The Numerical Simulation of Double-Diffusive Laminar Mixed Convection Flow in a Lid-Driven Porous Cavity

SHASHAK MISRA, A. SATHEESH*, C.G. MOHAN AND P. PADMANATHAN
School of Mechanical and Building Sciences
VIT University, Vellore – 632014
Tamil Nadu, INDIA
Corresponding author: Tel.: +91-416-224-2164; mob: +91-959-787-2825; satheesh.a@vit.ac.in

Abstract: - This paper presents the numerical investigation of double-diffusive mixed convective flow in an impermeable enclosed cavity. The uniform temperature and concentration is imposed along the vertical walls and the horizontal walls are considered as insulation. The flow behavior is analyzed when the top wall moves left side at a constant velocity (Uo) and the other walls kept remains stationary. Transport equations are solved using finite volume technique. The pressure and velocity terms are coupled by SIMPLE algorithm. The third order deferred Quadratic Upwind Interpolation for Convection Kinematics (QUICK) scheme and second order central difference scheme are applied at the inner and boarder nodal points, respectively. The present numerical simulation is compared with the reported literature and found that are in good agreement. The heat and mass transfer results are presented in the form of iso-temperature and concentration. The Lewis number (Le) and aspect ratio are varied over a wide range to analyse non-dimensional horizontal (U) and vertical velocities (V), stream line contours, temperature and concentration gradients. The present analysis is carried out at constant Prandtl (Pr=0.7), Richardson (Ri=1.0), Darcy (Da=1.0) and Reynolds (Re=100) numbers. Results show that the fluid flow and heat transfer increase with the increase in Lewis Number and low aspect ratio. The local and average Nusselt and Sherwood numbers are also presented for the different Aspect ratio and Lewis number.

Key-Words: - Double-diffusive; mixed convection; numerical study; aspect ratio; Lewis number

1 Introduction

Heat and mass transfer induced by double-diffusive mixed convection in fluid saturated porous media have practical importance in many engineering applications. This aspect of fluid dynamics is gaining attention towards researchers all over the world. Migration of moisture in fibrous insulation, drying processes, chemical reactors, transport of contaminants in saturated soil and electro-chemical processes are some examples of double-diffusive natural convection phenomena [1]. Double diffusion occurs in a very wide range of fields such as oceanography, astrophysics, geology, biology, and chemical processes, as well as in many engineering applications such as solar ponds, natural gas storage tanks, crystal manufacturing, and metal solidification processes. Due to the importance of double-diffusive convection in various engineering and geophysical problems, it has been studied extensively now-a-days. Al-Amiri and Khanafer [2,3] research groups extensively studied a double-diffusive mixed convection in a two-dimensional, horizontal annulus cavity. Transport equations are solved by Finite Element Galerkin Weighted Residual method. The effects of heat and mass transfer are investigated in a cylindrical geometry for a wide range of non-dimensional parameters. They extended their investigation for a mixed convection flow in a square cavity whose top wall is moving from left to right with the combined buoyancy effects of heat and mass diffusion. Nusselt and Sherwood numbers at the bottom wall are studied for the selected non-dimensional parameters. Khanafer and vafai [4] presented an unsteady laminar mixed convection heat transfer in a lid-driven cavity. The forced convective flow inside the cavity is attained by a mechanically induced sliding lid, which is set to oscillate horizontally in a sinusoidal fashion. The natural convection effect is sustained by subjecting the bottom wall to a higher temperature than its top counterpart. The two vertical walls are kept constant. Chen and Cheng [5] numerically investigated the Periodic behavior of the mixed convective flow in a rectangular cavity with a vibrating lid.

Kuznetsov and Sheremet [6] studied the transient thermosolutal convection in a three dimensional cubical enclosure having finite thickness walls filled with air, subjected to temperature and concentration gradients. Nikbhakti et al. [7] determined the maximum and minimum heat and mass transfer by varying thermal and solute buoyancy forces. Recently, Mahapatra et al. [8] analyzed the effect of buoyancy ratio and, uniform and non-uniform heating of walls by allocating staggered grid finite difference method. It is observed from the literature that the double-diffusive natural convection was reported extensively, very few work was presented in the mixed convection flow. However, they did not consider the flow behavior in porous medium. Teamah et al. [9] simulated a laminar double-diffusive natural convection flow in an inclined cavity. They presented that the increase in buoyancy ratio found to enhance Nu and Sh significantly for both cases of lid movement. Therefore, in the present study, attention will be focused on the problem concerning the steady state, laminar flow double-diffusion mixed convection in a porous media. The effect of velocities, stream line contours, temperature and concentration gradients are investigated over the range of Lewis number and geometrical aspect ratio.

