Improvement Of Ranking Method Based On Effectiveness Of Units In Society By Common Weights Approach In DEA

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Abstract: The aim of this paper is to modify the suggested method by Noura et al. [25], which is a ranking method based on the effectiveness of each unit in society. They utilized the assigned weights by manager for ranking decision making units (DMUs) on the basis of the effectiveness in society, however, this is not a conventional method for determining the weights. This paper proposes common weights approach for improving its method. A multi-objective linear fractional is derived and then it was converted to a multi-objective linear programming by Taylor series. The model is solved by Max Min method. Based on the obtained optimal solution, common weights are acquired and then DMUs will be ranked. The proposed method is illustrated by ranking Taiwan forests after reorganization.

Key–Words: Data envelopment analysis, Super efficiency, Effectiveness, Multi-objective linear fractional programming, Common weights, Taylor series

1 Introduction

Data envelopment analysis (DEA) which provides a measurement for the efficiency of a homogeneous group of decision making units (DMUs) was proposed by Charnes et al. [1]. Their proposed model to evaluate DMUs is called CCR model. DEA used in many different sciences, including management, economics, industry, health and so on.

DEA classifies DMUs in to the efficient DMUs and inefficient DMUs. For the category of inefficient DMUs, the efficiency scores are used for a comparison among them. However, DMUs in the efficient category have the efficiency score equal to one. It is not desirable to say that they have the same performance in real practices. To remove this difficulty, a variety of methods has been proposed for ranking the efficient DMUs. They are divided into three basic groups: super efficiency, cross efficiency and common weights methods.

The first super efficiency method which was presented by Andersen and Petersen [2] evaluates that the efficiency of DMUs possibly exceeds the conventional score 1.0, by comparing the evaluated DMU with a linear combination of other DMUs, while the evaluated DMUs exclude from the observations. Based on their nature, the efficiency score of non-extreme DMUs are not changed. In other word, they have still the efficiency score one and so cannot be ranked. The extreme efficient DMUs, on the other hand, are discriminated by using different super efficiency scores larger than 1.0. This method has three basic problems which are infeasibility for some cases, instability, and inability to rank non-extreme efficient DMUs. Infeasibility of super efficiency was firstly studied by Seiford and Zhu [3]. Chen [4] measured the super efficiency in the presence of infeasibility. In 2011, Lee et al. [5] and Chen and Liang [6] extended the super efficiency model of Chen [4]. The two mentioned papers were modified to measure the super efficiency with nonnegative data. According to the super efficiency approach introduced by Andersen and Petersen [2], many super efficiency methods such as works by Mehrabian et al. [7], Tone [8], Jahanshahloo et al. [9] and Rezai Balf et al. [10] have been suggested for measuring the distance of evaluating DMU from the efficient frontier, when the evaluating DMU ignores from the collection of DMUs. These methods have no problem of instability. All of the super efficiency models have the problem of ranking non-extreme efficient DMUs. Although, Gholam Abri et al. [11] proposed a method for ranking the non-extreme efficient DMUs based on the representation theorem, their method is infeasible from the computational point of view.

Sexton et al. [12] offered cross efficiency method for ranking the efficient DMUs. In their method, the performance of the efficient DMUs in comparison with an evaluating DMU is measured by using the optimal weights obtained from the CCR model of the evaluating DMU. A pairwise comparison matrix called cross efficiency matrix is then obtained and corresponding priority vector is calculated by the average of components of each row of cross efficiency matrix which is consider for ranking the efficient DMUs. Cross efficiency method has three basic problems. The first problem is choosing a set of weights from the collective alternative optimal solutions of the cross efficiency model. Secondary goals were posed to restrict the collective of the optimal solutions. Doyle and Green [13], Jahanshahloo et al. [14], and Wang and Chin [15] discussed models for abating weights in cross efficiency method. Another problem of cross efficiency method is inconsistency in the cross efficiency matrix which is usually led to unrealistic ranking for DMUs. This difficulty was carefully studied by Wu [16]. Finally,existing zero weights is as another problem in cross efficiency which was pointed out by Wang and Chin [15].

