Numerical investigation of the reflection of asymmetric shock waves in steady flow: Transitions (RR-MR) and Hysteresis Phenomenon

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Abstract: - The analytical study and experimental research on the reflection of the asymmetric shock waves (RR-MR interference), in the stationary supersonic flows been reported for the first time by H. Li and al [1]. The purpose of this research is to perform numerical simulations (CFD) on the Interference shock waves in the supersonic flows and more particularly being studied of the transition regular reflection (RR) to Mach reflection (MR) and vice versa – phenomenon hysteresis. This last process was largely studied numerically in the case of the symmetrical reflections during these last years. The analysis of the results obtained was based on experimental results H. Li and al [1], and compared analytically by the theory of tripled shock (shock polar). The theoretical study allowed the identification of the criteria of transition, and clearly shows the existence of a hysteresis phenomenon in the transition (RR) - (MR), from the asymmetric shock waves similar to that existing in interaction of symmetrical shock waves.

Key-Words: - Shock wave, Shock Interferences, regular reflection, Mach Reflection, shock polar.

1 Introduction

The reflexion is met and the interaction of shock waves in a great number of situations practise, for example, from the shock interaction of the various components of a launcher or the interaction of shocks induced by the fuselage and bearing surfaces of a reentry vehicle. Furthermore, the design of a propulsion system is a major technological problem in the definition of future hypersonic vehicles. In this regard, the entry of air, serving to slow the flow entering the supersonic combustion chamber through a series of shock waves, plays a crucial role it is necessary that the flow reaches it maximum efficiency and therefore a minimal loss of stopping the flow pressure. A determinant of the state of the flow at the inlet of the combustion chamber is the successive interactions of shocks that occur in the diffuser of the air inlet. This means tackling the fundamental problem of the interaction of two oblique shocks. By its theoretical formulation, the case of the interaction of two shocks of equal intensity but different families is equivalent to the case of reflection of a shock on a solid surface in the absence of the boundary layer. Indeed, the two symmetric shocks are reflected on a mirror plane. The phenomenon of MR was first observed by Mach (1878). A considerable number of studies have been made for when and how both types of reflection occur. Von Neumann (1943, 1945) developed the three-shock theory and proposed two transition criteria from RR to MR and vice versa: the reflected shock wave detachment criterion and the mechanical equilibrium criterion, or von Neumann criterion. These two criteria are separated by a zone now called dual solution domain, where both RR and MR are possible Henderson & Lozzi [2][3]. Hornung & Robinson [4]. Chpoun et al.[5][6]. Vuillon, Zeitoun & Ben-Dor [7]. Teshukov [8]. Li & Ben-Dor [9]. Whether it is an RR or an MR in the dual solution domain depends on the history that the actual steady flow is built. Chpoun et al [6]. Ivanov, Gimelshein & Beylich [10]. Shirozu & Nishida [11]. Ben-Dor, Elperin & Vasiliev [12].This dual zone H. Hornung led to suggest the existence of a hysteresis in the transition regular reflection => Mach reflection. However, the various attempts in the 70 and 80 (Hornung, Henderson) could not confirm the hypothesis Hornung. Subsequently, the experimental study of this phenomenon has been taken to Aero-thermodynamic Laboratory of CNRS at Meudon in a wind tunnel (SH2) of the continuous type and free vein [1]. In these studies the hysteresis phenomenon suggested by H. Hornung [13], has been observed as well in the case of the reflection of a shock to a plane of symmetry than in the case of the interaction of two shocks Asymmetric which is the subject of this presentation.

2 Interaction of Asymmetric Shocks

The reflection of a shock wave on a flat surface is altered by the presence of the boundary layer on the wall. The shock wave can cause separation of the boundary layer and the actual configuration is removed from the configuration predicted by the theory of ideal fluid. Similarly, the interaction of perfectly symmetrical shock is less likely. However the actual flows (air inlet, external flow) are often the shock interaction seat intensities and different families and the scope of their study is considerable. The experimental works in this area are few or nonexistent. As in the case of the interaction of two symmetrical shocks, there are two configurations of interaction: interaction Mach (MR) and the regular interaction (RR). Configuration interaction (MR) and (RR) and the corresponding geometric ratings are shown in Figure 1. Regular interaction consists of two incidents shocks (i_1) and (i_2) and two shock reflected (r_1) and (r_2).

