

Mesoscale Numerical Method for Prediction of Thermal Fluid Flow through Porous Media

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Abstract: - In this paper, the lattice Boltzmann method, a mesoscale numerical tool based on particle distribution function is used to simulate thermal fluid flow in porous media. The key point is to combine the simplest four and nine lattice velocity model to represent the temperature and density distribution functions respectively. Wide range of Rayleigh numbers and material's porosity has been applied to study their effects on the thermal fluid flow in the enclosure. Our numerical experiments demonstrate excellent agreements when the computed results are compared with those predicted by the finite element solution to the Brinkmann-Forcheimer equation and the conventional lattice Boltzmann scheme. This indicates the applicability of the present approach in realistic simulation of thermal fluid flow in porous media.

Key-Words: - Lattice Boltzmann, Double population, Natural convection, Porous media

1 Introduction

As The interaction between fluid flow behavior and heat transfer mechanism can be seen not only in almost all industrial processes such as metal furnace, power plants, jet engine, etc, but also in everyday situation such as ventilation, air conditioning, hair dryer and so on. In some applications, such as micro-electro mechanical systems (MEMS), the detail understanding of the fluid flow and heat transfer phenomenon is unrelentingly required in order to achieve the most effective method of microchips cooling [1]. On the other hand, lack of understanding in this problem can result in huge cost lost and inefficiency repercussions. For instance, inaccurate prediction of heat transfer and fluid flow can also leads to loss of human lives in the reentry of space shuttle due to the great heat involved in this activity.

Among the main three types of heat transfer mechanism, the convection type has a more pronounced effect on fluid flow. In fact, the convective heat transfer dominates the heat transfer mechanism in most cases when interact with surrounding fluid. This mechanism is very difficult to measure because of the effect on fluid flow only appears when dealing at severe conditions such as high Rayleigh or Grashof numbers. Furthermore, when the contact fluid is gas, it becomes difficult to visualize this flow configuration experimentally.

Flow driven by buoyancy force is a kind of flow resulted from convective heat transfer. This type of flow can be found in certain engineering applications within insulation technologies, in everyday situation such as roof ventilation or in academic research where it may be used as a benchmark problem for testing newly developed numerical methods. A classic example is the case where differentially heated walls of the cavity boundaries induce the flow. Two vertical walls with constant hot and cold temperature is the most well defined geometry and was studied extensively in the literature. A comprehensive review was presented by David [2]. Other examples are the work by Azwadi and Tanahashi [3], Davis [4] and Tric [5].

The analyses of flow and heat transfer in a differentially heated side walls was extended to the inclusion of porous media in the enclosure. Cheng [6] provides an extensive review of literature on natural convection in fluid saturated porous media with regard to applications in geothermal systems. Nield and Bejan [7] gives an excellent summary of the subject. Other works are [8-11] and [12]. Due to the complexity of porous structure and fluid interaction with the boundaries, most of the mentioned researchers preferred numerical approach to understand the fluid flow behavior in the system. Interestingly, many of them applied conventional numerical schemes based on discretizations of the

semiempirical models as their numerical tools.

Currently, numerical solutions to the fluid flow problem can be divided into three scales, which are macro, meso and microscale solutions. Macroscale solution considers the Navier-Stokes equation as its governing equation and applies one or a combination of discretization methods to be solved using digital computer. However, due to the nonlinear nature of the equation, greater attention has to be paid during preprocessor step to determine suitable mesh size, criteria of computational stability, error propagation, etc.

There are few numerical solutions that simulate the evolution of fluid flow at microscopic scale. Among them are direct simulation Monte Carlo [13] and Molecular Dynamics methods [14]. In these methods, the trajectories of every particle together with their position in the system are predicted using the second Newton's law. But remember, a cup of water contains 10^{23} number of molecules. Even when a gas is being considered where there are fewer molecules and a larger time-step can be used, because of the longer mean free path of the molecules, the number of molecules that can be considered is still limited. However, the question is, do we really need to know the behavior of each molecule or atom? The answer is no. It is not important to know the behavior of each particle, it is important to know the function that can represent the behavior of many particles (mesoscale). Therefore, in current study, we bring the so-called lattice Boltzmann method [15] as our numerical tool. The evolution of two distribution functions is considered to predict the velocity and temperature fields in the system. After showing how the formulation of mesoscale particle fits in to the framework of lattice Boltzmann simulations, a mathematical formulation is developed in order to investigate the effect of buoyancy force and the presence of porous media within the solution manifold.

The current study is summarized as follow: two-dimensional fluid flow and heat transfer in porous media filled in square cavity is investigated numerically. The two sidewalls are maintained at different temperatures while the top and bottom walls are set as an adiabatic wall. Here, we fix the aspect ratio to unity. The flow structures and heat transfer mechanism are highly dependent upon the porosity of the medium. By also adopting the Rayleigh and Darcy numbers as continuation parameters, the flow structure and heat flow represented by the streamlines and isotherms lines can be identified as a function of porosity. Comparisons of results among those published in

literature are carried out in terms of a computed averaged Nusselt number. Section two of this paper presents the governing equations for the case study in hand and introduces the numerical method, which will be adopted for its solution. Meanwhile section three presents the computed results and provide detailed discussions. The final section of this paper concludes the current study.

