Introduction to the Rhombus Trigonometry in Euclidian 2D-space with simulation of four Rhombus trigonometric functions *RhJes, RhJes-x, RhMar and RhRit*

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Abstract: - The Rhombus Trigonometry is an original study introduced in the mathematical domain by the author. Trigonometry is a branch of mathematics that deals with relations between sides and angles of triangles. It has some relationships to geometry, though there is disagreement on exactly what is that relationship. For some scientists, trigonometry is just a subtopic of geometry. The trigonometric functions are very important in technical subjects like Astronomy, Relativity, Science, Engineering, Architecture, and even Medicine. In this paper, the Rhombus trigonometry is introduced in order to be a part of the General Trigonometry topic. Thus, the definition of this original part is presented, and the Rhombus trigonometric functions are also defined. The importance of these functions is by producing multi signal forms by varying some parameters of a single function. Different signals and forms are analyzed and discussed.

The concept of the Rhombus Trigonometry is completely different from the traditional trigonometry in which the study of angles is not the relation between sides of a right triangle that describes a circle as the previous one, but the idea here is to use the relation between angles and sides of a rhombus form with the internal and external circles formed by the intersection of the rhombus form and the positive parts of x'ox and y'oy axis in the Euclidian 2D space and their projections. This new concept of relations will open a huge gate in the mathematical domain and it can resolve many complicated problems that are difficult or almost impossible to solve with the traditional trigonometry, and it can describe a huge number of multi-form periodic signals.

Key-words: - Mathematics, geometry, trigonometry, angular function, multi form signal, power electronics.

1 Introduction

The traditional trigonometry is a branch of the mathematics that deals with relationships between sides and angles of a right triangle [3], [4], and [5]. Six principal functions (e.g. cosine, sine, tangent, cosec, sec, and cotangent) are used to produce signals that have an enormous variety of applications in all scientific domains [6], [7], [8], [9]. It can be considered as the basis and foundation of many domains as electronics, signal theory, astronomy, navigation, propagation of signals and many others… [10], [11], [12], [13]. Now it is time to introduce a new concept of trigonometry in the Euclidean 2D-space, and this trigonometry opens new gates and new challenges by the reconstruction of the science [14], [15], [16].

In this paper, the new concept of the Rhombus trigonometry is introduced and few examples are shown and discussed briefly. Figures are drawn and simulated using AutoCAD and Matlab. The main goal of introducing this trigonometry is to form multi form periodic signals using just one function, this is not the case of the traditional trigonometry. The Rhombus trigonometry can be applied in all scientific domains and especially in signal theory and signal processing in which we can form a single circuit that produce more than 14 different signals; this is not the case of the traditional trigonometry in which one circuit can't produce more than one signal.

In the second section, the angular functions are defined [20], these functions have enormous applications in all domains, and it can be considered as the basis of this trigonometry, similar to other introduced trigonometry by the author [1], [2], [19], and [21]. The definition of the Rhombus trigonometry is presented and discussed briefly in the third section. In the fourth section, a survey on the Rhombus Trigonometric functions is discussed and four different functions are simulated with brief examples. A survey on the application of the Rhombus trigonometry in engineering domain is presented in the section 5. Finally, a conclusion about the Rhombus trigonometry is presented in the section 6.

2 The angular functions

Angular functions are new mathematical functions that produce a rectangular signal, in which period is function of angles. Similar to trigonometric functions, the angular functions have the same properties as the precedent, but the difference is that a rectangular signal is obtained instead of a sinusoidal signal [14],[15],[16] and moreover, one can change the width of each positive and negative alternate in the same period. This is not the case of any other trigonometric function. In other hand, one can change the frequency, the amplitude and the width of any period of the signal by using the general form of the angular function.

In this section three types of angular functions are presented, they are used in this trigonometry; of course there are more than three types, but in this paper the study is limited to three functions.

