Brazilian REITs performance: an analysis of higher moments and time scales influence

BRUNO MILANI*
PAULO SERGIO CERETTA**
MARI ELDIONARA ROSA MACHADO***
Post-Graduation in Administration Program
Federal University of Santa Maria
Roraima Ave, 1000, Building 74C, Santa Maria – RS
BRAZIL
*brunoprofess@gmail.com, **ceretta10@gmail.com, ***mari_eldionara@yahoo.com.br

Abstract: Real Estate investments have been used to provide diversification without increasing portfolio risk, although direct real estate investment has several disadvantages, such as low liquidity and high transaction costs, and Real Estate Investment Trusts (REITs) are an alternative investment aim to overcome these difficulties. This paper analyzes the REITs performance, comparing it to Ibovespa (proxy of market) and Ifix (Brazilian REITs proxy). For a better analysis we used the wavelet method, that allows to compare the results of different scales of time. Analyzing the wavelet transform, the REITs have significant coefficients when compared to the market, index Ibovespa, but considering the relationship between the REITs and the real estate funds, Ifix, influence happens in the long run. Market volatility has a negative effect in REITs returns in the long run, probably because a REITs investor tries to avoid risk. That's because maybe in the long run, REITs investors can change their investments to direct Real Estate if those are presenting better returns, what would reduce REITs demand and consequently, their price. It was noted that REITs returns are influenced by Real Estate companies returns in different scales, indicating that Brazilian REITs returns depend on market volatility. In a general way, this study showed that the relationships between REITs returns, Real Estate Companies returns and Market returns depend on the time scale analyzed, but they tend to be stronger at large scales. Differently of what was found by Milani and Ceretta (2014), Brazilian REITs returns can be better explained by Real Estate Sector Companies returns than by market returns, in the long run. Also, higher moments influence their returns and should be part of a model that tries to explain REITs returns.

*Key Words***:** Real Estate, REITs; wavelets; multiscale analysis; higher moments; Brazilian market;

1. Introduction

Real Estate investments have been considered a good instrument to provide diversification without increasing portfolio risk, especially considering a portfolio containing stocks, although direct real estate investment has several disadvantages such as low liquidity and high transaction costs. Real Estate Investment Trusts (REITs) are a well-known investment alternative to many investors who aim to overcome these difficulties (Parker, 2011). REITs have been able to minimize the liquidity problem, since they have traded shares.

However, the fact that REITs have traded shares raises the following question: are these shares driven by a "real estate factor" or they simply follow the overall market variation? As a consequence, one could question if a REIT share effectively improves portfolio performance. A similar kind of fund has been the target of the same academic inquiry: the Exchange Traded Funds (ETF), which frequently shows that their shares depend more on the market return than on their underlying assets return.

REITs of emerging markets have received little academic attention, although they have become increasingly important for investors, due to their fast growing economies. In Brazil, REITs are a relatively new type of asset. The existing REITs, in general, have been active for a short period of time and present few daily trades, what hampers to analyze their return.

Since January 1st, 2011, BM&FBOVESPA, the main Brazilian stock exchange, has been calculating the Ifix index, whose objective is to be a Brazilian REITs proxy. Also, since 2010, BM&FBOVESPA has been presenting the Imob index, a real estate sector index which serves as a proxy for the real estate companies performance. Considering the previous questioning and the data availability, this paper aims to discover whether Brazilian REITs returns depend on the real estate companies returns or they follow the overall market, or any of the alternatives. The answer to this question would help to define if a Brazilian REIT share adds value to a portfolio.

However, the dynamics of the relationship between returns and risk factors are likely to vary depending on the investor's time horizon, resulting in the need of incorporating different time scales. A relatively new approach known as wavelet analysis might help to reduce this problem. To comply with that, we have wavelet decomposed the series and then estimated OLS regressions for each time scale, besides OLS regressions with the original series, for comparison purposes.

We have organized the paper as follows: Section 2 brings a review of previous studies on REITs; Section 3 brings basic explanations about wavelets; Section 5 defines our data and method; Section 6 presents and discusses the results and finally, Section 7 will stand for our final considerations.

2. Previous Studies

Huang and Zhong (2013) analyzed the diversification benefits of Commodities, REITs and Treasury Inflation-Protected Securities (TIPS), using data from 1970 to 2010. Using the Dynamic Conditional Correlation (DCC) model of Engle (2002), they pointed that these asset classes are not substitutes, but their diversification benefits vary over time. Correlation between REITs and U.S. Equity (used as a benchmark) has increased from 0.5 in 2007 to 0.8 in 2009, impacting portfolio rebalances. Using DCC in asset allocations, investors would hold substantial portions of REITs in their portfolios before subprime crisis, but they would start unloading them and loading US Bond on the onset of the crisis.

They also examine the out-of-sample performance of portfolio strategies including these asset classes, concluding that the benefits of the three asset classes should be examined in a dynamic setting and investors need to appropriate correlation estimates to adjust for time variation. DCC was chosen as the best correlation estimate due to adjust to the time variation of diversification benefits.

Boudry et al. (2012) used a cointegration approach in an attempt to gain further insight into the complex interactions between REIT markets and other financial markets, as well as between REIT returns and direct real estate returns. Using transaction rather than appraisal based data, they have found significant evidence that REITS and the underlying real estate markets are cointegrated. This relationship appears to be stronger at larger horizons and it holds in the aggregated as well as in the property type level. But if the securitized and unsecuritized real estate get out of equilibrium, both adjust back towards the equilibrium path, indicating that financial markets informationally lead the real estate markets.

Case, Yang and Yildirim (2012) analyzed the Dynamic Conditional Correlation between REIT and Stock returns. Using Engle (2002) DCC model, they have found that the REIT-stock correlation forms three distinct periods. In the first period, before 1991, correlations were high, never dipping below 59% and with no trend. The second period ended in 2001 when REITS were included in broad stock market indexes, correlations declined to around 30%, enabling higher portfolio allocations without increasing volatility. During the third period, correlations increased steadily, reaching 59% in late 2008.

Fei, Ding and Deng (2010) explore asymmetries in conditional correlation based on the multivariate asymmetric dynamic conditional correlation (AD-DCC) GARCH. They found that there is a little asymmetry between the correlation among REITs direct real estate and stocks and that the time-varying correlation can be explained by macroeconomic variables. Also, when the correlation between REITS and S&P500 is the lowest, the future performance of REITS is the best.

Hoesli and Oikarinen (2012) examined whether securitized real estate reflect direct real estate returns or general stock markets returns using international data for the U.S., U.K. and Australia. Based on sectorial data level, they estimated Vector Error Correction models and investigated the forecast error variance decomposition and impulse responses. Both techniques suggest that the long-run REIT market performance is much more closely related to the direct real estate market than to the general stock market. Consequently, they should be relatively good substitutes in a long-horizon investment portfolio.

