Design of Multirate Output Feedback Controllers Based On Generalised Predictive Control

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Abstract: - In this work, Multirate Output Feedback (MROF) controllers namely Fast Output Sampling (FOS) controller and Periodic Output Feedback (POF) Controller are designed using Generalized Predictive Control (GPC). The performances of the designed controllers are evaluated by applying the same for the active vibration control of a smart structure. The closed loop responses of the controllers are obtained in simulation.

Key-Words: - multirate output feedback control, fast output sampling control, periodic output feedback control, generalized predictive control, smart structure

1 Introduction

The static output feedback problem is one of the most investigated problems in control theory however, complete pole assignment and guaranteed closed-loop stability is not obtained [1,2]. The dynamic output feedback controller involves more dynamics and is complex to design. The other approach to pole placement problem is MROF control, where sampling of control input and output of the system are at different rates. MROF can guarantee closed loop stability, a feature not assured by static output feedback. In MROF, states are directly computed in one sampling time, unlike that of the observer based design, with piecewise constant output gains. MROF maintains structural simplicity of static output feedback [3-6]. Fast Output Sampling feedback is a kind of MROF control in which the rank of the measurement matrix is lifted by sampling the output signal, the system output is sampled at a faster rate as compared to the control input [7]. Periodic Output Feedback control is also a kind of MROF control in which the rank of the control input matrix is lifted by sampling the control signal. The output is measured at a slower rate and the control is applied at a faster rate.

Generalized Predictive Control is an advanced method of control that relies on dynamic models of the system to be controlled. GPC is a type of Model Predictive Control (MPC), whose main advantage is the ability to handle constraints in an optimal fashion, non-minimum phase processes, changes in system parameters and its straightforward applicability to large, multivariable processes [8,9]. In this work, active vibration control of a smart structure is performed by a fast output sampling feedback controller and periodic output feedback controller, which are designed using GPC. This tuning results in the single step design of the controller to deliver optimal closed loop responses, which would otherwise be possible by trail and error method.

This paper is organized as follows: The smart structure system used in this work and its mathematical model are given in Section 2. A brief review of GPC is given in Section 3. Section 4 deals with design of FOS and POF controllers. Closed loop performance evaluation of controllers is covered in Section 5. Conclusions are drawn in Section 6.

2 System

The smart structure considered in this work is a clamped free smart aluminum beam with piezo sensing and actuation is shown in Fig.1. Piezo ceramic sensor is surface bonded on the bottom surface of the beam at a distance of 10 mm from the fixed end. A pair of piezo patches is surface bonded on the top surface of the beam, one at a distance of 10 mm and another at a distance of 375 mm from the fixed end, to act as control and disturbance actuators respectively.

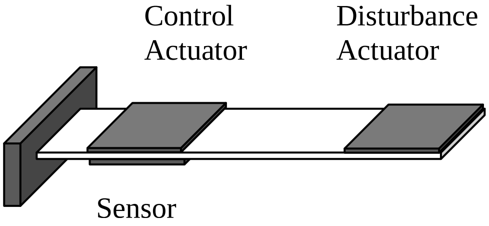


Fig. 1. Schematic of the piezo actuated structure

Table 1 Properties and Dimensions of Aluminum Beam

Length (m)	0.40
Width (m)	0.0135
Thickness (m)	0.001
Young’s modulus (Gpa)	71
Density (kg/m^3)	2700
First natural frequency (Hz)	5.07
Second natural frequency (Hz)	32.48

Table 2 Properties and Dimensions of Piezoceramic Sensor/Actuator

Length (m)	0.0765
Width (m)	0.0135
Thickness (m)	0.0005
Young’s modulus (Gpa)	47.62
Density (kg/m^3)	7500
Piezoelectric strain constant (mV^{-1})	-247x10^{-12}
Piezoelectric stress constant (VmN^{-1})	-9x10^{-3}

