Effects of MHD Visco-Elastic Fluid Flow Past a Moving Plate with Double Diffusive Convection in Presence of Heat Generation

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Abstract:-During the course of present investigation, effects of first order homogeneous chemical reaction and thermal radiation on two dimensional MHD flow of a visco-elastic fluid past a moving porous plate by considering double diffusive convection in presence of heat generation has been analyzed. A uniform magnetic field is applied in the direction normal to the moving plate. Also, the plate is considered to be moving with a constant velocity along the flow field. The present theoretical study has been carried out under perturbation approximation. Also, the expressions for velocity, temperature, concentration, shearing stress, rate of heat transfer and rate of mass transfer have been obtained. The velocity profiles and shearing stress for both heated and cooled plate are depicted for various values of flow parameters to observe the visco-elastic effects. The rate of heat transfer i.e. Nusselt number and the rate of mass transfer i.e. Sherwood number are not significantly affected by the visco-elastic parameter.

Key words:-visco-elastic, Walters liquid (model B′), Hartmann number, Prandtl number, Grashof number, Soret number.

2010 Mathematics Subject Classification 76A10

1 Introduction:

Free convection flow due to radiative heat and mass diffusion in a composite porous medium has gained interest owing to its application in the fields of geophysics, geohydrology, environmental engineering etc. It has also found its application in drying processes, moisture migration in fibrous insulation, nuclear waste disposal and in the control of pollutant spread in ground water or in thermal oil recovery where the process involved the flow of heated fluids. Further, the magneto convection plays an important role in the control of mountain iron flow in the steady industrial liquid metal cooling in nuclear reactors and magnetic separation of molecular semi conducting materials, MHD bearings, radio propagation through the ionosphere etc. Workers like Cowling [1], Singh et al. [3], Geindreau et al. [4], Alam et al. [5], Makinde [6-7] have investigated the effects of magnetic field on natural convection flow past a vertical surface. MHD flows assume greater significance in several biological and engineering systems when the flow is considered visco-elastic over a permeable boundary (Misra et al. [8], Choudhury et al. [9], [10], [11] etc).

Diffusion rates can be changed tremendously with chemical reaction. Many practical diffusive operations such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler involve the molecular diffusion of a species in the presence of chemical reaction within or in the boundary layer. Because of wide application of such flows, numerous scholars have paid their attention. Das et al. [12], Muthucumaraswamy et al.[13] , Ibrahim et al. [14], Chambre [15], Cussler [16], Prasad [17] have studied the effects of chemical reaction on heat and mass transfer flow past a moving vertical surface. Heat and mass diffusing simultaneously give rise to the cross-diffusion effect. The mass transfer caused by the temperature gradient is referred to as the Soret effect, while the heat transfer caused by the concentration gradient is called the Dufour effect. Investigators like Alam et al. [18] , Postelnicu [19] have also shown that Soret mass flux and Dufour energy flux have appreciable and at times significant effect on heat and mass transfer rates. Double diffusive convection driven by buoyancy force due to temperature and concentration gradients in presence of chemical reaction has been studied by many researchers, among them Kafoussias et al.[20] and Mohamed et al. [21] who have studied the effects of MHD double-diffusive convection boundary layer flow past a radiative hot vertical surface in porous media in the presence of chemical reaction. Mohammad [22] has investigated the radiation interaction on unsteady MHD flow past a vertical plate with heat generation including Soret effect due to double diffusive convection. Reddy et al. [23] have considered the MHD flow over a vertical moving porous plate with heat generation by considering double diffusive convection.

In this study, we propose to investigate the effects of MHD visco-elastic fluid flow past a moving plate with double diffusive convection in presence of heat generation by considering chemical reaction and thermal radiation. In the course of analysis it is assumed that the plate is embedded in composite porous medium with constant velocity. Solutions are presented in graphical forms. It is expected that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

The visco-elastic fluid flow is characterized by Walters liquid (Model B'**)**.

The constitutive equation for Walters liquid (Model B') is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik}$$

$$\sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \tag{1.1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \tag{1.2}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \tag{1.3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)d\tau, \tag{1.4}$$

N (τ) being the relaxation spectrum. This idealized model is a valid approximation of Walters's liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau, \; n \geq 2 \tag{1.5}$$

have been neglected.

2 Mathematical Analysis:

The unsteady MHD, visco-elastic fluid flow due to double diffusive convection past a moving porous plate with heat generation has been considered. The flow is two dimensional where x'-axis is along the plane of moving plate and y'- axis is normal to it, respectively. We assume that the surface is moving continuously with constant velocity in the positive x'- direction. A uniform magnetic field of strength B_0 is imposed transversely to the plate. In addition, there is no applied electric field so the Hall effects are neglected. Since the magnetic Reynolds number is very small for most fluids used in industrial applications, we have assumed that the induced magnetic field is negligible. Also, the fluid is assumed to be gray, absorbing radiation and emitting, but non-scattering medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x'-direction is considered negligible in comparison with the y'-direction. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation) and the concentration of species far from the wall is infinitesimally small and the viscous dissipation term in the energy equation is neglected (as the fluid velocity is very low). Taking into consideration the assumptions made above, the governing equations can be written as follows:

Equation of continuity:

$$\frac{\partial v'}{\partial y'} = 0$$

$$\Rightarrow v' = -V_0\left(1 + \varepsilon A e^{i\omega' t'}\right) \qquad (2.1)$$

where A is the suction parameter such that $\varepsilon A \ll 1$ and V_0 is scale of suction velocity. The negative sign indicates that the suction is towards the plate.

