Class of non-linear process models

The class of continuous non-linear dynamic systems of the following state-space form is considered in this work:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in U \subset \mathbb{R}^m$ denotes the manipulated input vector, and $w(t) \in W \subset \mathbb{R}^l$ is the disturbance vector. The energy available to the inputs are restricted to a non-empty convex set $U = \{u \in \mathbb{R}^m \mid u_i^{min} \leq u_i \leq u_i^{max}, i = 1, \dots. m\}$. The vector field $f: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^l$ is locally Lipschitz and the disturbance vector is considered to be bounded by some positive constant w_p(i.e., $w(t) \leq w_p$).

The integrated control scheme of dynamic process economics and feedback MPC drives the system of Eq.(1) to operate on the time varying operating policies. The explicitly time dependant economic cost function is assumed to be in the form: $L_e(t, x(t), u(t))$.

The optimal steady states obtained from convex optimization of the above cost function is denoted as $x_s(t)$. For the system of Eq.(1), the existence of a equilibrium point for each computed optimal steady states, $x_s(t)$, denoted by I (i.e., $I = \{x_s \in \mathbb{R}^n \mid u_s \in U$ subject to $f(x_s, u_s, 0) = 0\} \subset \mathbb{R}^n$) is assumed to guarantee the track ability of the time-varying reference trajectory. The deviation of the actual state trajectory from the time varying optimal trajectory is denoted as

$$e(t) = x(t) - x_s(t) \quad (2)$$

2.3 Stability Assumption

In this approach, the economic MPC optimizes directly in real time the economic performance of the process thereby increasing the problem of reachability and controllability under uncertainties. Thus, to specify the system performance set theoretic approach is used for stability consideration. Firstly, the presence of a Lyapunov controller $h(x, x_s)$ for each predicted state $x_s \in I$ is assumed that provides asymptotically stability to x_s of the non-linear system of Eq.1 under consecutive operation. Therefore, a Lyapunov function $V(x, x_s)$ is formulated such that it satisfies the following assumptions.

$$\alpha_1(|e|) \leq V(x, x_s) \leq \alpha_2(|e|) \quad (3a)$$

$$\frac{\partial V}{\partial x} f(x, h(x, x_s), 0) \leq -\alpha_3(|e|) \quad (3b)$$

$$\left|\frac{\partial V}{\partial x}\right| \leq \alpha_4(|e|) \quad (3c)$$

$$h(x, x_s) \in U \quad (3d)$$

where, $\alpha_i(\cdot)$, i=1,2,3,4 are continuous functions belonging to class K and $e \in D$ for each $x_s \in I$, D is a proximity to the origin. Secondly, for each $x_s \in I$, the corresponding lyapunov controller forms a stability region $\Omega_{\rho(x_s)}$. A compact and invariant set $\mathcal{X}$ is constructed by unifying all the stability regions, i,e., $\mathcal{X} = \bigcup_{x_s \in I} \Omega_{\rho(x_s)}$. Lastly, considering the boundedness of the input variables and continuous differentiable property of the Lyapunov function, the occurrence of certain positive constants $M, L_x, L_w, L_x^{'}, L_w^{'}$ are assumed such that

$$f(x,u,w) \leq M \quad (4a)$$

$$|f(x,u,w)| - |f(x^{'},u,0)| \leq L_x|x - x^{'}| + L_w|w| \quad (4b)$$

$$\left|\frac{\partial V(x,x_s)}{\partial x} f(x,u,w) - \frac{\partial V(x^{'},x_s)}{\partial x} f(x^{'},u,0)\right| \leq L_x^{'}|x - x^{'}| + L_w^{'}|w| \quad (4c)$$

all $u \in U . w \in W$, and $x, x^{'} \in \mathcal{X}$.