2.1 Angular function $ang_x(x)$

The expression of the angular function related to the (ox) axis is defined, for $K \in \mathbb{Z}$, as:

$$ang_x(\beta(x+\gamma)) = \begin{cases} +1 \ \ for \ \ (4K-1)\frac{\pi}{2\beta}-\gamma \le x \le (4K+1)\frac{\pi}{2\beta}-\gamma \\ -1 \ \ for \ (4K+1)\frac{\pi}{2\beta}-\gamma < x < (4K+3)\frac{\pi}{2\beta}-\gamma \end{cases} \quad (1)$$

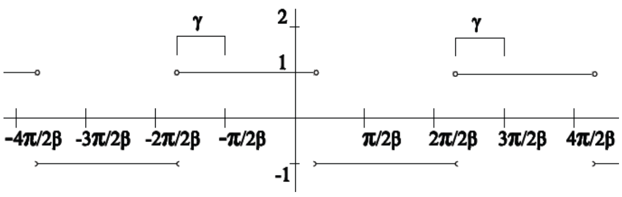


Fig. 1: The $ang_x(\beta(x+\gamma))$ waveform.

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$ang_x(x) = \begin{cases} +1 \ \ for \ \ \cos(x) \ge 0 \\ -1 \ \ for \ \ \cos(x) < 0 \end{cases}$$

2.2 Angular function $ang_y(x)$

The expression of the angular function related to the (oy) axis is defined, for $K \in \mathbb{Z}$, as:

$$ang_y(\beta(x+\gamma)) = \begin{cases} +1 \ \ for \ \ 2K\pi/\beta-\gamma \le x \le (2K+1)\pi/\beta-\gamma \\ -1 \ \ for \ \ (2K+1)\pi/\beta-\gamma < x < (2K+2)\pi/\beta-\gamma \end{cases} \quad (2)$$

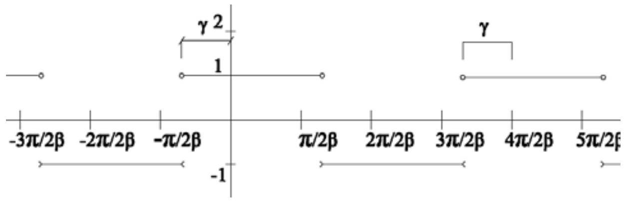


Fig. 2: The $ang_y(\beta(x+\gamma))$ waveform.

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$ang_y(x) = \begin{cases} +1 \ \ for \ \ \sin(x) \ge 0 \\ -1 \ \ for \ \ \sin(x) < 0 \end{cases}$$

2.3 Angular function $ang_\alpha(x)$

α (called firing angle) represents the angle width of the positive part of the function in a period. In this case, we can vary the width of the positive and the negative part by varying only α. The firing angle must be positive.

$$ang_\alpha(\beta(x+\gamma)) = \begin{cases} +1 \ \ for \\ (2K\pi-\alpha)/\beta-\gamma \le x \le (2K\pi+\alpha)/\beta-\gamma \\ \\ -1 \ \ for \\ (2K\pi+\alpha)/\beta-\gamma < x < (2(K+1)\pi-\alpha)/\beta-\gamma \end{cases} \quad (3)$$

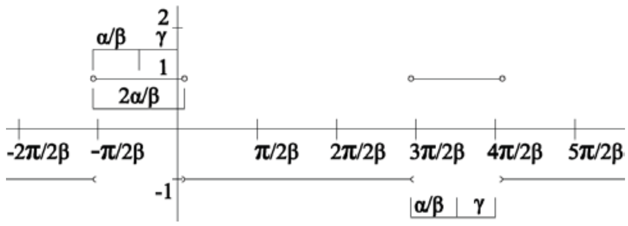


Fig. 3: The $ang_\alpha(\beta(x+\gamma))$ waveform.

3 Definition of the Rhombus Trigonometry

3.1 The Rhombus Trigonometry unit

The Rhombus Trigonometry unit is a rhombus (lozenge) form with a center O (x = 0, y = 0).

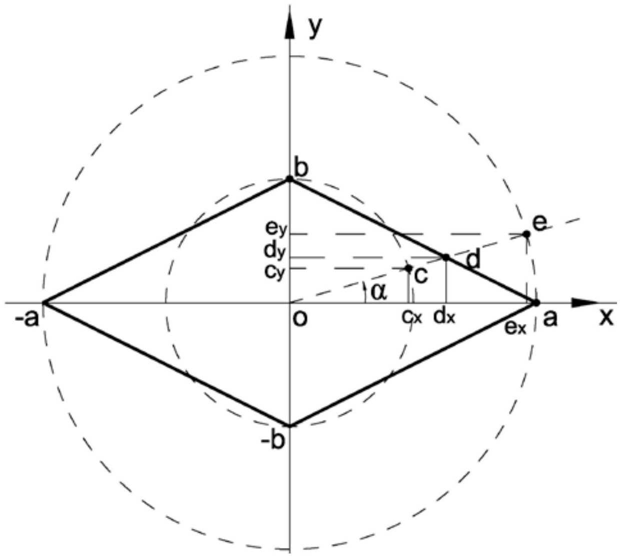


Fig. 4: The Rhombus trigonometry unit.