The effect of monetary policy stance changes in US equity real estate investment trust (EREIT) returns was analyzed by Chen et al. (2012). They found that bull markets changes in monetary policy have negative effect on EREIT when investors have lower expectations of real estate price increases, but they are not effective when investors have higher expectation about it. During bear and volatile markets, EREIT returns are not sensitive to changes in monetary policy stance.

Chiang, Tsai and Sing (2013) investigated the time-varying relationship between REITs and the stock markets of several Asian countries using a multivariate GARCH-vech model to capture the time-varying correlation. Their results show that the conditional risks have increased abruptly after the subprime mortgage crises. Besides, REITs have been positively correlated with stock markets since the subprime crisis unfolded, suggesting that they are not as defensive as they are in times of stable markets and might not be good shelter during financial chaos.

Zhou and Anderson (2013) investigate the herding behavior of the stock market in U.S equity REIT market. The authors used the quantile regression method and analized the herd behavior in up and down markets and the influence of the financial crisis. For that, they used the CRSP/Ziman Real Estate Data series. The results show that herding is stronger in turbulent markets than in emergent ones. Considering the period after crisis, the behavior changed, the investors show herding in moderate turbulent markets and this is more perceptible in inexperienced investors that are led by others investors.

In turn, Zhou and Kang (2011) evaluated the REIT volatility used models, forecasting how the variations of the GARCH model and ARFIMA (Fractional Integrated ARMA) work, both long memory models. The data of the study are daily total return indexes (with dividends) for REIT, between 1999 and 2008. The results show that the long-memory models are more indicated for model REIT volatility and also for forecast volatility.

Huang and Wu (2013) analyzed the economic benefits and determinants of extreme dependences between REIT and stock returns, investigated to cross-asset linkages during extraordinary periods post and pre real estate crisis, using data from 2000 to 2010. The study used the FTSE_REIT index obtained from Datastream Database and CRSP value-weighted Cap-Based Portfolio from CRSP Database. Other variables are used of REIT-stock extreme 87+ dependences: the default spread, the mortgage spread, the term spread, the S&P500 stock volatility index, the Amihud-version illiquidity index, and the treasury constant maturity rates. The method utilized is the copula, used to describe the contemporaneous dependences structure of the assets. As results, we found that the dependence dynamics supportive evidence for closer co-movements between REIT and stock returns in downward movements is consistent with low diversification benefits in unfavorable times.

Lu, Tse and Williams (2013) examined the daily cross-market return interactions and downside risk between a US REIT returns index and the returns indexes of twelve international REIT markets, considering period of normal REIT market conditions as well as periods of inflating and collapsing REIT prices. Data fom US and international REIT returns were used and also the VAR model for the analysis. The results show that the portfolio managers should embrace international REIT diversification opportunities. It is important not only to consider market interactions as measured by returns correlations and causality, but also the level of risk contribution as measured by Value at Risk.

Wiley (2014) investigate the illiquidity risk in non-listed funds. For that he utilized a list of public non-listed REITs of the SEC Edgar database, in order to gain insights into the role of managerial incentives and liquidity risk. Many characteristics of the REITs were analyzed and the results showed that the operating efficiency ratio and the ratio of operating cash flow to revenue were discovered to have a deterministic impact on the non-listed fund performance.

Abugri and Dutta (2014) compared the model of the multifactor REIT returns and the Fama–French three-factor model to estimate and compare the REIT idiosyncratic volatility. The data used 138 REITs obtained from Datastream from 2006 to 2012. The authors also used a bivariate EGARCH model to estimate conditional betas and observed a positive and significant relationship between expected beta and REIT returns in the cross-section. The results of estimate show that larger REITs have significantly higher average returns, once conditional idiosyncratic volatility is introduced as a control in the cross-sectional regression. They observed mild evidence of persistence of past idiosyncratic risk, which is short-lived, thereby suggesting that past idiosyncratic risk has a short-term impact on future realized idiosyncratic risk.

Bianchi and Guidolin (2014) analyzed whether and how simple linear predictability models of the vector autoregressive (VAR) type may be extended to capture the bull and bear patterns, typical of many asset classes, including REITs. For the analysis they used the U.S. monthly asset returns, from 1972 to 2009. Then, the data was analyzed in a simple VAR and after that they investigate the evidence of switching regimes, to show whether the behavior of the REITs is bull or bear state. Finally, the study presented that nonlinearities are so extensive that it is impossible for a large family of VAR models to either produce similar portfolio weights or to yield realized OOS long-horizon performances that may compete with those typical of MRSMs.

Already, Brauers, Thomas and Zietz (2014) analyzed the existence of rational bubbles in REITs. For this, they applies a complex systems approach to test for the presence of bubbles in the equity REITs market. They used hazard rate model give the log-periodic power-law (LPPL) function of the price trajectory as a function of time t before and endogenous collapse, considering data of equity REITs index covers all companies with a REIT structure that are listed on US stock exchanges, for the period of the 1989 to 2011. The results show that the price peak in the all equity REITs market was preceded by a rational bubble starting in 2003/2004. They also find evidence for a rational bubble in residential REITs market during the same period, but find no bubble in regime in the office REITs market for the sample period.

Rees and Selcuk-Kestel (2014) analyzed the cointegration structure between an within different types of REIT and investigate the influence of cointegrated assets on portfolio indicators. For this, used the monthly asset prices of equity REITs from 1995 to 2008, based on the retail and residential assets traded and listed on NYSE are retrieved from the Thompson Financial data stream. According the results about the cointegrated asset prices active investors rebalance the portfolio, se the temporary deviations of asset prices from the long-term equilibrium take decades to move towards the commom stochastic trend, the presence of cointegration is of little significance for an investor.

Milani and Ceretta (2014) estimated the dynamic conditional correlation between Brazilian REITs returns, Real Estate Sector Companies returns and Ibovespa returns. Their results showed that both correlations were not significant, although the correlation between REITs and Ibovespa appear to be higher than the correlation between REITs and Imob, both static as the dynamic. Also the log-likelyhood of the DCC between REITs and Ibovespa is higher, indicating a better model fit, although this does not overcome the fact that the relationship is not significant. These results, combined with the fact that Ifix index presents higher average return and smaller standard deviation, indicate that there may be interesting for an investor to include a Brazilian REIT share in his portfolio, since it would contribute to increase return with low standard deviation and low volatility correlation with the market.

3 About wavelets

Stock Market participants are a diverse group, which operate in different time scales, associated with different time horizons. However, most previous studies focus on only two scales: short-run and long-run. This has happened mainly because of the lack of an empirical tool. Recently, wavelet analysis has attracted attention as a mean to fill this gap (In and Kim, 2014).

Wavelets are small “waves” that grow and decay in a limited time period. The wavelets transforms decomposes a time series in terms of some elementary functions, called the daughter wavelets or, simply, the wavelets ($\psi_{\tau,s}(t)$). These wavelets are new time series resulting from a mother wavelet $\psi(t)$ that can be expressed as a function of the time position τ (translation parameter) and the scale s (dilatation parameter), which is related to the frequency.