2 Mathematical Modeling and Numerical Solution

P Fig.1 illustrates the two-dimensional square cavity consisting of four walls. Left and right walls are kept constant temperature and concentration. The cavity is filled with incompressible Newtonian fluid. Forced convection is provided by moving the top wall in the left direction with velocity (Uo) in –x direction as shown in the Fig. 1. Right wall is at low temperature (TC) and concentration (CC), while the left wall is at high temperature (TH) and concentration (CH).

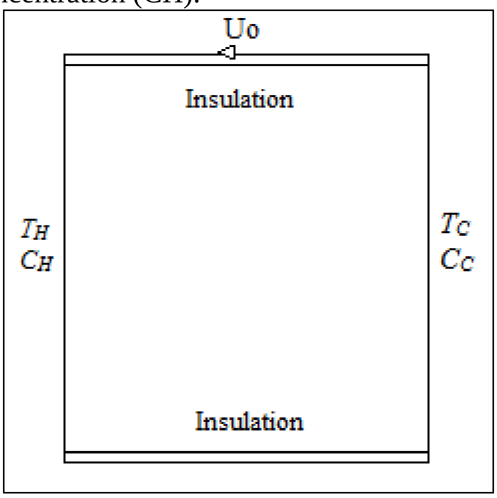


Fig. 1: Schematic of Physical problem

The fluidic porous media inside the cavity is assumed to be homogenous, isotropic and in state of thermodynamic equilibrium. The properties of the fluid such as thermal conductivity, diffusivity are kept constant except density because of the buoyancy effect. The density is calculated from the Boussinesq approximation as

$$\rho = \rho_o\left[1-\beta_T(T-T_L)-\beta_C(C-C_L)\right] \quad (1)$$

where,

$$\beta_T = -\frac{1}{\rho_o}\left(\frac{\partial\rho}{\partial T}\right)_{P,C} \qquad \beta_C = -\frac{1}{\rho_o}\left(\frac{\partial\rho}{\partial C}\right)_{P,T} \quad (2)$$

The governing equations for laminar steady two-dimensional mixed convection, after invoking Boussinesq approximation and neglecting the viscous dissipation, can be expressed in the non-dimensional form as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (3)$$

$$\frac{1}{\varepsilon^2}\left[U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right]=-\frac{\partial P}{\partial X}+\frac{1}{\varepsilon \,\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) -\frac{1}{Da\,\mathrm{Re}}U-\frac{F_c}{\sqrt{Da}}U\sqrt{U^2+V^2} \quad (4)$$

$$\frac{1}{\varepsilon^2}\left[U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right]=-\frac{\partial P}{\partial Y}+\frac{1}{\varepsilon \,\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) +\frac{Gr}{\mathrm{Re}^2}(\theta+NC)-\frac{1}{Da\,\mathrm{Re}}V-\frac{F_c}{\sqrt{Da}}V\sqrt{U^2+V^2} \quad (5)$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \quad (6)$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{\mathrm{Re}\,Sc}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \quad (7)$$

Reynolds number is $\mathrm{Re}=\frac{V_o L}{\nu}$; thermal Grashof number is $Gr_T=\frac{g\beta_T\Delta T L^3}{\nu^2}$; solutal Grashof number is $Gr_C=\frac{g\beta_C\Delta C L^3}{\nu^2}$; Prandtl number is $\mathrm{Pr}=\frac{\nu}{\alpha}$;

Schmidt number is $Sc = \frac{\nu}{D}$; buoyancy ratio is $N = \frac{Gr_T}{Gr_C}$ and Darcy number is $Da = \frac{K}{L^2}$.

