Assets Value Change in the Accounting Equation

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Abstract: The purpose of the study is to analyze the accounting equation and the relationship between assets and claims on assets (liabilities plus stockholders' equity), based on the dual concept of monetary units. The method is rationalistic, analytical and deductive, focusing on the implications that the dual concept has for the accounting equation. The analysis consists of the application of the identity and characteristic functions along with a coordinate transformation, to show that the assets-claims on assets relationship involves a change in assets value. The identity function represents the dual concept of monetary unit and the characteristic function relates both sides of the equation linking accounts whose units are partially identical. A reformulation of the accounting equation is made; the new equation has different number of terms on each side. Finally, a coordinate transformation for the assets side terms is done. Results show that the accounting equation consists of a series of addition functions, which take into account the dual concept of monetary units. A different number of coordinate dimensions arises on each side of the final equation, and a coordinate transformation is applied to have both sides of the equation with two dimensions. This transformation results in a change in the value of assets.

Keywords: Dual aspects, accounting transactions, accounting equation, balance sheet

1 Introduction

This paper addresses the issue of the equality of assets to claims on assets in the balance sheets of financial statements. The idea that assets are equal to liabilities plus stockholders´ equity (claims on the assets) is crucial in financial statements. The bases for this idea are the dual concept of monetary units, the dual aspect of the accounting transactions, the double-entry bookkeeping, and the accounting equation.

An accounting transaction must be recorded in two accounts with different signs in a double classification system [1]—i.e. the recording of monetary units in two accounts with opposite signs. That is the recognition of its duality: every single transaction has two different qualities, such as being a debit and a credit simultaneously.

In everyday practice, the double-entry bookkeeping system is the way to register the accounting transactions in financial statements, based on their dual aspects.

On the other hand, the dual concept is the recognition of the duality of monetary units: every monetary unit is an asset and a claim on asset simultaneously. Finally, the accounting equation is a checkpoint of the balance sheet correspondence with the accounting assumption and practice; besides, it is the ultimate expression of the dual concept. In this regard, the accounting equation could be a formula or the expression of relevant concepts that represent real world relationships [2], and its relevance is recognized in many practice operations [3] and education [4, 5, 6].

The accounting equation expresses the assets claims on assets equality and is a consequence of the dual concept of monetary units, the dual aspects of the accounting transactions and the double-entry bookkeeping. In fact, the accounting equation can be rewritten to represent the double-entry system [7].

One can consider the following conceptual distinctions: a) The dual concept of monetary units (duality principle, duality concept or duality assumption, as it is also known). It is an assumption or axiom that expresses the assets claims on assets equality; b) The dual aspects of the accounting transactions: it is a convention and a consequence of the duality concept. It is also a definition in the axiomatic system; and c) The double-entry bookkeeping system. It is a set of rules governing the accounting practice.

The axiomatic method was used to point out the importance of the duality approach [8, pp. 101–105] and also to build new systems [1, 8, 9, 10, 11, 12, 13, 14, see 15, 16 for some discussion], retaining the dual aspect of accounting transactions [see 9].

Other approaches to financial statements provided a different perspective on accounting information, such as quantum accounting [see 17, 18] and triple-entry bookkeeping [19, 20, 21]; however, they did not question the dual concept, the dual aspect of accounting transactions or the accounting equation.

Another way of elaborating financial statements is by fair value accounting, which introduces a market based analysis into the accounting information [22, 23, 24, 25; see 26 for a critique of fair value accounting]. It is somehow a detractor of the dual aspects of accounting transactions, the dual concept, and the accounting equation, but without making it explicit.

Despite the actual accounting equation formulation being crucial for the balance sheet, its logical foundations have been revisited, proposing a paraconsistent relationship between assets and claims on assets [see 27, 28, 29]. Besides, using the axiomatic method, it is shown that the dual concept of monetary units and the dual aspects of the accounting transactions, based on the different structures that assets and claims on the assets have, leads to a relationship between these sets that is neither equal nor equivalent [see 30, 31, 32 for a successive test development].

In previous researches [31, 32], the dual aspects of accounting transactions were taken as the defining aspect of the assets-claims on assets relationship; sometimes, in the axiomatic method, the credit-debit equality was used instead of assets claims on assets equality [see 8 as an example].