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{\partial U_\infty{}'}{\partial t'} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} + \frac{\nu}{K'}\left(U_\infty{}' - u'\right) - \frac{k_0}{\rho}\left(\frac{\partial^3 u'}{\partial y'^2\,\partial t'} + v'\frac{\partial^3 u'}{\partial y'^3}\right) \qquad (2.2)$$

Energy equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho C_p}(T' - T'_\infty) - \frac{1}{\rho C_p}\frac{\partial q'_r}{\partial y'} \qquad (2.3)$$

Concentration equation :

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D_M\frac{\partial^2 C'}{\partial y'^2} + D_T\frac{\partial^2 T'}{\partial y'^2} - K'_l(C' - C'_\infty) \qquad (2.4)$$

Here u', v' are the components of velocity in x' and y' directions respectively, g is the acceleration due to gravity, T' is the temperature of the fluid near the plate, $T_\infty{}'$ is the free stream temperature, C' is concentration, $C_\infty{}'$ is free stream concentration, β and β^* are the thermal and concentration expansion coefficients respectively, K_T is the thermal conductivity, $U_\infty{}'$ is the free stream velocity, C_p is the specific heat of constant pressure, B_0 the magnetic field, ρ the density , q'_r is the radiative heat flux, σ is the magnetic permeability of fluid, V_0 is the constant suction velocity, ν is the kinematic viscosity, D_M is the molecular diffusivity, D_T is the thermal diffusivity , K'_l is chemical reaction parameter, Q_0 is heat generation parameter, K' is the porosity parameter and k_0 is the visco-elastic parameter.

The relevant boundary conditions are

$$y' = 0 \ : \ u' = U'_p,\ T' = T'_w + (T'_w - T'_\infty)e^{i\omega' t'}$$

$$C' = C'_w + (C'_w - C'_\infty)e^{i\omega' t'}$$

$$y' \to \infty : \ u' \to U_\infty{}' = U_0\left(1 + \varepsilon e^{i\omega' t'}\right)$$

$$T' \to T'_\infty,\ C' \to C'_\infty \qquad (2.5)$$

where U'_p is the velocity, T'_w and C'_w the temperature and concentration of the wall respectively.

By using the Rosseland diffusion approximation, the radiative heat flux q'_r is given by

$$q'_r = -\frac{4\sigma^*}{3K_s}\frac{\partial T'^4}{\partial y'} \qquad (2.6)$$

where σ^* and K_s are the Stefan Boltzmann constant and the Rosseland mean absorption coefficient respectively. We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of temperature and

$$T'^4 \approx 4T_\infty{}'^3 T' - 3T_\infty{}'^3 \qquad (2.7)$$

Using (2.6) and (2.7) in the last term of equation (2.3), we obtain

$$\frac{\partial q'_r}{\partial y'} = -\frac{16\sigma^* T_\infty{}'^3}{3K_s}\frac{\partial^2 T'}{\partial y'^2} \qquad (2.8)$$

We now introduce the following non-dimensional quantities:

$$u' = uU_0,\ y' = \frac{y\nu}{V_0}, U'_\infty = U_\infty U_o,\ U'_p = U_p U_o, t' = \frac{t\nu}{V_0{}^2}, \omega' = \frac{V_0{}^2\omega}{\nu}, \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty}, \varphi = \frac{C'-C'_\infty}{C'_w-C'_\infty}, K' = \frac{K\nu^2}{V_0{}^2}$$

$$Sc = \frac{\nu}{D_M}, So = \frac{D_T(T'_w-T'_\infty)}{\nu(C'_w-C'_\infty)}, Pr = \frac{\nu\rho C_p}{K_T}, M = \frac{\sigma B_0^2}{\rho}\frac{\nu}{V_0{}^2}$$

$$Gr = \frac{\nu g\beta(T'_w - T'_\infty)}{V_0{}^2 U_0}, Gm = \frac{\nu g\beta'(C'_w - C'_\infty)}{V_0{}^2 U_0},$$

$$K = \frac{K' U_0{}^2}{\nu^2}, k = \frac{k_0 V_0{}^2}{\rho\nu^2}, \nu = \frac{\eta_0}{\rho}, K'_l = \frac{\gamma V_0{}^2}{\nu},$$

$$Q = \frac{\nu Q_0}{\rho V_0{}^2 C_p}, R = \frac{4\sigma^* T_\infty{}'^3}{K_s}$$

where Pr is the Prandtl number, Gr is the thermal Grashof number, M is the magnetic parameter, S_0 is the Soret number, Ec is the Eckert number, K is the permeability parameter, k is the visco-elastic parameter. Sc is the Schmidt number, Gm is the molecular Grashof number, γ is the chemical reaction parameter, R is the absorption of radiation parameter, Q is the heat source parameter.

The non-dimensional forms of the equations (2.2) - (2.4) are given by

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = Gr\theta + Gm\varphi + \frac{\partial^2 u}{\partial y^2} + \left(M + \frac{1}{K}\right)(U_\infty - u) + \frac{dU_\infty}{dt} - k\left(\frac{\partial^3 u}{\partial y^2\,\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\right) \tag{2.9}$$

$$\frac{\partial\theta}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\left(1 + \frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2} + Q\theta \tag{2.10}$$

$$\frac{\partial\varphi}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial\varphi}{\partial y} = \frac{1}{Sc}\frac{\partial^2\varphi}{\partial y^2} - \gamma\varphi + S_0\frac{\partial^2\theta}{\partial y^2} \tag{2.11}$$

subject to boundary conditions:

$$y = 0 : \ u = U_p, \theta = 1 + \varepsilon e^{i\omega t}, \varphi = 1 + \varepsilon e^{i\omega t}$$

$$y \to \infty : \ u = U_\infty \to \left(1 + \varepsilon e^{i\omega t}\right), \theta \to 0, \varphi \to 0 \tag{2.12}$$

3 Method of solution:

Equations (2.9)-(2.11) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. To solve these equations we make the following assumptions for the non-dimensional free stream velocity U_∞, velocity profile u , temperature profile θ and concentration profile φ.

$$\left.\begin{aligned} U_\infty &= 1 + \varepsilon e^{i\omega t} + o(\varepsilon^2), \\ u(y,t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + o(\varepsilon^2), \\ \theta(y,t) &= \theta_0(y) + \varepsilon e^{i\omega t}\theta_1(y) + o(\varepsilon^2) \\ \varphi(y,t) &= \varphi_0(y) + \varepsilon e^{i\omega t}\varphi_1(y) + o(\varepsilon^2) \end{aligned}\right\} \tag{3.1}$$

