On Cognitive Processes of Analysis and Synthesis in Cognitive Computing

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Abstract: - Analysis and synthesis are a pair of fundamental cognitive processes at the inference layer of the brain. The former is an inference process that deductively decomposes an object or a system into its constituting attributes and components. The latter is an inference process that inductively composes individual attributes of components into a complex whole. This paper presents the cognitive foundations of analysis and synthesis in cognitive inferences. A set of mathematical models of analysis and synthesis is created. Based on the cognitive and mathematical models, the cognitive processes of analysis and synthesis are formally described in Real-Time Process Algebra (RTPA), which enable a rigorous explanation of the cognitive mechanisms of mental inferences in cognitive computing and cognitive robotics.

Key-Words: - Cognitive informatics, brain science, LRMB, cognitive models, cognitive processes, mathematical models, denotational mathematics, RTPA, cognitive computing, computational intelligence, AI, inference, reasoning, cognitive robotics

1. Introduction

Cognitive informatics (CI) is a transdisciplinary enquiry on the internal information processing mechanisms and processes of the brain and minds in order to reveal the principles of natural intelligence and engineering applications [20, 22, 23, 25, 27, 31-34, 37-45, 50-60]. A *Layered Reference Model of the Brain* (LRMB) is developed in CI [57], which reveals that the brain can be formally modelled as 52 fundamental cognitive processes at seven layers known as the *sensation*, *action, memory*, *perception*, *cognitive*, *inference*, and *intelligence* layers from the bottom up. In this view, any other complex mental process or human behavior is a contingent composition of the fundamental processes of LRMB [57].

According to LRMB, the *inference processes* are a category of fundamental reasoning mechanisms that derive a causation from given premises based on empirical observations, logical truths, mathematical equivalence, and/or statistical norms [1-4, 7, 9-11, 13, 15, 17, 19, 25, 26, 35, 36, 40, 48, 52, 62, 63]. The inference layer encompass a set of eight advanced cognition processes known as those of deduction, induction, abduction, analogy, causation, analysis, synthesis, and recursion. The inference processes are higher level cognitive processes underpinning Layer 7, the intelligence layer, on the basis of lower functions throughout Layer 5 to Layer 1 known as the cognition, perception, memory, action, and sensation processes [5, 6, 8, 12, 14, 15, 16, 54, 61].

In the inference processes at Layer 6 of LRMB, analysis is an inference process that deductively decomposes an object or a system into its constituent attributes or components in order to examine or determine its detailed configuration and relationship. Synthesis is an inference process that inductively composes individual attributes or components into a complex whole. Recursion is a combination of analysis and synthesis that forms a closed loop of the *holistic analytic methodology* [26, 46, 47] of cognitive inference.

This paper presents a set of formal models of the cognitive processes of analysis and synthesis in formal inferences. In the remainder of this paper, Section 2 explores the cognitive foundations of analysis and synthesis as a pair of inverse inference processes. Section 3 introduces a formal model of general system layouts of structures and behaviours. In this context of system analytics, a set of mathematical models for cognitive analyses and syntheses is created. On the basis of the principles and mathematical models, the cognitive processes of analyses and syntheses are formally explained in Section 4, respectively, in Real-Time Process Algebra (RTPA) [21, 28, 29].

2. Cognitive Foundations of Analyses and Synthesis

The inference processes at Layer 6 of LRMB encompass the basic reasoning processes of *deduction, induction, abduction, analogy, causation, analysis, synthesis,* and *recursion* [57] as follows:

$$\begin{aligned}&\text{LRMB\_Layer\_6: The Inference Processes (IP)} \triangleq\\ &(\ \text{IP1: } \mathit{Deduction}|\text{PM}\\ &\ \|\ \text{IP2: } \mathit{Induction}|\text{PM}\\ &\ \|\ \text{IP3: } \mathit{Abdusction}|\text{PM}\\ &\ \|\ \text{IP4: } \mathit{Ana}\log y|\text{PM}\\ &\ \|\ \text{IP5: } \mathit{Causation}|\text{PM}\\ &\ \|\ \text{IP6: } \mathit{Analysis}|\text{PM}\\ &\ \|\ \text{IP7: } \mathit{Synthesis}|\text{PM}\\ &\ \|\ \text{IP8: } \mathit{Recursion}|\text{PM}\\ &)\end{aligned} \tag{1}$$

In the inference layer of LRMB, the first five cognitive processes such as deduction, induction, abduction, analogy, and causation are logic-oriented inferences. Correspondingly, algebraic counterparts are equivalent to interpolation, extrapolation, regression, pattern matching, and correlation, respectively. Similarly, analysis, synthesis, and recursion can be logic- or algebraic-oriented cognitive processes.