However, it is neccesary to redefine this postulate. The reasons for that are that the dual aspects of accounting transactions rely on credit-debit accounts which are on both sides of the equation: both assets and claims on assets have credit-debit accounts. In contrast, assets and claims on assets are clearly separated in the balance sheet. Besides, the assets-claims on assets equality is the final result of the operations associated with the dual aspects of accounting transactions, and not the other way around. It is the assets claims on assets equality which guides the credit-debit operations.

Accordingly, it is appropriate to distinguish the dual concept of the monetary unit as an axiom and the dual aspects of accounting transactions as a defintion. The consequence of redefining the category of these concepts is an axiom-equation congruence, which facilitates the analysis.

Nevertheless, conceptual differences are relevant to the analysis: the accounting equation is a mathematical expression, the dual concept of monetary units is an axiom and the dual aspects of accounting transactions is a definition, so they require different analytical methods. The mathematical formulation of the accounting equation requires analytical methods other than the axiomatic method [see 33 for a preliminary mathematical analysis]. The latter is appropriate to the analysis of axioms and principles of accounting, as well as definitions. Doing it in this way would simplify the understanding of the levels of accounting explanation, resulting in more solid research conclusions.

The accounting equation is the ultimate expression of the monetary unit dual concept, so this axiom must be taken into consideration when analyzing the accounting equation.

Therefore, the purpose of this research is to analyze the accounting equation, taking into consideration the different structures that assets and claims on assets have and the dual concept of monetary units.

2 Problem Formulation

The assets and claims on assets sides of the accounting equation have different item structures; they do not contain the same accounts. However, by the dual concept, they have the same monetary units. It is somehow weird that a single capital can have different structures at the same time and yet continue to be the same capital. This quality must have an impact on the accounting equation analysis.

Therefore it seems sensible to review what the relationship between assets and claims on assets is, considering the uniqueness of the monetary units located on both sides of the accounting equation and the different structures on these sides.

2.1 Methodology

This research uses a rationalistic, analytical and deductive method. It is based on the analysis of theoretical assumptions. It focuses on the accounting equation using its basic mathematical operations. An identity function accounts for the dual concept of monetary units. A characteristic function relates both sides of the equation, identifying the accounts whose monetary units are equal. These functions, as well as the rest of the analysis, make claims on assets as the domain and assets as the range whenever they are defined for the function.

A reformulation of the accounting equation is made based on the analyses. As a result of the new accounting equation, assets must be translated from a three-dimensional coordinate system to a two-dimensional coordinate system. This coordinate transformation shows the change in value in assets between the two systems that come up in the reformulated accounting equation.

In this analysis, the type of accounting valuation method assumed is the book value method.

3 Problem Solution

3.1 The dual concept of monetary units

This analysis will only use the lowest level accounts in a hierarchy of items in the balance sheet; for example, $I_{iLn} \subseteq I_{iLn-1}, \ldots, \subseteq I_{iL3}, \subseteq I_{iL2}, \subseteq I_{iL1}$ is the hierarchy and every I_i account is included in the next higher level account, and so on; I_{iLn} represents the lowest level accounts and I_{iL1} represents the highest level accounts.

This structure has been defined with the axiomatic method [30, 31] and the I_i accounts of the lowest level are those that have monetary units u_i, with no aggregation into higher order accounts.

The lowest level accounts of the equation claims on assets side include accounts C_i (claims on assets accounts with monetary units u_{ic}), and the lowest level accounts of the equation assets side contain accounts A_i (assets accounts with monetary units u_{ia}).

In correspondence with this structure, it must be pointed out that claims on assets do not comprise the typical aggregated liabilities and stockholders' equity items or any other aggregation account, but only the lowest level items located beneath them. Doing it in this manner does not change the result and provides a clearer explanation.

The accounts A_i and C_i are already ordered, because their sequence in the balance sheet follows national or international standards. Thus, an injective function f exists from the accounts of C (claims on assets) and A (assets) on the natural numbers $\mathbb{N}$, f: $C \to \mathbb{N}$, f: $A \to \mathbb{N}$; in this regard, the lowest level accounts $A_i \in A$, and $C_i \in C$, and with $i \in \mathbb{N}$, are defined as finite and countable; however, there must be discussion of this assertion. Therefore, the accounts $(A_1, A_2 \ldots A_n) \in A$ and $(C_1, C_2 \ldots C_n) \in C$ are ordered. Initially, monetary units located in the accounts A_i or C_i do not need to follow any order, but in a later stage of the analysis, they will.