2.4 Construction of stable set

To embellish the idea of stability, a compact invariant set $\mathcal{X}$ is constructed by integrating all the stability regions of individual steady states i.e. $\mathcal{X} = \bigcup_{x_s \in I} \Omega_{\rho(x_s)}$ as shown in Fig 1. The stability region for each $x_s \in I$ is computed offline by an algorithm given below:

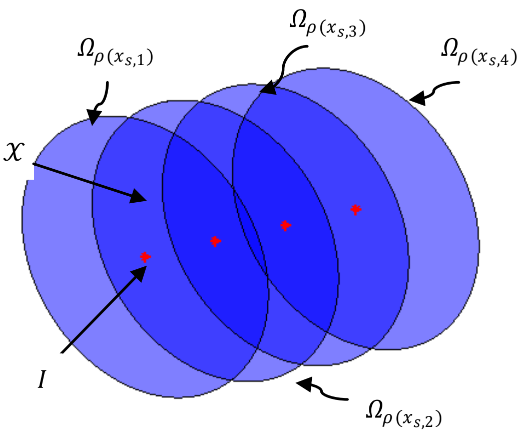

Fig.1. Representation of stable regions evolution and designing of set $\mathcal{X}$.

3 Two Layer Integrated Control Strategy

This section emphasis on the integrated architecture of optimizing time-varying process economics and Lyapunov based feedback MPC. The use of economic objective in the dynamic regulation layer directly motivates the system to transient through the high profit regions which was not possible in the traditional two layer control scheme as shown in Fig. 2.

However, from the operator's perspective there is increase in non-steady operation. To deal with the boundedness of the closed loop states, set theoretic approach is implemented to ensure stability.

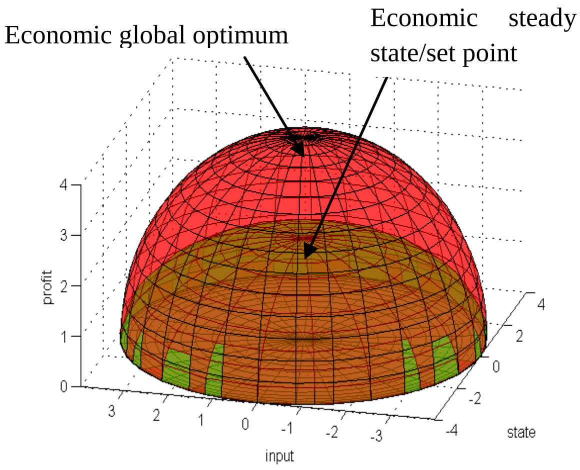

Fig. 2. Distinguishing between economic steady optimum and economic global optimum [3].

3.1 Application Scheme

The combined EMPC framework has two hierarchical internal layers. The top layer in the hierarchy carries out process economics optimization subject to the process dynamics and reachability of the states in the lower layer. As mentioned earlier, the explicitly time dependant economic cost function is represented in the form $L_e(t, x(t), u(t))$, the optimization problem (Eq.8a) can be constructed as;

$$minimize_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} L_e(\tau, \breve{x}(\tau), u(\tau))\, d\tau \quad (5a)$$

subject to,

$$\dot{\breve{x}}(t) = f(\breve{x}(t), u(t), 0) \quad (5b)$$

$$\breve{x}(t_k) = x(t_k) \quad (5c)$$

$$\breve{u}(t) \in U \;\forall\, t \in [t_k, t_{k+N}) \quad (5d)$$

$$\breve{x}(t_k) \in \mathcal{X} \;\forall\, t \in [t_k, t_{k+N})\; if\; x(t_k) \in \mathcal{X} \quad (5e)$$

where, $S(\Delta)$ is a class of piece-wise functions with constant values with Δ as the sampling period, N is the prediction horizon, $\breve{x}(\tau)$ is the reference trajectory, $u(\tau)$ represents the manipulated variable, $x(t_k)$ is the current state measurement at time (t_k), $\breve{x}(t_k)$ denote steady state computed with the manipulated variable $\breve{u}(t_k)$ at time t_k and $\mathcal{X}$ is the formulated stable region.

The first constraint Eq.(5b) is imposed on the process dynamics to evaluate the future states of the process. The second constraint Eq.(5c) defines the initial condition retrieve from the process. The bound to available energy of all the control inputs is defined in the third constraints (i.e. Eq.(5d)). The forth constraint (Eq.(5e)) is for stability purpose, the computed states stays within the stable set $\mathcal{X}$.