Tab.1: Boundary conditions of the physical problem

Boundary Conditions	T	C	U	V
Left Wall (X=0; 0≤Y≤1.0)	T_H	C_H	0	0
Right Wall (X=L; 0≤Y≤1.0)	T_C	C_C	0	0
Top wall (0≤X≤1.0; Y=L)	$\frac{\partial \theta}{\partial Y}=0$	$\frac{\partial C}{\partial Y}=0$	-1	0
Bottom wall (0≤X≤1.0; Y=0)	$\frac{\partial \theta}{\partial Y}=0$	$\frac{\partial C}{\partial Y}=0$	0	0

3 Methodology

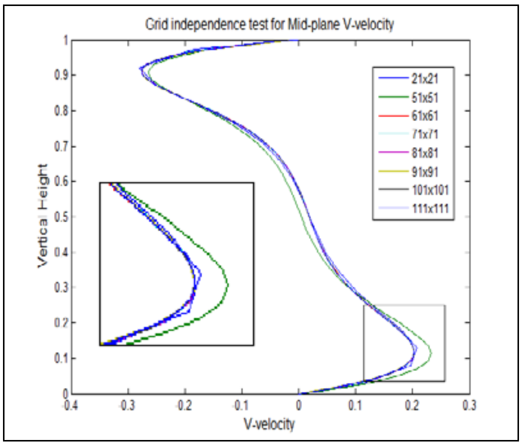
The Finite Volume Method (FVM) is used to obtain numerical solutions of the governing equations on a staggered grid arrangement. In the discretization, the third-order differed QUICK scheme and a second order central difference scheme are used for convection diffusion terms in the inner and boarder nodes, respectively. The SIMPLE algorithm is chosen to numerically solve the governing equations. The pressure correction equation is derived from the continuity equation to enforce the conservation of mass. The discretized equations are solved using line by line procedure, combining the tri-diagonal matrix algorithm (TDMA). The iterative procedure is repeated until the following condition is satisfied,

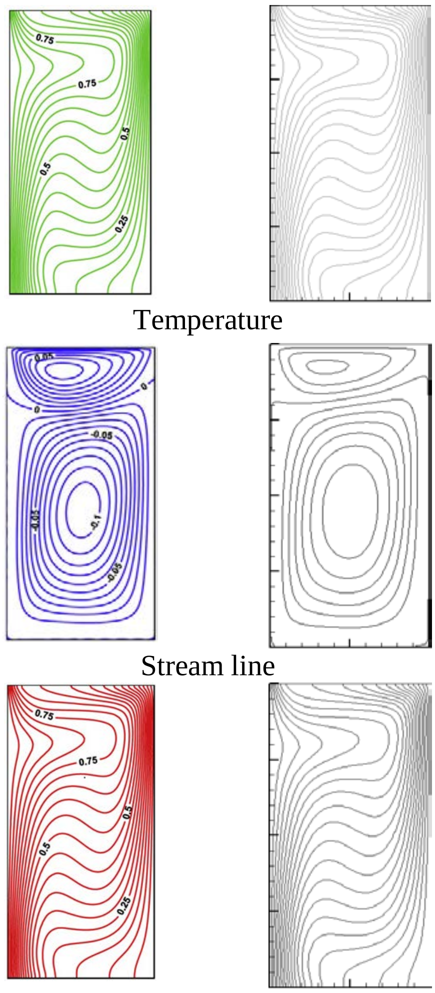
$$\frac{\sum_i \sum_j |\Phi_{i,j}^{n+1} - \Phi_{i,j}^{n}|}{\sum_i \sum_j |\Phi_{i,j}^{n+1}|} \leq 10^{-6}$$

Where, Φ denotes for U, V, T and C. The subscript i and j indices denote grid locations in the x and y directions, respectively. It is observed from the present numerical results that the decrease of convergence criteria 10-6 does not cause any significant change in the final results.

4 Results and Discussion

The grid independent test for the present analysis is performed at mid vertical plane V velocity as shown in Fig. 2. It is investigated with different grid sizes of 21x21, 51x51, 61x61, 71x71, 81x81, 91x91, 101x101 and 111x111 and it was found that the values of V velocity along the vertical mid plane did not have significant change in their values after the increase in grid size from 81x81. Therefore, further numerical analysis is carried out using 81x81 grid sizes. The code is also validated with the results of Tameah et al. (2010) as shown in Fig. 3. The results obtained from the present code are in good agreement with the literature, thereby giving credibility and validation to the result of present FVM code.