The value of the monetary unit in A_i or C_i is irrelevant; it can be the legal tender or any other, as long as it remains the same for all of the assets and claims on assets accounts in the balance sheet.

By virtue of the dual concept of the monetary unit, which is an accounting axiom [see 31, 32], every monetary unit u_{ia} in an account A_i, is simultaneously located in an account C_i as u_{ic}, but it is still the same monetary unit, so $u_{ia} = u_{ic}$. Therefore, there must be a function that creates a relationship between the monetary units u_{ia} of the accounts A_i and the monetary units u_{ic} of the accounts C_i, based on that characteristic.

Due to the fact that u_{ia} and u_{ic} are the same monetary unit, and each monetary unit is unique and different to the others, even having the same value [see 30, 31, 32], an identity function f_I exists, so that, by this function, for every monetary unit u_{ia} of A_i and every u_{ic} of C_i, $f_I(u_{ic}) = u_{ia}$.

In fact, the accounting equation requires that assets be equal to claims on assets. Its mathematical expression is $A = C$, with C being liabilities (L) plus stockholders' equity (E): $C = L + E$.

The identity function can take two directions. The first takes claims on the assets as the domain and assets as the range: F: $C \to A$. The second one takes assets as the domain and claims on the assets as the range: F: $A \to C$. Neither direction is more relevant, and the dual concept of monetary units does not privilege any direction. Nevertheless, for the purposes of this research, the analysis will assume the function F: $C \to A$.

3.2 The dual concept of monetary units and the assets-claims on assets inequality

The identity function f_I relating claims on assets to assets takes each element u_{ci} of C_i as the first component, and each element u_{ai} of A_i as the second component of the pair (u_{ci}, u_{ai}). To every u_{ci} corresponds an image u_{ai} by the identity function, and $f_I(u_{ci}) = u_{ai}$, with $u_{ci} = u_{ai}$; therefore, there exists a unique monetary unit u_i, named u_{ci} in C_i and u_{ai} in A_i, located in a claim on assets and assets accounts simultaneously. That is in accordance with the dual concept of monetary units, so the identity function denotes that a monetary unit is equal to itself despite its location in different and opposite places simultaneously.

Additionally, the monetary units u_{ci} of a single C_i are distributed in several A_i, i.e. the range of the function f for a C_i is not in a single A_i. That is so because financial statements do not classify monetary units by identity groups, but by accounts. Hence, it might be that part of the image of any C_i is in A_i and part in A_j.

Another issue is that the classification is different on the two sides of the equation because assets and claims on assets have different accounts. The number of accounts, their denominations and what they represent are totally different in assets compare to claims on assets. Therefore, to obtain all the images of a C_i, f_I must be a family of functions for that C_i, each function having some (but not all) of the monetary units u_{ci} of C_i (domain) and some (but not all) of the monetary units u_{ai} in an A_i (range). Collecting all the identity images of all u_{ci} for every C_i requires many functions in the form:

F_1: $f_I(u_{c1, \dots,} u_{cn}) = (u_{a1, \dots,} u_{an})$, with $(u_{c1, \dots,} u_{cn}) \subset C_i$ and $(u_{a1, \dots,} u_{an}) \subset A_i$

F_2: $f_I(u_{cn+1, \dots,} u_{cm}) = (u_{a1, \dots,} u_{an})$, with $(u_{cn+1, \dots,} u_{cm}) \subset C_i$ and $(u_{a1, \dots,} u_{an}) \subset A_j$

F_3: $f_I(u_{cm+1, \dots,} u_{cp}) = (u_{a1, \dots,} u_{an})$, with $(u_{cm+1, \dots,} u_{cp}) \subset C_i$ and $(u_{a1, \dots,} u_{an}) \subset A_k$

And so on, until the full domain of C_i is mapped on several *A*s: A_i, A_j, A_k, etc.