The common weights approach in DEA is another method for ranking the efficient DMUs introduced by Cook et al. [17] and Roll et al. [18] in order to evaluate highway maintenance units. They minimized the distance between upper and lower limits of the weights to obtain a set of weights for ranking DMUs. Ganley and Cubbin [19] acquired the common weights from a model maximizing the sum of efficient ratios of all the units. They used the obtained weights for ranking all DMUs. Instead, Jahanshahloo et al. [20] proposed using maximization of minimum of efficiency ratio of the efficient units to produce common weights. Liu and Peng [21] presented a linear model to calculate the common weights. They took number one as a benchmark of all units. Chiang et al. [22] introduced a linear model with a separation vector to obtain the common weights in the DEA problems. The common set of weights methods, however, have some problems such as incapability in ranking all the DMUs, sometimes, because there is usually more than one unit with score one. The common weights models may also have alternative optimal solution and if so the ranking of DMUs based on the different set of weights will be different. This default was fully discussed by Payan et al. [23].

All of the ranking methods have been considered the weighted sum of outputs to the weighted sum of inputs to evaluate the efficient DMUs in different procedures. This seems as if this ratio, in classical DEA models, could be differentiated between the efficient DMUs, then ranking them was not be required. Since this event does not happen, the alternative methods have been proposed for evaluating the efficient DMUs. Unfortunately, these methods are also used forgoing ratio for evaluating efficient DMUs. Recently, other criteria are suggested to evaluate the efficient DMUs. For example, Wang et al. [24] utilized the distance of DMUs from the ideal and anti-ideal DMUs in order to construct cross efficiency matrix, and Noura et al. [25] proposed to use the effectiveness of the efficient DMUs in society for ranking them. In this paper we use the suggested criterion by Noura et al. [25] in common weights method. From the management point of view, it is difficult to manage the large group, furthermore, large groups are more effective in society [25]. Noura et al. [25] assigned the weights to inputs and outputs such that these weights are determined by judgement of the manager. But according to theoretical science and management this has a problem. Because, if decision makers are a board of directors, then they may have different idea about the value of the indices of DMUs. The other problem appears when manager (or board of directors) assigned the unsuitable weights to DMUs that cause reduction in effectiveness of units in the society. To overcome these difficulties we propose the common weights method to determine the weights in the method of Noura et al.[25].

In so doing,we introduced a multi-objective linear fractional programming problem (MOLFP) to produce a set of weights appropriate to DMUs and then apply the Taylor series approach to transform MOLFP to multi-objective linear programming (MOLP). By using the weights and determining the indices, which large amounts or small amounts of them are beneficial for society, the performance of efficient DMUs is calculated.

Based on the above discussion, this paper is organized as: The next section includes the preliminaries of DEA. The proposed method complained in section 3. In section 4, a numerical example illustrates the method. Conclusions are provided in the last section.

2 Preliminaries

Assume there are n DMUs with m inputs and s outputs, each DMU$_j$ $(j = 1, ..., n)$ consumes inputs x_{ij} $(i = 1, ..., m)$ to produce outputs y_{rj} $(r = 1, ..., s)$. Charnes et al. [1] for evaluating the performance of DMUs provided the following model, which is known as multiplier CCR model:

$$\max \theta_p = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}, \qquad (1)$$

$s.t.$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n,$$
$$u_r \geq 0, \qquad r = 1, \ldots, s,$$
$$v_i \geq 0, \qquad i = 1, \ldots, m.$$

In this model, multipliers v_i $(i = 1, ..., m)$ and u_r $(r = 1, ..., s)$ are respectively the weights of inputs and outputs. This model can be transformed to an equivalent linear programming problem by conventional transformation variable of Charnes and Cooper [26] as follows:

$$\max \theta_p = \sum_{r=1}^{s} u_r y_{rp} \tag{2}$$

$s.t.$

$$\sum_{i=1}^{m} v_i x_{ip} = 1,$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n,$$
$$u_r \geq 0, \qquad r = 1, \ldots, s,$$
$$v_i \geq 0, \qquad i = 1, \ldots, m.$$