***Tameah et. al* (*2010*)** **Present study**

Fig. 3 Validation of present numerical results with Tameah et al. (2010)

The effects of Aspect Ratio (A) depicts in Fig. 4, 5 and 6. The effect on Iso-concentration for individual Lewis number (Le) and varying aspect ratio are discussed as per observations. The increasing Lewis number symbolizes that the decrease in mass diffusivity with increasing thermal diffusivity which can be physically explained that the mass flow pattern is reducing. The mixing of fluid is decreasing and the hotter fluid and cold fluid reach a steady state of equilibrium which can be specified by stating that hotter (lighter) fluid is circulating on the top half and the colder (denser) fluid at the bottom half of the cavity for A = 1 & 2 and Le = 10 & 50. For A = 0.25 & 0.5 the fluid undergoes thorough mixing as there was no separation of hot and cold fluid due to restriction of space along the vertical height or y-direction.

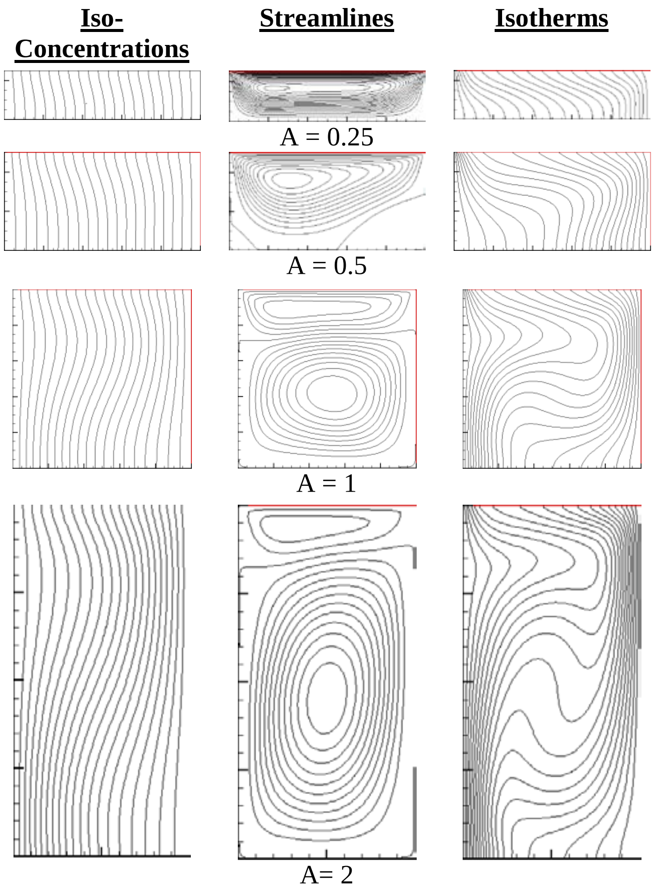


Fig. 4 Comparison of varying Aspect Ratios for *Le* = 0.1, *Pr*=0.7 and *Ri* = 1.

The effects on Streamlines are observed and it is clearly seen how the flow patterns develop stronger with increase in Le showing more mixing with increase for Aspect Ratio (A) 0.25. For Aspect Ratios increasing from 0.5 to 2, a progressive generation of two streamlines, top streamlines flowing in counter-clockwise direction with hot fluid whereas the lower streamlines flowing in the clockwise direction with cold fluid is observed. With increase in Le, the flow patterns become stronger and dominant in their respective regions. These streamlines show the direction of the fluid movement and also the effect of Aspect Ratio in development of the same. Lewis number characterizes the flow for the concentration of the fluid filled in the cavity and mixing occurring due to its variations.