On substituting the equations (3.1) into the equations (2.9) - (2.11) and equating the harmonic and non-harmonic terms, and neglecting the higher order terms of $o(\varepsilon^2)$, we obtain

$$ku_0''' + u_0'' + u_0' - \left(M + \tfrac{1}{K}\right)u_0 = -Gr\theta_0 - Gm\varphi_0 - \left(M + \tfrac{1}{K}\right) \tag{3.2}$$

$$ku_1''' + u_1'' - i\omega k u_1'' + u_1' - \left(M + \frac{1}{K} + i\omega\right)u_1 = -Gr\theta_1 - Gm\varphi_1 - \left(M + \tfrac{1}{K} + i\omega\right) - Au_0' - kAu_0''' \tag{3.3}$$

$$\theta_0'' + L\theta_0' + LQ\theta_0 = 0 \tag{3.4}$$

$$\theta_1'' + L\theta_1' + (Q - i\omega)L\theta_1 = -AL\theta_0' \tag{3.5}$$

where $L = \left(\frac{3Pr}{3+4R}\right)$

$$\varphi_0'' + Sc\varphi_0' - \gamma Sc\varphi_0 = -ScS_0\theta_0'' \tag{3.6}$$

$$\varphi_1'' + Sc\varphi_1' - (\gamma Sc + i\omega Sc)\varphi_1 = -ASc\varphi_0' - ScS_0\theta_1'' \quad (3.7)$$

Here prime denotes the differentiation with respect to y.
The modified boundary conditions are:

$$y = 0 : u_0 = U_p, u_1 = 0, \theta_0 = 1, \theta_1 = 1, \varphi_0 = 1, \varphi_1 = 1,$$
$$y \to \infty : u_0 \to 1, u_1 \to 1, \theta_0 \to 0, \theta_1 \to 0, \varphi_0 \to 0, \varphi_1 \to 0 \quad (3.8)$$

Solving equations (3.4)-(3.7) under the boundary condition (3.8) we get the zeroth order and first order solutions for temperature and concentration distribution as follows

$$\theta_0 = e^{-\alpha_2 y}$$
$$\theta_1 = a_1 e^{-\alpha_2 y} + (1 - a_1)e^{-\alpha_4 y}$$
$$\varphi_0 = a_2 e^{-\alpha_2 y} + (1 - a_2)e^{-\alpha_6 y}$$
$$\varphi_1 = A_8 e^{-\alpha_8 y} + a_3 e^{-\alpha_6 y} + a_5 e^{-\alpha_4 y} + a_7 e^{-\alpha_2 y} \quad (3.9)$$

Using (3.9) in (3.1), we get the temperature and concentration distribution as follows

$$\theta = e^{-\alpha_2 y} + \varepsilon e^{i\omega t}\{a_1 e^{-\alpha_2 y} + (1 - a_1)e^{-\alpha_4 y}\}$$

$$\varphi = a_2 e^{-\alpha_2 y} + (1 - a_2)e^{-\alpha_6 y} + \varepsilon e^{i\omega t}\{A_8 e^{-\alpha_8 y} + a_3 e^{-\alpha_6 y} + a_5 e^{-\alpha_4 y} + a_7 e^{-\alpha_2 y}\}$$

Again, in order to solve the equations (3.2) and (3.3), we use multi-parameter perturbation technique in terms of visco-elastic parameter and assuming $k \ll 1$, as the visco-elastic parameter is considered to be less than unity for small shear rate (Nowiski and Ismail [2]), thus we write

$$u_0(y) = u_{00}(y) + k\,u_{01}(y) + o(k^2)$$
$$u_1(y) = u_{10}(y) + k\,u_{11}(y) + o(k^2) \quad (3.10)$$

Using (3.10) in the equations (3.2) to (3.3) and equating the coefficients of like powers of k, we get the following set of differential equations:

$$u_{00}'' + u_{00}' - \left(M + \frac{1}{K}\right)u_{00} = -Gr\theta_0 - Gm\varphi_0 - \left(M + \frac{1}{K}\right),$$

$$u_{01}'' + u_{01}' - \left(M + \frac{1}{K}\right)u_{01} = -u_{00}''',$$

$$u_{10}'' + u_{10}' - \left(M + \frac{1}{K} + i\omega\right)u_{10} = -Gr\theta_1 - Gm\varphi_1 - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right) - Au_{00}',$$

$$u_{11}'' + u_{11}' - \left(M + \frac{1}{K} + i\omega\right)u_{11} = i\omega u_{10}'' - u_{10}''' - Au_{01}' - Au_{00}'''$$

(3.11)

The relevant boundary conditions are:

$$y = 0 : u_{00} = U_p, u_{01} = 0, u_{10} = 0, u_{11} = 0,$$
$$y \to \infty : u_{00} \to 1, u_{01} \to 0, u_{10} \to 1, u_{11} \to 0 \quad (3.12)$$

Now, solving the differential equations (3.11) subject to the boundary conditions (3.12), we get the solutions of zeroth order and first order for velocity profile as

$$u_0 = (1 + A_{10}e^{-\alpha_{10}y} + a_8 e^{-\alpha_2 y} + a_9 e^{-\alpha_6 y} + a_{10}e^{-\alpha_2 y}) + k\{A_{12}e^{-\alpha_{10}y} + a_{11}ye^{-\alpha_{10}y} + a_{12}e^{-\alpha_2 y} + a_{13}e^{-\alpha_6 y} + a_{14}e^{-\alpha_2 y}\}$$

$$u_1 = (1 + A_{14}e^{-\alpha_{12}y} + a_{23}e^{-\alpha_{10}y} + a_{64}e^{-\alpha_2 y} + a_{65}e^{-\alpha_6 y} - a_{21}e^{-\alpha_4 y} - a_{17}e^{-\alpha_8 y}) + k\{A_{16}e^{-\alpha_{12}y} + a_{58}ye^{-\alpha_{12}y} + a_{59}e^{-\alpha_{10}y} + a_{60}e^{-\alpha_2 y} + a_{61}e^{-\alpha_6 y} - a_{62}e^{-\alpha_4 y} - a_{63}e^{-\alpha_8 y} + a_{44}ye^{-\alpha_{10}y}\}$$

Substituting u_0 , u_1 in (3.1), we get the expression for main flow velocity u, but not presented here for the sake of brevity.