Definition 1 *If the optimal value of the above model to evaluate DMU$_p$ is equal to one and there is optimal weights such that $v_i^* > 0$ $(i = 1, ..., m)$ and $u_r^* > 0$ $(r = 1, ..., s)$, then the evaluated DMU is CCR efficient, otherwise it is inefficient [27].*

Suppose $E = \left\{ j \mid \theta_j^* = 1 \right\}$ is the set of all efficient DMUs. Usually, $|E| > 1$. In the other word, there is more than one efficient DMU. Therefore we need a method to compare them. One of the successful methods for ranking efficient DMUs is super efficiency method which was firstly introduced by Andersen and Petersen [2]. The proposed method is to delete the efficient DMU from the set of observed DMUs and then evaluate the impact of its removing by the following model as:

$$\max \theta_p = \sum_{r=1}^{s} u_r y_{rp} \tag{3}$$

$s.t.$

$$\sum_{i=1}^{m} v_i x_{ip} = 1,$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n, \ j \neq p,$$
$$u_r \geq 0, \qquad r = 1, \ldots, s,$$
$$v_i \geq 0, \qquad i = 1, \ldots, m.$$

The optimal value of the above model is considered as the supper efficiency of DMU$_p$.

Theorem 2 *The supper efficiency of DMU$_p$ is more than or equal to one if and only if DMU$_P$ is CCR efficient [2].*

3 Proposed Method

Based on the proposed methods by Noura et al. [25] the inputs and outputs of DMUs are divided into the following groups:

- D_I^+= {inputs at larger amount are beneficial for society}

  D_I^-= {inputs at smaller amount are beneficial for society}
- D_O^+= {outputs at larger amount are beneficial for society}
- D_O^-= {outputs at smaller amount are beneficial for society}

According to this classification, the performance of DMU$_j$ $(j \in E)$, denoted by $f_j(u, v)$, can be calculated by the following formula:

$$f_j(u, v) = \frac{\sum_{i \in D_I^+} v_i x_{ij} + \sum_{r \in D_O^+} u_r y_{rj}}{\sum_{i \in D_I^-} v_i x_{ij} + \sum_{r \in D_O^-} u_r y_{rj}} \tag{4}$$

The inputs and outputs that large amounts of them are beneficial for society are considered in the numerator and the inputs and outputs that small amounts of them are beneficial for the society are put in the denominator. The weights in the above formula are not determined. Noura et al. [25] have mentioned that the weights can be determined by a manager. This does not seems logical because the manager may determine inappropriate weights. In the other situation, if there is a board of directors, then they may have different idea about the value of indices . Hence, we need to present a method that automatically determines the value of indices. This paper suggests using common weights method in DEA for removing this problem. Thus, following model is proposed to determine the weights as:

$$\max\{f_j(u,v), j \in E\} \tag{5}$$

$s.t.$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, \ldots, n, \qquad (5-1)$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \qquad (5-2)$$

$$u_r \geq \varepsilon, \;\; r = 1, \ldots, s, \qquad (5-3)$$

$$v_i \geq \varepsilon \;, \;\; i = 1, \ldots, m. \qquad (5-4)$$

In this model, the constraints (5-1) construct feasible region of the CCR model and the constraint (5-2) normalizes the weights of inputs and outputs and constraints (5-3) and (5-4) avoid that the inputs and outputs become zero and ε is a non-archimedean positive number. Model (5) is an MOLFP. Recently, Guzel and Sivri [28] provided a method to convert MOLFP to MOLP by using first-order Taylor polynomial series. We use their method to solve MOLFP (5).