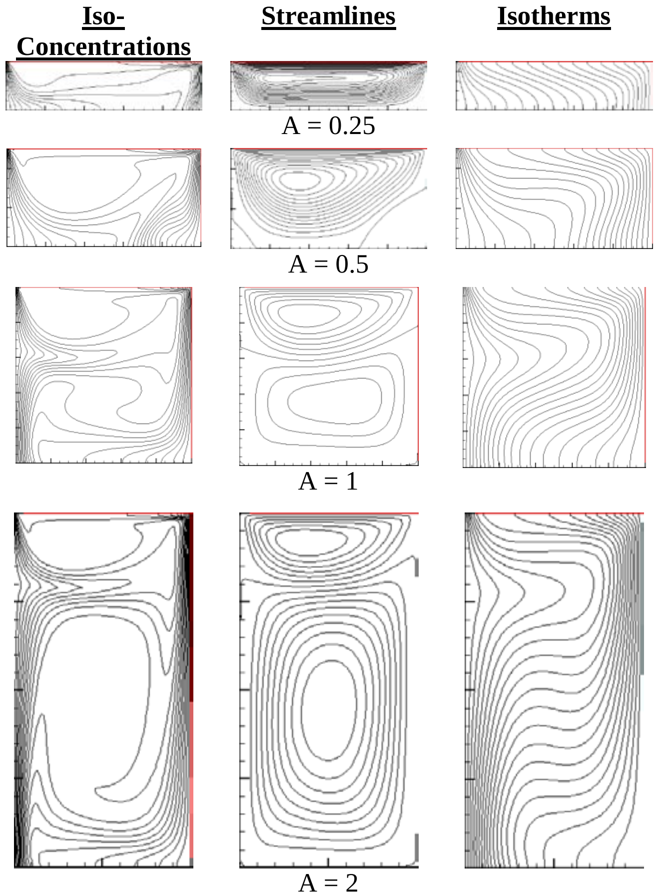


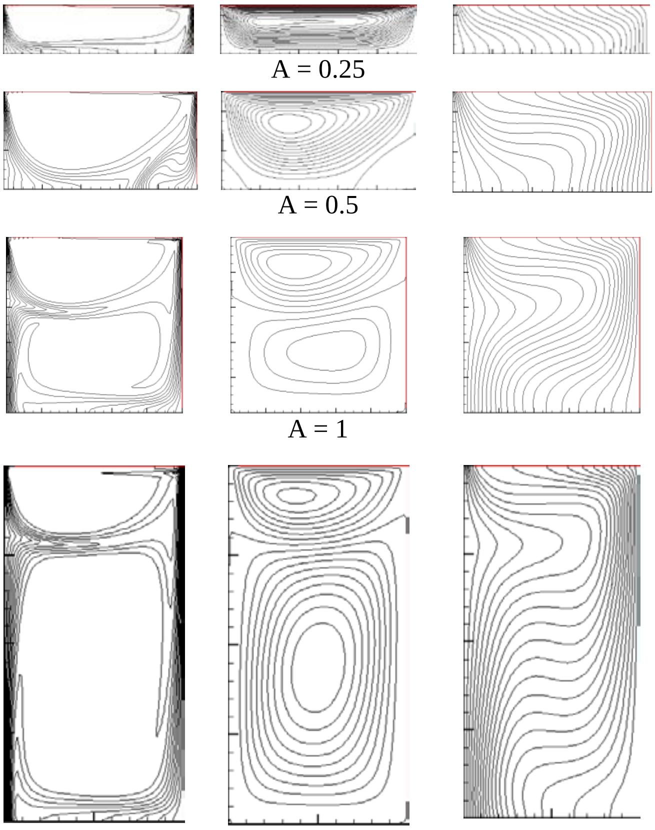
Fig. 5 Comparison of varying Aspect Ratios for *Le* = 10, *Pr*=0.7 and *Ri* = 1.

The effect on isotherms as observed in the given results clearly shows that there is no significant difference. The heat transfer flow remains same for a given Aspect Ratio irrespective of the increasing Lewis number (Le). The results for Lewis Number (Le) = 1 are not presented here as the Le value itself signifies that the thermal diffusivity and mass diffusivity are equal to each other thereby their patterns are exactly identical.

The effect on u-velocity for different Lewis numbers with varying aspect ratios is presented in Fig. 7. It is clearly seen that the velocities change direction in the lower half with increasing Aspect

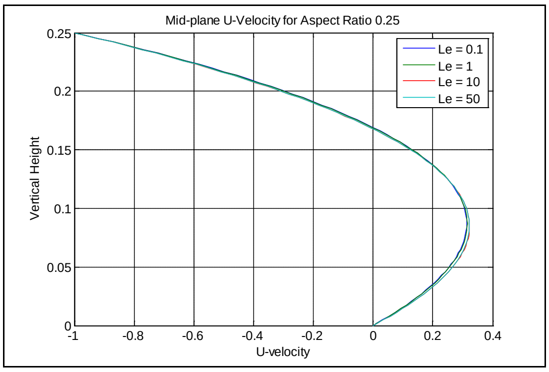
Ratio. For A = 0.25 and 0.5, the mid plane u-velocity is positive from the bottom wall and gradually decreases to zero, then, it reaches the maximum velocity (u = 1) at the top wall. It is also noted that the magnitude of velocities are decreasing with increasing Lewis number for a particular Aspect Ratio. Thus, we can state that the mass diffusivity decreases the velocities also increase thereby increasing the mixing for a particular aspect ratio.