Analyses and syntheses are a pair of inverse and complement approaches to formal reasoning. The former is a deductive process of top-down inferences; while the latter is an inductive process of bottom-up inferences.

Definition 1. *Analysis* is an inference process that deductively divides a physical or abstract system into a set of constituent components and their relations.

Definition 2. *Synthesis* is an inference process that inductively composes and aggregates a set of related components into a coherent system via the configurable relations obtained in the phase of analysis.

Analysis is typically embodied by a series of system decompositions that incrementally reduces a complex system onto the terminal level where all individual attributes and properties are known or determinable. Synthesis is usually embodied by a series of system compositions that incrementally produces a more complex system from that of lower level components.

The taxonomy of analyses/syntheses may be classified into the categories of relational, logical, fuzzy, causal, and cognitive analyses and syntheses according to the cognitive levels of inference [3, 10, 25, 40, 48, 50, 52, 55, 62]. It may be classified as structural, functional, and hybrid analyses/syntheses according to the facet of orientation. It may be classified as logic and algebraic analyses/syntheses according to their mathematical means. It may also be classified as system, pattern, model, problem, characteristic, and element analyses/syntheses according to the target objects. The analysis and synthesis processes can be divided into two phases known as the generic and specific analysis/synthesis as shown in Table 1. The central focus of formal inferences is to seek causations implied in a thread of thought beyond the semantics of natural language expressions. A coherent framework of formal inferences reveals how human reasoning may be formalized and how machines may rigorously mimic the mechanism of human inference.

Table 1. The Framework of Cognitive Analyses/Syntheses

No.	Category	Phase	Description
1	Analysis	General	To set up the entire layout of a system between all objects and attributes
2		Refinement	To reduce each of the objects in the general layout of the system to a subset of related attributes
3	Synthesis	Component	To represent each object in the system by the specific attributes yielded in refinement analyses
4		System	To aggregate all component-level syntheses into a coherent system

One of the latest findings in logic, cognitive informatics, and system science is the *recursive inference*, which is a closed loop inference concatenating analysis and synthesis in system design and modeling [24, 46, 47].

3. Mathematical Models of Cognitive Analyses and Synthesis

On the basis of the cognitive foundations and mathematical models of system layouts explored in preceding sections, formal models of cognitive analysis and synthesis are rigorously elaborated in this section.

3.1 The Layout of Formal Systems

As the context of system analytics, the layout of abstract systems can be modeled as a hierarchical structure as shown in Fig. 1. At each layer of the hierarchical system, there is a set of objects and attributes. The objects, O_k, at the kth layer of the system are a set of subsystems or components; while the attributes of the system, A_k, at the kth layer are objects at the k-1th layer, i.e., O_{k-1}, which denote detailed characteristics of each object O_k.

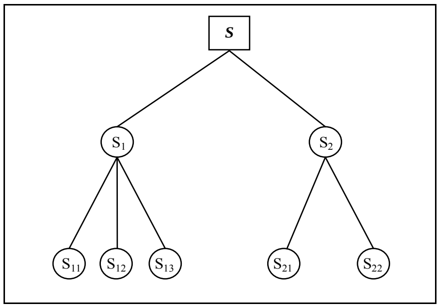

Fig. 1. An abstract system with a hierarchical structure

Definition 3. The *abstract model of hierarchical system*, *S*, is deductively described by a series of refinements from the top down where each *k*th layer of it, S^k, in the system hierarchy is represented by its next layer, S^{k-1}, until the terminal layer S^0 is reached as known objects, i.e.:

$$\begin{aligned} S &\triangleq \mathop{R}_{k=n}^{1} S^k(S^{k-1}),\ \ S^0 = O_0 \times A_0 \\ &= S^n(S^{n-1}(...(S^1(S^0)))) \end{aligned} \quad (2)$$

where the *big-R* notation is a mathematical operator that is used to denote: a) a set of indexed behaviors, b) a set of recurring structures, or c) a set of repetitive behaviors [24, 30].