The boundary conditions for the configuration (RR) are [1]:

$$\theta_1 - \theta_3 = \theta_2,$$

$$\theta_4 = \delta \qquad (1)$$

$\delta = 0$ for when, $\theta_1 = \theta_2$, that is, when the interaction is symmetric.

The Mach interaction comprises more incidents bumps and shocks reflected, a quasi normal strong shock connecting the triple points (T_1) and (T_2). Two flow lines (s_1) and (s_2) supplement the shock system. The boundary conditions for a Mach interaction are [14].

$$\theta_1 - \theta_3 = \delta_1 \text{ and, } \theta_2 - \theta_4 = \delta_2 \qquad (2)$$

$\delta_1 = \delta_2$ for when $\theta_1 = \theta_2$.

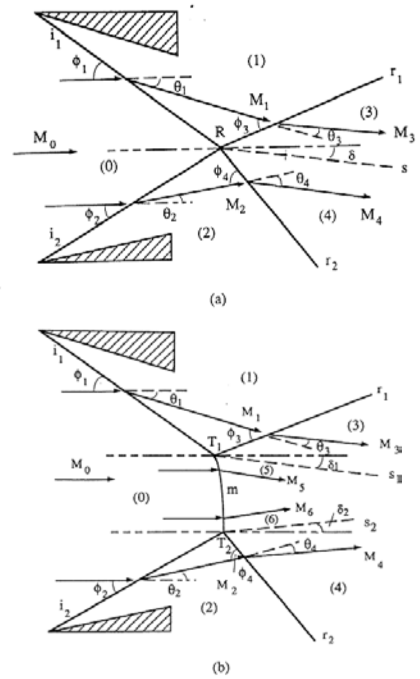


Fig. 1: (a) Schematic of regular interaction. (b) Schematic of a Mach interaction [14].

The main interest in the shock interaction studies is the determination of transition criteria. It turns out that the triple shock theory gives satisfactory results regarding the transition criteria in the case of the interaction of two symmetric shocks. We'll use this theory to the determination of transition criteria in the asymmetric case. The boundary conditions configurations (RR) and (MR) in the asymmetric case is different from the symmetric case, some peculiar to the asymmetric configuration will be discussed. The use of shock polar is very convenient to analyze the different possible configurations of shock interactions [15]. Depending on the position of the intersection of the incident shock polar (P-I) and the reflected shock (P-R), one can distinguish three types of Mach interaction Figure 2. Mach direct interaction (DiMR) defined by the intersection (point A in the figure) of the polar (P-R_1) with the right side of the polar (P-I). The Mach inverse interaction (InMR) results from the intersection of the polar (P-R_3) with the left side of the polar (P-I) (point c in the figure). When the intersection is located on the pressure axis (point b in the figure), we say that the Mach interaction is stationary (StMR). In the case of the direct interaction of Mach, the flow forms a current converging tube Figure 3., while in the case of a Mach inverse interaction forms a split flow tube. In the case of a stationary Mach interaction streamlines are parallel behind the strong shock. For a uniform flow characterized by a Mach number *M*, pressure ratios across an oblique shock according to the flow deflection angle represent the polar shock whose expression is given by Han and Yin [16].

$$\theta = \pm f(\gamma, M\xi) \quad (3)$$

With

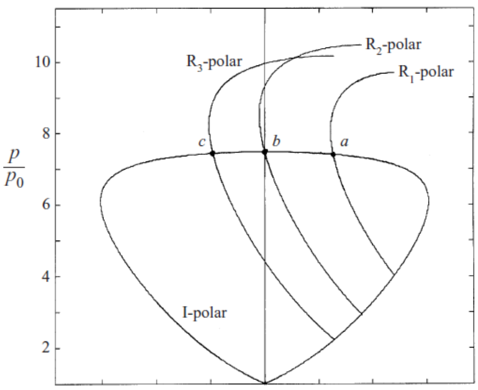
$$f(\gamma, M\xi) = ar\tan\left(\frac{(\zeta-1)^2[2\gamma(M^2-1)-(\gamma+1)(\zeta-1)]}{[\gamma M^2-(\zeta-1)]^2[2\gamma+(\gamma+1)(\zeta-1)]}\right)^{1/2} \quad (4)$$

