An Evaluation of Teaching Performance: The Fuzzy AHP and Comprehensive Evaluation Approach

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Abstract: - Teaching performance evaluation is one of the main instruments to improve teaching quality and plays an important role in strengthening management of higher education institutions. In this paper, we present a framework for teaching performance evaluation based on fuzzy AHP and fuzzy comprehensive evaluation methods. First, the teaching performance index system was determined and then the factor and sub-factor weights were calculated by the fuzzy AHP method. Employing the fuzzy AHP in group decision making facilitated a consensus of decision-makers and reduces uncertainty. Fuzzy comprehensive evaluation was then employed to evaluate teaching performance. This paper also used a case application to illustrate the proposed framework. The application of this framework can obtain a scientific and objective evaluation result. It is expected that this work may serve as a tool for educational institution managers that improves teaching performance quality.

Key Words: - Evaluation, Higher education, Fuzzy AHP; Fuzzy comprehensive evaluation, Fuzzy theory, Teaching performance

1 Introduction

Due to the trends of internationalization and globalization, universities face increased competition from many higher education institutions. To improve their competitiveness level in the higher education system, universities should provide their best services to meet social needs. According to Weber [1], a university can only provide the best services to the community if it commits to continuous improvement in the quality of services and activities. Teaching is always one of the major tasks of most universities. Therefore, quality teaching is one of the primary objectives of higher education institutions, and consequently, there is a need to evaluate teaching performance. Evaluating teaching performance is not an easy task as it involves human decision making which is imprecise, vague, and uncertain. Hence, using a scientific method to evaluate teaching performance comprehensively and effectively plays a crucial role in determining the teaching performance quality level.

Many studies related to teaching performance evaluation can be found in literature. However, most of them concentrated on strategies, while few were devoted to the quantitative analysis of an evaluation index system. In recent years, the fuzzy logic techniques have been successfully applied in comprehensive evaluations to reduce the subjectivity and imprecision of evaluation results. Additionally, a number of researchers have focused on teaching performance by employing the fuzzy theory. He et al. [2] presented an approach for teaching performance evaluation based on the Analytic Hierarchy Process (AHP) and fuzzy comprehension evaluation. Their method used AHP to determine the weights and the results of the evaluation reflected teaching quality more objectively. Dong and Dai [3] combined fuzzy and neuron networks to evaluate teaching quality. They used historical data as a standard indicator to train the neuron network. The combined method has been a good application of fuzzy theory in evaluating teaching performance. Ramli et al. [4] proposed a method for teaching performance evaluation with outlier data using the fuzzy approach. Their method provided an accurate evaluation of teaching performance. In addition to the above studies, there are other achievements in the application of fuzzy theory in teaching performance evaluation. These studies all provided good applications of mathematical models in performance evaluation.

The studies we examined did not give adequate consideration to the design of a scientific evaluation index system. In contrast, our study concentrates on the establishment of a teaching performance evaluation index system with reasonable and objective factor weights. Determining the weight of a factor is related to the multiple-criteria decision making problem, and decision-makers usually feel more confident giving linguistic variables rather than expressing their judgments in the form of numeric values. Hence, fuzzy set theory is a useful tool to deal with imprecise and uncertain data. AHP, proposed by Satty [5] in the middle of the 1970s, is a practical decision-making method that solves multi-target and multi-layer decision-making problems. It can deal with the weights with respect to many factors and alternatives. Being an extension of AHP, fuzzy AHP is able to solve hierarchical fuzzy decision-making problems. The fuzzy AHP method has been widely used by various researchers to solve different decision making problems. For example, Chang et al. [6] used fuzzy AHP to construct an expert decision making process. Their system was used to assist decision makers in assessing the feasibility of a digital video recorder system. Gungor et al. [7] proposed a personnel selection system based on fuzzy AHP that evaluated the best and most adequate personnel dealing with the rating of both qualitative and quantitative criteria. Chou et al. [8] employed fuzzy AHP to evaluate the weighting for each criterion in human resources for science and technology. Apart from the aforementioned applications, there are many other studies that used fuzzy AHP to solve different managerial problems. These studies revealed the high applicability of fuzzy AHP for practical purposes. Therefore, fuzzy AHP is appropriate for determining the weights in the performance evaluation index system. In this paper, the fuzzy AHP method [9, 10] was utilized to obtain the factor weights of a teaching performance evaluation index system. On the basis of the system, the fuzzy comprehensive evaluation is applied to evaluate teaching performance.

The application of fuzzy AHP for determining the weights in the teaching performance evaluation system can be briefly described as follows. First, the hierarchical structure of the system was developed. A group of decision-makers was then formed and invited to evaluate the factors. The decision-makers' comparison of the importance of one factor over another can be done with the help of a questionnaire, which is in the form of a linguistic assessment. The linguistic assessments of each decision-maker in the group were converted to triangular fuzzy numbers. After that, these triangular fuzzy numbers were used to build the comparison matrices of individual decision-makers based on the pair-wise comparison technique. Once their consistency ratios were checked and accepted, the matrices of all decision-makers were used to build a representative comparison matrix of the group. The factor and sub-factor weights can now be calculated from this representative comparison matrix by the fuzzy AHP method.