Wavelets are similar to sine and cosine functions because they oscillate around zero, but differ because they are localized both in the time and frequency domains. In contrast to Fourier analysis, wavelets are compactly supported, because all projections of a signal onto the wavelet space are essentially local, not global, and thus it doesn’t need to be homogeneous over time. In fact, wavelet analysis can be seen as a refinement of Fourier analysis.

Wavelets are flexible in handling a variety of non-stationary signals, considering the non-stationarity as an intrinsic property of the data rather than a problem to be solved. Basic wavelets are characterized into father and mother wavelets. A father wavelet (scaling function) represents the smooth baseline trend, while the mother wavelets (wavelet function) are used to describe all deviations from trends. Formulations (1) and (2), respectively represents the father and mother wavelets.

$$\phi_{j,k}(x) = 2^{\frac{j}{2}}\phi(2^j x - k). \quad (1)$$

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k). \quad (2)$$

Where $j, k \in \mathbb{Z}$, for some coarse scale j_0, that will be taken as zero. j=1, in a j-level decomposition. The father wavelet integrates to one and reconstructs the trend component (longest time scale component) of the series. The mother wavelets integrate to zero and describe all deviations from the trend. In order to compute the decomposition, wavelet coefficients at all scales representing the projections of the time series onto the basis generated by the chosen family of wavelets need to be calculated first. They are $D_{j,k}$ (smooth; mother wavelet) and $S_{j,k}$ (detailed; father wavelet), as expressed by the formulation (3), that generates an orthonormal system. For any function *f* that belongs to this system we may write, uniquely:

$$f(x) = \sum_k S_{0,k}\, \phi_{0,k}(x) + \sum_{j\geq 0} \sum_k D_{j,k} \psi_{j,k}(x). \tag{3}$$

In (3), $S_{0,k} = \int f(x)\phi_{0,k} dx$ and $D_{j,k} = \int f(x)\psi_{j,k}\, dx$ are the Smooth and Detail component wavelet coefficients. We could also understand that $f(x)$ is reconstructed, containing the separate components of the original series at each frequency j. After we decompose the function $f(x)$ into j crystals, the crystals d_j are recomposed into a time domain. Formulation (3), thus, represents the entire function $f(x)$, where $\sum_k D_{j,k}\psi_{j,k}(x)$ is the recomposed series in the time domain from the crystal d_j and $\sum_k S_{0,k}\, \phi_{0,k}(x)$ is the recomposition of the residue. In this sense, $\sum_k D_{j,k}\psi_{j,k}(x)$ represents the contribution of frequency j to the original series.

Considering a time series $f(t)$ that we want to decompose into various wavelet scales. Given the father wavelet, such that its dilates and translates constitute an orthonormal basis for all subspaces that are scaled versions of the initial subspace, we can form a Multiresolution Analysis for $f(t)$. The wavelet function in formulation (3) depends on two parameters, scale and time: the scale or dilation factor j controls the length of the wavelet, while the translation or location parameter k refers to the location and indicates the non-zero portion of each wavelet basis vector.

The Discrete Wavelet Transform (DWT) is the usual approach for this multiresolution analysis, but it is restricted to sample sizes to a power of 2, i.e., for j levels we must have a sample of size 2^j. In order to overcome this difficulties, in this study we adopt the Overlap Discrete Wavelet Transform (MODWT), which can handle data of any length, not just powers of two; it is translation invariant, i.e., a shift in the of scales (Gençay, Selçuk,Whitcher, 2001). This way, giving up of orthogonality, MODWT gains attributes that are more desirable in economic applications.

3.1 Previous studies exploring multiscale analysis in financial time series

Gençay Selçuk,Whitcher (2005) proposed the multiscale measurement of systematic risk, decomposing the traditional Beta into wavelets. The excess log-return of US, UK and Germany markets were individually analyzed with a different range of time for each one, but all of them with daily data. Their results showed that the higher the scale, the stronger the relationship between portfolio return and its beta, which means that the beta was higher at low frequencies (64-128 days dynamics).

Fernandez (2006) formulates a time-scale decomposition of an international version of CAPM that accounts for both market and exchange-rate risk, considering stock indexes of seven emerging countries of Latin America and Asia, for the sample period of 1990-2004. With daily data of the MSCI world index and the MSCI emerging markets index, two approaches are analyzed: the first consists in decomposing each index and recomposing its crystals by DWT and then estimate an OLS regression. The second approach is based on wavelet-variance analysis, which determines estimates for the slopes and the goodness of fit of the model (R^2) by the MODWT variance and covariance formulas. Both methods were used to estimate Beta. The results depended on which world index was used, although the emerging markets appear to depend more on the other emerging markets than the developed ones.

Cifter e Özün (2007) decomposed the variance and returns of 10 stocks of ISE-30 by the MODWT method and then estimated a CAPM model to six scales. Their results showed that the return-risk maximization of the portfolio with these 10 stocks may be achieved at the scale of 32 days and the risk will be higher in the portfolios established at the scales different than 32 days. Rhaeim, Ammoudn and Mabrouk (2007) estimated the systematic risk at different scales in the French stock market, with a sample composed of twenty-six actively traded stocks over 2002-2005 periods. Individual stocks and market returns were decomposed into 6 scales. Thus, Beta was estimated by OLS regression. The relationship between excess return and market portfolio becomes stronger at higher scales because beta increases as the scale increases.

Rua and Nunes (2012) illustrated the use of wavelets method assessing the risk of an investor in emerging markets over the last twenty years, using the monthly percentage returns of Morgan Stanley Capital International (MSCI), all country world index and the MSCI emerging markets index, expressed in US Dollars. Using the variance as a measure of total risk, the wavelet spectrum analysis shows that the volatility of monthly stock returns is concentrated at high frequencies, which means that short-term fluctuations dictate the variance of the series. In fact, frequencies associated with movements longer than one year are almost negligible in terms of contributions to total variance. They identify changes in variance across different time-scales in each country, which are clearly linked to well-documented crisis, although there is no evidence of an upward or downward trend in the volatility of emerging countries.

The overall beta of emerging countries is 1.17, seeming to be more stable over time at low frequencies and more time-varying at high frequencies. At high frequencies, one can identify regions in the time-frequency space where the beta is near 3. Given that, their conclusions oppose others like Gençay, Selçuk,Whitcher (2005), Fernandez (2006) and Rhaeim, Ammoudn and Mabrouk (2007). However, the periods where the beta is high include several crises, which mean that if the crises effects were controlled, these results could not hold.