Which $\gamma, \zeta, and\ \theta$ are respectively the specific heat ratio, the pressure ratio across the shock and the flow deflection angle. Finally the local slope of the pole of shock is obtained by differentiating the above function, that is to say

$$\frac{d\theta}{d\zeta} = \pm g(\gamma, M, \zeta) \quad (5)$$

$$g(\gamma, M, \zeta) = \frac{4\gamma(M^2-1)-(\gamma+1)(4-M^2)(\zeta-1)-(\gamma+1)(\zeta-1)^2}{[2\gamma(M^2-1)-(\gamma+1)(\zeta-1)]^{1/2}[2\lambda+(\gamma+1)(\zeta-1)]^{1/2}} \quad (6)$$

$$x\left\{2\gamma M^2 - \left[4-(\gamma+1)M^2\right](\xi-1) - 2(\xi-1)^2\right\}^{-1}$$



a- Match direct interaction DiMR

b- Mach stationary interaction StMR

c- Mach inverses interaction InMR

Fig. 2: combinations of the shock polar showing the various types of Mach interaction [17].

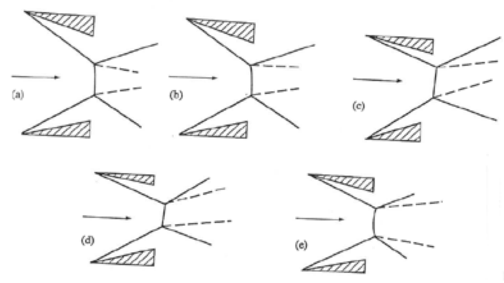


Fig. 3: The different flow patterns downstream of the Mach disk [17].

3 Numerical Studies

3.1 Geometry and Flow Parameters

The geometry used Figure 4. to reproduce the hypothesis of hysteresis is similar to those used experimentally in the wind tunnel SH2 Aerothermodynamics of the CNRS laboratory H. Li and al [1]. This is a configuration that contains two dihedrals (shock generator) sections free form of a convergent nozzle 60 mm in length (*l*), the distance (*h*) between the two edges of the generators has been maintained at 70 mm. It is noted that these dimensions lead to an aspect ratio (*h* / *l*) = 1.16 [18], this report is important enough to prevent edge effects can affect the flow in the central part of the plates. The upper shock generator is rotational movement with respect to the rotation axis "O_1", using the technique of the moving grid, by collision against the bottom of the generator is fixed.

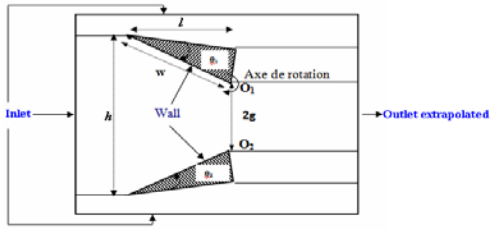


Fig. 4: Geometry and flow parameters of the problem

Achieving an appropriate mesh to the problem addressed is one of the most important steps in numerical simulations. The adaptation of the mesh to the boundary conditions of the problem is essential in this type of calculation. A structured mesh is used in the calculations made. This type of mesh to control rigorously the thickness of the mesh in specific locations (near the walls for example). The only problem encountered in this type of mesh compared to unstructured mesh is the increased number of stitches in his refinement in sensitive areas to study. The total computational domain is divided into several blocks; the refinement of each block depends on the size of the areas studied. In our study, for example, we are interested in the flow in the part where the interference shock waves will be located (RR and MR reflections).

3.1 Modelling and Numerical Solution

For the numerical solution, two research codes were used, CFD-GEOM (mesh generator) and CFD-FASTRAN (solver), both provided by ESI CFD GROUPS. The numerical approach to CFD-FASTRAN code is based on solving the Navier-Stokes equations by the finite volume method. The flow field is divided into small controlled volumes. The conservation equations are applied to each volume taking into account the flow variables across each face of the control volume. Each variable is calculated at the center of each cell and assumed to be constant throughout the volume control. In our case the fluid used is air, considered ideal gas (specific heat $\gamma = 1.4$). The admission requirements (if the flow limits) are represented in the Table 1. Scheme Roe FDS less dissipative chose been associated with the Minmod limiter based on the calculation of the minimum slope gradients associated with physical parameters of the flow taken on the faces of the control volume. The implicit scheme was also used for the time integration. The time step is controlled by the number of CFL (Courant-Fredrichs-Lewy). The acceleration of the convergence is achieved by varying the number of CFL linearly from its generally low initial value to a final value, on a number of steps specified time. The convergence of iterative calculations is obtained (quasi-stationary solution) when the specified value of the residual values is reached.