The linear time invariant continuous time model of the structure which is taken from [10] is

$$\dot{x} = Ax + Bu + er; y = Cx \tag{1}$$

with

$$A = \begin{bmatrix} 76.9893 & 71.5731 & -45.5632 & 71.9048 \\ -136.1042 & 6.1271 & 116.6837 & -116.7537 \\ 115.7932 & -116.2021 & -6.5425 & 136.6781 \\ -70.8876 & 45.1268 & -71.2168 & -77.5364 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.2046 \\ 0.1955 \\ -0.4427 \\ -0.0299 \end{bmatrix} \quad e = \begin{bmatrix} 0.0029 \\ 0.0265 \\ -0.0664 \\ 0.0588 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

where $x \in \Re^n, u \in \Re^m, y \in \Re^p$, with A being the system matrix, b the control input vector, e the disturbance vector, C the output matrix, x the state vector and y the system output.

3 Review of Generalised Predictive Control

The GPC is a receding-horizon method, which depends on predicting the system output, based on assumptions about future control actions. It is assumed that there is a control horizon, beyond which all control increments become zero, which is shown to be beneficial both in terms of robustness and for providing simplified calculations [11-15].

The system given in (1) is discretized and transformed into regular form by applying suitable transformation. The control input

$$U(k) = \left[u(k+1), u(k+2), \cdots, u(k+N_2)\right]^{\mathrm{T}} \tag{2}$$

Is chosen to minimize the quadratic cost function

$$J = \sum_{j=N_1}^{N_2} \left[\hat{y}(k+j) - y_d(k+j)\right]^2 + \lambda \sum_{j=1}^{N_u} \left[\Delta u(k+j-1)\right]^2 \tag{3}$$

where $\hat{y}$ is the predicted output, y_d the desired output, $\lambda \geq 0$ the control input weighting. The parameters N_1, N_2 and N_u are minimum prediction horizon, maximum prediction horizon and control horizon. N_1 is equal to (1+ the system’s delay) if delay exists, N_2 is chosen to satisfy N_2 T_s is equal to the response time of the system, where T_s is the sampling time, N_u and λ considerably affect the stability of the system.

The time derivative of the system output is

$$v(k) = \dot{y}(k) = Gx(k) + HU(k) \tag{4}$$

where

$$U(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \cdots \\ u(k+n-1) \end{bmatrix}$$

$$H = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ CA^{n-1}B & CA^{n-2}B & \cdots & CB \end{bmatrix} \quad G = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^n \end{bmatrix}$$

The control law design is to find $U(k)$, which minimizes $J(k)$ and regulates v(k) to zero, in finite time. This implies

$$U(k) = 2H^T Gx(k) + 2H^T HU(k) + 2\lambda U(k) \tag{5}$$

$$U(k) = K_{GPC}x(k) = -(H^T H + \lambda I)^{-1} H^T Gx(k) \tag{6}$$

The first row of $U(k)$ is the control law K_{GPC}, using which the eigen values of the closed loop system $(A+BK_{GPC})$ is obtained. These values are also the closed loop poles, which are taken as the closed pole values for calculating the state feedback gain for the fast output sampling feedback controller and for calculating the stabilizing output injection gain for the periodic output feedback controller.

4 Design of FOS and POF Controllers

4.1 Fast Output Sampling Feedback Controller

Theory of fast output sampling feedback control is presented in [7], and its application to smart structure control can be seen in [16]. The discrete time system obtained by sampling the system in (1), with a sampling interval of 0.01 seconds is

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k) + E_\tau d(k) \qquad (7)$$
$$y(k) = Cx(k)$$

where

$$\Phi_\tau = \begin{bmatrix} 0.99 & 1.0 & 0.0 & 0.0 \\ -0.27 & 0.0 & 1.0 & 0.0 \\ 0.98 & 0.0 & 0.0 & 1.0 \\ -0.99 & 0.0 & 0.0 & 0.0 \end{bmatrix} \Gamma_\tau = \begin{bmatrix} 0.0037 \\ -0.0021 \\ -0.0020 \\ -0.0005 \end{bmatrix}$$

$$E_\tau = 10^{-3} \begin{bmatrix} 0.2955 \\ -0.3649 \\ -0.0657 \\ 0.3733 \end{bmatrix}$$