Iso-Concentrations **Streamlines** **Isotherms**



A = 2

Fig. 6 Comparison of varying Aspect Ratios for *Le* = 50, *Pr*=0.7 and *Ri* = 1.



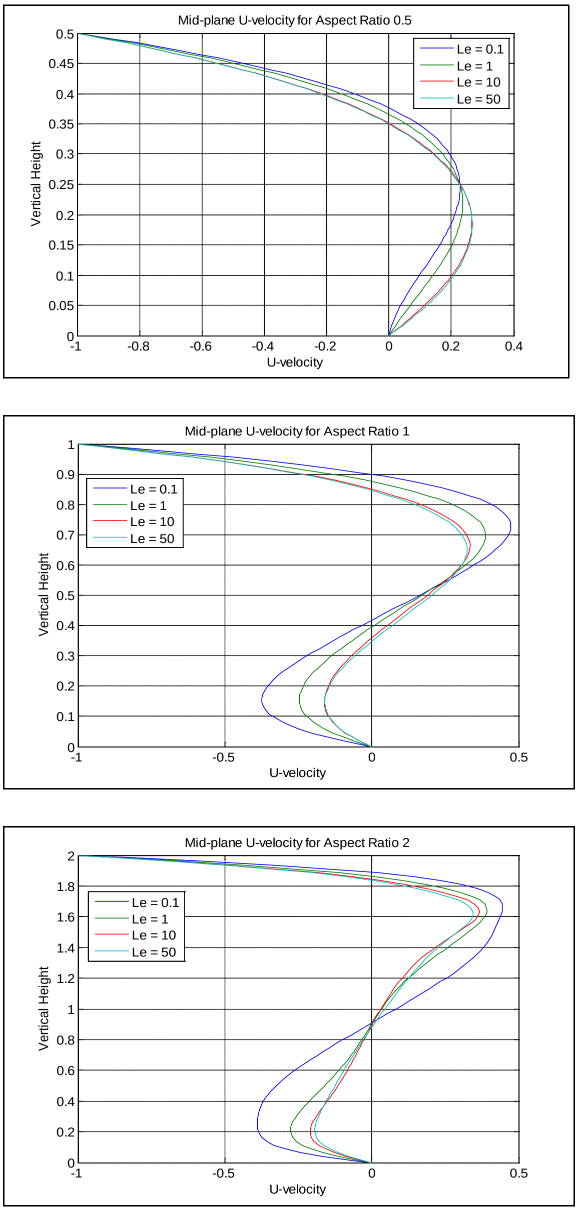


Fig. 7: Effects on Lewis Number at different Aspect Ratio in Mid-plane U-velocity.

In Fig. 8, the effect on v-velocity profile for Lewis numbers at different Aspect Ratios (A) is presented. For the aspect ratio of 0.25, the distance between left and right walls is high. Therefore, the v-velocity remains zero over a horizontal distance of 0.4 to 0.6 as shown in Fig. 8(a). It is also observed that the variation of v- velocity with Le is found to be insignificant for aspect ratio 0.25. Results show that the velocities change direction near the right wall with increasing Aspect Ratio. This shows reversal of velocity and creation secondary stream lines near the centre and lower half with increase in Aspect Ratio. It is also noted that the magnitude of velocities are decreasing with increasing Lewis number for a particular Aspect Ratio.