In the feedback control level, a Lyapunov based proportional controller is designed to track the computed steady states. The given model of Eq.(1) is solved repeatedly in sample and hold manner to maintain the actual states on the steady states $x_s(t)$ with the control input $u(t)$ obtained by minimizing the following optimization problem:

$$minimize_{u\in S(\Delta)} \int_{t_j}^{t_{j+t'}} |\check{e}(\tau)|_{Q_c} + |u(\tau) - \check{u}(\tau)|_{R_c}\, d\tau \quad (6a)$$

subject to,

$$\dot{\check{x}}(\tau) = f(\check{x}(t), u(t), 0) \quad (6b)$$

$$u(t) \in U \quad (6c)$$

$$\check{e}(t_j) = x(t_j) - x_s(t) \quad (6d)$$

$$x(t_j) \in \mathcal{X}\ \forall\, t \in [\, t_j, t_{j+t'}) \ if\ x(t_k) \in \mathcal{X} \quad (6e)$$

$$\frac{\partial V(x,x_s)}{\partial x} f(x(t_j), u(t_j), 0) \le \frac{\partial V(x,x_s)}{\partial x} f\left(x(t_j), h\left(x(t_j), x_s(t_j)\right), 0\right) \quad (6f)$$

where, $S(\Delta)$ is a class of piece-wise functions with constant values with Δ as the sampling period, t' is the prediction horizon, $x_s(t)$ is the reference trajectory, $x(t_j)$ is the current state at time (t_j) obtained by applying the manipulated variable, $u(t)$, $\mathcal{X}$ represents the formulated stable region, $V(x,x_s)$ is the Lyapunov function and $h(x(t_j), x_s(t_j))$ is the Lyapunov controller.

The control objective is to minimise the control input $u(t)$ by solving the optimization problem represented by Eq. (6a) such that the process states reaches the optimal steady states. The system of equations (Eq. (1)) is solved continuously to optimize the manipulated values constrained by its dynamics (Eq. (6b)). The control inputs have bounded amount of available is shown in Eq. (6c). The third constraint (Eq. (6d)) defines the starting conditions of the optimization process. The fourth constraint (Eq. (6e)) is to ensure that the derived states of the process lies in the reachable set $\mathcal{X}$. The last constraint signifies the stability of the process, as the Lyapunov controller $h(x(t_j), x_s(t_j))$ acts upon the process, the Lyapunov function $V(x,x_s)$ should decrease with time to reach a minimal equilibrium point.

3.2 Implementation Procedure

The combined structure of process economics and control, commonly known as EMPC, works in a receding horizon fashion unaffected by uncertainties and random disturbances. The optimization problem is solved for a sampling period of Δ for both the layers as summarised below:

Step 1: The internal upper layer receives the state feedback from the plant and computes the steady states.

Step 2: The computed steady states are projected on the stable set $\mathcal{X}$, to check the reachability of the process states.

Step 3: The feedback MPC at the lower level tracks these steady states while maintaining its operation within the stability region $\mathcal{X}$ subject to process dynamics.

3.3 Stability Analysis

To ensure stability of the process states, the stable set $\mathcal{X}$ is designed and constructed using set theoretic methods. In this section, the closed-loop process states are maintained inside the set $\mathcal{X}$ for any initial condition which is proved by providing the sufficient condition for closed loop stability in the following theorem.

Theorem 1: Considering the system of Eq. (1) and designing a Lyapunov controller $h(x,x_s)$ satisfying the set of equations of Eq. (3). Suppose $\epsilon_w > 0, \Delta > 0, \rho(x_s) > \rho_e(x_s) \ge \rho_{e,min} > 0$ for all $x_s \in I$ such that

$$0 < \rho_{e,min} = min_{x_s\in I}\{\max\{\rho_e(x_s) \mid \Omega_{\rho_e(x_s)} \subseteq \mathcal{X}\}\} \quad (7)$$

and, $$-\alpha_3(\alpha_2^{-1}(\rho_{e,min})) + L'_x M\Delta + L'_w w_p \le -\epsilon_w/\Delta \quad (8)$$

if the initial condition $x(t_0) \in \mathcal{X}$ and $N \ge 1$, then the actual states from the process is also bounded within $\mathcal{X}$.

Proof: The above theorem is proved in two divisions. The 1st part is to prove the reachability of the states calculated by optimizing the objective function of Eq.5. The 2nd part is to ensure the boundedness of the actual states of the process within the stable set $\mathcal{X}$.