The remainder of this paper is organized as follows. Section 2 discusses fuzzy AHP and some related concepts. Section 3 then presents the framework for designing the evaluation index system. Section 4 deals with establishing the teaching performance evaluation index system and determining the factor and sub-factor weights. Section 5 presents an application of the proposed evaluation index system based on a comprehensive evaluation method; finally, conclusions are then given in Section 6.

2 Fuzzy Analytic Hierarchy Process (Fuzzy AHP)

In this section, the fuzzy sets and fuzzy numbers are briefly introduced, and then the fuzzy AHP method is presented.

2.1. Fuzzy Sets and Fuzzy Numbers

Fuzzy set theory was first introduced by Zadeh [11] to deal with the uncertainty due to imprecision or vagueness. A fuzzy set, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, is a set of ordered pairs and X is a subset of the real numbers, R, where $\mu_{\tilde{A}}(x)$ is called the membership function which assigns to each object, x, a grade of membership ranging from zero to one. Since its introduction, fuzzy set theory has been widely applied to address real-world problems in which decision makers need to analyze and process information that is imprecise. A fuzzy number is a special case of a convex normalized fuzzy set [12]. It is possible to use different fuzzy numbers in various particular situations. Triangular and trapezoidal fuzzy numbers are usually adopted to deal with the vagueness of decisions related to the performance levels of alternative choices with respect to each criterion. When the two most promising values of a trapezoidal fuzzy number are the same number, it becomes a triangular fuzzy number (TFN). This means that a TFN is a special case of a trapezoidal fuzzy number. Because of its intuitive appeal and computational efficiency, the TFN is the most widely used membership function for many applications. TFNs are usually employed to capture the vagueness of the parameters related to the decision-making process. In order to reflect the fuzziness which surrounds the decision makers when they conduct a pairwise comparison matrix, the TFN is expressed with boundaries instead of crisp numbers. A triangular fuzzy number, denoted as $\tilde{A} = (l, m, u)$, has the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-l)/(m-l), & l \le x \le m \\ (u-x)/(u-m), & m \le x \le u \\ 0 & otherwise \end{cases} \quad (1)$$

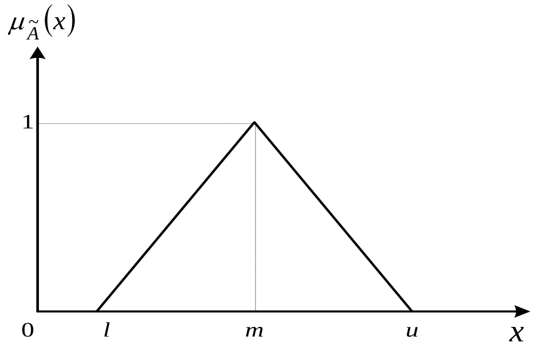

Fig. 1: A triangular fuzzy number, $\tilde{A} = (l, m, u)$

The triangular fuzzy number $\tilde{A}$ is shown in Figure 1. The parameter m is the most promising value. The parameters l and u are the smallest and the largest possible value, respectively; they limit the field of possible evaluation. When $l=m=u$, the triangular fuzzy number becomes a non-fuzzy number. The triplet (l, m, u) can be used to describe a fuzzy event.

Consider two TFNs, $\tilde{A}_1$ and $\tilde{A}_2$, $\tilde{A}_1 = (l_1, m_1, u_1)$ and $\tilde{A}_2 = (l_2, m_2, u_2)$. The main operational laws [13] for two triangular fuzzy numbers, $\tilde{A}_1$ and $\tilde{A}_2$, are as follows:

Addition of the fuzzy number

$$\tilde{A}_1 \oplus \tilde{A}_2 = (l_1+l_2, m_1+m_2, u_1+u_2) \quad (2)$$

Multiplication of the fuzzy number

$$\tilde{A}_1 \otimes \tilde{A}_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2) \text{ for } l_i > 0, m_i > 0, u_i > 0, i = 1, 2 \quad (3)$$

Division of the fuzzy number

$$\tilde{A}_1 / \tilde{A}_2 = (l_1/u_2, m_1/m_2, u_1/l_2) \text{ for } l_i > 0, m_i > 0, u_i > 0, i = 1, 2 \quad (4)$$

Reciprocal of the fuzzy number

$$\tilde{A}_1^{-1} \approx (1/u_1, 1/m_1, 1/l_1) \quad \text{for } l_1 > 0, m_1 > 0, u_1 > 0 \quad (5)$$

2.2 The Fuzzy AHP method

The AHP decision-making process uses pairwise comparisons and matrix algebra to identify and estimate the relative importance of elements. This method questions relevant experts using a nine-point scale. AHP has the power to solve complex decision-making problems. However, ambiguous problems can limit the power of pure AHP. Fuzzy AHP, an extension of the AHP model, has been applied to fuzzy decision-making problems. In fuzzy AHP, by using fuzzy arithmetic, the weights of evaluative elements are determined. The weight calculation steps at a given level are as follows:

A matrix $\tilde{A}$ is constructed according to fuzzy pairwise comparisons.