Table.1 : Physical flow parameters.

Working conditions	
Fluid of work	Air
Mach number	4.96
Temperature	76.5 K
Pression	1683 Pa
CFL	0.5

4 Validation of the results

Figure 5. Represents the iso-density contours. It shows a case of validation of our calculation with the results of Li H and al [1]. We see that both configurations are almost the same, especially Mach Stem disks, and also the direct Mach reflection is well marked.

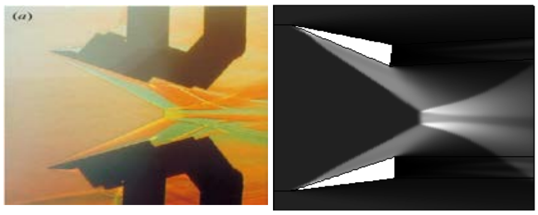


Fig. 5: Comparison of our results (θ_1 = 28.44 °, θ_2 = 24.14 °) (right) with the work [1] (θ_1 = 28 °, θ_2 = 24 °) (left), under the same conditions. M0 = 4.96.

4 Results and Discussion

To reproduce the hysteresis sequences, a fixed angle θ_2, and the angle θ_1 has been decreased from a configuration of Mach until a regular pattern. This operation was repeated in reverse, i.e. the regular configuration to the Mach configuration. The hysteresis sequences were propagated for six θ_1 values equal to 12.5 °, 15 °, 20 °, 26 °, 30 ° and 36 °. The sequence Figure 6. Shows the iso-density curves of numerical results. The figure shows a complete sequence hysteresis, transition angles, different areas of existence of interactions RR and MR and dual zone. The angle θ_2 is 25 ° while the angle θ_1 increases from 12.5 ° to 36 ° and then decreases. We see that the transition to RR, MR (detachment criterion) takes place in a brutal way between 30 ° and 36 ° and that in contrast MR transition, RR (criterion of Von Neumann) is between 15 ° and 12.5 °. In other words, in the dual area, for the same geometry of the plates θ_1 = 26 °, the two configurations of interaction are obtained. On the other hand the analytical study suggests the existence of an inverse Mach reflection stationary supersonic flow. This shock pattern was numerical observed during this study for the first time Figure 6-E. This configuration has sparked renewed interest in the problem and research on various aspects of this phenomenon is underway around the world.