Part 1: Any computed state $x(t_k) \in \mathcal{X}$ lies in the stability region $\Omega_{p(x_s)}$ of the Lyapunov controller $h(x, x_s)$ for some steady state x_s. This signifies that the existence of an input trajectory $u(t_{K+j}) = h(x_{K+j}, x_s), j = 0,1, \dots \dots N-1$ which is a feasible solution for the system of Eq. (5a) confirms the occurrence of an input trajectory in the lower layer which is a feasible solution. This ensures the stability of the Lyapunov controller $h(x, x_s)$ when operated in sample and hold manner.

Part 2: As $x(t_k) \in \mathcal{X}$, a steady state $x_s \in I$ is found such that the actual process state $x(t_k) \in \Omega_{p(x_s)}$, since the stability region $\Omega_{p(x_s)}$ is computed by the Lyapunov controller for every state x_s. Let us consider the Eq.(6f) exist such that for some $x_s \in I$ with optimal control input $u_s(t_k)$ obtain by optimising Eq. (5a). The term on the right side of Eq. (6f) can be written using Eq. (3b) as follows:

$$\frac{\partial V(x, x_s)}{\partial x} f\left(x(t_j), u(t_j), 0\right) \leq -\alpha_3 \left(|e|\right) \tag{9}$$

The rate of change of Lyapunov function along the state trajectory for $\tau \in [t_k, t_{k+1})$ is

$$\dot{V}(x(\tau), x_s) = \frac{\partial V(x(\tau), x_s)}{\partial x} f\left(x(\tau), u_s(\tau), w(\tau)\right) \tag{10}$$

Addressing the bounds on Eq. (9) and Lipschitz properties of Eq. (4c), adding and subtracting $\dot{V}(x(\tau), x_s)$ the following bound can be imposed on the Lyapunov function to decrease due to control action $u_s(\tau)$ for $\tau \in [t_k, t_{k+1})$:

$$\dot{V}(x(\tau), x_s) \leq -\alpha_3 \left(|e|\right) + L_x^{'}|e| + L_w^{'}|w(\tau)| \tag{11}$$

The deviation of the actual state from the optimal steady state for a single sampling period $\tau \in [t_k, t_{k+1})$ can be bounded as;

$$|x(\tau) - x_s(t_k)| \leq M\Delta \tag{12}$$

Considering Eq. (11) and Eq. (12), the bound on the derivation of the Lyapunov function for $\tau \in [t_k, t_{k+1})$ can be given as:

$$\dot{V}(x(\tau), x_s) \leq \alpha_3 \left(|e|\right) + L_x^{'}|M\Delta| + L_w^{'}|w(\tau)| \tag{13}$$

As $I \in \mathcal{X}$, the difference between the actual and optimal states is always greater than zero. Now, to derive the lower and upper bounds on the Lyapunov function we consider the largest level set of $V(x, x_s)$ i.e. $\Omega_{\rho_e(x_s)}$ existing in $\mathcal{X}$ for each $x_s \in I$. The smallest level set among all is $\rho_{e,min}$. Thus, the lower bound on the Lyapunov function can be given as:

$$V(x, x_s) \geq \rho_{e,min} \tag{14}$$

for all $x_s \in I$ and $x \in \mathcal{X}$.
Now, using Eq.(3a) and Eq.(14), the difference between actual and optimal states can be written as:

$$\alpha_2^{-1}(\rho_{e,min}) \leq |x(\tau) - x_s(t_k)| \tag{15}$$

for all $x_s \in I$ and $x \in \mathcal{X}$. Applying Eq. (14) on Eq.(13) we have the following bound:

$$\dot{V}(x(\tau), x_s) \leq -\alpha_3 \left(\alpha_2^{-1}\left(\rho_{e,min}\right)\right) + L_x^{'}|M\Delta| + L_w^{'}|w(\tau)| \tag{16}$$

Using Eq. (8) and Eq. (16) we have:

$$\dot{V}\left(x(\tau)\right) \leq -\epsilon_w/\Delta \tag{17}$$

for all $\tau \in [t_k, t_{k+1})$.