Definition 4. The *layout of a system S*, Ω_S, is the entire state space of the system embodied by all potential compositions between the set of *objects* O_S and set of *attributes* A_S of the system, i.e.:

$$\Omega_S \triangleq O_S \times A_S = \mathop{R}_{i=0}^{n} \mathop{R}_{j=0}^{m} \omega_{ij} = \left[\begin{array}{c|cccc} O_S \times A_S & a_1 & a_2 & \cdots & a_m \\ \hline o_1 & \omega_{11} & \omega_{12} & \cdots & \omega_{1m} \\ o_2 & \omega_{21} & \omega_{21} & \cdots & \omega_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ o_n & \omega_{n1} & \omega_{n2} & \cdots & \omega_{nm} \end{array} \right] \quad (3)$$

where ω_{ij} denotes an element in the layout of the system as a pair yielded by the Cartesian product $O_S \times A_S$.

Example 1. A 7-segment digit system, S_1, is given in Fig. 2. In S_1 the set of attributes A_{S_1} represents the seven segments of digits, and the set of objects O_{S_1} represents the ten digits to be displayed by the 7-segment LED device, i.e.:

$$\begin{cases} O_{S_1} = \mathop{R}_{i=0}^{9} o_i = \{0,1,2,3,4,5,6,7,8,9\} \\ A_{S_1} = \mathop{R}_{j=0}^{7} a_j = \{a,b,c,d,e,f,g\} \end{cases}$$

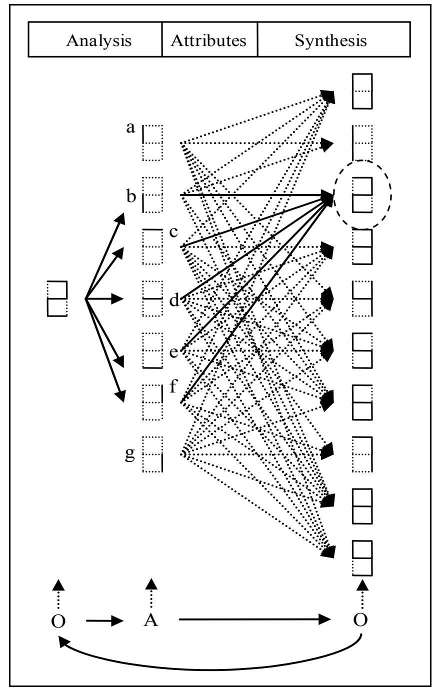

Fig. 2. The 7-segment digit system for cognitive analysis and synthesis

According to Definition 4, the refined layout $\Omega_{S_1'}$ of this system can be derived based on the general layout Ω_{S_1} tailored by the characteristic matrix R_{S_1} as follows:

$$\Omega_{S_1'} \triangleq R_{S_1} \bullet \Omega_{S_1},\ \ r_{ij} \in \{0,1\} \in R_{S_1}$$

$$= \begin{bmatrix} 1&1&1&0&1&1&1 \\ 1&1&0&0&0&0&0 \\ 0&1&1&1&1&1&0 \\ 0&0&1&1&1&1&1 \\ 1&0&0&1&0&1&1 \\ 1&0&1&1&1&0&1 \\ 1&1&1&1&1&0&1 \\ 0&0&1&0&0&1&1 \\ 1&1&1&1&1&1&1 \\ 1&0&1&1&1&1&1 \end{bmatrix} \bullet \left[\mathop{R}_{i=0}^{n} \mathop{R}_{j=0}^{m} \omega_{ij} \right]$$

=

$O_{S_1} \times A_{S_1}$	*a*	*b*	*c*	*d*	*e*	*f*	*g*
0	1	1	1	0	1	1	1
1	1	1	0	0	0	0	0
2	0	1	1	1	1	1	0
3	0	0	1	1	1	1	1
4	1	0	0	1	0	1	1
5	1	0	1	1	1	0	1
6	1	1	1	1	1	0	1
7	0	0	1	0	0	1	1
8	1	1	1	1	1	1	1
9	1	0	1	1	1	1	1

The tailored layout obtained in Example 1 represents the conceptual model of the 7-segment digit number system.

3.2 The Mathematical Model of Formal Analyses

The mathematical models of cognitive analyses can be classified into the categories of general and refinement analyses from the top down according to the cognitive models as described in Section 2.