Counterpointing results are also found by Masih, Alzahrani and Al-Titi (2010), who estimates beta at different time scales in the context of the emerging Gulf Cooperation Council (GCC) equity markets by applying wavelet analysis, finding a multiscale tendency. They analyzed companies of the Saudi stock market (88), Muscat Securities Market (114), Kuwait stock exchange (189), Bahrain stock exchange (43), Doha securities market (38), Abu Dhabi securities market (61) and Dubai financial market (46), in different time ranges, comprising February 2007 to April 2008, with daily data. Each return series is separated into its components multiresolution (multihorizon) constituents using orthogonal Haar wavelet transformation. Then, an OLS estimation is ran to each stock and for each frequency, generating several multiscale Betas. They found that Beta and its variability increase between lowest and highest scale, which makes long-term investors more exposed to systematic risk than short-term investors. Also, R^2 decreases when moving to higher scales (longer interval), which means that market return is more able to explain individual stock return at higher frequencies, similarly as the study of Rua and Nunes (2012).

Additionally, Rua and Nunes (2012) also computed the wavelet of R^2 as a multiplication of the country's conditional Beta by the wavelet of market return divided by the country return, analogously to the traditional R^2. This is due to the importance of the systematic risk in explaining total risk, since the overall value of R^2 was near 0.5, but changing considerably over time and frequencies. In low frequencies, 80% of total variance is explained by the systematic risk, but in high frequencies, only 30%.

Deo and Shah (2012) applied the multiscale Beta estimation approach based on wavelet analysis to all stocks comprising BSE-Sensex, using the wavelet decomposition from the maximal overlap discrete wavelet transform (MODWT). With daily data from the BSE-30 (a representative index of the thirty biggest companies of the Indian stock market) from 5 January 2010 to 31st march 2012 (562 observations), they separate out each return series into its constituent multi-resolution (multi-horizon) components. The MODWT was chosen because giving up orthogonality, they gain attributes that are more desirable in economic applications, as the possibility to handle data of every length, not just powers of two; it is translation invariant – that is, a shift in the time series results in an equivalent shift in the transform; it has increased resolution at lower scales since it oversamples data; the choice of a particular wavelet filter is not so crucial; it is slightly affected by the arrival of new information.To each scale of stock return series, two equations are estimated by the OLS method, one with the conventional Beta and other with two coefficients analogous to Beta, one associated to a short periodicity series and the other to a long-periodicity series of market returns.The market index is also decomposed and the Beta coefficient estimated in each level. Beta coefficients were significantin all cases but, they observed that the R^2 is higher at lower scales, implying that major part of market portfolio influence on individual stocks is between medium to higher frequencies. If market risk is concentrated at the medium and higher frequencies, the model predictions would be more relevant at medium to long-run horizons as compared to short time horizons.

Conlon, Crane and Ruskin (2008) explored multiscale analysis for Hedge Funds, due to their wide acceptance by institutional investors because their seemingly low correlation with traditional investments and attractive returns. The Hedge Funds correlation and market risk scaling properties are analyzed by the MODWT, with monthly data from April 1994 to October 2006, tracking over 4500 funds holding at least US$ 50 million under management. They found that both correlation and market risk level with respect to S&P500 varies greatly according to the strategy and time scale examined. The correlation between Convertible Arbitrage, Fixed Income and Multi-strategy, besides the S&P500 and the Hedge Fund Composite Index was found to increase as the time scale increases.

But the correlation between Dedicated Shorted Bias, Equity Market Neutral, Global Macro and Managed Futures strategies correlation with S&P500 and the Hedge Fund Composite Index was found to decrease as the time scale increases. Also, the market risk level held by different Hedge Funds strategies varies according to the time horizon studied. The level of market risk of convertible Arbitrage, Emerging Markets, Event-Driven and Long/Short Equity was found to increase as the time scale increased. The market risk of Dedicated Short Bias, Global Macro and Managed Futures was found to decrease as the time scale increased.

Milani and Ceretta (2014b) used wavelet decomposition to verify the differences in scale of the risk pricing in emerging markets, based on international CAPM model. They verified a Beta tendency to increase at lower frequencies, as well as the model goodness-of-fit (R^2). Their results were consistent with Rua and Nunes (2012) in the sense that the emerging market dependency to the world market is higher at large scales.

Thus, in general, there is certain consensus among the studies, in the sense that betas are higher at low frequencies (large scales), pointing that an asset (or a fund) dependency on the market is stronger and easily verified in the long-run analysis. R^2 are also higher at low frequencies, showing that the market return is more able to explain a stock return in the long run, which may be due to a high degree of speculative behavior at the short-run.

Reboredo and Rivera-Castro (2014) study the impact of oil prices on stock returns in the Europe and USA using the wavelet decomposition. Data used are divided into two periods: june 2000 to june 2008, before the financial crisis, and july 2008 to july 2011, later the financial crisis, at either the aggregate or sectoral level. Results showed that for the first period, that oil price changes had no effect on stock market returns in the pre-crisis period at either the aggregate or sectoral level (with the exception of oil and gas company stock). Considering the levels, with the onset of the financial crisis we found evidence of contagion and positive interdependence between these markets. Since the onset of the financial crisis, oil price leads stock prices and vice versa for higher frequencies, where as for lower frequencies oil and stock prices lead each other.

Zheng and Chen (2014) investigate the stock market forces, used the data influential factors of Dow Jones Industrial Average - DJIA and Shanghai Stock Exchange Composite Index - SSE and the influential factors of the US and China markets are also compared to find differences between the developed market and the emerging market, for period January 2008 to November 2011 considering the after global financial crisis in 2007. The study shows that influential factors are market-dependent and frequency-dependent. The interest rate, oil price, VXD, BDI and EUR/JPY are found to Granger cause the external force of DJIA while SSE is only affected by USD/CNY and international stock markets which are represented by BS&P 500 and HSI. Comparing the US market with the China market, they also found the differences between a developed market and an emerging market. Influential factors tend to be complicated and hard to find in the emerging market due to its immaturity.

Andries, Ihnatov and Tiwari (2014) analyze the relationship between interest rate, stock prices and exchange rates in India, for period of July 1997 and December 2010. They results show that the three variables are linked, and the cross wavelet results show that stock price movements are lagging both to the exchange rate and interest rate fluctuations. The interest rate lead over the stock price movements is even clearer, especially after 2006, and it suggests that the stock market follows the interest rate signals. Comparing results o wavelet coherence and cross wavelet transform, we find very clear results of phase difference of lead–lag relationship between stock prices, exchange rates and interest rates.

Addo, Billio and Guégan (2013) used the wavelet method for detecting financial crisis in stock markets. For this, they used S&P 500 Index and Nasdaq Composite, are consider the daily adjusted closing price, for the period of 2nd January, 1990 to 31st August, 2012. The results of study provides a proposed outline on how to anticipate these rare events and even their impacted before occurrence. The findings from the data analysis with recurrence plots, shows that these plots are robust to extreme values, non stationarity and to the sample, are replicable and transparent, are adaptive to different time series and finally, can provide better chronology of financial cycles since it avoids revision of crisis dates through time.