2 The Governing Equations

Following Nithirasu [16], the generalized model for athermal incompressible fluid flow in porous media can be expressed by the following equations

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \left(\frac{\mathbf{u}}{\varepsilon} \right) = -\frac{1}{\rho} \nabla(\varepsilon p) + \nu_e \nabla^2 \mathbf{u} + \mathbf{F} \quad (2)$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u} T) = \chi \nabla^2 T \quad (3)$$

where ν_e is the effective viscosity, ε is the porosity of the media and χ is the thermal diffusivity. \mathbf{F} represents the total body force due to the presence of a porous medium and buoyancy force resulted from differentially heated sidewalls, and is given by

$$\mathbf{F} = -\frac{\varepsilon \nu}{K} \mathbf{u} - \frac{1.75}{\sqrt{150 \varepsilon K}} |\mathbf{u}| \mathbf{u} + \varepsilon \mathbf{G} \quad (4)$$

where ν is the kinematic viscosity and K is the permeability of which can be related to non-dimensional parameter of Darcy number Da as follow

$$K = Da \times H^2 \quad (5)$$

where H is the characteristic length. In present study, the Boussinesq approximation is applied to the buoyancy force term where

$$\mathbf{G} = \beta g_0 (T - T_m) \mathbf{j} \quad (6)$$

Here, β is the thermal expansion coefficient, g_0 is the acceleration due to gravity, T_m is the averaged temperature and \mathbf{j} is the vertical direction opposite to that of gravity.

2.1 The Lattice Boltzmann Method

In recent years, the lattice Boltzmann method (LBM) has been developed into an alternative and promising numerical technique of Computational Fluid Dynamics (CFD) [17-20]. Unlike conventional numerical schemes based on the discretization of partial differential equations describing macroscopic conservation laws, the LBM is based on solving the discrete-velocity Boltzmann equation in statistical physics. In this work, the governing equations of thermal fluid flow in porous media are solved indirectly, i.e., by using LBM with second-order accuracy.

Our literature study found that there are several investigations have been conducted using the LBM to understand the problem in hand [21-23]. However, most of them applied the same lattice model to predict the evolution of velocity and temperature fields in the system. Combination of nine-lattice model for the density and also the same model for the temperature distribution functions is the most common approach by the previous researchers. Currently, one of present authors has developed the simplest lattice model to predict the evolution of temperature field [24]. Unfortunately, the developed model was found not in good agreement with the literature studies when predicting thermal fluid flow at high Rayleigh numbers. This was due to the limitation of the model where unable to capture high speed of fluid flow in the system [24]. The presence of the porous medium is expected to decelerate the flow although depended on the magnitude of the porosity. Therefore, the objective of present paper is to reconsider the newly developed model and predict the fluid and thermal flow in an enclosure filled with porous medium at high Rayleigh numbers. To see this, we start with the evolution equations of density and temperature distribution functions, given as [24]

$$f_i(\mathbf{x} + \mathbf{c}_i \Delta \mathbf{x}, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau_v} [f_i - f_i^{eq}] + F \quad (7)$$

$$g_i(\mathbf{x} + \mathbf{c}_i \Delta \mathbf{x}, t + \Delta t) - g_i(\mathbf{x}, t) = -\frac{1}{\tau_g} [g_i - g_i^{eq}] \quad (8)$$

where density distribution function f_i is used to calculate the density and velocity fields and temperature distribution function g_i is used to calculate the macroscopic temperature field. Note that Bhatnagar-Gross-Krook (BGK) collision model

[25] with a single relaxation time is used for the collision term. For the D2Q9 model, the discrete lattice velocities are defined by

$$\begin{aligned} \mathbf{c}_0 &= (0, 0) \\ \mathbf{c}_{1-4} &= (\pm 1, 0), (0, \pm 1) \\ \mathbf{c}_{5-8} &= (\pm 1, \pm 1) \end{aligned} \quad (9)$$

The equilibrium function for the density distribution function f_i^{eq} for the D2Q9 model is given by

$$f_i^{eq} = \rho \omega_i \left[1 + 3\mathbf{c}_i \cdot \mathbf{u} + \frac{9(\mathbf{c}_i \cdot \mathbf{u})^2}{2\varepsilon} - \frac{3\mathbf{u}^2}{2\varepsilon} \right] \quad (10)$$

where the weights are $\omega_0 = 4/9$, $\omega_i = 1/9$ for $i = 1-4$ and $\omega_i = 1/36$ for $i = 5-8$. According to Azwadi and Tanahashi [24], the simplest four-lattice velocity model can be applied to represent the temperature distribution function of g_i where $i = 1-4$. The equivalent equilibrium function for temperature distribution function is given as

$$g_{1,2,3,4}^{eq} = \frac{T}{4} [1 + \mathbf{c}_i \cdot \mathbf{u}] \quad (11)$$

The effective viscosity ν_e and the diffusivity χ are determined by $\nu_e = 1/3(\tau_v - 1/2)$ and $\chi = \tau_c - 1/2$, respectively.