Let $(u^{p^*},\ v^{p^*})$ is the optimal solution of the *P*th efficient DMU that is obtained by the following problem:

$$\max f_p(u,v) \tag{6}$$

$s.t.$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r \geq \varepsilon, \qquad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon, \quad i = 1, \ldots, m.$$

Taylor series expansion of the *P*-th objective for the optimal solution $(u^{p^*},\ v^{p^*})$ of the above problem is as follows:

$$f_p\left(u,v\right) \simeq \hat{f}_p\left(u,v\right)$$

$$= f_p\left(u^{p^*}, v^{p^*}\right)$$

$$+ \sum_{i \in D_I^+} \left(v_i - {v_i}^{p^*}\right) \frac{\partial f_p\left(u^{p^*}, v^{p^*}\right)}{\partial v_i}$$

$$+ \sum_{r \in D_O^+} \left(u_r - {u_r}^{p^*}\right) \frac{\partial f\left(u^{p^*}, v^{p^*}\right)}{\partial u_r}$$

$$+ \sum_{i \in D_I^-} \left(v_i - {v_i}^{p^*}\right) \frac{\partial f\left(u^{p^*}, v^{p^*}\right)}{\partial v_i}$$

$$+ \sum_{r \in D_O^-} \left(u_r - {u_r}^{p^*}\right) \frac{\partial f\left(u^{p^*}, v^{p^*}\right)}{\partial u_r}$$

In this way the MOLFP (5) will be transformed to an MOLP as follows:

$$\max\{\widehat{f}_j(u,v), j \in E\} \tag{7}$$

$s.t.$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r \geq \varepsilon, \qquad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon, \quad i = 1, \ldots, m.$$

Using the maximin method to solve the above MOLP, model (7) is converted to a problem as:

$$\max \min_{j \in E} \widehat{f}_j(u,v), \tag{8}$$

$s.t.$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r \geq \varepsilon, \quad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon, \qquad i = 1, \ldots, m,$$

by setting $t = \min_{j \in E} \widehat{f}_j(u,v)$, an equivalent linear programming problem is obtained as follows:

$$\max\ t, \tag{9}$$

$s.t.$

$$t \leq \widehat{f}_j(u,v), j \in E,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r \geq \varepsilon, \quad r = 1, \ldots, s,$$

$$v_i \geq \varepsilon, \qquad i = 1, \ldots, m.$$

By solving this problem a set of weights is obtained and used for calculating the performance of the efficient DMUs. One probable problem in the above model is the existence of alternative optimal solution. In other words, after solving problem (9), there exist more than one set of weights for evaluating the performance of the DMUs. In this situation, rank of DMUs according to different set of weights may not be identical. Also in a common set of weights, some weights of indices have very small value. Therefore, corresponding index has a little influence over the performance of DMUs and often has no influence over the performance of DMUs.

To overcome the above problems, we partition the set of weights to h subset $S_1, \ldots, S_h$ according to a priority determined by the manager. This is clear that $S_i \cap S_j = \emptyset \ (i \neq j)$ and $S_i \neq \emptyset \ \ (i = 1, \ldots, h)$. After solving problem (9), h models must be subsequently solved which the k-th problem for determining a unique set of weights is as:

$$\max \min\{u_r, v_i\}, \qquad r, i \in S_k, \qquad (10)$$

$s.t.$

$$\begin{aligned}
& t^* \leq \widehat{f}_j(u, v), j \in E, \\
& \frac{\sum\limits_{r=1}^{s} u_r y_{rj}}{\sum\limits_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n, \\
& \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, \\
& u_r \geq \varepsilon \,, \qquad r = 1, \ldots, s, \\
& v_i \geq \varepsilon \,, \qquad i = 1, \ldots, m, \\
& u_r = u_r^*, \ \ v_i = v_i^*, \ \ \forall i, r \in S_l \\
& l = 1, \cdots, k-1,
\end{aligned}$$

where t^* is the optimal value of the problem (9) . The optimal solution of the last model is considered as a set of common weights. The performance of the efficient DMUs is then gained by using the formula (4) and based on them the DMUs can be ranked.

4 Numerical Example

To illustrate the proposed method, in this section, we use the data of the paper Kao [29],in where he considered 17 Taiwan forests after reorganization.

Taiwan is an island which more than half of that are forest lands. The national forests were divided into 13 districts by Taiwan Forestry Bureau (TFB). Kao [29] added four other forests of comparable size to the DMUs. The inputs of Taiwan forests are as follow:

- Land (X_1): area in hectares.
  Labor (X_2): number of employees.
- Expenditures (X_3): expenses per year in US dollars.
- Initial stock (X_4): volume of forest stock before the evaluation in cubic meters.