4 Higher Moments

We intend to use and extended version of CAPM, which incorporates co-skewness and co-kurtosis. If we cannot expect a perfectly normal distribution, the effect of skewness and kurtosis should be considered. Many authors worked in the construction of this model, as Kraus and Litzenberger (1976), Ang and Chua (1979) which included the co-skewness; and Fang and Lai (1997) and Chunhachinda et al. (1997) which included the co-kurtosis. The extended CAPM, which includes co-skewness and co-kurtosis, can be described by Equation (4):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + \gamma_i(r_{M,t} - r_{f,t})^2 + \delta_i(r_{M,t} - r_{f,t})^3 + \varepsilon_{i,t}. \quad (4)$$

Where $r_{i,t}$ is the return of portfolio i; $r_{f,t}$ is the risk-free asset return; $r_{M,t}$ is the market proxy return; α_i is the linear coefficient; β_i is the co-variance coefficient; γ_i is the co-skewness coefficient; δ_i is the co-kurtosis coefficient; $\varepsilon_{i,t}$ is the error term.

5 Data and Method

Our data consists in three time-series: the Ifix index (a Brazilian REITs share return index, used as a proxy), the Imob index (a Brazilian real estate sector index) and the return of Ibovespa Index, used as the market proxy. All the indexes were provided by BM&FBOVESPA, the largest Brazilian stock exchange, and they are dividend-adjusted. The sample period was chosen according to Ifix data availability and it ranges from January 2011 to July 2014, with daily observations.

Our objective is to discover whether Brazilian REITs returns depend on the real estate companies returns or they follow the overall market, or any of the alternatives. We will estimate OLS regressions to verify this dependence, using a model analogous to an extended CAPM, which will capture the possible effect of higher moments, as co-skewness and co-kurtosis. Also, we will wavelet the series, dividing them into scales, to allow us to consider the different relationships that may exist among variables in the different scales. This multiscale analysis will not exclude the original variables analysis, but will be used to improve our investigation.

This way, we will begin our estimations verifying if the Brazilian REITs returns depend on Ibovespa returns, with and extended-CAPM based on Equation (4). We will not use the waveleted series at this point, for comparison purposes. This model can be described by Equation (5).

$$r_{reits,t} = \alpha + \beta_1 r_{ibov,t} + \beta_2 r_{ibov,t}^2 + \beta_3 r_{ibov,t}^3 + \varepsilon_t. \quad (5)$$

Where $r_{reits,t}$ is the REITs return in time t; $r_{ibov,t}$ is the Ibovespa returns in time t; ε_t is the regression error in time t; α, β_1, β_2, β_3 are the linear, co-variance, co-skewness and co-kurtosis coefficients, respectively.

Similarly to Equation (5), we will verify if Brazilian REITs depend on the Imob returns, which represents the Real Estate Companies returns. The model is described by the Equation (6).

$$r_{reits,t} = \alpha + \beta_1 r_{imob,t} + \beta_2 r_{imob,t}^2 + \beta_3 r_{imob,t}^3 + \varepsilon_t. \quad (6)$$

Where $r_{imob,t}$ is the Imob return in time t.

Then, we will finally verify Brazilian REITs returns dependency in a multiscale analysis, using the waveleted series, according to the procedures described in Section 3, by Equations (1), (2) and (3). So, we will make new estimations, analogously to Equations (5) and (6), but with the waveleted series, in order to verify the coefficients differences between each scale. To verify if Brazilian REITs returns depend on Ibovespa returns in different scales, we will estimate Equation (7).

$$r_{reits,t}(\tau_n) = \alpha(\tau_n) + \beta_1 r_{ibov,t}(\tau_n) + \beta_2 r_{ibov,t}^2(\tau_n) + \beta_3 r_{ibov,t}^3(\tau_n) + \varepsilon_t(\tau_n). \quad (7)$$

Where $r_{reits,t}(\tau_n)$ is the REITs returns in time t; in scale n; $r_{ibov,t}(\tau_n)$ is the Ibovespa returns in time t, in scale n; ε_t is the regression error in time t, in scale n; $\alpha(\tau_n)$, β_1, β_2, β_3 are the linear, co-variance, co-skewness and co-kurtosis coefficients, in scale n, respectively.

Similarly, we will verify if Brazilian REITs returns depend on Imob returns in different scales using Equation (8).

$$r_{reits,t}(\tau_n) = \alpha(\tau_n) + \beta_1 r_{imob,t}(\tau_n) + \beta_2 r_{imob,t}^2(\tau_n) + \beta_3 r_{imob,t}^3(\tau_n) + \varepsilon_t(\tau_n). \quad (8)$$

Where $r_{imob,t}(\tau_n)$ is the return of Real Estate Sector companies in time t, in scale n. Section 6 will present and discuss our results.

6 Results

In order to initiate the results discussion, Table (1) presents the summary statistics of the Ifix, Imob and Ibovespa return series. We will also present the Sharpe Index in the same table to simplify the analysis and reduce the quantity of tables.

Table 1 shows that the Ifix series is the only one with positive average return. Also, it presented the smaller standard deviation, what clearly makes that Ifix have the best performance among the analyzed series. This can be verified by the Sharpe Index, which confirms that Ifix have the best performance, followed by Ibovespa and Imob, in this order.

Nonetheless, Ifix presents the higher minimum and the smaller maximum points, showing that the index does not oscillate as much as Imob or Ibovespa. These results mean that the Brazilian REITs in general have been able to comply with the objective of reducing risk of a portfolio, but without decreasing its return. They undoubtedly presented better performance than the Brazilian market proxy, during the studied period.

Table 1 – Summary Statistics for the log-returns for variables Ifix, Imob and Ibov in period January 2011 to July 2014.

Variable	Mean	Minimum	Maximum	Std Deviation	Skewness	Ex. Kurtosis	Sharpe Index
Ifix	0.0004	-0.0266	0.0177	0.0042	-0.2397	339342	0.0913
Imob	-0.0005	-0.0735	0.0733	0.0164	-0.0096	114175	-0.0306
Ibov	-0.0002	-0.0843	0.0498	0.0138	-0.2045	195771	-0.0177

However, it is important to consider that Ifix presented the largest negative skewness coefficient and the largest excess kurtosis, indicating that their returns may be more influenced by these characteristics than Imob and Ibovespa. This situation confirms that it is relevant to verify the influence of higher moments, like co-skewness and co-kurtosis, like it was proposed by Equation (5).

Before we analyze the multiscale regressions, we will present the original series regressions (Equations (5) and (6)), for comparison purposes, in Table 2.

Table 2 – Estimated Coefficients of Equations (5) and (6) for the dependent variable REITs.

Equation	Independent Variable	Coefficient	t-value	p-value	Adjusted R^2
(5)	Constant	0.0005	2.7433	**0.0062**	0.0119
	r_{ibov}	0.0235	1.8445	0.0654	
	$r^2{}_{ibov}$	-0.2960	-0.6325	0.5272	
	$r^3{}_{ibov}$	8.9597	1.0205	0.3078	
(6)	Constant	0.0003	2.0450	**0.0412**	0.0093
	r_{imob}	0.0155	1.3475	0.1782	
	$r^2{}_{imob}$	0.2425	0.8117	0.4172	
	$r^3{}_{imob}$	9.5072	1.4011	0.1615	

The OLS regressions with the original series generated significant ($p<0.05$) but very small linear coefficients. This is reflected by the small R^2 coefficients, which confirms that the model has a poor explanatory power. So, we cannot consider this explanation relevant from an economic perspective. It is also important to point that Ibovespa explains Ifix better than Imob, showing that the Brazilian REITs performance is more influenced by the overall spot market than by the real estate companies return.