Integrating the bounds on the interval, we have:

$$V(x(t_{k+1}) \leq V\left(x(t_k)\right) - \epsilon_w \tag{18}$$

Thus, for all $\tau \in [t_k, t_{k+1})$,

$$V(x(\tau)) \leq V\left(x(t_k)\right) \forall\, x_s \in I \text{ and } x \in \mathcal{X}. \tag{19}$$

Hence, the plant operation is proved to be maintained in the constructed stable set □

4 Case Study

The integrated set up of EMPC is implemented and demonstrated on optimal operation of a fossil fuel power unit (FFPU) [18]. There is no type-cast for the continuous economic cost function used in the upper layer.

The pivotal objective of a FFPU is to maintain the productivity of electric power with invariable voltage and frequency constantly in accordance to the variation in load demand at all times. Traditionally, the unit load demand provided at the top of the hierarchy is converted by a fixed non-linear mapping to a desired set point for the lower level pressure control loop shown in Fig. 3. However, there is no process optimization for circumstances when the operating conditions differ from the original signals. In this approach, much emphasis is given on achieving accurate feedback control without doubting on the reachability and adequacy of the set points.

The integrated control paradigm i.e. EMPC is designed for this FFPU to calculate the reachable set points to meet the variable load demand thereby maximizing the net return. The process dynamics of a 160 MW oil fired drum-type boiler-turbine-generator unit is transformed into a state-space model of order three as given in Eq. (20).

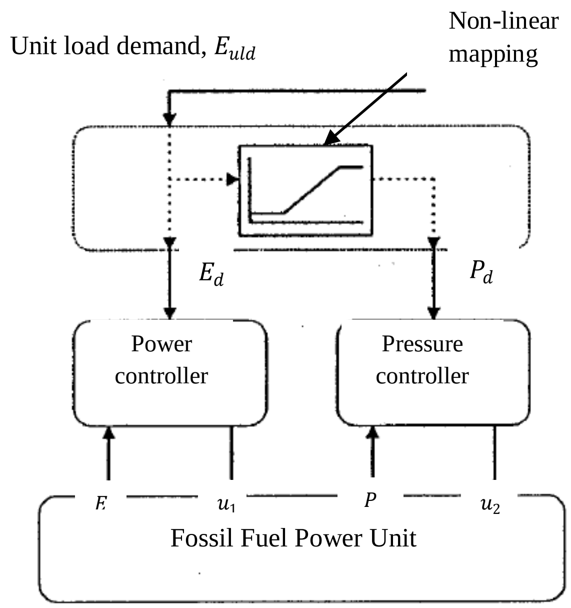

Fig. 3 Traditional coordinated control of FFPU [18]

$$\frac{dP}{dt} = 0.9u_1 - 0.0081u_2\, P^{\frac{9}{8}} - 0.15u_3 \quad (20a)$$

$$\frac{dE}{dt}\left((0.73u_2 - 0.16)P^{\frac{9}{8}} - E\right)10 \quad (20b)$$

$$\frac{d\rho_f}{dt} = \frac{141\ u_3 - (1.1u_2 - 0.9)P}{85} \quad (20c)$$

The control input signals u_1, u_2 and u_3 are given to the actuators for manipulating the fuel flow rate (in pu), the flow of steam to the turbine (in pu) and feedwater flow to the drum (in pu) respectively. The process states are $x = [P\ E\ \rho_f]^T$.

where,

P is steam presume in the drum (in kg/cm^2).

E represents the electric power (in MW).

ρ_f is the density of the steam-water(kg/m^3).

The inputs available to the control actuators are constraint in the region [0, 1] and the rate of change of control inputs are bounded as follows:

$$-0.007 \leq \frac{du_1}{dt} \leq 0.007 \quad (21a)$$

$$-2.0 \leq \frac{du_2}{dt} \leq 0.02 \quad (21b)$$

$$-0.05 \leq \frac{du_3}{dt} \leq 0.05 \quad (21c)$$

The equilibrium points of all the computed steady states lies within the region defined as I. The economic objective function for the optimal control of the FFPU is formulated to credit optimal load tracking and heat rate, minimizes fuel usage and penalise the throttling losses in the control values of the main stream flow and feedwater flow Eq. (22). The time-varying cost function addressing the above mentioned process economics is as follow:

$$J_e = E_{uld} - E_{ss} \quad (22)$$

The variable load demand is denoted as E_{uld} (in MW) and the corresponding power generated is E_{ss} (in MW) obtained from the steady state model of Eq. (20).