It is essential to note that 'a' and 'b' must be positive. In this paper, 'a' is fixed to 1. We are interested to vary only a single parameter which is 'b'.

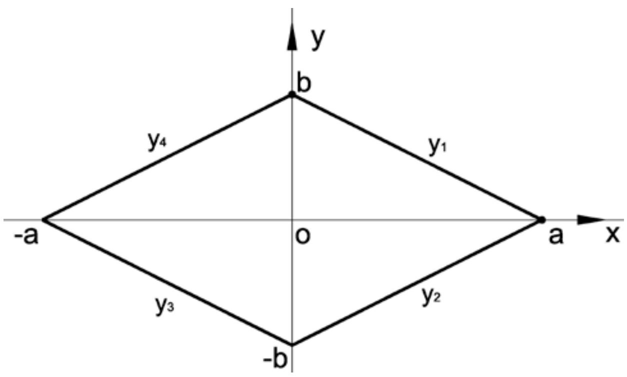


Fig. 4.1: represents four equations that form the Rhombus.

We obtain four equations that describe the rhombus as following:

$$y_1 = \frac{-b}{a}x + b \text{ with } x \in [0, a] \quad (4)$$

$$y_2 = \frac{b}{a}x - b \text{ with } x \in [0, a] \quad (5)$$

$$y_3 = \frac{-b}{a}x - b \text{ with } x \in [-a, 0] \quad (6)$$

$$y_4 = \frac{b}{a}x + b \text{ with } x \in [-a, 0] \quad (7)$$

3.2 Intersections and projections of different elements of the Rhombus Trigonometry on the relative axes

From the intersections of the Rhombus with the positive parts of the axes (ox) and (oy), define respectively two circles of radii $[oa]$ and $[ob]$ (refer to figure 4). These radii can be variable or constant according to the form of the Rhombus.

The points of the intersection of the half-line $[od)$ (figure 4) with the internal and external circles and with the Rhombus and their projections on the axes (ox) and (oy) can be described by many functions that have an extremely importance in creating plenty of signals and forms that are impossible to be created in the traditional trigonometry.

Definition of the letters in the Figure 4:

a: is the intersection of the Rhombus with the positive part of the axe (ox) that gives the relative circle of radius "a". It can be variable.

b: is the intersection of the Rhombus with the positive part of the axe (oy) that gives the relative circle of radius "b". It can be variable.

c: is the intersection of the half-line $[od)$ with the circle of radius b.

d: is the intersection of the half-line $[od)$ with the Rhombus.

e: is the intersection of the half-line $[od)$ with the circle of radius a.

c_x: is the projection of the point c on the ox axis.

d_x: is the projection of the point d on the ox axis.

e_x: is the projection of the point e on the ox axis.

c_y: is the projection of the point c on the oy axis.

d_y: is the projection of the point d on the oy axis.

e_y: is the projection of the point e on the oy axis.

α: is the angle between the (ox) axis and the half-line$[od)$.

o: is the center (0, 0).

3.3 Definition of the Rhombus Trigonometric functions $\boldsymbol{Rhfun(\alpha)}$

The traditional trigonometry contains only 6 principal functions: Cosine, Sine, Tangent, Cosec, Sec, Cotangent [6], [16], [17]. But in the Rhombus Trigonometry, there are 32 principal functions and each function has its own characteristics. These functions give a new vision of the world and will be used in all scientific domains and make a new challenge in the reconstruction of the science especially when working on the economical side of the power of electrical circuits, the electrical transmission, the signal theory and many other domains [15],[18].