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \ldots & \tilde{a}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \ldots & 1 \end{bmatrix}, \quad (6)$$

where $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ is the fuzzy comparison value of element i to element j.

The fuzzy weights of each element are calculated as

$$\tilde{r}_i = (\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \ldots \otimes \tilde{a}_{in})^{1/n}, \text{ for } i = 1,2,\ldots,n, \quad (7)$$

$$\tilde{w}_i = \frac{\tilde{r}_i}{\tilde{r}_1 \oplus \tilde{r}_2 \oplus \ldots \oplus \tilde{r}_n}, \text{ for } i = 1,2,\ldots,n, \quad (8)$$

where $\tilde{r}_i$ is the geometric mean of the fuzzy comparison value of element i to each element, and $\tilde{w}_i$ is the fuzzy weight of the ith element.

The fuzzy weight vector $\tilde{W}$ is constructed as

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)^T. \quad (9)$$

3 The Proposed Framework for Designing a Performance Evaluation Index System based on Fuzzy AHP

In order to search for a consensus, it is necessary to establish a representative and democratic decision-making process when designing the evaluation index system. The proposed framework is composed of the following steps:

3.1. Developing a Hierarchical Structure of the Evaluation Index System

The hierarchical structure is constructed by combining all the factors and sub-factors specific to the research problem. Based on the identified factors and sub-factors, the hierarchical structure for evaluation is obtained. In the system, the objective is in the first level, and factors and sub-factors are in successive levels. Regarding the number of elements, most decision-makers cannot simultaneously handle more than seven to nine factors when making a decision [14].

3.2. Establishing a Group of Decision-makers

A group of decision-makers is formed. The members of the group are experts who have experiences in the research field. The decision-makers are required to provide the relative importance of each factor and sub-factor.

3.3. Determining the Linguistic Variables and Fuzzy Conversion Scale

The decision-makers make pair-wise comparisons of the importance or preference between each pair of factors. Consider a problem at a level with n elements. Each set of pair-wise comparisons for a level requires $n(n-1)/2$ judgments, which are further used to construct a positive fuzzy reciprocal comparison matrix. The comparison of one factor over another can be done with the help of questionnaires, which are in the form of linguistic variables. A linguistic variable is a variable whose values are words or sentences in a natural or artificial language [15]. In this paper, TFNs are used to represent subjective pair-wise comparisons of decision-makers, namely, “just equal”, “equally important”, “weakly more important”, “strongly more important”, “very strongly more important” and “absolutely more important”. The triangular fuzzy conversion scales and linguistic scales, which are proposed by Kahraman et al. [16], are used to convert such linguistic values into fuzzy scales, as is demonstrated in Figure 2 and Table 1.

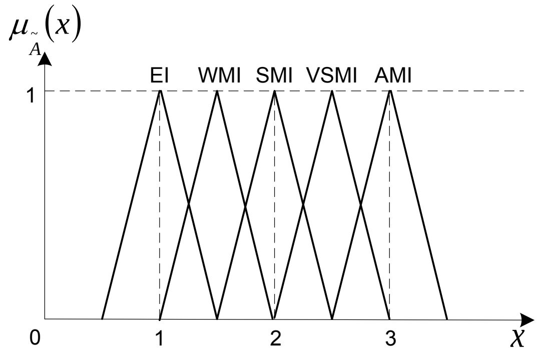

Fig. 2: Linguistic scale for relative importance.

Table 1: Linguistic scales and fuzzy scales for importance

Linguistic scale for importance	Triangular fuzzy scale
Just equal	(1,1,1)
Equally important (EI)	(1/2,1,3/2)
Weakly more important (WMI)	(1,3/2,2)
Strongly more important (SMI)	(3/2,2,5/2)
Very strongly more important (VSMI)	(2,5/2,3)
Absolutely more important (AMI)	(5/2,3,7/2)

3.4. Establishing Comparison Matrices

Consider a problem at one level with n factors, where the relative importance of factor i to j is represented by triangular fuzzy numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. One decision-maker considers factor i is strongly more important as compared with factor j; he/she may set $\tilde{a}_{ij}$ = (3/2, 2, 5/2). If factor j is thought to be strongly more important than factor i, the pair-wise comparison between i and j could be presented by $\tilde{a}_{ij}$ = (2/5, 1/2, 2/3).