Definition 5. A *general analysis, Λ*, is a universal function f_A in the domain of a given system layout Ω_S that maps the entire set of *objects* O_S into that of *attributes* A_S, which determines the entire state space by a Cartesian product $O_S \times A_S$, i.e.:

$$\begin{aligned}\Lambda(S) &\triangleq f_{\Lambda_S}: O_S \to A_S,\ O_S \sqsubset \Omega_S \wedge A_S \sqsubset \Omega_S \\ &= O_S \times A_S = \Omega_S\end{aligned} \tag{4}$$

where $\sqsubset$ denotes a dimensional or subset inclusion to a hyperstructure.

Example 2. According to Definition 5, the general analysis for the layout of the 7-segment digits system S_1, $\Lambda(S_1)$, as given in Fig. 2 can be formally described as follows:

$$\begin{aligned}\Lambda(S_1) &\triangleq f_{\Lambda_1}: O_1 \to A_1 \\ &= f_{\Lambda_1}: \{0,1,...,9\} \to \{a,b,c,...,g\}\end{aligned}$$

On the basis of the mathematical model of general analysis for a given system, the entire sets of objects, attributes, and their relations are identified within the layout of the system. Then, refinement analysis as a subset of the general state space can be formally derived as follows.

Definition 6. A *refinement analysis, Λ′*, is a set of instantiation functions $\overset{|O_S|}{\underset{i=0}{R}} f_{\Lambda_i}$ tailored from the general analysis f_A in the tailored system layout $\Omega_{S'}$ that maps a specific object $o_i,\ o_i \in O$, into a subset of the identified attributes $A_i^{'},\ A_i^{'} \subseteq A$, i.e.:

$$\begin{aligned}\Lambda_S^{'} &\triangleq \overset{|O_S|}{\underset{i=0}{R}} f_{\Lambda_i}: o_i \to A_i^{'},\ \ o_i \in O_S \sqsubset \Omega_S^{'} \wedge A_i^{'} \subseteq A_S^{'} \sqsubset \Omega_S^{'} \\ &= \overset{|O_S|}{\underset{i=0}{R}} [f_{\Lambda_i}: o_i \to \overset{|A_S|}{\underset{j=0}{R}} (a_j \mid (o_i, a_j) \neq \varnothing)]\end{aligned} \tag{5}$$

where each subset of attributes, $A_i^{'}$, is selected by the *i*th row of the characteristic matrix or the tailored layout of the system $\Omega_S^{'}$.

Example 3. According to Definition 6, the refinement analysis for the set of ten specific object-attribute relations in the layout of the 7-segment digit system S_1, $\Lambda'(S_1)$, is as follows based on the general analysis result obtained in Example 2:

$$\Lambda_{S_1}^{'} \triangleq \overset{9}{\underset{i=0}{R}} f_{\Lambda_i}: o_i \to A_i^{'},\ \ o_i \in O_{S_1} = \{0,1,..,9\} \sqsubset \Omega_{S_1},$$
$$A_i^{'} \subseteq A_{S_1} = \{a,b,c,d,e,f,g\} \sqsubset \Omega_{S_1}$$
$$= \begin{cases} f_{A_0}: 0 \to \{a,b,c,e,f,g\} \\ f_{A_1}: 1 \to \{a,b\} \\ f_{A_2}: 2 \to \{b,c,d,e,f\} \\ f_{A_3}: 3 \to \{c,d,e,f,g\} \\ f_{A_4}: 4 \to \{a,d,f,g\} \\ f_{A_5}: 5 \to \{a,c,d,e,g\} \\ f_{A_6}: 6 \to \{a,b,c,d,e,g\} \\ f_{A_7}: 7 \to \{c,f,g\} \\ f_{A_8}: 8 \to \{a,b,c,d,e,f,g\} \\ f_{A_9}: 9 \to \{a,c,d,e,f,g\} \end{cases}$$

3.3 The Mathematical Model of Formal Syntheses

The mathematical models of cognitive syntheses can be classified into the categories of component and system syntheses from the bottom up according to the cognitive models as described in Section 2.