The functions $Cjes(\alpha), Cmar(\alpha), Cter(\alpha)$ and $Cjes_y(\alpha)$, which are respectively equivalent to cosine, sine, tangent and cotangent. These functions are particular cases of the "*Circular Trigonometry*". The names of the cosine, sine, tangent and cotangent are replaced respectively by *Circular Jes*, *Circular Mar*, *Circular Ter* and *Circular Jes-y*.

$Cjes(\alpha) \Leftrightarrow cos(\alpha)$; $Cmar(\alpha) \Leftrightarrow sin(\alpha)$

$Cter(\alpha) \Leftrightarrow tan(\alpha)$; $Cjes_y(\alpha) \Leftrightarrow cotan(\alpha)$.

The Rhombus Trigonometric functions are denoted using the following abbreviation "$Rhfun(\alpha)$":

-the first two letters "Rh" are related to the Rhombus trigonometry.

-the word "$fun(\alpha)$" represents the specific function name that is defined hereafter: (refer to Figure 4).

• **Rhombus *Jes* functions:**

-$Rhjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe}$ (8)

-$Rhjes_x(\alpha) = \frac{od_x}{oe_x} = \frac{Rhjes(\alpha)}{Cjes(\alpha)}$ (9)

-$Rhjes_y(\alpha) = \frac{od_x}{oe_y} = \frac{Rhjes(\alpha)}{Cmar(\alpha)}$ (10)

• **Rhombus *Mar* functions:**

-$Rhmar(\alpha) = \frac{od_y}{ob} = \frac{od_y}{oc}$ (11)

-$Rhmar_x(\alpha) = \frac{od_y}{oc_x} = \frac{Rhmar(\alpha)}{Cjes(\alpha)}$ (12)

-$Rhmar_y(\alpha) = \frac{od_y}{oc_y} = \frac{Rhmar(\alpha)}{Cmar(\alpha)}$ (13)

• **Rhombus *Ter* functions:**

-$Rhter(\alpha) = \frac{Emar(\alpha)}{Ejes(\alpha)}$ (14)

-$Rhter_x(\alpha) = \frac{Rhmar_x(\alpha)}{Rhjes_y(\alpha)} = Rhter(\alpha) \cdot Cter(\alpha)$(15)

-$Rhter_y(\alpha) = \frac{Rhmar_y(\alpha)}{Rhjes_x(\alpha)} = \frac{Rhter(\alpha)}{Cter(\alpha)}$ (16)

• **Rhombus *Rit* functions:**

-$Rhrit(\alpha) = \frac{od_x}{ob} = \frac{od_x}{oc} = \frac{Rhmar(\alpha)}{Cter(\alpha)}$ (17)

-$Rhrit_y(\alpha) = \frac{od_x}{oc_y} = \frac{Rhrit(\alpha)}{Cmar(\alpha)}$ (18)

• **Rhombus *Raf* functions:**

-$Rhraf(\alpha) = \frac{od_y}{oa} = Cter(\alpha).Rhjes(\alpha)$ (19)

-$Rhraf_x(\alpha) = \frac{od_y}{oe_x} = \frac{Rhraf(\alpha)}{Cjes(\alpha)}$ (20)

• **Rhombus *Ber* functions:**

-$Rhber(\alpha) = \frac{Rhraf(\alpha)}{Rhrit(\alpha)}$ (21)

-$Rhber_x(\alpha) = \frac{Rhraf_x(\alpha)}{Rhrit_y(\alpha)} = Rhber(\alpha) \cdot Cter(\alpha)$(22)

-$Rhber_y(\alpha) = \frac{Rhraf_y(\alpha)}{Rhrit_x(\alpha)} = \frac{Rhber(\alpha)}{Cter(\alpha)}$ (23)

3.4 The reciprocal of the Rhombus Trigonometric function

$Rhfun^{-1}(\alpha)$ is defined as the inverse function of $Rhfun(\alpha)$. $(Rhfun^{-1}(\alpha) = 1/Rhfun(\alpha))$. In this way the reduced number of functions is equal to 32 principal functions.