Outputs are considered as follows:

- Timber production (Y_1): timber harvested each year in cubic meters.
  Soil conservation (Y_2): forest stock for conserving soil in cubic meters.
- Recreation (Y_3): visitors served by forests every year in number of visits.

Input and output data are respectively shown in Tables 1 and 2.

Table 1: Inputs of Taiwan forests [29]

Districts	**Inputs**			
	Land ($10^3 ha$)	Labor (persons)	Expend. (\$$10^6$)	Init.S. ($10^6\ m^3$)
Wen Shan	60.85	270.0	4.11	5.04
Chu Tung	108.46	597.9	9.30	13.45
Ta Chia	79.06	421.4	6.35	8.27
Ta Su Shan	59.66	860.1	12.28	10.95
Pu Li	84.50	271.0	4.33	9.93
Luan Ta	127.28	592.0	10.45	13.36
Yu Shan	98.80	863.0	12.15	8.14
Nan Nung	123.14	852.0	8.84	10.86
H Chung	86.37	285.0	5.35	8.62
K Shan	227.20	216.1	5.87	24.04
Yu Li	146.43	205.0	4.08	15.76
Mu Kua	173.48	774.9	12.60	23.03
Lan Yang	171.11	2722.7	14.51	17.84
ForExp. B.	93.65	1399.0	128.94	17.58
For.R. Inst	13.65	350.9	0.91	1.42
Taiw Univ	33.52	165.0	1.73	0.38
C H Univ.	8.23	49.0	0.30	1.59

Table 2: Outputs of Taiwan forests [29]

Districts	Outputs		
	Harvest (10^3m^3)	Stock (10^6m^3)	Visitors (10^3 $visits$)
Wen Shan	15.85	5.17	14.57
Chu Tung	47.19	18.86	7.00
Ta Chia	21.57	10.48	33.73
Ta Su Shan	8.41	11.71	9.64
Pu Li	39.04	12.25	0.00
Luan Ta	57.11	13.81	0.00
Yu Shan	42.81	12.43	399.83
Nan Nung	55.20	9.18	7.56
H Chung	39.24	6.88	1081.89
K Shan	44.08	27.28	0.00
Yu Li	37.30	19.30	0.00
Mu Kua	9.63	23.53	41.86
Lan Yang	19.73	18.86	84.00
ForExp. B.	42.11	17.30	0.00
For.R. Inst	19.07	1.58	0.00
Taiw Univ	13.57	0.50	1061.48
C H Univ.	3.86	1.57	67.73

Efficient DMUs are determined by the CCR model (2). The second column of Table 5 shows the CCR efficiency of the Taiwan forests. As we see Chu Tung, Ta Su Shan, Pu Li, Yu Shan, H Chung, K Shan, Yu Li, For.R. Inst, Taiw Univ, C H Univ are CCR efficient. In this example we rank these ten DMUs by using the proposed method in this paper and then compare its results with the super efficiency method.

In this example the first input is not beneficial for society because the more area needs the more protection and the more protection needs the more cost. The second input will be beneficial. Its reason comes from the fact that increasing number of employees leads to more people that occupied jobs and have positive effect over the society. As we see, the third input won't be beneficial, because the goal is reducing expenses then put it in denominator.

The first output is not good for environment and consequently has bad effect over the society. The second output is beneficial for the society because soil is an important part of the forest. The third output has cultural effect over the society.

Therefore we define D_I^+, D_I^-, D_O^+ and D_O^- as follows:

$$D_I^+ = \{X_2\}, D_I^- = \{X_1, X_3\},$$
$$D_O^+ = \{Y_2, Y_3\}, D_O^- = \{Y_1\}.$$

Thus, the function of the proposed method to evaluate the efficient DMUs in the society is as follows:

$$f_j(u,v) = \frac{v_2x_{2j} + u_2y_{2j} + u_3y_{3j}}{v_1x_{1j} + v_3x_{3j} + u_1y_{1j}}, j \in E$$

The optimal input and output weights of the problem (6) in order to construct the Taylor series for the efficient DMUs are respectively presented in Table 3 and 4.