In addition, to get some monetary units in each function, the analysis uses the characteristic function 1_{Ci}. For every C_i and A_i, the characteristic function 1_{Ci} assigns '1' to the u_{ai} of every A_i, which are identity images of an u_{ci} located in C_i, and '0' to those that are not. It means that the images of the domain C_i are indexed by '1' in any A_i.

Afterward, those images that are indexed '1' by the characteristic function are multiplied by the value of u_{ai} of A_i, which is the value of the monetary unit in the financial statements and is the same throughout the financial statements. The full expression including the characteristic function and its multiplication by u_{ai} is:

$$u_{ai}(1_{\mathrm{Ci}}(u_{ci})) = u_{ai}(1 \mid u_{ci} = u_{ai}; 0 \mid u_{ci} \neq u_{ai}) \qquad (1)$$

Since the images of all of the monetary units of each C_i are in several A_is, each C_i range, comprising u_{a1}, u_{a2}, u_{a3} … u_{an}, is distributed in various A_is. It is uncommon to find one particular A_i with all the images of a specific domain C_i. The domains and the ranges show no one-to-one correspondence.

Let us use an example: given a C_i with n u_{ci} monetary units, there might exist an A_1 with m_1 u_{ai} and another A_2 with m_2 u_{ai} monetary units. Some of the A_1 monetary units are images of some monetary units of C_1 and some of the monetary units of A_2 are images of some monetary units of C_1.

Based on these different ranges, the C_i n u_{ci} monetary units are divided into n_1 and n_2, with $n = n_1 + n_2$, each one corresponding with each range in A_1 and A_2. However, A_1 has more monetary units than those that are images of some of the monetary units of C_i and so the monetary units of A_1 are $m_1 = m_{1c} + m_{1x}$, with m_{1c} being images of some monetary units of C_i and m_{1x} being no images of C_i. Similarly, A_2 has more monetary units than those that are images of some of the monetary units of C_i and so the monetary units of A_2 are $m_2 = m_{2c} + m_{2x}$, with m_{2c} being images of some monetary units of C_i and m_{2x} being no images of C_i.

In this way, the images of the monetary units of C_i are spread over several partial ranges (m_{1c} of A_1 and m_{2c} of A_2).The rest of the monetary units of A_1 and A_2, m_{1x} and m_{2x}, are not taken into consideration for C_i, because they are not images of any monetary unit u_{ci} of C_i; however, they are images of another C_j.

The standard accounting equation adds all the asset values and all the claims on assets values resulting in $A = C$; however, one can add domains, one by one, on the claims on assets side of the equation, and the asset ranges of each domain, one by one, on the other side. This means adding according to the identity of monetary units.

The linear accumulation of a single domain C_i with n monetary units is:

$$SC_i = \sum_{i=1}^{n} u_{ci} \qquad (2)$$

To every C_i there are several A_is with partial ranges of the function f_I, so we can choose the first A_i in the order they are arranged in the balance sheet, with some u_{ai} images of the monetary units u_{ci}; the linear accumulation of the images u_{ai} of this A_i is:

$$SA_i = \sum_{i=1}^{n} u_{ai} \qquad (3)$$

It must be noted that all the monetary units of A_i that are not images of C_i were removed in this A_i by the characteristic function (1), which results in $SA_i < SC_i$ for each A_i range of a domain C_i. Therefore, for every SC_i, SC_i is not equal to any of its partial ranges SA_i.

The addition of the same monetary units—those that are the same in assets and claims on assets, of a domain and its range—is not possible when considering a single domain C_i and a sole partial range A_i for that C_i; that is so because there are other A_i ranges for the domain C_i and other functions f_I are required to have them added, as previously shown.

Extending the previous computation made for a C_i to all the n C_i domains with m elements in each domain, their sum SC_T is:

$$SC_T = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} \tag{4}$$

Applying again one single function f_I that takes every domain C_i and one single partial range A_i but not all of them for each domain, for every domain C_i, the sum SA_p is:

$$SA_p = \sum_{h=1}^{k} \sum_{i=1}^{\exists! n} \sum_{j=1}^{\exists! m} u_{ahij} \tag{5}$$

with k = sequence number for C_i; n = sequence number for A_i; m = number of images in an A_i of some of the monetary units in a C_i; $\exists! m$ = a unique range in a unique A_i for a C_i exists and is found; and $\exists! n$ = the function f_I takes on in a single range A_i for every domain C_i. This formula means that for every C_i, only a single A_i with u_{ai} images of the monetary units u_{ci} of C_i is selected, and only the u_{ai} images of the monetary units u_{ci} of C_i are added up. Hence, the other partial ranges of that C_i are not included in the addition.