As in the traditional AHP, the comparison matrix $\tilde{A} = \{\tilde{a}_{ij}\}$ can be constructed as:

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ 1/\tilde{a}_{12} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ 1/\tilde{a}_{1n} & 1/\tilde{a}_{2n} & \dots & 1 \end{bmatrix} \quad (10)$$

3.5. Calculating the Consistency Index and Consistency Ratio of the Comparison Matrix

To assure a certain quality level of a decision, the consistency of an evaluation has to be analyzed. Saaty [5] proposed an index to measure consistency. This index can be used to indicate the consistency of the pair-wise comparison matrices. To investigate their consistency, the fuzzy comparison matrices need to be converted into crisp matrices [17]. The fuzzy mean and spread method [18] is utilized to defuzzify the fuzzy numbers. This method ranks fuzzy numbers according to the probabilities of fuzzy events. Assume that $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ is a TFN with uniform distribution. Its mean $x(\tilde{a}_{ij})$ is calculated as

$$x(\tilde{a}_{ij}) = (l_{ij}+m_{ij}+u_{ij})/3. \quad (11)$$

After all the elements in the comparison matrix are converted from triangular fuzzy numbers to crisp numbers. The consistency index, CI, for a comparison matrix can be computed with the use of the following equation:

$$CI = \frac{\lambda_{\max} - n}{n-1}, \quad (12)$$

where $\lambda_{\max}$ is the largest eigenvalue of the comparison matrix, and n is the dimension of the matrix.

The consistency ratio (CR) [5] is defined as a ratio between the consistency of a given evaluation matrix and consistency of a random matrix:

$$CR = \frac{CI}{RI(n)}, \quad (13)$$

where *RI(n)* is a random index [19] that depends on n, as shown in Table 2.

Table 2: Random index (*RI*) of random matrices

n	3	4	5	6	7	8	9
RI(n)	0.58	0.9	1.12	1.24	1.32	1.41	1.45

If the CR of a comparison matrix is equal to or less than 0.1, it can be acceptable. When the CR is unacceptable, the decision-maker is encouraged to repeat the pair-wise comparisons. In this step, the MATLAB package can be employed to calculate the eigenvalues of all comparison matrices.

3.6. Constructing the Representative Matrix of all Decision-makers

Each individual judgment matrix represents the opinion of one decision-maker. Aggregation is necessary to achieve a group consensus of decision-makers. In the conventional AHP, there are two basic approaches for aggregating the individual preferences into a group preference, namely, aggregation of individual judgments (AIJ) and aggregation of individual priorities (AIP) [20]. The concepts and ideas employed in the conventional AHP can also be utilized in the fuzzy AHP. In the AIJ approach, the group judgment matrix is obtained from the individual judgment matrices. This means that the group judgment matrix is considered as the judgment matrix of a "new individual" and the priorities of this individual are derived as a group solution. However, in the AIP approach, the group members act individually. Specifically, from the individual judgment matrices, we obtain the individual priorities, and then from these, the group priorities. AIJ is most often performed using geometric mean operations, whereas, AIP is typically performed using arithmetic mean operations. Geometric mean operations are commonly used within the application of the AHP for aggregating group decisions [21], and only the geometric mean satisfies the Pareto principle (unanimity condition) and homogeneity condition [22]. Hence, in this research, the AIJ approach is utilized for the aggregation of group decisions.

Consider a group of K decision-makers involved in the research: they make pairwise comparisons of n elements. As a result of the pairwise comparisons, we get a set of K matrices, $\tilde{A}_k = \{\tilde{a}_{ijk}\}$, where $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$ represents a relative importance of element i to j, as assessed by the expert k. The triangular fuzzy numbers in the group judgment matrix can be obtained by using the following equation [23]:

$$l_{ij} = \min_{k=1,2,..K} \left(l_{ijk}\right)$$

$$m_{ij} = \sqrt[K]{\prod_{k=1}^{K} m_{ijk}}$$

$$u_{ij} = \max_{k=1,2,..K} \left(u_{ijk}\right) \qquad (14)$$

3.7. Calculating the Weights

Through the use of the fuzzy AHP method, factor and sub-factor fuzzy weights can be calculated from group decision matrices.

4 Establishing the Teaching Performance Evaluation Index System

It is necessary to design the index system from different angles and levels in order to reveal teaching performance accurately. However, designing teaching evaluation indexes is not an easy task [24] because there are many factors that impact teaching performance and they are at different levels with different relative levels of importance. In order to make a correct and objective evaluation, the sources of data used to evaluate teaching performance are students, colleagues, and teachers themselves [25]. Hence, the index system used in the evaluation process should have the capability of getting their opinions of teaching performance quality. Some organizations have produced guidelines which provide criteria for teaching performance evaluation in universities [26]. In this study, the hierarchical structure of the teaching performance evaluation index was derived from the criteria proposed by Brooker et al. [27]. The selected factors and sub-factors were discussed and revised by managers and experienced lecturers at educational institutions. The final hierarchical structure was then achieved, as shown in Table 3. It consists of six factors: "Planning and preparation", "Communication and interaction", "Teaching for learning", "Managing the learning environment", "Student evaluation" and "Professionalism", each of which is divided into two or more sub-factors.