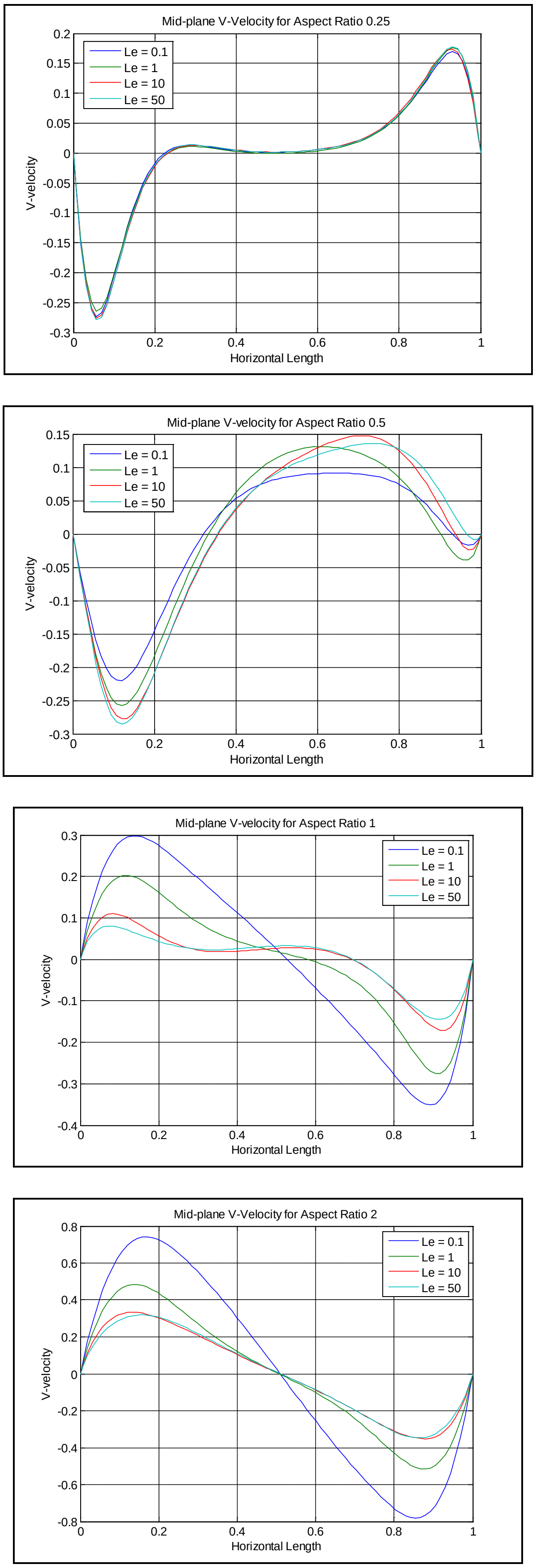


Fig. 8: Effects on Lewis Number at different Aspect Ratio in Mid-plane V-velocity

Thus, we can state as mass diffusivity decreases the velocities also decrease thereby reducing the mixing for a particular aspect ratio.

The variation of Local Nusselt Number (Nu) on the left and the right wall are presented in Figs. 9(a) and 9(b). For the selected aspect ratio of 1, the effect of Le on the heat transfer is negligible. However, the mass diffusion is predominant which is clearly observed from Figs.9 and 10.

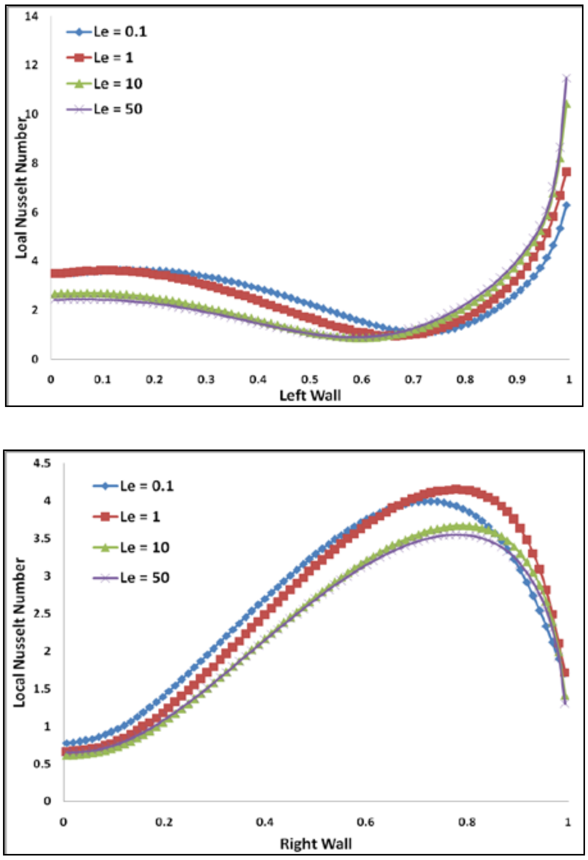


Fig. 9: Local Nusselt Number for Aspect Ratio 1 with varying Lewis Number on left and right walls

Fig. 11 shows the average Nusselt Number ($\overline{Nu}$) and Sherwood Number ($\overline{Sh}$) variation with respect to Lewis Number and Aspect Ratio. It is observed that the average Sherwood number value decrease with increase in Aspect Ratio which shows to provide more space for convective as well as diffusive mass transport. In case of average Nusselt Number increment in lower aspect ratio while the trend reverses for higher aspect ratios with increment in Lewis Number. Thus, for A = 0.5, we can say that the conductive heat transport is dominant for increasing Lewis Number. Whereas for A = 2, the convective heat transfer is dominates.