$$E_{ss} = \frac{0.73u_2 - 0.16}{0.0018u_2}(0.9u_1 - 0.15u_3) \quad (23)$$

The sampling time for optimizing the process economics in the upper layer EMPC is fixed as 0.01 h and prediction horizon is taken as 60 steps. The actual process states from the lower layer is updated every 0.5 h. Hence, the initial condition to the optimization problem in the upper layer changes for two times in every 1 h resulting to compute a new optimal trajectory after every 50 steps. The updated lower layer states are first projected on the set I to ensure the process operation within the bounds. For e.g. if the actual states of the process is [270 $kg/cm^2, 160MW$] which is exterior to set I then the constrained optimization problem forces it to [$kg/cm^2, 160MW$].

In the feedback control level, two Lyapunov based proportional controllers $h(x, x_s)$ are designed to track the dynamic computed steady states for a stabilizing control law Eq. (24).

$$h(x) = \begin{Bmatrix} -K_1\left(x_1(t) - x_{s,1}(t)\right) + u_{s,1}(t) \\ -K_2\left(x_2(t) - x_{s,2}(t)\right) + u_{s,2}(t) \\ -K_3\left(x_3(t) - x_{s,3}(t)\right) + u_{s,3}(t) \end{Bmatrix} \quad (24)$$

where, the gains of the controller are K_1= 100 , K_2=10, K_3=100 and $u_{s,1}, u_{s,2}, u_{s,3}$ are the manipulated inputs that drives the process states ($x_1(t)$ and $x_2(t)$) to follow the optimal steady states $x_{s,1}(t)$ and $x_{s,2}(t)$. In this application, we consider a Lyapunov function $V(x, x_s)$ in quadratic form to determine the stability of the process during its operation. The Lyapunov function is in the form $V(x, x_s) = e^T Pe$, where P is the Lyapunov matrix, Eq. (25), and e is the deviation of the actual process states to the steady states (i.e. $e = x(t) - x_s(t)$).

$$P = \begin{bmatrix} 10 & 1 \\ 1 & 100 \end{bmatrix} \quad (25)$$

The lower level Lyapunov controller is also designed with sampling time 0.01h, prediction horizon of 5 steps and weighing matrices $Q_C = P$ and $R_C = diag[5 \times 10^3 \; 5 \times 10^4 \; 5 \times 10^2]$.To ease our task of not re-computing the optimal trajectory we choose the same time partition (i.e. 0.01 h).

4.1 Preliminary Results

To understand the dynamics of the process, we used the MATLAB function *ode45*. This routine applies the variable step Runge-kutta method for numerically solving the differential equations. The process is simulated for time duration of 4 h and sampling time of 0.01 h. The state variables for the FFPU $x = [P \; E \; \rho_f]^T, P$ is steam presume in the drum (in kg/cm^2),E represents the electric power (in MW) and ρ_f denotes the density of the steam-water(kg/m^3). The initial conditions for the states are considered as [$30 kg/cm^2$, 10MW, $1000 kg/m^3$]. The manipulating inputs u_1, u_2 and u_3 are given to the ontrol actuators to manipulating the fuel flow rate (in pu), the flow of steam to the turbine (in pu) and feedwater flow to the drum (in pu) respectively. From the output of the power plant shown in Fig.4, we can infer that it is a double integrating system.

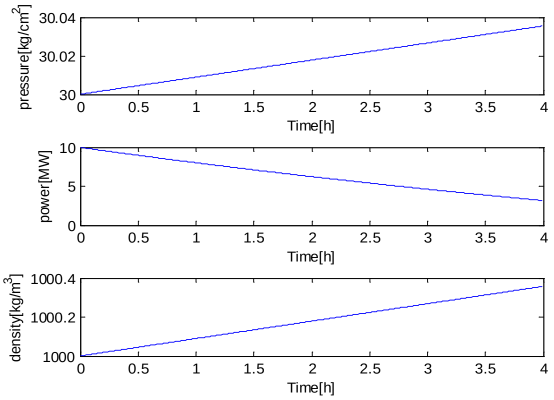

Fig.4. Open loop process output of power plant by solving with *ode45*.