For example, we write the Taylor expansion of Chu Tung as:

$$\hat{f}_2(u,v) = f_2(u^*,v^*)$$
$$+(v_2 - v_2^*)\frac{x_{22}}{(v_1^*x_{12} + v_3^*x_{32} + u_1^*y_{12})^2}$$
$$+(u_2 - u_2^*)\frac{y_{22}}{(v_1^*x_{12} + v_3^*x_{32} + u_1^*y_{12})^2}$$
$$+(u_3 - u_3^*)\frac{y_{32}}{(v_1^*x_{12} + v_3^*x_{32} + u_1^*y_{12})^2}$$
$$+(v_1 - v_1^*)\frac{-x_{12}}{(v_1^*x_{12} + v_3^*x_{32} + u_1^*y_{12})^2}$$
$$+(v_3 - v_3^*)\frac{-x_{32}}{(v_1^*x_{12} + v_3^*x_{32} + u_1^*y_{12})^2}$$
$$+(u_1 - u_1^*)\frac{-y_{12}}{(v_1^*x_{12} + v_3^*x_{32} + u_1^*y_{12})^2}$$

where the optimal weights are determined from the corresponding row of Tables 3 and 4 and are as:

$$v_1^* = 0.0087,\ v_2^* = 0.9863,\ v_3^* = 0.01,\ v_4^* = 0.001$$
$$u_1^* = 0.001,\ u_2^* = 0.001,\ u_3^* = 0.001$$

When model (9) is solved, we see that the model has alternative optimal solution. Two of the optimal solutions of the model are as:

$$(t^*, v_1^*, v_2^*, v_3^*, v_4^*,\ u_1^*, u_2^*, u_3^*) =$$
$$(0.96298, 0.001, 0.001,\ 0.001, 0.94, 0.001, 0.001, 0.001)$$

and

$$(t^*, v_1^*, v_2^*, v_3^*, v_4^*,\ u_1^*, u_2^*, u_3^*) =$$
$$(0.96298, 0.001, 0.94, 0.001, 0.001, 0.001, 0.001, 0.001)$$

Table 3: The optimal input weights of model (6) for the efficient DMUs

Districts	v_1^*	v_2^*	v_3^*	v_4^*
Chu Tung	.0087	.9863	.001	.001
Ta Su Shan	.016	.979	.001	.001
Pu Li	.0113	.9837	.001	.001
Yu Shan	.0956	.9854	.001	.001
H Chung	.011	.823	.001	.001
K Shan	.0042	.9908	.001	.001
Yu Li	.0065	.9885	.001	.001
For.R. Inst	.001	.9433	.001	.001
Taiw U-niv	.0294	.8054	.001	.001
C H U-niv.	.121	.7172	.001	.001

Table 4: The optimal output weights of model (6) for the efficient DMUs

Districts	u_1^*	u_2^*	u_3^*
Chu Tung	.001	.001	.001
Ta Su Shan	.001	.001	.001
Pu Li	.001	.001	.001
Yu Shan	.001	.001	.001
H Chung	.001	.001	.1619
K Shan	.001	.001	.001
Yu Li	.001	.001	.001
For.R. Inst	.001	.001	.001
Taiw Univ	.001	.001	.1612
C H U-niv.	.001	.001	.1578

Based on the concepts of linear programming theory, we know that any point in the convex combination of the above-mentioned solutions is also optimal solution of the problem and so there is a vague to select one of them. Therefore we need a method to choose an optimal solution among the alternative optimal solutions. Based on the suggested method in this paper and in order to gain this goal, we must firstly partition the set of weights $\{v_1,\ v_2,\ v_3,\ v_4,\ u_1,\ u_2,\ u_3\}$ into number of its subsets. Now we consider the following partition to acquire a unique set of weights as:

$$S_1 = \{\ v_4,\ u_2, u_3\}$$

$$S_2 = \{v_1, v_2, v_3, u_1\}$$

Therefore, to obtain the unique optimal set of weights, we need to solve two subsequent models, similar to model (10), as follows:

$$\max\min\{u_2, u_3, v_4\}, \tag{11}$$

$$\begin{aligned}
s.t.\quad & t^* \leq\ \hat{f}_j(u,v), j \in E,\\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n,\\
& \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,\\
& u_r \geq \varepsilon\ , \qquad r = 1, \ldots, s,\\
& v_i \geq \varepsilon\ , \qquad i = 1, \ldots, m,
\end{aligned}$$

and then

$$\max\min\{u_1, v_1, v_2, v_3\}, \tag{12}$$

$$\begin{aligned}
s.t.\quad & t^* \leq\ \hat{f}_j(u,v), j \in E,\\
& \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1, ..., n,\\
& \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,\\
& u_r \geq \varepsilon\ , \qquad r = 1, \ldots, s,\\
& v_i \geq \varepsilon\ , \qquad i = 1, \ldots, m,\\
& u_2 = u_2^*,\ u_3 = u_3^*,\ v_4 = v_4^*,
\end{aligned}$$

The optimal solution of the second problem (Model (12)) is as follows:

$$v_1^* = 0.001,\ \ v_2^* = 0.001,\ \ v_3^* = 0.001,\ \ v_4^* = 0.94$$

$$u_1^* = 0.001,\ u_2^* = 0.001,\ u_3^* = 0.001$$

Table 5: The score of DMUs based on different methods

Districts	CCR Efficiency	Sup eff	$f_j = (u, v)$
Wen Shan	0.7337761	-	-
Chu Tung	1	1.1509	3.7815
Ta Chia	0.8961212	-	-
Ta Su Shan	1	1.0191	10.9701
Pu Li	1	1.1903	2.2151
Luan Ta	0.8813878	-	-
Yu Shan	1	1.0751	8.2938
Nan Nung	0.7657179	-	-
H Chung	1	1.1704	10.49
K Shan	1	1.3289	0.8782
Yu Li	1	1.1163	1.1943
Mu Kua	0.8192824	-	-
Lan Yang	0.7438508	-	-
For Ex-p.B	0.9745701	-	-
For.R. Inst	1	3.047	10.4811
Taiw Univ	1	22.1532	25.1327
C H Univ.	1	1.633	9.548

Table 6: Ranking DMUs with the AP and proposed methods

Districts	Sup eff	$f_j = (u, v)$
Wen Shan	17	17
Chu Tung	7	7
Ta Chia	12	12
Ta Su Shan	10	2
Pu Li	5	8
Luan Ta	13	13
Yu Shan	9	6
Nan Nung	15	15
H Chung	6	3
K Shan	4	10
Yu Li	8	9
Mu Kua	14	14
Lan Yang	16	16
For Exp.B	11	11
For.R. Inst	2	4
Taiw Univ	1	1
C H Univ.	3	5

Table 5 represents the performance of DMUs based on the three methods, CCR, super efficiency and the proposed method in this paper. The performance of DMUs, based on the suggested method and according to the above weights, is provided in the last column of Table 5.

The ranking of DMUs is represented in Table 6. The results of Table 6 show that DMU16 (Taiw Univ) has the first rank in both methods. According to Table 5, this unit has very similar performance scores 22.1532 and 25.1327 based on the super efficiency and our proposed methods, respectively. As we observe, there are two DMUs which have equal ranks with the super efficiency and our methods. These units are Chu Tang and Taiw Unive. Very different ranking is occurred for DMU4 (Ta Su Shan) and DMU10 (K Shan) in these two methods. For example, DMU4 has the tenth rank based on the super efficiency method, while it has the second rank in our proposed method. This difference in ranking usually exists in all ranking method, because the nature of different methods is different.

5 Conclusion

In this paper, the method for overcoming the problem of determining the weights of indices by manager (decision maker) in the proposed method by Noura et al. [25] for ranking DMUs. This method was suggested based on the effectiveness of units in society. The offered method uses common set of weights method in DEA to determine the weight indices. An MOLFP was introduced and then converted to an LP by using Taylor series. The optimal solution of this problem was considered as the weights of indices. Another advantage of this method arises from the fact that the proposed method is linear, consequently, the the calculation process will be reduced. As an expansion, the data could be considered as fuzzy numbers and the units would be evaluated based on the effectiveness of them in society.

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