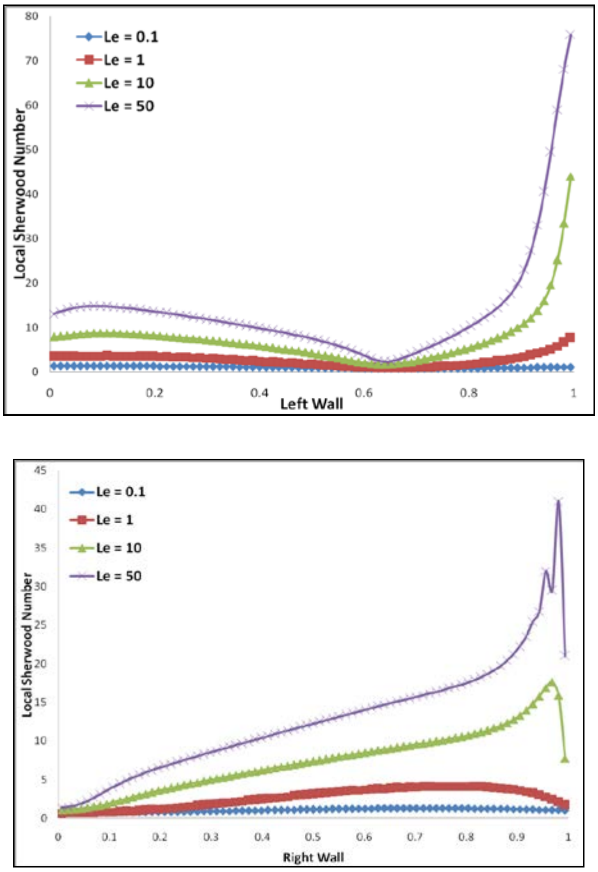


Fig. 10: Local Sherwood Number for Aspect Ratio 1 with varying Lewis Number on left and right wall

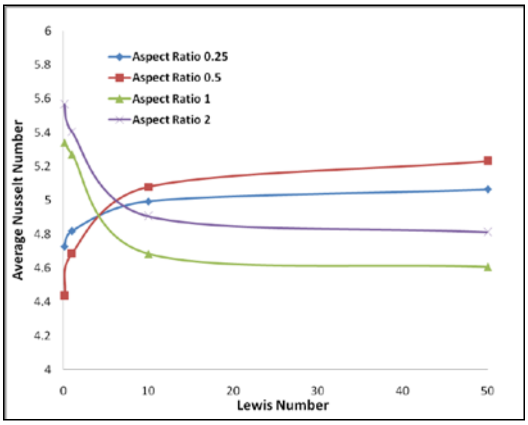


Fig. 11: Effects on Average Nusselt Number with varying Lewis Number

5 Conclusion

The fluid behavior in a two sided lid driven closed square porous cavity due to double diffusive mixed convection is investigated. The effect of aspect ratio and Lewis Number on stream line contours, temperature and concentration gradient are presented. The heat and mass diffusion are also presented in terms of local and average Nusselt and Sherwood numbers. For low aspect ratio (A=0.25), the variation of u and v- velocities with Le is found to be insignificant. There was good agreement between the present numerical results and the literature. The u-velocity magnitude is decreasing with increasing Lewis number for higher aspect ratios. Results show that the fluid flow increases with the increasing Lewis Number (Le). The effect of thermal gradient is negligible for the low Le. From this analysis, it is concluded that the temperature and concentration are strong function of aspect ratio and Lewis number.

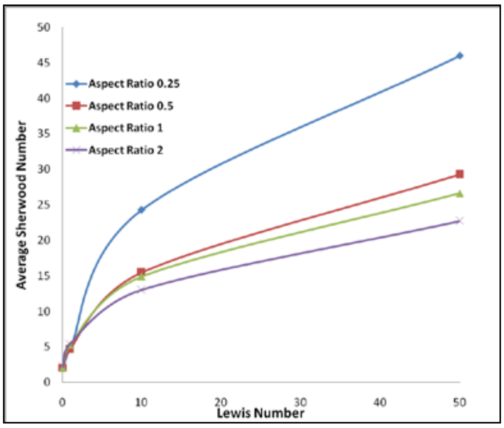


Fig. 12: Effects on Average Sherwood Number with varying Lewis Number

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