E.g.: $Rhjes^{-1}(\alpha) = \frac{1}{Rhjes(\alpha)}$

3.5 Definition of the Absolute Rhombus Trigonometric functions $\overline{Rh}fun(\alpha)$

The *Absolute Rhombus Trigonometry* is introduced to create the absolute value of a function by varying only one parameter without using the absolute value "| |". The advantage is that we can change and control the sign of a Rhombus Trigonometric function without using the absolute value in an expression. Some functions are treated to get an idea about the importance of this new definition. To obtain the Absolute Rhombus Trigonometry for a specified function (e.g.: $Rhjes(\alpha)$) we must multiply it by the corresponding Angular Function (e.g.: $(ang_x(\alpha))^i$) in a way to obtain the original function if i is even, and to obtain the absolute value of the function if i is odd (e.g.: $|Rhjes(\alpha)|$).

If the function doesn't have a negative part (not alternative), we multiply it by $(ang_x(\beta(\alpha-\gamma))^i$ to obtain an alternating signal which form depends on the value of the frequency "β" and the translation value "γ". By varying the last parameters, one can get a multi form signals using only one function.

• $\overline{Rh}jes_i(\alpha) = (ang_x(\alpha))^i \cdot Rhjes(\alpha)$ (24)

$$= \begin{cases} \left(ang_x(\alpha)\right)^1 \cdot Rhjes(\alpha) = |Rhjes(\alpha)| & if\ i = 1 \\ \left(ang_x(\alpha)\right)^2 \cdot Rhjes(\alpha) = Rhjes(\alpha) & if\ i = 2 \end{cases}$$

• $\overline{Rh}jes_{i,x}(\alpha) = \left(ang_x(\alpha-\gamma)\right)^i \cdot Rhjes_x(\alpha)$ (25)

$$= \begin{cases} ang_x(\alpha-\gamma) \cdot Rhjes_x(\alpha) & if\ i = 1 \\ Rhjes_x(\alpha) & if\ i = 2 \end{cases}$$

• $\overline{Rh}jes_{i,y}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Rhjes_y(\alpha)$ (26)

$$= \begin{cases} ang_y(2\alpha) \cdot Rhjes_y(\alpha) = |Rhjes_y(\alpha)| & if\ i = 1 \\ Rhjes_y(\alpha) & if\ i = 2 \end{cases}$$

• $\overline{Rh}mar_i(\alpha) = \left(ang_y(\alpha)\right)^i \cdot Rhmar(\alpha)$ (27)

• $\overline{Rh}mar_{i,x}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Rhmar_x(\alpha)$ (28)

• $\overline{Rh}mar_{i,y}(\alpha) = \left(ang_x(\alpha-\gamma)\right)^i \cdot Rhmar_y(\alpha)$(29)

• $\overline{Rh}rit_i(\alpha) = \left(ang_x(\alpha)\right)^i \cdot Rhrit(\alpha)$ (30)

And so on…

4 A survey on the Rhombus Trigonometric functions

As previous sections, a brief study on the Rhombus Trigonometry is given. Four functions of 32 are treated with examples to show multi form signals made using the characteristic of this trigonometry.

For this study the following conditions are taken:

- $a = 1$
- $b > 0$ is the height of the Rhombus from the center.

4.1 Determination of the Rhombus *Jes* function

The Rhombus form in the figure 4 is written as the equations (4-7). Thus, we can divide the study into four regions in order to form the formula of the Rhombus *Jes* as following:

• First region for y_1

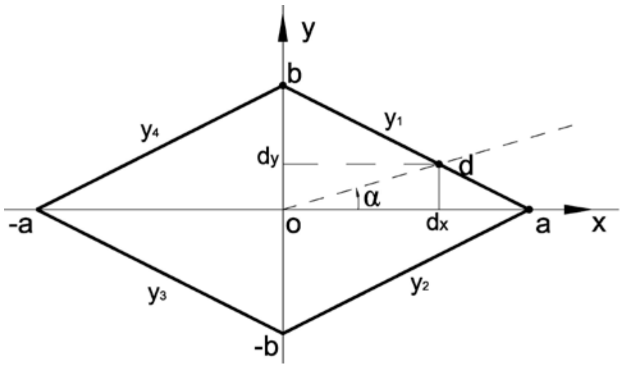


Fig. 6: represents the first region with y_1 that form the Rhombus.