4.2 Closed loop simulation of economic MPC

The optimization problems in the upper layer and lower layer of EMPC are solved by a optimization toolbox present in MATLAB, *fmincon* and the algorithm used for solving the objective function is interior point algorithm. To demonstrate the closed loop performance of the integrated system of time-varying process economics and regulatory feedback control, simulations are performed on the FFPU.

Firstly, an offline construction of the reachable stable set is done and posed as a constraint for computing the optimal states as well as maintaining the process actual states within its bound as shown in Figure 6.6. Secondly, the computed optimal steady states are displayed. Thirdly, the trackability of the lower layer states maintained within the bounds to the steady states is demonstrated. Lastly, minimisation of the deviation of the actual states from the steady states is shown.

We have considered a 160 MW oil fired drum-type boiler-turbine-generator unit and the bounds on the states (i.e. x_1 = pressure in $\frac{kg}{cm^2}$ and x_2 = power in MW) are given as

$$I := \left\{ \begin{matrix} x \in R^2 \mid 10 \le x_1 \le 250 \frac{kg}{cm^2}, \\ 0 \le x_2 \le 160\text{MW} \end{matrix} \right\} \quad (26)$$

The limitations posed by Eq. (26) is used for constructing the reachable set $\mathcal{X}$ as shown in Fig.5. The cost function of the upper layer of EMPC designed for FFPU is the direct conversion of variable load demand to time varying economic objective. The plant is started at an initial steam pressure of $40 \frac{kg}{cm^2}$ and power generation of 10 MW. The obtained optimal trajectory is shown in Fig.6.

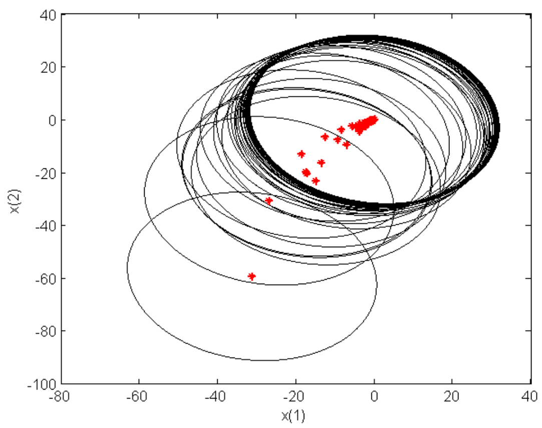

Fig.5. Constructing the reachable set for FFPU.

$$x_s(t) = \left\{ \begin{matrix} [71.2914 Kg/cm^2, 68MW]^T, t < 1.0h \\ \left[89.9715 \frac{Kg}{cm^2}, 90MW\right]^T, 1.0h < t < 2.0h \\ \left[33.2507 \frac{Kg}{cm^2}, 140MW\right]^T, 2.0h < t < 3.0h \\ \left[141.6783 \frac{Kg}{cm^2}, 150MW\right]^T, 3.0h < t < 4.0h \end{matrix} \right\}$$

These output trajectories shown in Fig. 6 are obtained from the inputs derived by optimising the objective function of Eq. (22) with a prediction horizon of 60 steps. The corresponding optimal input trajectories shown in Fig.7 (in pu) are:

$$u_s(t) = \begin{cases} [0.3149 \quad 0.9857 \quad 0.4521]^T, t < 1.0h \\ [0.4126 \quad 1.0000 \quad 0.5807]^T, 1.0h < t < 2.0h \\ [0.6346 \quad 1.0000 \quad 0.8600]^T, 2.0h < t < 3.0h \\ [0.6787 \quad 1.0000 \quad 0.9144]^T, 3.0h < t < 4.0h \end{cases}$$

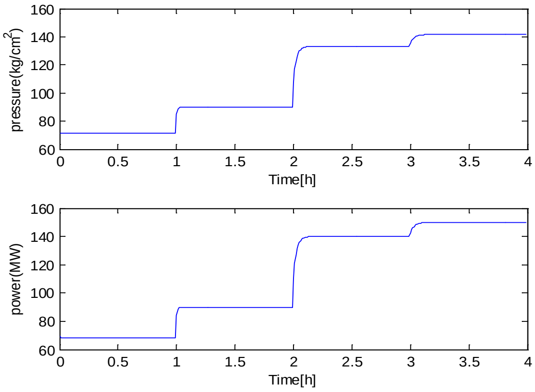

Fig.6. EMPC trajectories for FFPU.