Therefore, the question about what drives the REITs performance cannot be answered by these variables, at least not if they are not divided into time scales. The wavelets transformation allowed us to bring a new perspective into the problem, considering the different variable relationships may exist according to different scales.

To extend our analysis, the Ifix, Imob and Ibovespa time series were divided into 7 time scales, which ranges from 2-4 days (high frequency/small scale) to 128-256 days (low frequency/large scale). Also, we included the co-skewness and co-curtosis variables in the model, according to Equation (4). Table (3) presents the estimated coefficients of the model that explains Ifix return by the Ibovespa return, as described by Equation (7).

Table 3 – Estimated coefficients of Equation (7) for the dependent variable REITs.

	Independent Variable	Coefficient	t-value	p-value	Adjusted R^2
D1 (2-4 days)					
$r_{reits} w_1$	$\text{Constant}_{(w1)}$	0.0000	-0.0000	1.0000	0.0112
	$r_{ibov}(w_1)$	0.0001	0.0001	0.9999	
	$r^2_{ibov}(w_1)$	0.0233	0.9257	0.3549	
	$r^3_{ibov}(w_1)$	17.6479	4.4130	**0.0001**	
D3 (8-16 days)					
$r_{reits} w_3$	$\text{Constant}_{(w3)}$	0.0000	-0.0000	1.0000	0.0244
	$r_{ibov}(w_3)$	0.0466	2.0881	**0.0371**	
	$r^2_{ibov}(w_3)$	-0.0097	-0.3489	0.7273	
	$r^3_{ibov}(w_3)$	-4.3469	-0.8503	0.3954	
D5 (32-64 days)					
$r_{reits} w_5$	$\text{Constant}_{(w5)}$	0.0000	-0.0000	1.0000	0.0578
	$r_{ibov}(w_5)$	0.0645	1.3543	0.1760	
	$r^2_{ibov}(w_5)$	-0.0831	-1.6201	0.1056	
	$r^3_{ibov}(w_5)$	19.7407	1.5917	0.1118	
D7 (128-256 days)					
$r_{reits} w_7$	$\text{Constant}_{(w7)}$	0.0000	-0.0000	1.0000	0.2292
	$r_{ibov}(w_7)$	0.2515	4.6133	**0.0001**	
	$r^2_{ibov}(w_7)$	-0.0695	-1.7301	0.0840	
	$r^3_{ibov}(w_7)$	-46.7894	-2.8041	**0.0052**	

Only four of the seven crystals generated by the wavelet estimation were used to estimate the Equation (7) coefficients. We choose to use only the intermediate wavelet series to reduce the amount of information to analyze.

In the D1 crystal, which represents the smaller scale (2-4 days), the estimation of Equation (7) generated only one significant coefficient ($\beta_{3,ibov}(\tau_1)$). In the short run, Ifix returns cannot be explained directly by Ibovespa returns nor by co-skewness, but only by the co-kurtosis. So, in the short run Brazilian REITs return does not react proportionally to the market movements, but they may react highly to the most accentuated movements. We can assume that, in this case, Brazilian REITs cannot be priced by Ibovespa, but by its volatility.

The regressions with the D3 crystal variables also generated a significant coefficient: $\beta_{1,ibov}(\tau_3)$, what means that in that scale (8-16 days) Brazilian REITs are explained by the market. The D5 (32-64) crystal regressions did not generated significant coefficients. It is important to note that the regressions in these scales generated low R^2 coefficients, due to their poor explanatory power.

However, in the largest scale, represented by the D7 crystal, the R^2 coefficient is 0.2292, indicating that in the long run Equation (7) is more efficient to explain the Brazilian REITs return. There is two significant coefficients: $\beta_{1,ibov}(\tau_7)$ and $\beta_{3,ibov}(\tau_7)$ (negatively), showing that in the long run Brazilian REITs return can in fact be explained by Ibovespa return, in a considerable proportion (0.25). Real Estate investments are not completely independent from the stock market and the market risk may influence them, especially because these REITs have traded shares.

The co-kurtosis coefficient is negative in this scale, showing that in the long run REITs react negatively to market volatility, differently from other kinds of funds. This is possibly related to the REITs investor’s profile, which try to avoid risky investments. The fact that Brazilian REITs are priced by the stock market presents a counterpoint: market volatility influences their price negatively.

It is interesting to note that the estimation of Equation (7) did not generated linear coefficients, as it was in the estimation of Equation (5) with the non-waveleted series. We believe that the wavelets decomposition showed some relations that were masked before, when we analyzed the original variables. Now we can understand that what was considered linear before is actually explained by a multiscale analysis.

Table 4 will present the estimated coefficients of Equation (8), which explains Brazilian REITs returns by the Real Estate sector companies returns.

Table 4 – Estimated coefficients of Equation (8) for the dependent variable REITs.

Dependent Variable	Independent Variable	Coeff.	t-value	p-value	Adjusted R^2
D1 (2-4 days)					
	Constant$_{(w1)}$	0.0000	11.0013	1.0000	
	$r_{imob}(w_1)$	-0.0015	-0.1002	0.9202	0.0027
$r_{reits}w_1$	$r^2_{imob}(w_1)$	-0.0142	-0.6684	0.5041	
	$r^3_{imob}(w_1)$	11.0013	1.7298	0.0840	
D3 (8-16 days)					
	Constant$_{(w3)}$	0.0000	-0.0000	1.0000	
	$r_{imob}(w_3)$	-0.0235	-2.1814	**0.0294**	
$r_{reits}w_3$	$r^2_{imob}(w_3)$	-0.0033	-0.1651	0.8689	0.0325
	$r^3_{imob}(w_3)$	-2.8246	-0.6503	0.5157	
D5 (32-64 days)					
	Constant$_{(w5)}$	0.0000	-0.0000	1.0000	
	$r_{imob}(w_5)$	0.0753	1.6973	0.0900	0.1105
$r_{reits}w_5$	$r^2_{imob}(w_5)$	-0.1093	-2.1267	**0.0337**	
	$r^3_{imob}(w\ )$	6.4480	0.3106	0.7562	
D7 (128-256 days)					
	Constant$_{(w7)}$	0.0000	-0.0000	1.0000	
	$r_{imob}(w_7)$	-0.3732	-5.6015	**0.0001**	0.3810
$r_{reits}w_7$	$r^2_{imob}(w_7)$	0.1029	4.1234	**0.0001**	
	$r^3_{imob}(w_7)$	528.982	8.0983	**0.0001**	

The regressions with the D1 crystal variables did not generated any significant coefficients, showing that the Real Estate sector companies returns does not explain REITs returns in the short run. However, the regressions with the D3 crystals variables generated one significant coefficient: ($\beta_{1,imob}(\tau_3)$), which is small and negative. Also, the regressions with the D5 crystal variables generated only one significant coefficient: ($\beta_{2,imob}(\tau_5)$), which influences REITs returns negatively.