The shearing stress at the plate in the dimensionless form is given by

$$\tau = \left(\frac{\tau'}{\rho {U_0}^2}\right)_{y=0}$$

$$= \left\{\frac{\partial u}{\partial y} - k\left[\frac{\partial^2 u}{\partial y\,\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial^2 u}{\partial y^2}\right]\right\}_{\text{at } y=0}$$

$$= a_{66} + \varepsilon e^{i\omega t} a_{66} + k\left(a_{74} + \varepsilon e^{i\omega t} a_{75}\right)$$

Similarly, the rate of heat transfer at the plate in terms of Nusselt number is given by

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$= -\alpha_2 + \varepsilon e^{i\omega t}\{\alpha_4(a_1 - 1) - \alpha_2 a_1\}$$

The mass flux at the wall in terms of Sherwood number is given by

$$Sh = \left(\frac{\partial \varphi}{\partial y}\right)_{\text{at } y=0}$$

$$= -[\{\alpha_6(1 - a_2) + \alpha_2 a_2\}] + \varepsilon e^{i\omega t}\{\alpha_8 A_8 + \alpha_6 a_3 + \alpha_4 a_5 + \alpha_2 a_7\}$$

4 Results & Discussion:

The purpose of this study is to bring out the effects of visco-elastic parameter on the double diffusive convective flow over a moving vertical porous plate in presence of magnetic field and radiative heat transfer as well as mass transfer characteristics as the effects of other parameters has been discussed by Reddy et al. [18]. The non zero values of visco-elastic parameter (k) characterizes the visco-elastic fluid and the corresponding results for Newtonian fluid can be evaluated from the above results by putting k =0 and it is likely to mention that these results coincide with that of Reddy et al. [18]. The velocity field u has been plotted against y and we have considered here two types of flow i.e. one past a heated plate (Gr < 0) and other past a cooled plate (Gr > 0) under various parametric conditions such as M, Pr, Gm, Q, R, S_0 etc.

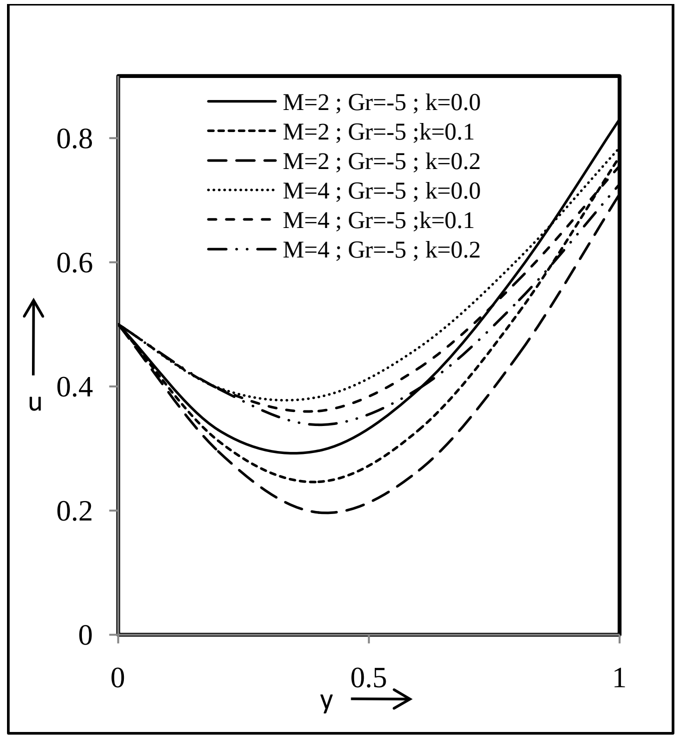

Figure 1: Velocity u against y for Pr=2, Gm=5, R=3, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

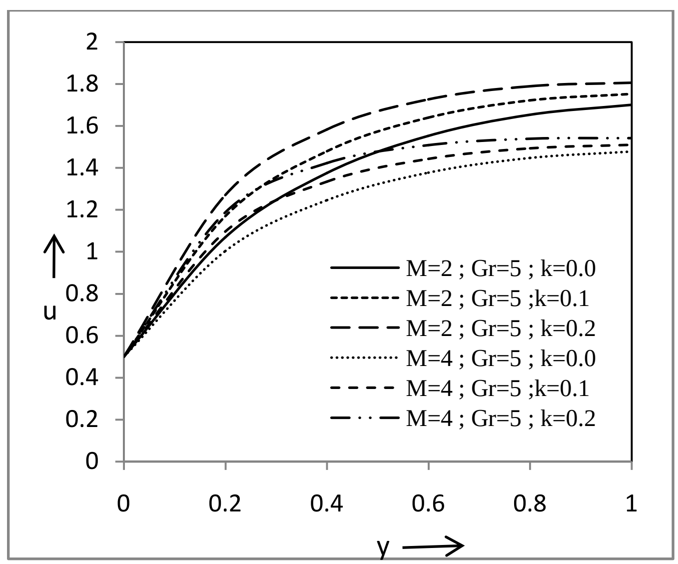

Figure 2: Velocity u against y for Pr=2, Gm=5, R=3, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

The influence of magnetic field parameter on the fluid velocity has been shown in figure 1 & 2, for fluid past a heated plate as well as fluid past a cooled plate. It is observed that with the growth of magnetic parameter, the velocity increases in case of Gr<0 whereas the behavior is found to be opposite totally for Gr>0. This implies that magnetic field tends to accelerate the fluid flow in case of Gr<0 and retard the same for Gr>0.

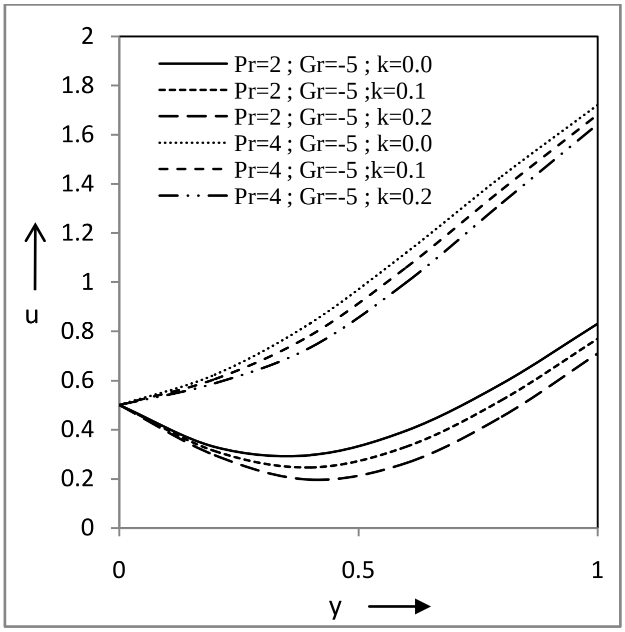

Figure 3: Velocity u against y for M=2, Gm=5, R=3, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