Therefore, as happens with a single C_i in (2) and (3), and in general:

$$A_s \neq C_s \tag{6}$$

Putting together (4) and (5) and expressing the inequality in sums:

$$\sum_{h=1}^{k} \sum_{i=1}^{\exists! n} \sum_{j=1}^{\exists! m} u_{ahij} \neq \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} \tag{7}$$

The inequality arises from the fact that the images of every C_i are distributed in partial ranges A_i and a run of the function f_I only takes one partial range in an A_i for each C_i. In doing so, the accounting equation in its standard formulation does not reflect the real relationship between assets and claims on assets, which is, actually, an inequality. The standard equation is:

$$A_s \neq C_s \tag{8}$$

$$C_s = L+E \tag{9}$$

$$A_s \neq L+E \tag{10}$$

Typically, $A_s < C_s$ because ranges are restricted to those obtained in a unique f_I for each C_i. For both sides to be equal, the range that was obtained should be multiplied by a coefficient to artificially increase its value.

However, this result means that, when introducing the dual concept of monetary units, an accounting equation is not a simple summation but a more complicated computation.

3.3 The assets claims on assets relationship in the accounting equation as a series of functions

First of all, to preserve the equality of the accounting equation, it must be noted that it is not possible to obtain the total value of assets by adding all A_i in the usual form of the standard accounting equation:

$$A_s = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ahij} \tag{11}$$

with n = sequence number for A_i; m = number of images of any C_i in a A_i. That is so because the addition based on the dual concept of the monetary unit must follow the financial statement's classification and its corresponding function domains and ranges, and not add all the ranges of different functions in one run. In other words, it is not possible to sum up all the u_{ai} monetary units of all the A_is as is done in the standard form in (11).

The addition of the ranges (a procedure with several runs) must stop in every run once the function finds a partial range in an A_i for a C_i. That is the equivalent to the standard form of the accounting equation, but taking into account the dual concept of monetary units, domains and ranges. The standard equation runs the sum once for every C_i and for every A_i.

To group together all the m images of each one of the n ranges for a particular C_i, the function f_I must be run n times, to include all the partial functions. In this way, the procedure picks up all the images for a C_i. The recursive procedure to get all the A_i images for all C_i is:

$$A_s = \sum_{h=1}^{k} \sum_{i=1}^{\exists n} \sum_{j=1}^{\exists m} u_{ahij} \tag{12}$$

with k = sequence number for C_i; n = sequence number for A_i; m = number of images in an A_i of the monetary units in a C_i; $\exists m$ = a range in an A_i for a C_i exists and is found, and $\exists n$ = the function f takes, if it exists, in a range in every A_i for every domain C_i. In other words, the procedure takes for every C_i the A_is with images u_{ai} of some monetary units u_{ci} of that particular C_i and adds them up; it is a recursive procedure that runs until all C_i and A_i are scanned.

The difference between equations (12) and (5) must be noted. In the latter, the equation states that a unique range exists in an A_i, so it stops once that range is found, while in (12) it states that a partial range exists in many A_i; the formula (12) allows for running the procedure multiple times for every C_i until all its ranges are found.

Finally, the equation:

$$A_s = C_s \tag{13}$$

results in:

$$\sum_{h=1}^{k} \sum_{i=1}^{\exists n} \sum_{j=1}^{\exists m} u_{ahij} = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} \tag{14}$$

where u_{ahij} is as stated in (1).

The equation (14) is the new expression for the accounting equation when taking into consideration the dual concept of monetary units. The computation results in a more complex series of sums to preserve the equality.

3.4 The assets value transformation in coordinate systems

One can take both sides of the equation (14) as coordinate systems with three assets side and two claims on assets side dimensions. In this manner, a three-dimensional coordinate system (assets) is put in correspondence with a two-dimensional coordinate system (claims on assets).