Definition 7. The *component synthesis, Θ′*, is a set of instantiation functions $\overset{|A_S|}{\underset{j=0}{R}} f_{\Theta_j}$ in the tailored system layout $\Omega_{S'}$ that maps each specific attribute $a_j,\ a_j \in A$, into a subset of related objects $O_j^{'},\ O_j^{'} \subseteq O$ i.e.:

$$\begin{aligned}\Theta_S^{'} &\triangleq \overset{|A_S|}{\underset{j=0}{R}} a_j \times O_j^{'},\ a_j \in A_S \sqsubset \Omega_S^{'} \wedge O_j^{'} \subseteq O_S^{'} \sqsubset \Omega_S^{'} \\ &= \overset{|A_S|}{\underset{j=0}{R}} [a_j \times \overset{|O_S|}{\underset{i=0}{R}} (o_i \mid (o_i, a_j) \neq \varnothing)]\end{aligned} \tag{6}$$

where each subset of objects, $O_j^{'}$, is projected by the *j*th column of the characteristic matrix or the tailored layout of the system $\Omega_S^{'}$.

Example 4. According to Definition 7, the set of individual component syntheses for the 7-segment digit system can be formally described by a set of specific synthetic functions based on the tailored system layout $\Omega_{S_1}^{'}$ as obtained in Example 1 and analyzed in Examples 2 and 3 as follows:

$$\Theta'_{S_1} = \overset{6}{\underset{j=0}{R}} [a_j \times (O'_j = \overset{9}{\underset{i=0}{R}} (o_i \mid (o_i, a_j) \neq \varnothing))]$$
$$a_j \in A_{S_1} = \{a,b,c,d,e,f,g\} \sqsubset \Omega'_{S_1},$$
$$O'_j \subseteq O'_{S_1} = \{0,1,..,9\} \sqsubset \Omega'_{S_1}$$
$$= \begin{cases} a \times \{0,1,4,5,6,8,9\} \\ b \times \{0,1,2,6,8\} \\ c \times \{0,2,3,5,6,7,8,9\} \\ d \times \{2,3,4,5,6,8,9\} \\ e \times \{0,2,3,5,6,8,9\} \\ f \times \{0,2,3,4,7,8,9\} \\ g \times \{0,3,4,5,6,7,8,9\} \end{cases}$$

Definition 8. *System synthesis, Θ*, is a universal function $f_\Theta = \overset{|A_S|}{\underset{j=0}{\uplus}} f_{\Theta_j}$ in the given system layout Ω_S that maps the entire set of *attributes A* into that of objects *O* by a set of synthesis operators $\uplus$ on the basis of results yielded in the phase of component synthesis, i.e.:

$$\begin{aligned} \Theta_S &\triangleq f_\Theta = \overset{|A_S|}{\underset{j=0}{\uplus}} f_{\Theta_j}: a_j \to O'_j,\ a_j \in A_S \sqsubset \Omega'_S \wedge O'_j \subseteq O'_S \sqsubset \Omega'_S \\ &= \overset{|A_S|}{\underset{j=0}{\uplus}} [f_{\Theta_j}: a_j \to \overset{|O_S|}{\underset{i=0}{R}} (o_i \mid (o_i, a_j) \neq \varnothing)] \end{aligned} \quad (7)$$

where the set of synthesis operators on system components is formally defined in Definition 9.

Definition 9. The *synthesis operators*, $\uplus$, is a set of relational operators $\mathbb{R}_\Theta$ between individual components that encompasses ten basic structural composition functions, i.e.: i.e.:

$$\uplus \triangleq \mathbb{R}_\Theta = \{\|, \to, \Uparrow, \Downarrow, \rightarrowtail, \vartriangleright, \vartriangleleft, \Rrightarrow, \lightning, \hookrightarrow\} \quad (8)$$

where the operators represent *parallel, sequential, aggregation, decomposition, embedded, input, output, channel, interrupt,* and *dispatch* synthesis, respectively [46].

In synthesis inference, only the first few structural operators such as parallel, sequential, aggregative, and decompositional relations are frequently used. Further details of the synthesis operators may refer to [24, 46].