Table 3: The teaching performance hierarchical structure

Factors	Sub-factors
Planning and preparation (U_1)	Clear goals and objectives (U_{11}) Clear, logical, and innovative documentations (U_{12})
Communication and interaction (U_2)	Proficiency in writing and oral language (U_{21}) Demonstrating the enthusiasm and supports (U_{22})
Teaching for learning (U_3)	Getting students' interest and curiosity (U_{31}) Using a variety of appropriate media/approaches to present content (U_{32}) Reducing the barriers (location, gender, cultural background,…) which affect learning (U_{33})
Managing the learning environment (U_4)	Enhancing students' responsibility and self-management (U_{41}) Resolving inappropriate behavior (U_{42})
Student Evaluation (U_5)	Using effective ways of evaluating students' learning (U_{51}) Providing appropriate feedback (U_{52})
Professionalism (U_6)	Making a contribution to the university endeavor (U_{61}) Following the policies, regulations, and procedures of the school (U_{62}) Reviewing learning – teaching process to achieve self-improvement (U_{63})

In developing a performance evaluation system, the weight of each of the criteria must be considered. Our study is related to the evaluation of higher education in Vietnam. To acquire the factor and sub-factor weights, a group of 17 decision-makers, including institution managers and experienced lecturers, was formed. Questionnaires were provided to get their viewpoints. Pairwise comparisons, which were derived from their assessments on the relative importance of one factor over another, were used to form the comparison matrices of each decision-maker. By employing Eq. (14), the geometric mean method was then applied to get the representative comparison matrix of the group. The representative comparison matrix of the group acquired when making pairwise comparisons of the factors is shown in Table 4.

Table 4: Comparison matrix of the factors

	U_1	U_2	U_3	U_4	U_5	U_6
U_1	(1,1,1)	(0.5,1.317,2.5)	(0.4,0.791,2)	(1,1.331,2)	(1,1.632,2.5)	(0.5,1.21,2)
U_2	(0.4,0.759,1)	(1,1,1)	(0.4,0.804,2)	(0.5,1,1.5)	(0.5,1,1.5)	(0.5,1,1.5)
U_3	(0.5,1.264,2.5)	(0.5,1.243,2.5)	(1,1,1)	(0.5,1.1,2)	(0.5,1.449,2.5)	(0.5,1.182,2)
U_4	(0.5,0.751,1)	(0.667,1,2)	(0.5,0.909,2)	(1,1,1)	(1,1.127,2)	(0.667,1,2)
U_5	(0.4,0.613,1)	(0.667,1,2)	(0.4,0.69,2)	(0.5,0.888,1)	(1,1,1)	(0.667,1,2)
U_6	(0.5,0.826,2)	(0.667,1,2)	(0.5,0.846,2)	(0.5,1,1.5)	(0.5,1,1.5)	(1,1,1)

In order to get the individual comparison matrices of the sub-factors, all sub-factors within a specific corresponding factor are compared. The representative matrices were then obtained and they are shown in Tables 5-10. The consistency test results of the individual comparison matrices and the representative matrices show that they are all less than 10%. Therefore, all matrices are acceptable.

Table 5: Comparison matrix of the sub-factors within "Planning and preparation" (U_1)

	U_{11}	U_{12}
U_{11}	(1,1,1)	(0.4,0.722,2)
U_{12}	(0.5,1.385,2.5)	(1,1,1)

Table 6: Comparison matrix of the sub-factors within "Communication and interaction" (U_2)

	U_{21}	U_{22}
U_{21}	(1,1,1)	(0.5,1.443,2.5)
U_{22}	(0.4,0.693,2)	(1,1,1)

Table 7: Comparison matrix of the sub-factors within "Teaching for learning" (U_3)

	U_{31}	U_{32}	U_{33}
U_{31}	(1,1,1)	(0.5,1,1.5)	(1,1.459,2)
U_{32}	(0.667,1,2)	(1,1,1)	(0.5,1,1.5)
U_{33}	(0.5,0.685,1)	(0.667,1,2)	(1,1,1)

Table 8: Comparison matrix of the sub-factors within "Managing the learning environment" (U_4)

	U_{41}	U_{42}
U_{41}	(1,1,1)	(1,1.556,2.5)
U_{42}	(0.4,0.643,1)	(1,1,1)

Table 9: Comparison matrix of the sub-factors within "Student Evaluation" (U_5)

	U_{51}	U_{52}
U_{51}	(1,1,1)	(0.4,0.582,1)
U_{52}	(1,1.717,2.5)	(1,1,1)

Table 10: Comparison matrix of the sub-factors within “Professionalism” (U_6)

	U_{61}	U_{62}	U_{63}
U_{61}	(1,1,1)	(0.667,1,2)	(0.333,0.422,0.667)
U_{62}	(0.5,1,1.5)	(1,1,1)	(0.4,0.641,2)
U_{63}	(1.5,2.372,3)	(0.5,1.561,2.5)	(1,1,1)

Fuzzy AHP was then employed to identify the weights of factors and sub-factors. Taking pairwise comparison matrix of the factors in Table 4 as an illustration, the weights of factors were acquired as follows:

Using Eq. (7), we determined the TFN values of the geometric mean for the fuzzy comparison value of factor C_1 to each factor, as can be seen in the following:

$\tilde{r}_1 = \left(\tilde{U}_{11} \otimes \tilde{U}_{12} \otimes \tilde{U}_{13} \otimes \tilde{U}_{14} \otimes \tilde{U}_{15} \otimes \tilde{U}_{16}\right)^{1/6}$ =((1*0.5*0.4*1*1*0.5)$^{1/6}$,(1*1.317*0.791*1.331*1.632*1.210)$^{1/6}$, (1*2.5*2*2*2.5*2)$^{1/6}$) = (0.681, 1.183, 1.919)

Similarly, we obtained $\tilde{r}_2$, $\tilde{r}_3$, $\tilde{r}_4$, $\tilde{r}_5$, and $\tilde{r}_6$

$\tilde{r}_2$ = (0.521, 0.921, 1.375)

$\tilde{r}_3$ = (0.561, 1.198, 1.992)

$\tilde{r}_4$ = (0.693, 0.957, 1.587)

$\tilde{r}_5$ = (0.573, 0.849, 1.414)

$\tilde{r}_6$ = (0.589, 0.942, 1.619)

Subsequently, the weight of each factor ($\tilde{w}_{Ui}$) can be calculated as follows:

$\tilde{w}_{U1} = \tilde{r}_1 \otimes (\tilde{r}_1 \oplus \tilde{r}_2 \oplus \tilde{r}_3 \oplus \tilde{r}_4 \oplus \tilde{r}_5 \oplus \tilde{r}_6)^{-1}$

=(0.681, 1.183, 1.919) $\otimes$ (1/(1.919+1.375+1.992+1.587+1.414+1.619), 1/(1.183+0.921+1.198+0.957+0.849+0.942), 1/(0.681+0.521+0.561+0.693+0.573+0.589))

= (0.069, 0.195, 0.53)

Likewise, $\tilde{w}_{U2}$ =(0.053, 0.152, 0.380), $\tilde{w}_{U3}$ =(0.057, 0.198, 0.55), $\tilde{w}_{U4}$ =(0.07, 0.158, 0.439), $\tilde{w}_{U5}$ =(0.058, 0.14, 0.391) and $\tilde{w}_{U6}$ =(0.059, 0.156, 0.447)

Thus, the fuzzy weight vector is as follows:

$\tilde{W} = (\tilde{w}_{U1}, \tilde{w}_{U2}, \tilde{w}_{U3}, \tilde{w}_{U4}, \tilde{w}_{U5}, \tilde{w}_{U6})^T$ =((0.069, 0.195, 0.53), (0.053, 0.152, 0.380), (0.057, 0.198, 0.55), (0.07, 0.158, 0.439), (0.058, 0.14, 0.391), (0.059, 0.156, 0.447))^T

The weight of each factor was calculated by employing the defuzzification procedure. Thus,

$W = (w_{U1}, w_{U2}, w_{U3}, w_{U4}, w_{U5}, w_{U6})$ = (0.265, 0.195, 0.268, 0.222, 0.196, 0.221)^T

We then normalized the weight vector and obtained the relative weights of the six factors.

$W = (w_{U1}, w_{U2}, w_{U3}, w_{U4}, w_{U5}, w_{U6})$ = (0.194, 0.143, 0.196, 0.163, 0.144, 0.161)^T

The calculation results show that the weight of “Teaching for learning (U_3)” is largest. This factor plays the most important part in teaching performance, followed by “Planning and preparation (U_1)”.

Following a similar calculation, the weight vectors W_1, W_2, W_3, W_4, W_5, W_6, of sub-factors at the successive levels were determined. They are as shown below:

The weight vector from Table 5 was calculated as $W_{U1} = (w_{u11}, w_{u12})$ = (0.458, 0.542)^T

The weight vector from Table 6 was calculated as $W_{U2} = (w_{u21}, w_{u22})$ = (0.545,0.455)^T

The weight vector from Table 7 was calculated as $W_{U3} = (w_{u31}, w_{u32}, w_{u33})$ =(0.359,0.339,0.303)^T

The weight vector from Table 8 was calculated as $W_{U4} = (w_{u41}, w_{u42})$ =(0.611,0.389)^T

The weight vector from Table 9 was calculated as $W_{U5} = (w_{u51}, w_{u52})$ =(0.381,0.619)^T

The weight vector from Table 10 was calculated as $W_{U3} = (w_{u61}, w_{u62}, w_{u63})$ =(0.248,0.302,0.450)^T

The above derived weights formed the evaluation matrix R for fuzzy comprehensive evaluation.

5 Fuzzy Evaluation of Teaching Performance

From the above teaching evaluation index system and acquired factor and sub-factor weights, the fuzzy comprehensive evaluation method was utilized to assess the teaching performance. In order to illustrate the method, we took a case application as an illustration.