These results, similarly to Equation (7) coefficients, show that when we divide our analysis in scales, it is possible to perceive some relationships between variables that were previously masked by the use of non-waveleted variables. We can also perceive that, similarly to Equation (7) coefficients, in the long run the dependent variables tend to explain the Brazilian REITs better than in the short run.

All the coefficients are significant in the estimations with the D7 crystal variables, except for the linear coefficients. This Estimation generated the highest R^2 coefficient of all, including the results presented in Tables 2 and 3. So, these are the equation and scale that best explain REITs return. Brazilian REITs returns can be explained by Real Estate Companies returns in the long run, better than by the stock market returns. In fact, Table 2 results made us to believe that the stock market returns were more important to explain REITs returns, showing that a multiscale analysis can open a new perspective above the problem.

It is interesting to note that the linear coefficient, which was significant in the estimation with the original variables, is not significant when we divide our analysis into scales, showing that assuming homogenous expectations, i.e., assuming that all investors have the same time horizon, can lead to erroneous conclusions.

To finish our paper, Section 7 will finally summarize our findings and to define some conclusions.

7 Final Considerations

This paper's objective was to discover whether Brazilian REITs returns depend on the real estate companies returns or they follow the overall market, or any of the alternatives. To improve our analysis, we used a model analogous to an extended-CAPM, with higher moments. Also, we divided our sample into scales, based on the wavelet transformation, in order to consider the possibility of different relationships between the variables in different time scales.

We used the Ifix index as a proxy for Brazilian REITs and we verified if its returns could be explained by Ibovespa returns and Imob returns, which represents the returns of the Real Estate sector companies.

The summary statistics showed that Brazilian REITs present better performance than the Ibovespa index and the Imob index, when we consider the mean returns, standard deviation and sharpe index. Furthermore, Ifix presented larger skewness and kurtosis coefficients, what means that REITs returns could be influenced by these characteristics.

The regressions with the original variables, which tried to explain Ifix returns by Ibovespa and Imob returns, did not generated significant coefficients, except for the linear coefficients. So, at first, it did not seem that Ifix could be influenced by Ibovespa or Imob. However, the multiscale analysis brought a new perspective onto the problem.

Equation (7) coefficients showed that in the short run (D1 crystal), there is a significant co-kurtosis coefficient, indicating that Brazilian REITs returns depend on market volatility; in th

e D3 crystal, Brazilian REITs returns depend on market return; in the D5 crystal there was no significant coefficient; in the D7 crystal Brazilian REITs depend on market return and market volatility. We should emphasize that in the large scale crystal (D7) the R^2 coefficient is 0,2292, i.e., the model has a considerably better explanatory power. Although it seems that the market did not influenced REITs returns, in the long run they do. Moreover, market volatility has a negative effect in REITs returns, in the long run, probably because a REITs investor try to avoid risk.

We also verify if REITs returns are influenced by Real Estate companies returns, in different scales. Equation (8) did not generate significant coefficients in the D1 crystal; in the D3 crystal, market returns influences REITs returns; in the D5 crystal, there is a significant and negative co-skewness coefficient. This may happens because Imob, Ibovespa and, mainly, Ifix presents a largely negative skewness coefficient. So, if there is negative skewness, co-skewness would be harmful for REITs returns.

In the D7 crystal we perceive that all coefficients were significant, except for the linear. In the long run, Real Estate Companies returns explains better REITs returns, what can be confirmed by the R^2 coefficient of 0,3810. It is interesting to note that $\beta_{1,imob}(\tau_7)$ coefficient is negative, maybe because in the long run, REITs investors change their investments to direct Real Estate if those are presenting better returns. This would reduce REITs demand and, consequently, their price.

In a general way, this study showed that the relationships between REITs returns, Real Estate Companies returns and Market returns depend on the time scale analyzed, but they tend to be stronger at large scales. Differently of what was found by Milani and Ceretta (2014), Brazilian REITs returns can be better explained by Real Estate Sector Companies returns than by market returns, in the long run. Also, higher moments influence their returns and should be part of a model that tries to explain REITs returns.

References:

[1] Abugri, B.; Dutta, S. 2014. Are we overestimating REIT idiosyncratic risk? Analysis of pricing effects and persistence. *International Review of Economics and Finance*, v. 29, pp. 249–259.
[2] Addo, P.; Billio, M.; Guégan, D. 2013. Nonlinear dynamics and recurrence plots for detecting financial crisis. *North American Journal of Economics and Finance*, v. 26, pp. 416–435.

[3] Andries, A.; Ihnatov, I.; Tiwari, A. 2014. Analyzing time–frequency relationship between interest rate, stock price and exchange rate through continuous wavelet. *Economic Modelling*, v. 41, pp. 227–238.

[4] Ang, J.; Chua, J. 1979. Composite measures for the evaluation of investment performance. *Journal of Financial and Quantitative Analysis,* v. 14, pp. 361–384.

[5] Bianchi, D.; Guidolin, M. 2014. Can linear predictability models time bull and bear real estate markets? Out-of-sample evidence from REIT portfolios. *of Real Estate Finance an Economic,* v. 49, pp. 116–164.

[6] Brauers, M.; Thomas, M.; Zietz, J. 2014. Are there rational bubbles in REITs? New evidence from a complex systems approach. *Journal of Real Estate Finance and Economics,* v. 49, pp. 165–184.

[7] Bollerslev,T., 1986.Generalized autoregressive conditional heterosdasticity. *Journal of Econometrics,* v. 31, pp. 307–327.

[8] Boudry, W. I.; Coulson, N. E.; Kallberg, J. G.; Liu, C. H. 2012. On the hybrid nature of REITs. *Journal of Real Estate Financial Economics*, v. 44, pp. 230–249.

[9] Case, B.; Yang, Y.; Yildirim, Y. 2012. Dynamic correlations among asset classes: REIT and stock returns. *Journal of Real Estate Financial Economics*, v. 44, pp. 298– 318.

[10] Chen, M.; Peng, C.; Shyu, S.; Zeng, J. 2012. Market States and the effect of equity REIT returns due to changes in monetary policy stance. *Journal of Real Estate Financial Economics*, v. 45, pp. 364–382.

[11] Chiang, M.; Tsai, I.; Sing, T. 2013. Are REITs a good shelter from financial crises? Evidence from Asian Markets. *Journal of Property Investment & Finance*, v. 31, i. 3, pp. 237–253.

[12] Chunhachinda, P.; Dandapani, K.; Hamid, S.; Prakash, A. 1997. Portfolio selection and skewness: evidence from international stock markets. *Journal of Banking & Finance,* v. 21, pp. 143–167.