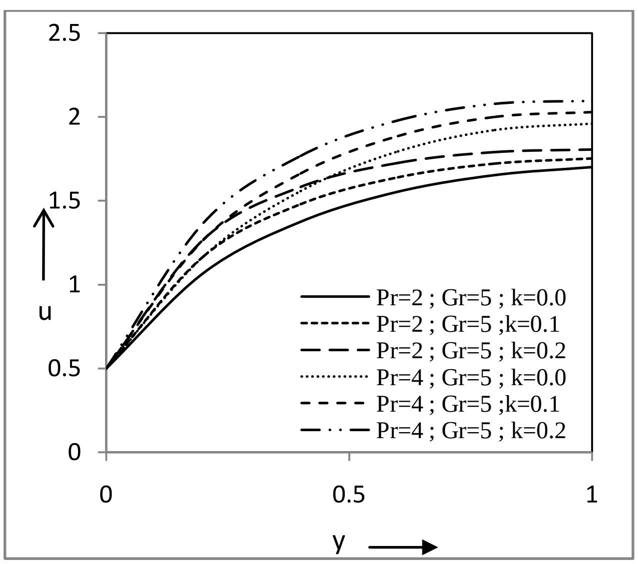

Figure 4: Velocity u against y for M=2, Gm=5, R=3, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

Figures 3-4 depict the effects of Prandtl number on fluid velocity under both the plate conditions. It is noticed that when Prandtl number increases, the fluid flow rises for both plate conditions. And the velocity of Newtonian fluid system for Gr>0 attains a minimum magnitude in comparison with visco-elastic fluid, on the other hand the velocity of Newtonian fluid is in amplified form as compared to visco-elastic fluid for Gr<0.

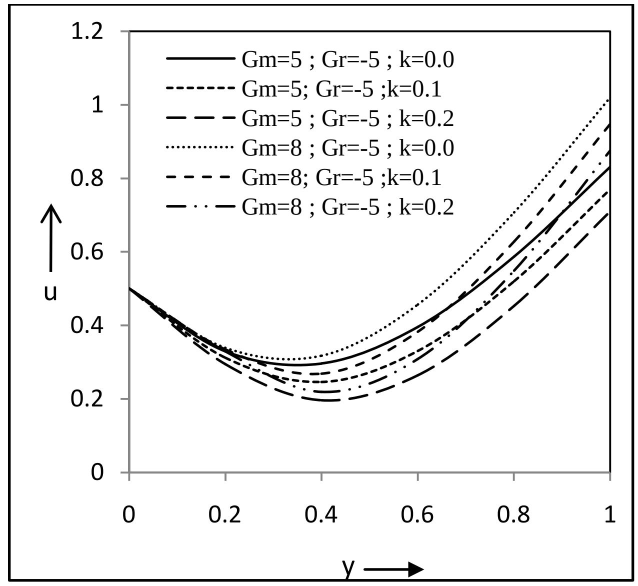

Figure 5: Velocity u against y for M=2, Pr=2, R=3, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

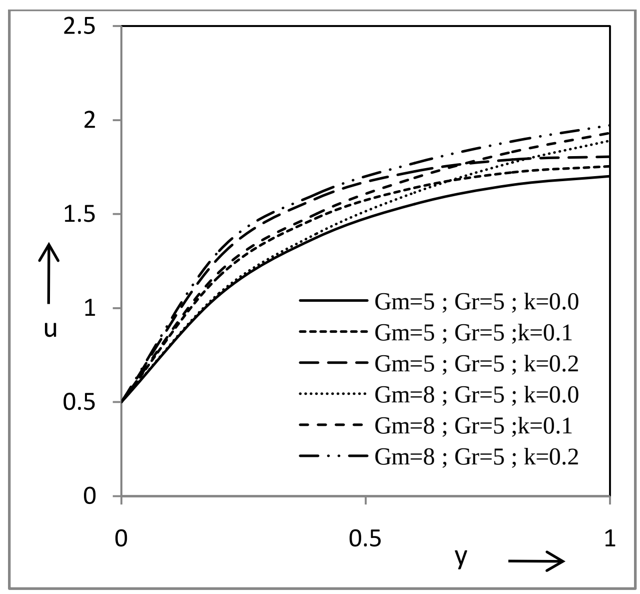

Figure 6: Velocity u against y for M=2, Pr=2, R=3, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

The influence of dimensionless Grashof number for mass transfer Gm on velocity profile has been demonstrated in figures 5 & 6, when both the fluid systems past a heated as well as cooled plate. Also, as the parameter rises, very slight increase of speed of fluid has been notified in the fluid velocity for flow past a heated plate (Gr<0) with the declined pattern of speed of visco-elastic fluid. However, in case of cooled plate when Gm increases the fluid velocity gains it amplification form with the inclined nature of speed of visco-elastic fluid along with the modification of visco-elasticity in comparison with the Newtonian fluid.

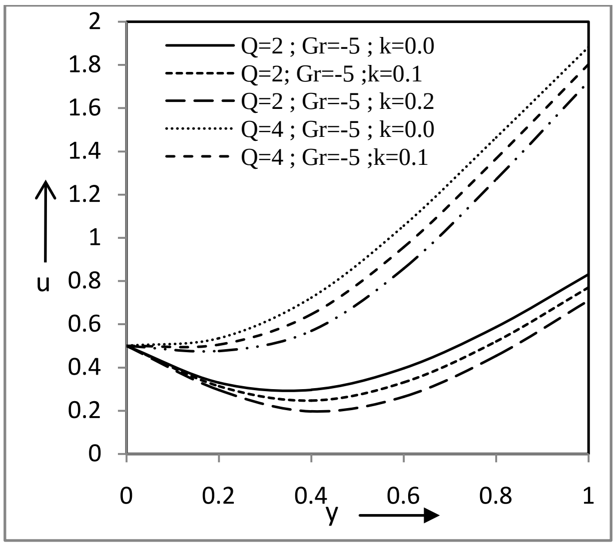

Figure 7: Velocity u against y for M=2, Pr=2, Gm=5, R=3, Sc= 5, S_0= 3, $\gamma = 2$, K=1