From this viewpoint, the issue that arises is to transform the three sum terms on the left side into two sum terms to make the coordinate systems equal in dimensions. This transformation could facilitate certain calculations. It must be pointed out that the system to be transformed is the assets three-dimensional coordinate system and not claims on assets, which has two dimensions. Therefore, the assets three-dimensional coordinate system is to be transformed into an assets two-dimensional coordinate system. The claims on assets side remains as it is.

Every dimension in the three-dimensional coordinate system has an order, as previously mentioned C_i accounts are ordered, and an injective function f existing from the C_i accounts, A_i ranges, and u_{ahij} monetary units on the natural numbers $\mathbb{N}$. The functions are as follows: 1) f_1: $C_i \to \mathbb{N}$, with C_i accounts ordered by location; 2) f_2: $A_i \to \mathbb{N}$, with A_i ranges ordered by the sequence of search for every C_i; and 3) f_3: $u_{ahij} \to \mathbb{N}$, with u_{ahij} monetary units in each A_i range; monetary units can be given a sequence order, due to the property that for each pair is (u_{ahij}, u_{ahik}), $u_{ahij} \neq u_{ahik}$ [see 30, 31, 32].

Now, a reference axis in the three-dimensional coordinate system must be chosen. That axis will be taken as the transformation reference; it is like positioning the projection parallel to that axis.

To perform this alignment with the axis in the three-dimensional coordinate system, the reference axis can be C_i; the alignment with the C_i axis allows it to be excluded from the computations. The reason for choosing this alignment is that the C_i axis is an index in the three-dimensional coordinate system of assets, and its characteristics are not subject to change; it is just a direction by which to collect information in the assets system.

The reference C_i axis is to be the y axis in the three-dimensional system. Therefore, it is necessary to project the values in the x (a_x^3 values) and z (a_z^3 values) axis of the three-dimensional system onto the x (a_x^2 values) and y (a_y^2 values) axis in the two-dimensional system. To effect this transformation, the computation must introduce scaling parameters s_i and c_i—slope and constant respectively. Let us define the transformation function g involving two linear equations. They are in the following form:

$$g(a^3) = a^2 = \begin{cases} \forall a_x^3, a_x^2 = s_x a_x^3 + c_x \\ \forall a_z^3, a_y^2 = s_z a_z^3 + c_z \end{cases} \tag{15}$$

Let us take the x (a_x^3 values) axis in the three-dimensional coordinate system as the sequence of all the ordered A_i ranges for every C_i; the z (a_z^3 values) axis is the ordered monetary units u_{ahij} for every range in that system too. Once the y axis (C_i accounts) is removed, and substituting in (15) the computations for each monetary unit, the equations are:

$$g(A^3) = A^2 = \begin{cases} \forall A_x^3, A_x^2 = s_x A_x^3 + c_x \\ \forall u_{ahij}^3, u_{ay}^2 = s_z u_{ahij}^3 + c_z \end{cases} \tag{16}$$

The transformation function g maps a three-dimensional coordinate system onto a two-dimensional coordinate system. To each point in the three-dimensional system, the function g: (A_x^3, u_{ahij}^3) → (A_x^2, u_{ay}^2) creates a set of parameters (s_x, c_x, s_z, c_z). Neverthess, all the monetary units in each pair (A_x^3, u_{ahij}^3) with a particular A_x^3 have the same A_x^2 parameter because they belong to the same partial function (A_{xn}^3, u_{ahij}) → (A_{xn}^2, u_{ay}), with n = a particular A_x^3. Moreover, this coordinate also belongs to a single C_i.

The scaling parameters for each A_x^2 in the two-dimensional coordinate system are s_x and c_x; they are position transformations in an ordered sequence of ranges, starting with the range 1 in A_1^2 for the first pair (A_1^3, u_{ahij}^3) and ending with the last range in an A_n^2 for the last pair (A_n^3, u_{ahij}^3). It is important to note that this transformation does not change the value of the monetary units but the position of the A_x^3 when they become A_x^2; however, that will have an impact when aggregating accounts.