5.1. Fuzzy Comprehensive Evaluation

Fuzzy comprehensive evaluation is an application of fuzzy mathematics. It uses the principles of fuzzy transformation and maximum membership degree, evaluating all relevant factors to make a comprehensive evaluation. This is an efficient evaluation method to evaluate objects that are affected by various factors. For objects that are influenced by a few factors, we can use one-layer models. If the objects are complicated and the number of the factors is large, we can use models with two or more layers. In this paper, we used a fuzzy comprehensive evaluation model with two layers as a tool for teaching performance evaluation. The application steps of fuzzy comprehensive evaluation [28] are as follows:

Step 1: Establishment of the evaluation index system

According to the nature of the characteristics of the evaluation index system, the factor set in the evaluating relationship is as follows:

$U= \{u_1, u_2, u_3,\ldots,u_n\}$

In Section 4, the teaching performance evaluation system was established and the factor and sub-factor weights were calculated.

Step 2: Determining of the set of comments

The evaluation comment set is as followed:

$V= \{v_1, v_2, v_3,\ldots,v_m\}$

In this research, we used five grades to set up the comments for evaluation: V= {excellent, very good, good, fair, poor}.

In order to make the index quantitative, we provided grade for the corresponding comment sheet.

$V = (100, 85, 70, 55, 40)$

Step 3: Establishing of the single-factor evaluation matrix R from U to V

Each factor u_i $(i \leq n)$ should be evaluated as a single-factor. As there are different types of evaluation levels, the evaluation result of each factor is a fuzzy set of evaluation set V which can be written as the fuzzy vector

$R_i=(r_{i1}, r_{i2}, r_{i3},\ldots,r_{im})$, $i=1,2,..,n$, $R_i \in \mu(V)$. The results of these evaluations meet the normalized conditions and the sum of the weight of the vector is 1, that is, for every i, there is: $r_{i1} + r_{i2} + r_{i3} + \ldots + r_{im} = 1$

All of the single-factor evaluations constitute the fuzzy relationship R from U to V: $R = (r_{ij})_{n \times m}$

That is,

$$R = \left(r_{ij}\right)_{n\times m} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1m} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2m} \\ \cdots & \cdots & \cdots & \cdots & \\ r_{n1} & r_{n2} & r_{n3} & \cdots & r_{nm} \end{pmatrix} \quad (15)$$

r_{ij} presents the grade of membership of factor u_i aiming at the comment v_j.

Step 4: Determining of the factor weighs

Weight means the proportion of each evaluation factor in the evaluation index system based on relative importance. If a weight is given to an element, the weight distribution set W can be seen as a fuzzy set of set U. How to determine the weight of each factor is the core task of the evaluation system. As discussed in Section 4, we employed fuzzy AHP to determine the factor and sub-factor weights in the evaluation index system.

Step 5: Producing the results of evaluation

The results of an evaluation can be obtained through multiplying the vector of the factor weight and the matrix R of single-factor evaluation:

$$B = W.R = (b_1, b_2, b_3,\ldots,b_m) \quad (16)$$

B is evaluation result based on all factors in index system U. The k-th element b_k is membership of the evaluation object with regard to k-th element in the comment set. The conclusion of the comprehensive evaluation can be obtained by the maximum membership principle.

5.2. A Case Application

We took a case application from one Vietnamese public university as an illustration. At every Vietnamese university, teaching performance evaluation is conducted every academic semester. The objective of teaching evaluation is to provide information and feedback to lecturers in order to improve the teaching quality. However, the existing evaluation method is mainly based on the acquisition of knowledge. This method has not been suitable for lifetime education or for deepening and widening education reform. In addition, the evaluation procedure is largely a formality, and lacks accuracy and objectivity. Inaccuracy of evaluation work is due to the lack of a standard accompanied by criteria as well as a method to evaluate the performance of each faculty member. Therefore, the need for a scientific evaluation method with objective and accurate results is essential.

The application was carried out in evaluating the teaching performance of a lecturer at the University of Transport Technology in Vietnam when his “Digital Technique” course for the 2011-2012 academic year finished. According to the evaluation index system and the comment set proposed in the previous sections, we collected opinions of students and colleagues about his teaching performance and investigated his teaching portfolio. The evaluation matrices of indexes were then formed. Taking the constructed matrix R_1 as an example, “Clear goals and objectives” was concerned, 27% of respondents rated it “excellent”, 44% of respondents rated it “very good”, 29% rated it “good”, and 0% rated it “fair” or “poor”; when “Clear, logical, and innovative documentation” was concerned, 23% of respondents rated it “excellent”, 51% of respondents rated it “very good”, 22% rated it “good”, 4% rated it “fair”, and 0% rated it “poor”. Hence, the matrix R_1 can be derived as follows:

$$R_1=\begin{bmatrix} 0.27 & 0.44 & 0.29 & 0 & 0 \\ 0.23 & 0.51 & 0.22 & 0.04 & 0 \end{bmatrix}$$