[13] Cifter, A.; Özün, A. 2007. Multiscale Systematic Risk: an application on ISE-30. *Munich Personal RePEc Archive.*

[14] Conlon, T.; Crane, M.; Ruskin, H. 2008. Wavelet mustiscale analysis for Hedge Funds: scaling and strategies. *Physica A*, v. 21, pp. 5197–5204.

[15] Deo, M.; Shah, A. 2012. Wavelet-based Beta estimation: applications to Indian stock market. *Kuwait Chapter of Arabian Journal of Business and Management Review,* v. 1, n. 11.

[16] Engle, R.F., 1982. Autoregressive conditional Heteroskedacity with the estimates of variance of the United Kingdon inflation. *Econometrica*, v. 50, n. 4, p. 987–1008.

[17] Engle, R.; Sheppard, K., 2001.Theoretical and Empirical properties of Dynamic Conditional Correlation Multivariate GARCH, *NBER Working Paper,* n. 8554.

[18] Engle, R.F., 2002. Dynamic Conditional Correlation: a simple class of multivariate garch models. *Journal of Business and Economic Statistics*, v. 20, pp. 339–350.

[19] Fang, H.; Lai, T. 1997. Co-Kurtosis and Capital Asset Pricing. *Financial Review,* v. 32, pp. 293–307.

[20] Fei, P.; Ding, L.; Deng, Y., 2010. Correlation and volatility dynamics in REIT returns: performance and portfolio considerations. *The Journal of Portfolio Management*, v. *36*, pp. 113–125.

[21] Fernandez, V. 2006. The international CAPM and a wavelet-based decomposition of value at risk. *National Bureau of Economic Research*. Cambridge, Massachusetts.

[22] Francq, C.; Zakoian, J. 2010. Inconsistency of the MLE and inference based on weighted LS for LARCH models. *Journal of Econometrics*, v. 159, i. 1, pp. 151–165.

[23] Gençay, R.; Selçuk, F.; Whitcher, B. 2001. An introduction to wavelets and other filtering methods in finance and economics. Academic Press, San Diego.

[24] Gençay, R.; Selçuk, F.; Whitcher, B. 2005. Multiscale systematic risk. *Journal of International Money and Finance*, v. 24, pp. 55–70.

[25] Glosten, L.; Jagannathan, R; Runkle, D. 1993.On the relation between the expected value and volatility of the nominal excess return on stocks. *Journal of Finance,* v. 48, pp. 1779–1801.

[26] Hoesli, M.; Oikarinen, E. 2012. Are REITS real estate? Evidence from international sector data. *Journal of International Money and Finance,* v. 31, pp. 1823–1850.

[27] Hsu, C. C.; Tseng, C. P.; Wang, Y. H., 2008.Dynamic hedging with Futures: a copula-based GARCH model. *The Journal of Futures Markets*, v. 28, pp. 1095–1116.

[28] Huang, J.; Zhong, Z. (Ken), 2013. Time variation in diversification benefits of commodity, REITs, and TIPs. *Journal of Real Estate Financial Economics,* v. 46, pp. 152–192.

[29] Huang, M.; Wu, C. 2013. Economic benefits and determinants of extreme dependences between REIT and stock returns. *Review of Quantitative Finance and Accounting,* v. 40, pp. 1 – 29.

[30] In, F.; Kim, S. 2014. An introduction to wavelet theory in finance: a wavelet multiscale approach. World Scientific Publishing.
[31] Kraus, A.; Lintzenberger, R. 1976. Skewness preference and the valuation of risk assets. *The Journal of Finance*, v. 31, pp. 1085–1100.
[32] Lu, C; Tse, Y.; Williams, M. 2013. Return transmissions, value at risk, and diversification benefits in international REITs: evidence from the financial crisis. *Review of Quantitative Finance and Accounting*, v.40, pp. 293–318.
[33] Masih, M.; Alzahrani, M.; Al-Titi, O. 2010. Systematic risk and time scales: new evidence from an application of wavelet approach to the emerging Gulf stock markets. *International Review of Financial Analysis*, v. 19, pp. 10–18.
[34] Milani, B.; Ceretta, P.S. 2013. Do Brazilian REITs depend on Real Estate sector companies or Overall Market? *Economics Bulletin*, v. 33, nº. 4, pp. 2948–2957.
[35] ________________. 2014. Dynamic Correlation between Share returns, NAV variation and Market proxy of Brazilian ETFs. *Inzinerine Ekonomika*, v. 25, pp. 21–30.
[36] _________________. 2014b. A multiscale approach to emerging market pricing. *Economics Bulletin*, v. 34, pp. 784–792.
[37] Parker, D. 2011. Global Real Investment Trusts: people, process and management. 1st ed. Blackwell, West Sussex.
[38] Reboredo, J.; Rivera-Castro, M. 2014. Wavelet-based evidence of the impact of oil prices on stock returns. *International Review of Economics and Finance*, v. 29, pp. 145–176.
[39] Rees, P.; Selcuk-Kestel, S. 2014. Analysis of portfolio diversification between REIT assets. *Journal of Computational and Applied Mathematics*, pp. 425–433.
[40] Rhaeim, N.; Ammoudn, S.; Mabrouk, A. 2007. Wavelet estimation of systematic risk at different time scales: application to French stock market. *The International Journal of Applied Economics and Finance*, v. 3, pp. 189–195.
[41] Rua, A.; Nunes, L. C. 2009. International Co-movement of stock market returns: A wavelet Analysis. *Journal of Empirical Finance*, v. 16, i. 4, pp. 632-639.
[42] Rua, A.; Nunes, L. C. 2012. A wavelet-based assessment of market-risk: The emerging markets case. *The Quarterly Review of Economics and Finance*, v. 52, pp. 84-92.
[43] Todorov, G.; Bidarkota, P. V. 2011. Time-Varying Risk and Risk Premiums in Frontier Markets. *SSRN*, 2011. Available at: http://ssrn.com/abstract=1947412.
[44] Tse, Y. K.; Tsui, A. K. C., 2002.A multivariate GARCH model with time-varying correlations.*Journal of Business and Economic Statistics*, v. 20, pp. 351–362.
[45] Wiley, J. 2014. Illiquidity Risk in Non-Listed Funds: Evidence from REIT Fund Exits and Redemption Suspensions. *Journal of Real Estate Finance and Economics*. v. 49, pp. 205–236.
[46] Zheng, X.; Chen, B. 2014. Identification of stock market forces in the system adaptation framework. *Information Sciences*, v. 265, pp. 105–122.
[47] Zhou, J.; Anderson, R. 2013. An Empirical Investigation of Herding Behavior in the U.S. REIT Market. *Journal of Real Estate Finance and Economic,*. v. 47, pp. 83–108.
[48] Zhou, J.; Kang, Z. 2011. A Comparison of Alternative Forecast Models of REIT Volatility. *Journal of Real Estate Finance and Economics*, v. 42, pp. 275–294.