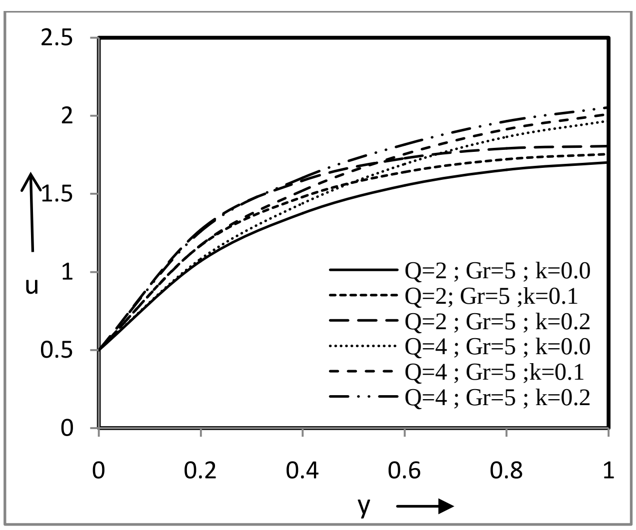

Figure 8: Velocity u against y for M=2, Pr=2, Gm=5, R=3, Sc=5, S_0=3, $\gamma = 2$, K=1

The contribution of the heat generation/absorption parameter on the fluid velocity is illustrated in figures 7 & 8. The effect seems to be more predominant for flow past a heated plate (Gr<0) than the flow past a cooled plate (Gr>0). For Gr<0 these profiles visualize that the speed of the Newtonian fluid grows and subsequently settles down to absolutely high level as compared to visco-elastic fluid with the increasing values of heat generation parameter. Whereas, for Gr>0, the modification of visco-elastic parameter increases the complex fluid as compared to simple fluid with Q.

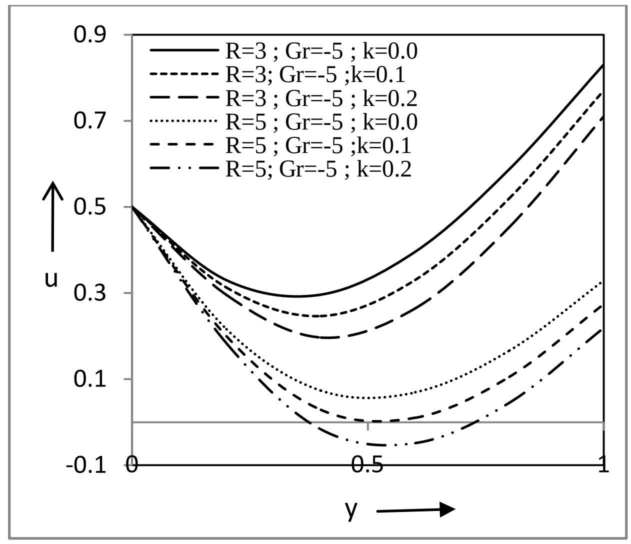

Figure 9: Velocity u against y for M=2, Pr=2, Gm=5, Sc= 5, S_0= 3, $\gamma = 2$, Q=2, K=1

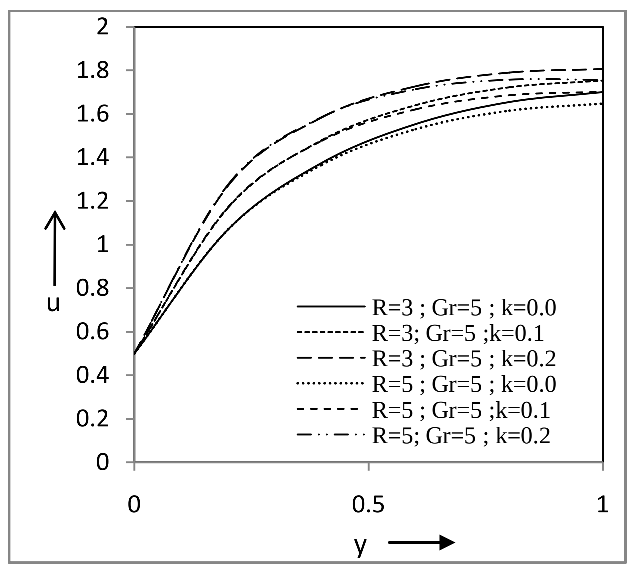

Figure 10: Velocity u against y for M=2, Pr=2, Gm=5, Sc= 5, S_0= 3, $\gamma = 2$,Q=2, K=1

Figures 9 & 10 demonstrate the effects of radiation parameter on fluid velocity u. It is observed that, for both the plate systems, velocity u decreases on increasing R. Also, from the figures, it can be concluded that the Newtonian fluid shows a rising trend as compared to visco-elastic fluid for both kind of surface systems. Further, slightly away from the plate the dispersion in the velocity profiles is considerable as compared to the initial stage.

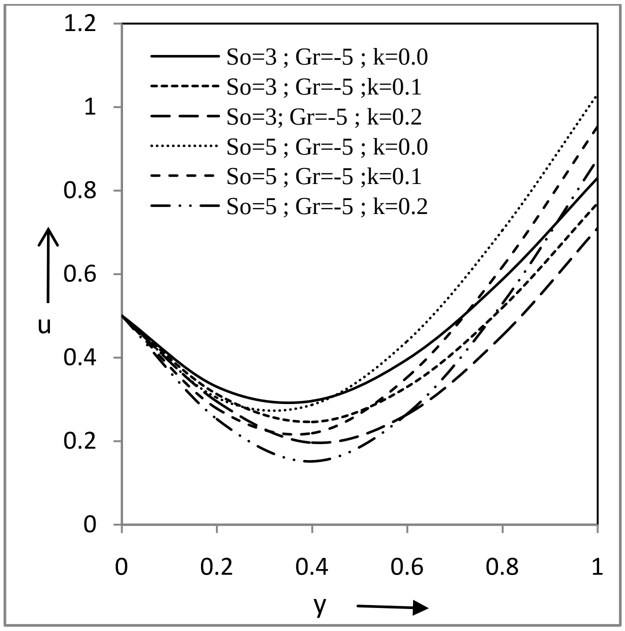

Figure 11: Velocity u against y for M=2, Pr=2, Gm=5, Sc=5, R =3, $\gamma = 2$, Q=2, K=1