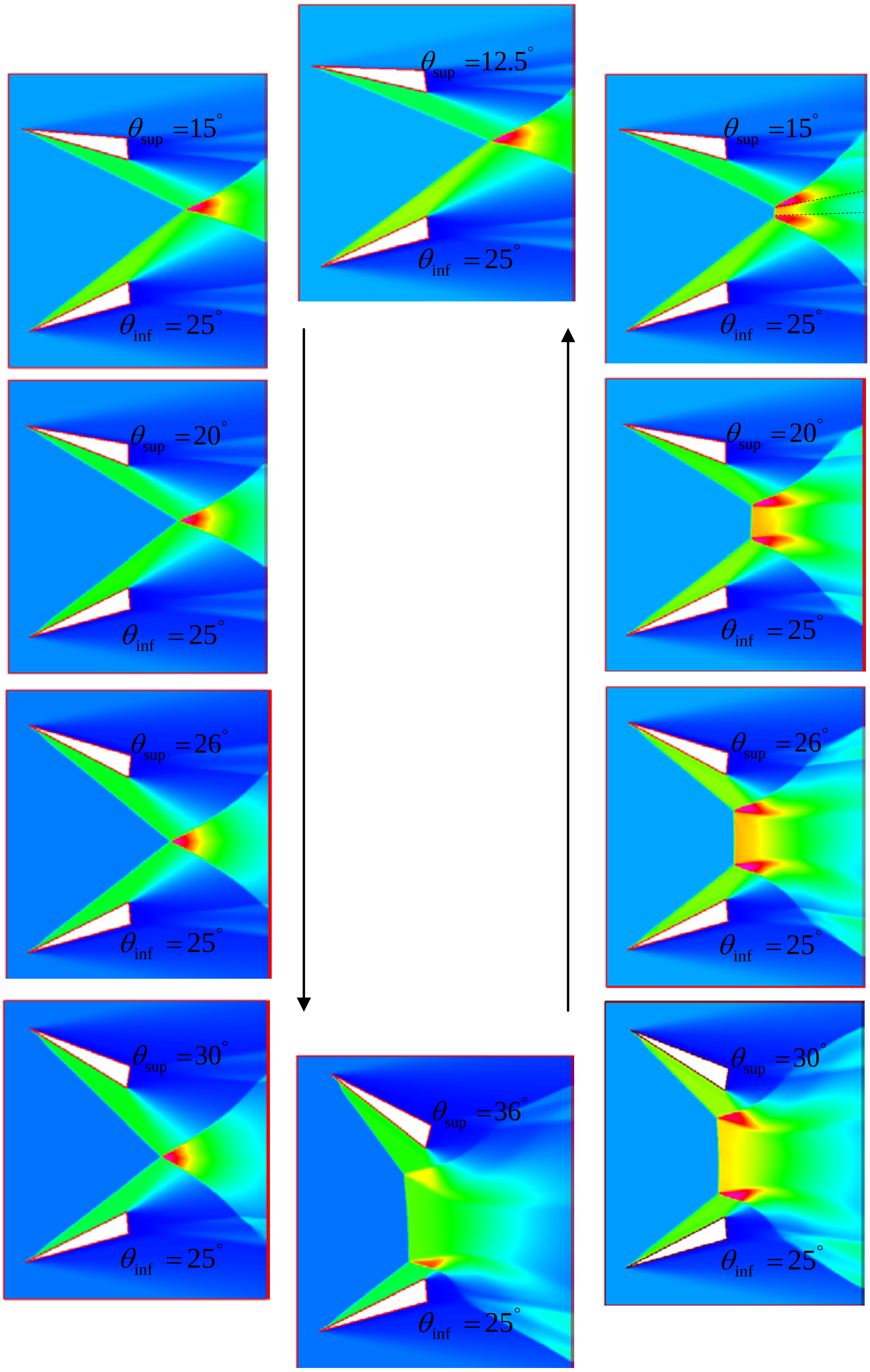


Fig. 6 : Phénomène d’hystérésis induit par variation de l’angle d’incidence de dièdre générateurs de chocs

The Figure 7. shows the different combinations of the shock polar showing the hysteresis induced by variation of the angle of incidence of shock generators dihedral : Where asymmetrical interference (Figures 7-a. to 7-f). are six possible combinations of polar shock. For all the Mach number is M_0 = 4.96, and the polar (P-I) is the same. Moreover by setting the deflection, polar (P-R_2), are identical. It is noted that the intersections of the polar (P-I) and (P-R2) predict a direct Mach reflection (DiMR) as shown in Figure 7. (Figure 7-a) to (Figure 7-f). Figure 7-a. shows the combination of the pole of a shock for deflection. As you can see, the intersection of polar (P-I) and (P-R_1) still defines direct Mach reflection. As a result the overall configuration of the interaction consists of two direct Mach reflections. This shock configuration is illustrated in Figure 6-A. Slip lines from triple points then form a convergent nozzle through which the flow is accelerated up to supersonic conditions. For an angle, the impact pattern is represented by the shock of the pole Figure 7-c. This also includes regular interaction and Mach interaction consists of two direct Mach reflections. This shock configuration is illustrated in Figure. 6-C. and Figure. 6-I. When the deflection is 20 °, with the polar (P-R1) intersects (PI) on the axis of the pressure Figure 7-d. This then results in three points defining a stationary Mach reflection (StMR), while the second triple point always sets a direct Mach reflection (DiMR). The pattern for this type of interaction is represented in Figure 6-D. After the slip line of reflection (StMR) is then parallel to the initial direction of flow. Two flow lines then form a current converging tube. For a smaller angle, for example, an extreme situation is obtained where the three polar intersect at the same point Figure 7-e. In this situation, the two solutions, that is to say, regular interaction and Mach interaction are combined. This is equivalent to the criterion of Von Neumann. However, the Mach interaction is now formed an inverse Mach reflection (InMR) and direct Mach reflection (DiMR). The diagram of this type of interaction is shown in Figure 6-E. Note that the resulting slip line of reflection (InMR) is positively oriented with respect to the direction of flow and the downstream flow tube remains strong shock converge. For smaller deflection angles, for example 12.5 °, is obtained a combination of the pole shown in Figure 7-f. the pattern of interaction is given in Figure 6-E. Here, the interaction is a regular interaction.