Also, the new u_{ay}^2 values in the y axis are the values of the u_{ay}^2 monetary units of each A_1^2 range in the two-dimensional system, and they are a transformation of the three-dimensional system's original u_{ahij}^3 values. The s_z and c_z scaling parameters are not the same as those of the s_x and c_x for the A_i^3 transformation. Their values come from the fact that the monetary units in any A_i^3 could be images of different C_is, and to collect them involves a family of functions for each C_i (see 12). Hence, the values of s_z and c_z depend on the function g: (A_i^3, u_{ahij}^3) → (A_x^2, u_{ay}^2) for every u_{ahij}^3 monetary unit, and all of these parameters might be different.

Bearing in mind all these considerations, the final accounting equation is as follows:

$$\sum_{x=1}^{n_1}\sum_{y=1}^{m_1} u_{axy}^2 + \cdots + \sum_{x=1}^{n_n}\sum_{y=1}^{m_n} u_{axy}^2 = \sum_{i=1}^{n}\sum_{j=1}^{m} u_{cij} \tag{17}$$

with (i =1, .., n) = sequence number for C_i; (i =1, .., m) = monetary units u_{cj} in each C_i; (x =1, .., n_1/ n_n) = sequence number for A_i^2 ranges in the two-dimensional system; (y =1, .., m_1/m_n) = monetary units u_{axy}^2 in each A_x^2 range in the two-dimensional system.

Substituting the monetary units in the new equation (17) for their coordinate transformation,

$$\sum_{x=1}^{n_1}\sum_{y=1}^{m_1}(s_z u_{ahij}^3 + c_z)_{xy} + \cdots$$
$$\ldots + \sum_{x=1}^{n_n}\sum_{y=1}^{m_n}(s_z u_{ahij}^3 + c_z)_{xy} = \sum_{i=1}^{n}\sum_{j=1}^{m} u_{cij} \tag{18}$$

with (i =1, .., n) = sequence number for C_i; (i =1, .., m) = monetary units u_c in each C_i; (x =1, .., n_1/n_n) = sequence number of $A_x^2 s$ in the two-dimensional coordinate system (which are A_i^2 = $s_x A_i^3$ +c_x); (y =1, .., m_1/m_n) = sequence number of monetary units u_{axy}^2s in the two-dimensional system (which are $u_{ay}^2 = s_z u_{ahij}^3 + c_z$; where u_{ahij} is the monetary unit value in the three-dimensional system).

The s_x, c_x, s_z and c_z parameter values need a definition for each A_x^2 and u_{ay}^2; they depend on every A_x^3 and u_{ahij}^3 and, indirectly, on every C_i for which the A_x^3s are ranges.

This computation results in a change of the assets value, as its monetary units are a linear transformation of the original ones in the three-dimensional coordinate system. In the new equation (18), claims on assets does not change its value, so to keep the equality the asset value must change. Some of the assets can increase their value and others can decrease it to maintain equality with claims on assets.

A transformation from a three-dimensional coordinate system into a two-dimensional coordinate system cannot retain the same scale in both systems. However, at this stage, it is impossible to know what the parameters of all the numerous linear equations are. The capital of a company can be huge, and to keep track of every single monetary unit will be impossible and also unproductive.

The procedure to find the final result must use analytical methods to reach an acceptable solution.

4 Conclusion

By introducing the identity and characteristic functions combined with a coordinate transformation, and based on the dual concept of monetary units, the results confirm that the accounting equation is more than just addition, but a sequence of sum functions with a value transformation. In spite of this conclusion, the method used in this research, led to a satisfactory reformulation of the accounting equation maintaining the assets-claims on assets identity; in this sense, the method was appropriate. Nevertheless, it was done by attributing a changing quality to the value of assets.

The sum functions of the reformulated equation are intended to follow the financial statements classification. This classification is crucial in financial statements and any operation with the accounts, should agree with the way items are ordered.

The reformulated mathematical equation, even though it is just a series of sums and linear transformations, introduces uncertainty about the value that every asset has. This is so because it seems not to be possible to identify the parameters needed for every monetary unit value transformation; as a consequence, the value of assets in the new system is not known at this stage.

The reformulation also introduces uncertainty about the value of the items or accounts included on the assets side of the equation. The only possible conclusion about this value is that assets value will match the claims on assets value, but the magnitude of the internal changes in every account and its monetary units is unknown.

In this regard, more complex analysis must be developed to reach an acceptable solution, and it might happen that this analysis takes into account a different assets and claims on assets nature, very different to the one that they have now.

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