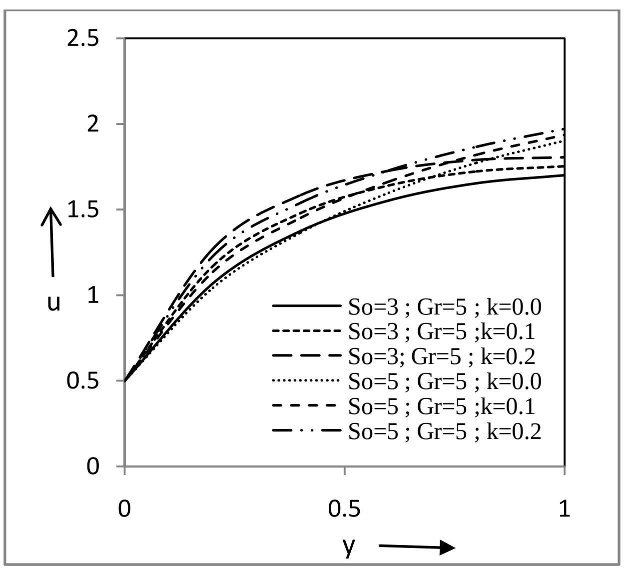

Figure 12: Velocity u against y for M=2, Pr=2, Gm=5, Sc=5, R =3, $\gamma = 2$, Q=2, K=1

The effect of Soret number on the fluid velocity profile has been discussed in figures 11 & 12. The Soret number has a propensity to enhance the speed of fluid velocity. Also it is likely to experience that slightly away from the plate the speed of flow decreases to some extent and thereafter as we move far away from the plate, the speed considerably increases for heated plate (Gr<0) but increasing pattern is observed for cooled plate (Gr>0). Furthermore, it is noticed that as the Soret number increases, there is inclined as well as declined in speed of visco-elastic fluid in comparison with Newtonian fluid for Gr>0 and Gr<0 respectively.

It is very important from practical point of view to know the effect of visco-elastic parameter on shearing stress. Figure 13-19 illustrate the shearing stress for visco-elastic fluid in comparison with the Newtonian fluid past a heated plate (Gr<0) and cooled plate (Gr>0) for various values of flow parameters. It is investigated from these figures that the magnitude of shearing stress experienced by cooled plate is highly predominant as compared to heated plate. Also, it is worth mentioning here that in case of cooled plate the magnitude of shearing stress in case of visco-elastic fluid is significantly less in comparison with Newtonian fluid but a totally opposite trend has been discovered for heated plate with the modification of visco-elastic parameter i.e k=0, k=0.1, k=0.2 etc.

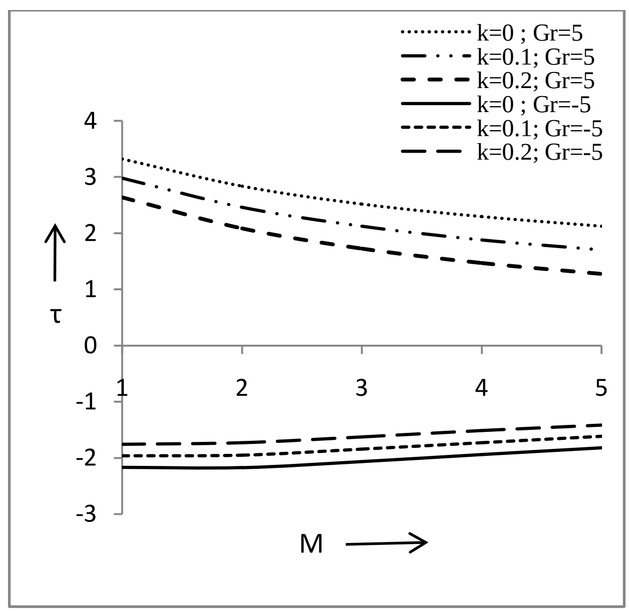

Figure 13: shearing stress against M for Pr=2, Gm=5, R=3, Sc=5, S_0=3, $\gamma = 2$, Q=2, K=1

Figure 13 shows the effect of magnetic parameter M on the shearing stress. It is depicted from the figures that the magnitude of shearing stress is higher in cooled plate than that of the heated plate. Also, the magnitude of the shearing stress for visco-elastic fluid is inclined for heated plate but declined in case of cooled plate as compared to the Newtonian fluid.

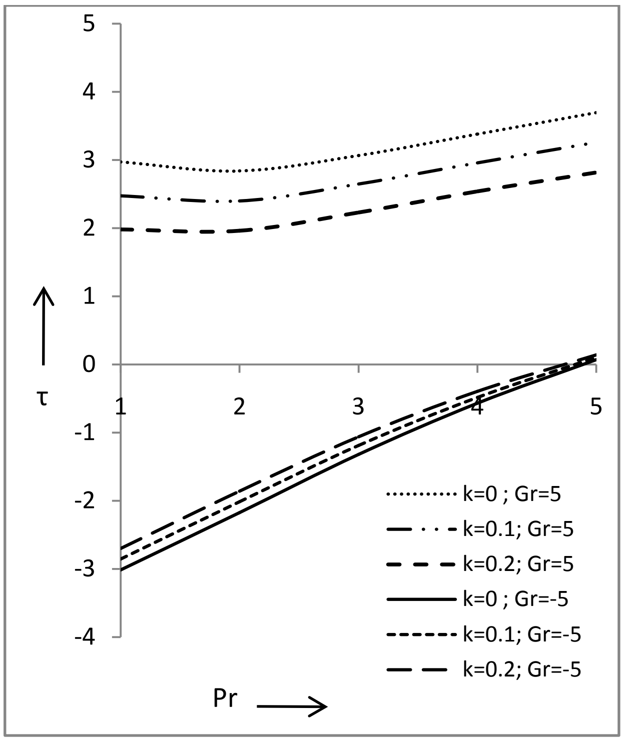

Figure 14: shearing stress against Pr for M=2, Gm=5, R=3, Sc=5, S_0=3, $\gamma = 2$, Q=2, K=1

The effect of Prandtl number (Pr) number on shearing stress is shown in figure 14. It is illustrated from the figure that for the fluid past a cooled plate the increasing value of Pr enhances the magnitude of shearing stress as compared to the shearing stress experienced by the heated plate.