Numerical Transition points were brought in Figure 8. comparing with those experimental and areas of existence of different types of theoretical shock setup for a Mach number M_0=4.96.The triangles represent the transitions MR/RR, and squares correspond to transitions RR/MR. This figure highlights not only the hysteresis phenomenon predicted in case of symmetric interference, but in addition it shows a very good agreement between the theoretical transition levels, experimental and numerical values. Considering the figure, a Mach interaction obtained for a given angle and a sufficiently large angle to be located beyond the detachment line. When decreases as the constant are maintained, we first reached the detachment line. In the dual-zone Mach interaction must be maintained. When you get to the line of Von Neuman transition to regular interaction must take place since beyond this line there is no theoretical solution for a Mach interaction. In the other direction, when increasing the line Von Neuman is reached first. The regular configuration must remain in the dual zone and the transition to Mach configuration must operate at the posting online since beyond this line there is no solution for regular interaction.

The different fields of existence of various configurations of interactions are represented, in Figure 9. In the plan (M_0, θ_1) for a deflection angle $\theta_2 = 25^\circ$. In this figure, the line represents the criterion of Von Neumann extended to the asymmetric case. The line θ_1^T represents the detachment criterion extended to the asymmetric case. The criterion of Von Neumann in the symmetrical case, shown by the dotted line θ_1^N. Line θ_1^D indicates the upper limit value beyond which an attached shock wave cannot be obtained. These four curves define five areas with different topologies interactions. For a given Mach number and by varying θ_1 it is clearly possible to obtain different configurations of reflections. For $0 < \theta_1 < \theta_1^T$ a regular reflection is obtained (RR) for $\theta_1^T < \theta_1 < \theta_1^N$ a regular reflection is obtained either (RR), an inverse Mach reflection (InMR) for $\theta_1^N < \theta_1 < \theta_1^E$, there is a regular reflection (RR) or direct Mach reflection (DiMR), and finally for $\theta_1^E < \theta_1 < \theta_1^D$ to was an inverse Mach reflection (InMR) or direct Mach reflection (DiMR).