The controllability index of the system $(\Phi_\tau, \Gamma_\tau, C)$ is four. Let (Φ, Γ, C) be the discrete time system obtained by sampling (A,B,C) at a rate of $\frac{1}{\Delta}$, where $\Delta = \frac{\tau}{N}$, and the number of subintervals N is chosen as 4. For a λ value of 0.95, the eigen values obtained from the GPC design is $-0.45 \pm 0.89i$, $0.95 \pm 0.30i$. The state feedback gain obtained is

$$F = \begin{bmatrix} -9.2859 & -10.7422 & -1.2536 & 1.2572 \end{bmatrix}$$

The LMI approach proposed in [18] is used to design the fast output feedback gain L such that, the spectral norms of L and LD_0-$F\Gamma_\tau$, as well as, the distance between LC and F can be controlled. The LMIs are solved with $\rho_1 = 1200$, $\rho_2 = 6000$, $\rho_3 = 2$ by minimizing the linear objective under LMI constraints using the solver mincx() in the LMI control tool box in MATLAB, where ρ_1, ρ_2, ρ_3 represent upper bounds on the spectral norms of L, LD_0- $F\Gamma_\tau$ and LC-F respectively. The fast output feedback gain obtained is

$$L = \begin{bmatrix} 226.6891 & -696.1215 & 758.5624 & -299.3471 \end{bmatrix}^T$$

4.2 Periodic Output Feedback Controller

Theory of periodic output feedback control is presented in [7,19], and its application to smart structure control is given in [20]. A stabilizing output injection gain is designed for the system $(\Phi_\tau, \Gamma_\tau, C)$, such that the eigen values of $(\Phi_\tau + GC)$ lie inside the unit circle. For a λ value of 0.005, the stabilizing output injection gain obtained from GPC design is

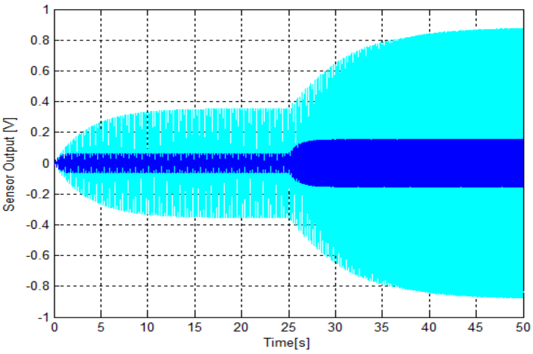
$$G = \begin{bmatrix} 1.3984 & -0.1849 & -0.1849 & 0.2084 \end{bmatrix}^T$$

The periodic output feedback gain is obtained by minimizing the performance index in [20], so that the amplitude of control signal required can be reduced. Periodic output feedback gain K obtained with the following performance index weight matrices $R=0.00001, \overline{Q} = 200I_4, \overline{P} = 165I_4$ is

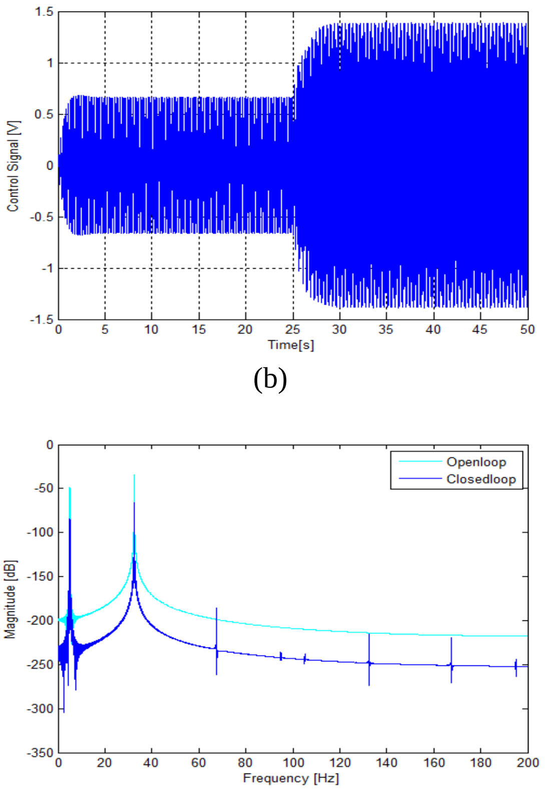
$$K = \begin{bmatrix} 490.7564 & 26.1736 & 219.1575 & -421.5495 \end{bmatrix}^T$$