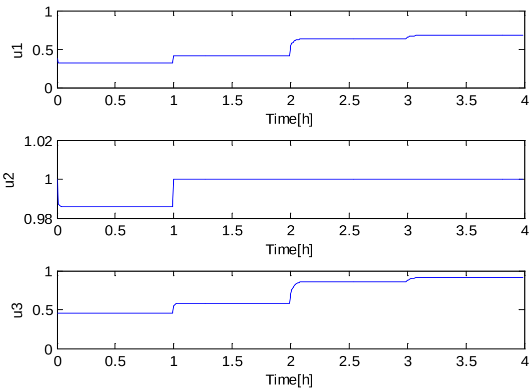

Fig.7. Optimal input trajectories of FFPU.

The lower feedback controller tracks the optimal state trajectory by solving the control objective with the help of a Lyapunov controller.

The Lyapunov controller minimises the error between the actual trajectory and the steady states. The controlled state trajectory is shown in Fig. 8.

The controlled inputs obtained by minimizing the control objective are given to the actual process. The input trajectories are shown in Fig. 9.

The rate of change of the Lyapunov function $V(x, x_s)$ minimizes along the state trajectory is shown in Fig. 10. The final value of the error reaches approximately equal to zero which implies that the process have attain stability.

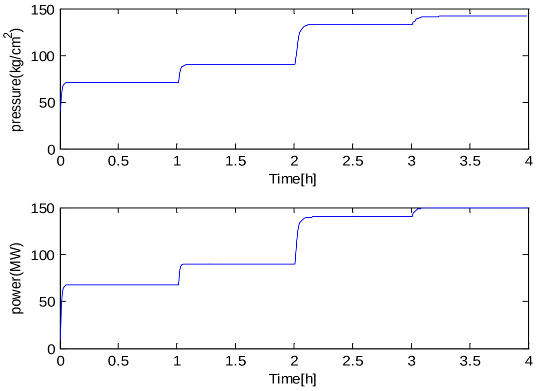

Fig.8. Controlled state trajectory from the feedback MPC controller.

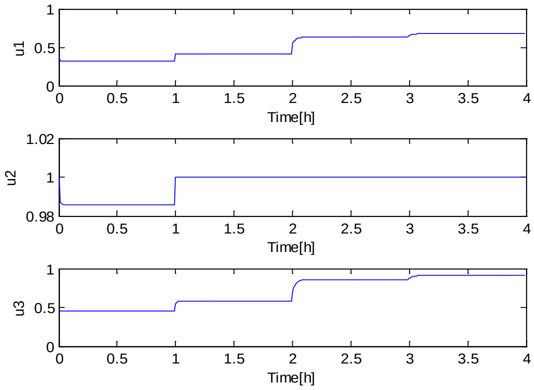

Fig.9. Controlled input trajectories of FFPU.

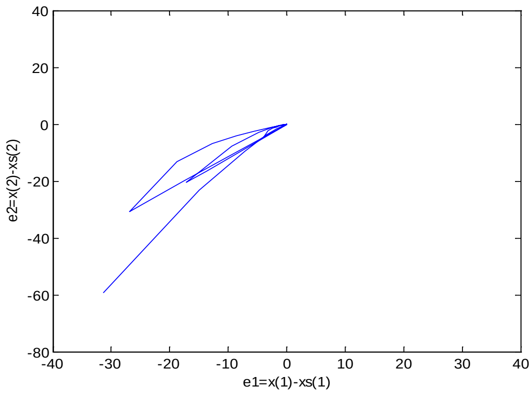

Fig.10. Lyapunov controller minimizing error along with state trajectory.

5 Conclusion

In this work we have implemented the combined two layer control scheme. This control framework is a recent development that addresses the dynamic process economics along with a better control performance. The integrated framework of Economic MPC can handle the real-time uncertainties and randomness in energy cost, market demands, and various other economic considerations. The upper layer was used to optimize the real time process economics while the obtained state trajectory was constrained inside a stable set designed using set theoretic method. The lower layer was allowed to track the reference trajectory subject to the process dynamics. However, the limitations on achieving the global economic optimum are overcome in this work. The major concern in this control scheme is to ensure the stability of the process. The simulation results of the case study proved that the process was stable and operating at a bounded region.

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