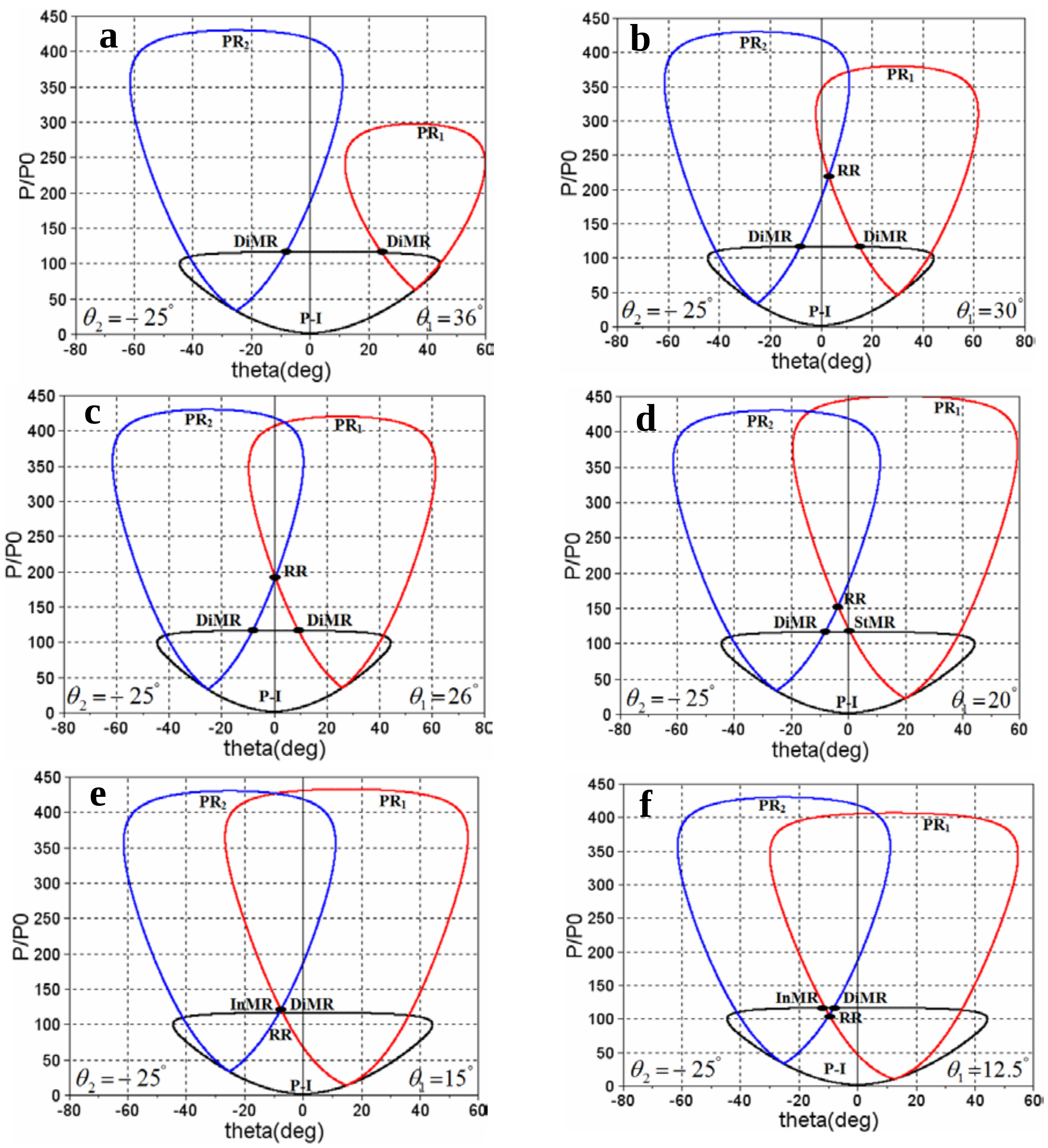


Fig. 7: The different combinations of the shock polar showing the hysteresis induced by variation of the angle of incidence of shock generators dihedral: the asymmetric case. M0 = 4.96.

1- (a) - the combination of shock pole corresponding to a Mach interaction formed by two Mach direct reflections (DiMR).

2- (b) - the combination of shock polar corresponding to a Mach interaction consists of two direct Mach reflections (DiMR) and regular interaction (RR) (a situation illustrating the dual zone).

3-(c) - the combination of shock polar corresponding to a Mach interaction consists of two direct Mach reflections (DiMR) and regular interaction (RR) (position illustrating the dual zone).

4-(d) - the combination of shock polar matching regular interaction (RR) and a Mach interaction consists of two direct Mach reflections (DiMR) and a stationary Mach reflection (StMR).

5-(e) - the combination of polar shock illustrating the superposition of a regular interaction and interaction Mach formed of a direct reflection of Mach (DiMR) and a Mach reverse reflection (InMR) (limit position corresponding to the Von Neumann. criterion)

6-(f) - combination of shock polar matching regular interaction (RR) and a Mach interaction (non-phasic solution) formed a direct Mach reflection (dimr) and an inverse Mach reflection (InMR).