5 Closed Loop Performance Evaluation of Controllers

The performance of the controllers is evaluated through simulation using MATLAB at resonant conditions. The structure is initially excited at its first mode frequency and later at its second mode frequency, for duration of 25 seconds each. The simulation results obtained for fast output sampling controller is shown in Fig. 2. The open loop and closed loop response in time domain, the control effort, open loop and closed loop response in frequency domain are shown in Fig. 2(a), 2(b) and 2(c) respectively.



(a)

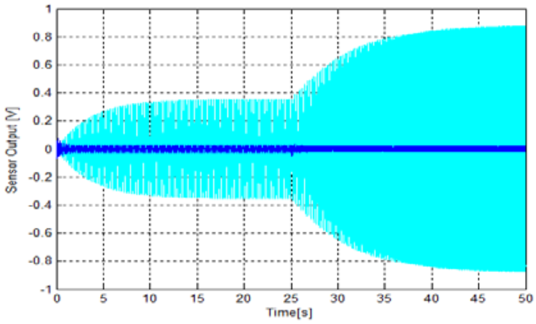


(b)

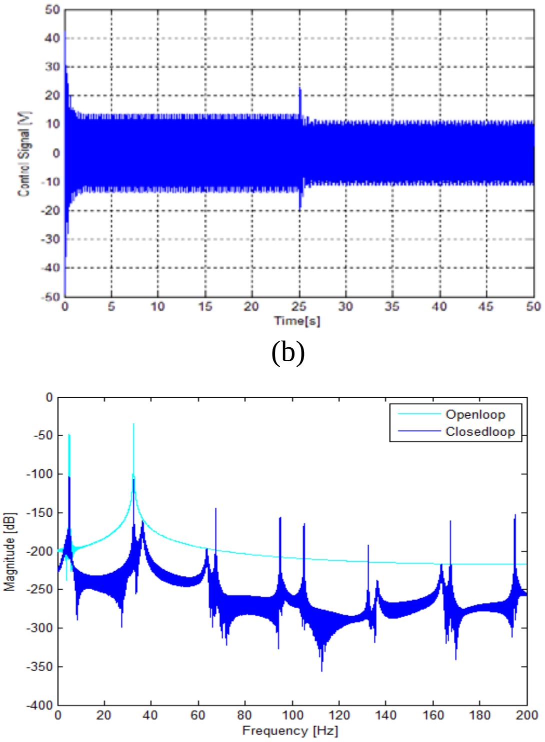
(c)

Fig. 2 Simulation results of fast output sampling controller
(a) Open loop response and Closed loop response in time domain
(b) Control signal
(c) Open loop response and Closed loop response in frequency domain

The simulation results obtained for periodic output feedback controller is shown in Fig. 3. The open loop and closed loop response in time domain, the control effort, open loop and closed loop response in frequency domain are shown in Fig. 3(a), 3(b) and 3(c) respectively.



(a)



(b)

(c)

Fig.3 Simulation results of periodic output feedback controller
(a) Open loop response and Closed loop response in time domain
(b) Control signal
(c) Open loop response and Closed loop response in frequency domain

6 Conclusions

From the simulation results of FOS controller, the percentage of vibration suppression obtained is 82.8 for first mode and 82.5 for second mode, and from the results of POF controller, the percentage of vibration suppression obtained is 92.8 for first mode and 97.1 for second mode. The above result is obtained by designing the FOS and POF controllers, by using GPC, which would otherwise be designed by trail and error method.

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