Example 5. According to Definitions 8, the general synthesis for the 7-segment digit system based on the individual component synthesis results obtained in Example 4 can be formally described as follows:

$$\Theta_{S_1} = \overset{6}{\underset{j=0}{\uplus}} [f_{\Theta_j}: a_j \to (O'_j = \overset{9}{\underset{i=0}{R}} (o_i \mid (o_i, a_j) \neq \varnothing))],$$
$$a_j \in A_{S_1} = \{a,b,c,d,e,f,g\} \sqsubset \Omega'_S,$$
$$O'_j \subseteq O'_S = \{0,1,..,9\} \sqsubset \Omega'_S$$
$$= f_{\Theta_0}: a \to \{0,1,4,5,6,8,9\}$$
$$\| f_{\Theta_1}: b \to \{0,1,2,6,8\}$$
$$\| f_{\Theta_2}: c \to \{0,2,3,5,6,7,8,9\}$$
$$\| f_{\Theta_3}: d \to \{2,3,4,5,6,8,9\}$$
$$\| f_{\Theta_4}: e \to \{0,2,3,5,6,8,9\}$$
$$\| f_{\Theta_5}: f \to \{0,2,3,4,7,8,9\}$$
$$\| f_{\Theta_6}: g \to \{0,3,4,5,6,7,8,9\}$$

where || denotes a parallel relation between individual synthesis functions which embody the system synthesis strategies in the given system.

The single layer synthesis as modeled in Definition 8 can be generally extended to that of multi-layer hierarchical systems.

Definition 10. The *general synthesis of a multi-layer system* S^h, Θ_S^h, *h* > 2, is a hierarchical synthesis of *h* single-layer syntheses $\Theta_S^k,\ 0 \le k \le h$, where Θ_S^k denotes the layout of the system at the *k*th layer, i.e.:

$$\begin{aligned} \Theta_S^h &= \overset{h}{\underset{k=0}{\uplus}} \Theta_S^k \\ &= \overset{h}{\underset{k=0}{\uplus}} [f_{\Theta_{kj}}: \overset{|A_{kj}|}{\underset{j=0}{R}} a_{kj} \to (O'_{kj} = \overset{|O_{kj}|}{\underset{i=0}{R}} (o_{ki} \mid (o_{ki}, a_{kj}) \neq \varnothing))] \end{aligned} \quad (9)$$

where the element ω_{ij}^k in a multi-layered system is identified by a triple (*k, i, j*) known as the numbers of layer, object, and attributes.

Example 6. The general synthesis of the 3-layer abstract system as given in Fig. 1 can be formally described according to Definition 10 as follows:

$$\Theta_{S_2}^2 = \overset{1}{\underset{k=0}{\uplus}} \Theta_{S_2}^k$$
$$= (s_1 \| s_2) \Uparrow S$$
$$\Uparrow \{\ (s_{11}, s_{12}, s_{13}) \Uparrow s_1$$
$$\| (s_{21}, s_{22}) \Uparrow s_2$$
$$\}$$

4. The Cognitive Processes of Analysis and Synthesis

The cognitive processes of analysis and synthesis for inferences can be formally design and implemented based on the mathematical models created in preceding sections. The formal cognitive processes of analysis and synthesis are useful not only for explaining the nature of the brain for inference and thinking, but also for serving as simulation algorithms for cognitive computing and computational intelligence.

4.1 The Cognitive Process of Analysis

The cognitive process of analysis is formally described as shown in Fig. 3 based on the mathematical models as given in Definitions 6 and 7. The formal analysis process, CognitiveAnalysis|PM, is described in Real-Time Process Algebra (RTPA) [21, 29, 30]. The input of the process is the layout of the given system (Ω|SM). Its output is a set of refined analyses (Λ′|SM). The global model (GM) as the context of the problem is the layout of the system (Ω|SM). A number of RTPA type suffixes [24, 29], such as |PM, |SM, |Ξ, and |N, are adopted to denote those of process model, structure model, set, and natural number, respectively.

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The Cognitive Process of Analysis

CognitiveAnalysis|PM (<I:: Ω|SM>, <O:: Λ′|SM>, <GM:: Ω|SM>) ≙
{ // I. General analysis – general layout and solutions Λ|SM
  → O|Ξ:= {o1|SM, o2|SM, …, on|SM} := Ω|SM.O|Ξ
  → A|Ξ = {a1|SM, a2|SM, …, am|SM} := Ω|SM.A|Ξ
  → Λ|SM = O|Ξ × A|Ξ

  // II. Property analysis – Identify the characteristic matrix R|SM
       n   m
  →    R   R  ( → (  � oi|SM × aj|SM ≠ ∅
      i=0 j=0
                          → rij|N = 1
                   | �~
                          → rij|N = 0
                 )
              → R|SM(i, j) := rij|N
           )
                  n   m
  → R|SM :=       R   R   rij|N
                 i=0 j=0