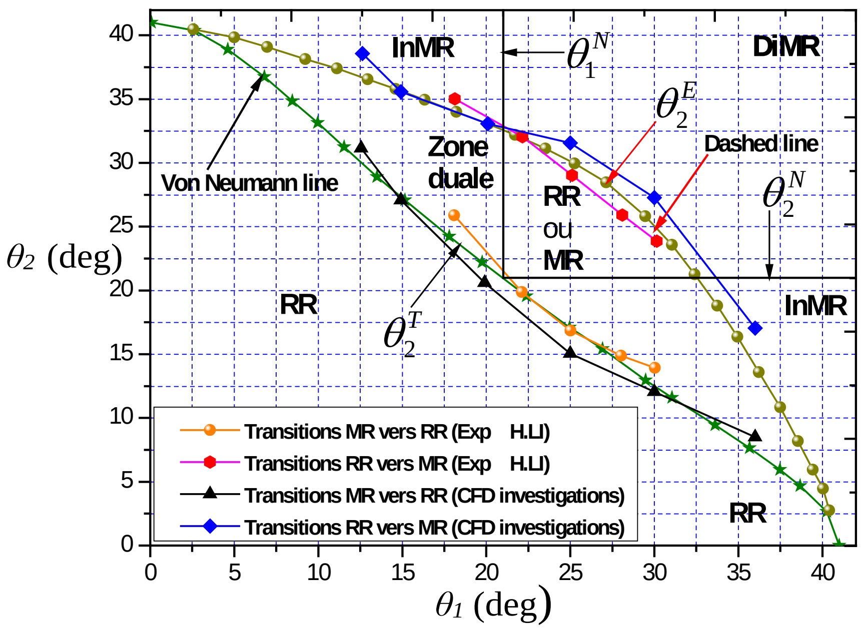


Fig. 8: The existence of areas of different types of shock pattern and the digital transition points for a Mach number (M0 = 4.96).

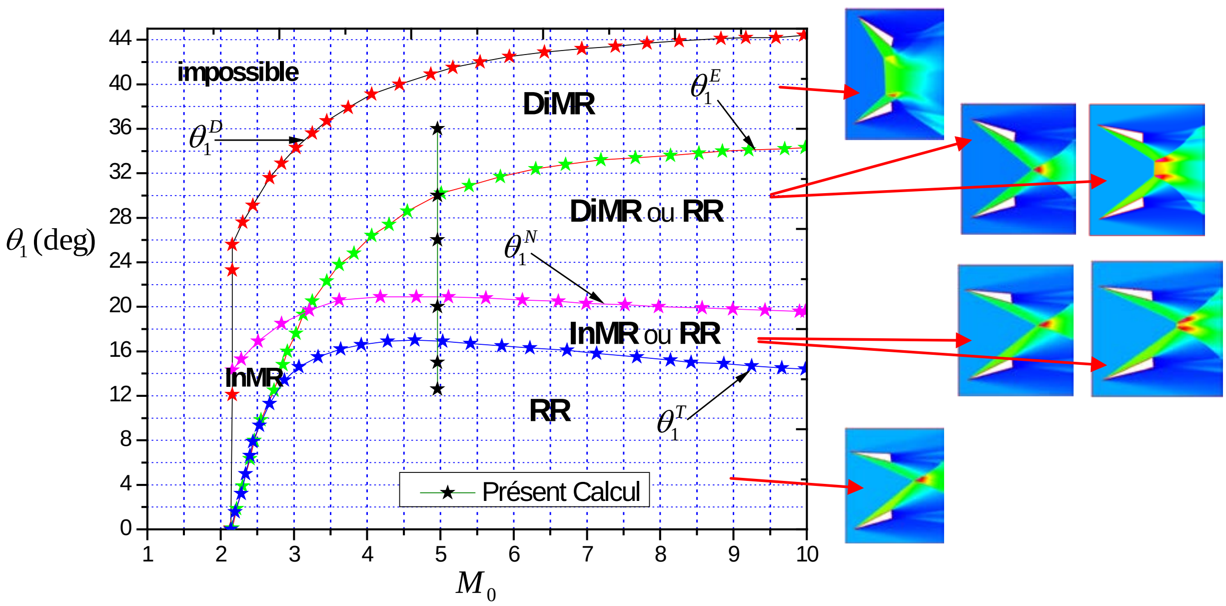


Fig. 9: Domains different theoretically possible reflections in the plan (M_0, θ_1) for a fixed angle $\theta_2 = 25^{\circ}$.

5 Conclusion

A numerical study concerning the interaction of two asymmetric oblique shocks was implemented. Although the study was 2 D. The study comply identifying criteria transitions (between the shock configuration RR and MR) with that found analytically. A dual zone where the two RR / MR shock configurations can breed was highlighted. The study suggests the existence of an inverse Mach reflection. The hypothesis of the existence of a hysteresis phenomenon due to the discharge memory effect caused by the interference between the shock waves similar to that existing in symmetrical shock interaction was formulated and tested numerically. An inverse Mach reflection was obtained numerically for the first time in steady supersonic flow.

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