Similarly, the matrix R_2, R_3, R_4, R_5, and R_6 were obtained. They are as shown below:

$$R_2=\begin{bmatrix} 0.16 & 0.67 & 0.17 & 0 & 0 \\ 0.13 & 0.62 & 0.19 & 0.06 & 0 \end{bmatrix}$$

$$R_3=\begin{bmatrix} 0.02 & 0.52 & 0.22 & 0.24 & 0 \\ 0.46 & 0.35 & 0.19 & 0 & 0 \\ 0.37 & 0.59 & 0.05 & 0 & 0 \end{bmatrix}$$

$$R_4=\begin{bmatrix} 0.06 & 0.38 & 0.56 & 0 & 0 \\ 0.21 & 0.75 & 0.05 & 0 & 0 \end{bmatrix}$$

$$R_5=\begin{bmatrix} 0.12 & 0.58 & 0.23 & 0.08 & 0 \\ 0 & 0.11 & 0.24 & 0.65 & 0 \end{bmatrix}$$

$$R_6=\begin{bmatrix} 0 & 0.73 & 0.13 & 0.13 & 0 \\ 0.1 & 0.63 & 0.12 & 0.15 & 0 \\ 0.05 & 0.41 & 0.35 & 0.19 & 0 \end{bmatrix}$$

Then, we can get the evaluation result of U_1

$$B_1=W_1.R_1=(0.458,0.542)\cdot\begin{bmatrix} 0.27 & 0.44 & 0.29 & 0 & 0 \\ 0.23 & 0.51 & 0.22 & 0.04 & 0 \end{bmatrix} = (0.248, 0.478, 0.253, 0.021, 0)$$

Similarly, we got the evaluation result of U_2, U_3, U_4, U_5, and U_6 through calculations

$B_2=W_2.R_2=$ (0.144, 0.645, 0.182, 0.029, 0)
$B_3=W_3.R_3=$ (0.272, 0.484, 0.159, 0.085, 0)
$B_4=W_4.R_4=$ (0.119, 0.523, 0.358, 0, 0)
$B_5=W_5.R_5=$ (0.044, 0.289, 0.235, 0.432, 0)
$B_6=W_6.R_6=$ (0.054, 0.556, 0.224, 0.166, 0)

We established the evaluation matrix R at the first level from the above matrices as follows:

$$R=\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix}=\begin{bmatrix} 0.248 & 0.478 & 0.253 & 0.021 & 0 \\ 0.144 & 0.645 & 0.182 & 0.029 & 0 \\ 0.272 & 0.484 & 0.159 & 0.085 & 0 \\ 0.119 & 0.523 & 0.358 & 0 & 0 \\ 0.044 & 0.289 & 0.235 & 0.432 & 0 \\ 0.054 & 0.556 & 0.224 & 0.166 & 0 \end{bmatrix}$$

The evaluation matrix R represents the membership degree values of each comment, which is correlative with each factor in the evaluation index. Hence, the comprehensive evaluation of his teaching performance is calculated as:

$$B=W.R=(0.194,0.143,0.196,0.163,0.144,0.161).\begin{bmatrix} 0.248 & 0.478 & 0.253 & 0.021 & 0 \\ 0.144 & 0.645 & 0.182 & 0.029 & 0 \\ 0.272 & 0.484 & 0.159 & 0.085 & 0 \\ 0.119 & 0.523 & 0.358 & 0 & 0 \\ 0.044 & 0.289 & 0.235 & 0.432 & 0 \\ 0.054 & 0.556 & 0.224 & 0.166 & 0 \end{bmatrix}=(0.156,0.496,0.234,0.114,0)$$

The result shows that the "very good" probability of the teaching performance is 0.496; the probability of "excellent", "good", "fair", and "poor" is 0.156, 0.234, 0.114, and 0, respectively. According to the maximum membership degree principle, the comprehensive evaluation result of the lecturer's teaching performance is "very good". Besides this, another implication from the distribution of B_1, B_2, B_3, B_4, B_5, and B_6 vector weights is that his achievements regarding the "Student evaluation" and "Professionalism" factors are not good as those for other factors. The evaluation result, which is based on the opinions of students and peers, also provided the lecturer with suggestions on how to improve his teaching quality. We interviewed the dean of the department and the lecturer about the evaluation result. They agreed that the result in the proposed evaluation method is transparent and objective. Moreover, the proposed method makes it easier to explain to the lecturer about the result and provides institution managers with useful information.

6. Conclusion

Teaching performance evaluation is an effective means to maintain the quality of teaching effectiveness. This paper presents an evaluation index system for teaching performance and develops a teaching performance evaluation framework based on fuzzy AHP and comprehensive evaluation methods. The application of fuzzy comprehensive evaluation to conduct teaching performance evaluations can not only reflect the overall teaching performance of lecturers, but also reflect their achievements regarding each evaluation factor. This helps the lecturers know what needs improvement in order to enhance teaching quality. One signification of this approach is the introduction of fuzzy AHP in determining the factor and sub-factor weights. Because the fuzzy AHP has the capability to capture the vagueness of human judgments, it makes the calculated weights in the index system more objective. This approach can reduce subjectivity in the evaluation process. A case application shows the applicability of this framework to providing a valuable tool in the teaching performance evaluation process. It is expected that this approach may provide an effective and objective measure to evaluate teaching performance in higher education institutions.

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