  // III. Refinement analysis – identify specific solutions Λ′|SM
  → Λ'|SM := R|SM • Λ|SM
            = R|SM • (O|Ξ × A|Ξ)
}
```

Figure 3. The cognitive process of analysis in RTPA

CognitiveAnalysis|PM encompasses three subprocesses known as those of general analysis, property analysis, and refinement analysis. The subprocess of general analysis explores the general layout and solutions of the given problem as a Cartesian product, $\Lambda|\mathrm{SM} = O|\Xi \times A|\Xi$, by identifying the sets of objects and attributes of the system. The subprocess of property analysis identifies the characteristic matrix of the problem R|SM. It examines each pairs of object and attribute in the general layout Ω|SM. If the relation between the pair is true, the corresponding element $r_{ij} \in R|\mathrm{SM}$ is set to 1; otherwise, it is set to 0. The subprocess of refinement analysis derives specific solutions to the problem by tailoring the general solution yielded in the subprocess of general analysis by the characteristic matrix, i.e., $\Lambda'|\mathrm{SM} = R|\mathrm{SM} \bullet \Lambda|\mathrm{SM} = R|\mathrm{SM} \bullet (O|\Xi \times A|\Xi)$. As a result, a set of specific solutions is obtained.

4.2 The Cognitive Process of Synthesis

The cognitive process of synthesis, CognitiveSynthesis|PM, is formally described as shown in Fig. 4 based on the mathematical models as given in Definitions 8 and 10. The inputs of the process are the refinement analysis results (Λ′|SM) and the characteristic matrix (R|SM) obtained in the previous analysis by CognitiveAnalysis|PM. Its output is a set of integrated functions for the synthesis of the system (Θ|SM). The global model (GM) of the process is the layout of the system (Ω|SM).

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The Cognitive Process of Synthesis

CognitiveSynthesis|PM (<I:: Λ′|SM, R|SM >, <O:: Θ|SM>, <GM:: Ω|SM>) ≙
{ // I. Component synthesis – integration of individual solutions
     |A|
  →   R  (O'j|Ξ := ∅
     j=0
               |O|
          →     R  (� R|SM.rij|N = 1
               i=0
                       → O'j|Ξ := O'j|Ξ ∪ oi|SM
                 )
          )
                  |A|
  → Θ'|SM :=       R  (aj|SM × O'j|Ξ)
                  j=0

  // II. System synthesis – integration of the system
              |A|
  → Θ|SM :=    ⊎  fΘj : aj|SM → O'j|Ξ
              j=0
          = A|Ξ × O'j|Ξ
}
```

Figure 4. The cognitive process of synthesis in RTPA

CognitiveSynthesis|PM encompasses two subprocesses known as those of component synthesis and system synthesis. The process first integrates a set of individual solutions for each object by the subset of corresponding attributes. As a result, a set of individual component syntheses, $\Theta'|\mathrm{SM} := \underset{j=0}{\overset{|A|}{R}} a_j|\mathrm{SM} \times O_j'|\Xi$, is obtained. Then, the process aggregates the component-level solutions to the system level represented by a coherent set of functions, $\Theta|\text{SM} := \underset{j=0}{\overset{|A|}{R}} f_{\Theta_j}: a_j|\text{SM} \to O_j^{'}|\Xi$, which map each attribute in $A|\Xi$ into related objects $O_j^{'}|\Xi$ in the layout of the system.

5. Conclusion

This work is a part of the formalization and simulations of the cognitive processes of the brain and a cognitive computer according to the Layered Reference Model of the Brain (LRMB). It has been recognized that analysis and synthesis play an important role in the inference processes of the brain and cognitive systems. This paper has formally explained the cognitive processes of analysis and synthesis with a set of cognitive, mathematical, and process models. The generic cognitive processes of analysis and synthesis have been rigorously described in Real-Time Process Algebra (RTPA). Applications of this work have been identified in cognitive informatics, cognitive computing, computational intelligence, cognitive machine learning, cognitive robotics, and cognitive knowledge bases.

Acknowledgment: The author would like to acknowledge the support in part by a discovery fund of the Natural Sciences and Engineering Research Council of Canada (NSERC). The author would like to thank the anonymous reviewers for their valuable suggestions and comments on the previous version of this paper.

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