In order to obtain the correct macroscopic governing equations, the forcing term F_i must be expressed in terms of medium porosity and buoyancy force as follows [16]

$$F_i = \omega_i \rho \left(1 - \frac{1}{2\tau_v} \right) \left[3\mathbf{c}_i \cdot \mathbf{F} + \frac{9(\mathbf{u}\mathbf{F} : \mathbf{c}_i \mathbf{c}_i)}{\varepsilon} - \frac{3\mathbf{u} \cdot \mathbf{F}}{\varepsilon} \right] \quad (12)$$

The macroscopic variables, density ρ , and temperature T can be evaluated as the moment to the distribution function

$$\begin{aligned} \rho &= \sum_i f_i \\ T &= \sum_i g_i \end{aligned} \quad (13)$$

In order to consider the effect of porous media, the fluid velocity \mathbf{u} must be calculated as follow

$$\mathbf{u} = \frac{\mathbf{v}}{c_0 + \sqrt{c_0^2 + c_1|\mathbf{v}|}} \quad (14)$$

where $\mathbf{v} = \sum_i \mathbf{c}_i f_i / \rho + \varepsilon G / 2$ is the temporal velocity, $c_0 = (1 + \varepsilon v / 2\kappa) / 2$ and $c_1 = 1.75\varepsilon / 2\sqrt{150\varepsilon^3\kappa}$. It is noted that, if we set $\varepsilon = 1$, the lattice Boltzmann equation reduces to the standard equation for free fluid flows.

3 Problem Solution

In this section, we begin with the validation of the thermal lattice Boltzmann model by setting $\varepsilon \rightarrow 1$. Table 1 shows the average Nusselt number computed by the LBM for $\varepsilon = 0.9999$, $Da = 10^7$ and $Ra = 10^3$ to 10^4 and comparisons with those by Davis [4] and Nithiarasu et al. [16].

Table 1. Comparison among Navier Stokes solver, finite element method and present model

Rayleigh Number	Davis [4]	Nithiarasu et. al [16]	Present
10^3	1.116	1.127	1.117
10^4	2.238	2.245	2.236

As can be seen from the table, the results predicted by current model agree well with the previous studies. This gives us confidence to apply the proposed method for simulation of thermal fluid flow in porous media.

We next extend our simulation study to predict the thermal and fluid flow characteristics with the presence of porous media at three values of porosity. For the sake of comparison, we bring the results predicted by LBM scheme using the combination of D2Q9 and D2Q9 [21], and solution to Brinkman-Forchheimer equation using finite element method [16]. The Darcy number and Prandtl number are set at constant value of 0.01 and 1.0 respectively. As can be seen from the table, the results predicted by present LBM model are in excellent agreement with those from FEM and conventional LBM. There are two interesting characteristics which can be drawn from the table; (1) for a fix porosity, the Nusselt number increases as the Rayleigh number increases and (2) for a fix Rayleigh number, the Nusselt number decrease linearly as the porosity decrease. These indicate the applicability of the combination of D2Q9 and D2Q4 lattice model for simulating thermal fluid flow in porous medium.

Table 2. Comparison of average Nusselt number among Navier Stokes solver using finite element method, LBM and present model.

		FEM[16]	LBM[21]	Present
$Ra = 10^3$	$\varepsilon = 0.9$	1.023	1.017	1.019
	$\varepsilon = 0.6$	1.015	1.012	1.012
	$\varepsilon = 0.4$	1.010	1.007	1.008
$Ra = 10^4$	$\varepsilon = 0.9$	1.640	1.659	1.721
	$\varepsilon = 0.6$	1.530	1.522	1.495
	$\varepsilon = 0.4$	1.408	1.393	1.313
$Ra = 10^5$	$\varepsilon = 0.9$	3.910	3.555	3.110
	$\varepsilon = 0.6$	4.030	3.555	3.110
	$\varepsilon = 0.4$	3.635	3.437	2.982

4 Conclusion

In this paper, the simplest combination of two-dimensional thermal lattice Boltzmann method is brought to predict natural convection in a square cavity filled with porous medium. We found that the present method correctly predicted the flow feature for different Rayleigh number and porosity, and gives excellent agreement with the results of previous studies. The results obtained demonstrate that this proposed approach in the thermal lattice Boltzmann model is very efficient procedure to study flow and heat transfer in a differentially heated square enclosure with the presence of porous medium. Computation at lower value of Darcy numbers to investigate the behavior of fluid flow at non-Darcian region will be our near future research topic.

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