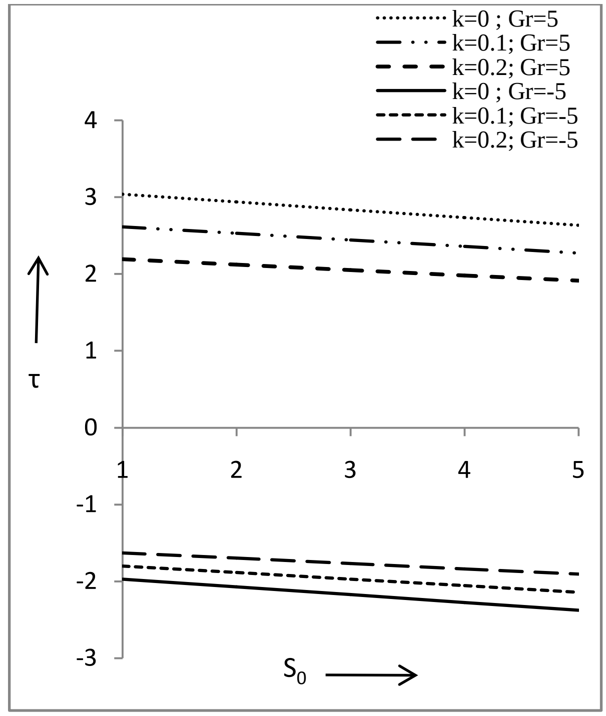

Figure 15: shearing stress against S_0 for M=2, Pr=2, Gm=5, R=3, Sc=5, $\gamma = 2$, Q=2, K=1

Figure 15 shows the effect of Soret number (S_0) on shearing stress and the figure implies that when Soret number increases, the magnitude of shearing stress experienced by both plates i.e., heated and cooled plate gradually decreases as we move far away from the plate.

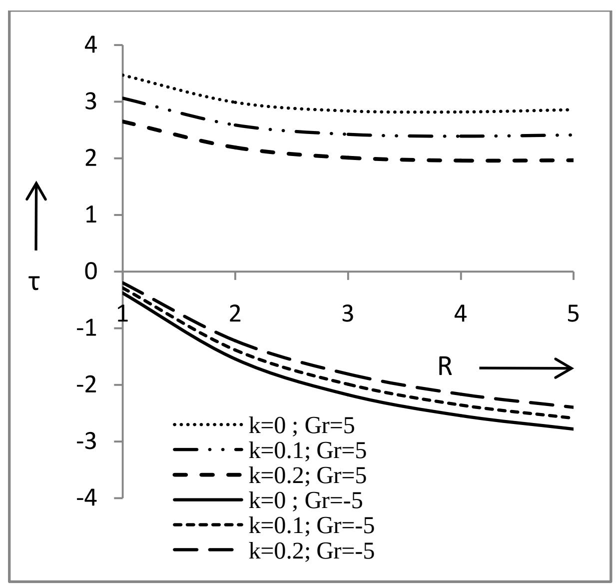

Figure 16: shearing stress against R for M=2,Pr=2, Gm=5, Sc=5, S_0=3, $\gamma = 2$, Q=2, K=1

For the flow past a heated plate the magnitude of shearing stress decreases rapidly with radiation parameter (R) than that of the shearing stress experienced by cooled plate (figure 17). Also, it is observed that, in case of cooled plate the dispersion of magnitude of shearing stress is considerably more than the magnitude of shearing stress experienced by heated plate.

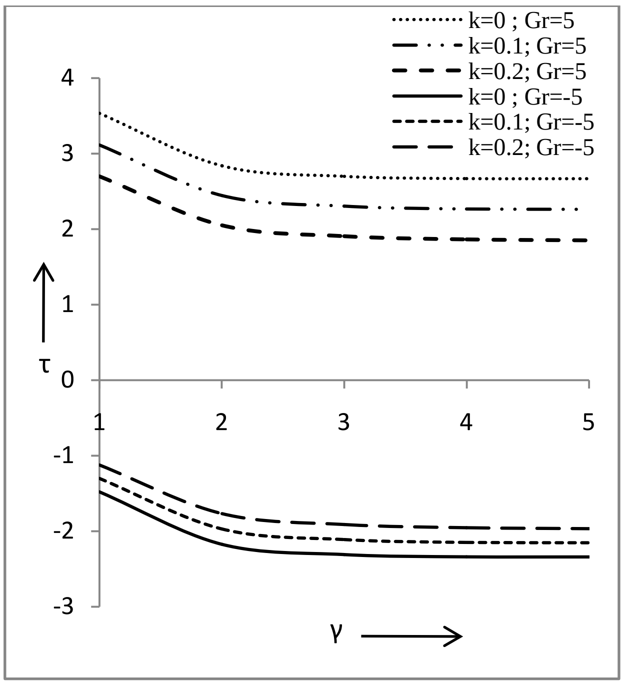

Figure 17: shearing stress against γ for M=2, Pr=2, Gm=5, R=3, Sc=5, S_0=3, Q=2, K=1

The influence of chemical reaction parameter (γ) on the shearing stress is exhibited in figure 17. It is noticed that as the chemical reaction parameter increases, there is declination in magnitude of shearing stress away from the plate. Also, the magnitude of shearing stress experienced by cooled plate is considerably higher than the magnitude experienced by heated plate.

5 Conclusion

In this study, an analytical solution of unsteady MHD visco-elastic flow with heat and mass transfer over a porous moving plate by considering double diffusive convection has been investigated. Some of the significant points of the present study are listed as below:

- The rate of heat transfer and the rate of mass transfer is not significantly affected by the visco-elastic parameter in the fluid flow region.
- Increasing values of Magnetic parameter, Prandtl number, Soret number, Grashof number for mass transfer and heat source parameter accelerate the fluid flow past a heated plate but a reverse behavior is experienced during the flow past a cooled plate.
- For heated plate, Newtonian fluid is predominant in nature than visco-elastic fluid while for cooled plate visco-elastic fluid is predominant than Newtonian fluid.
- Shearing stress experienced at the cooled plate is considerably higher in magnitude than the shearing stress experienced at the heated plate under various values of flow parameters such as magnetic parameter (M),Prandtl number (Pr), radiation parameter (R),chemical reaction parameter(γ) , Soret number (S_0) and porosity parameter(K).

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