We have the equation (8)

$$Rhjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe} \quad (8)$$

And we know from the figure 6 the following equalities:

$$\frac{b}{a} = \frac{od_y}{a - od_x} \quad (31)$$

$$\tan(\alpha) = \frac{od_y}{od_x} \quad (32)$$

By replacing the equation (32) in the equation (31) we obtain:

$$\frac{b}{a} = \frac{\tan(\alpha) \cdot od_x}{a - od_x}$$

$$\Rightarrow Rhjes(\alpha) = \frac{od_x}{a} = \frac{b}{a \cdot \tan(\alpha) + b} \quad (33)$$

• Second region for y_2

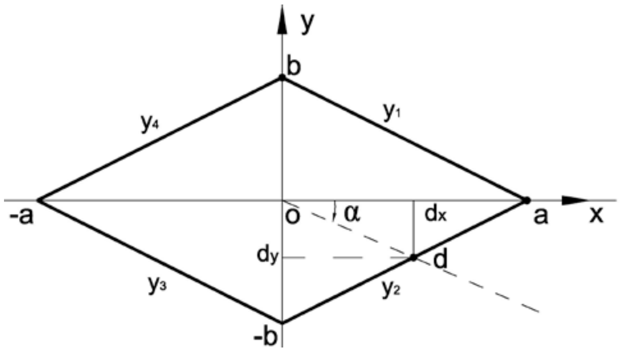


Fig. 7: represents the second region with y_2 that form the Rhombus.

We have the equation (8)

$$Rhjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe} \quad (8)$$

And we know from the figure 7 the following equalities:

$$\frac{-b}{a} = \frac{od_y}{a - od_x} \quad (34)$$

$$\tan(\alpha) = \frac{od_y}{od_x} \quad (35)$$

By replacing the equation (35) in the equation (34) we obtain:

$$\frac{-b}{a} = \frac{\tan(\alpha)\cdot od_x}{a - od_x}$$

$$\Rightarrow Rhjes(\alpha) = \frac{od_x}{a} = \frac{-b}{a\cdot\tan(\alpha) - b} \quad (36)$$

• Third region for y_3

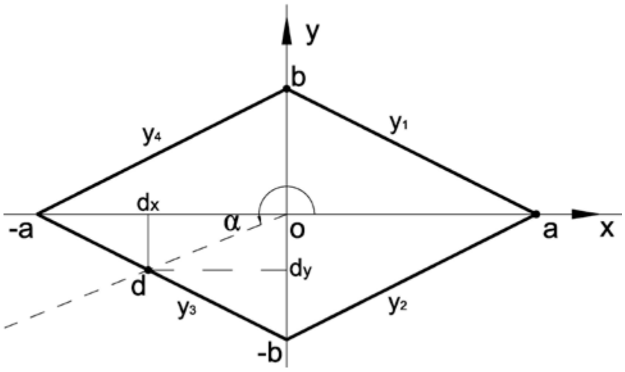


Fig. 8: represents the third region with y_3 that form the Rhombus.

We have the equation (8)

$$Rhjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe} \quad (8)$$

And we know from the figure 8 the following equalities:

$$\frac{-b}{-a} = \frac{od_y}{-a - od_x} \quad (37)$$

$$\tan(\alpha) = \frac{od_y}{od_x} \quad (38)$$

By replacing the equation (38) in the equation (37) we obtain:

$$\frac{-b}{-a} = \frac{\tan(\alpha)\cdot od_x}{-a - od_x}$$

$$\Rightarrow Rhjes(\alpha) = \frac{od_x}{a} = \frac{-b}{a\cdot\tan(\alpha) + b} \quad (39)$$

• Fourth region for y_4

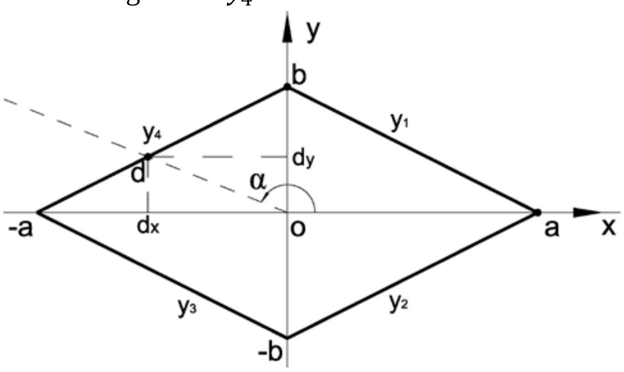


Fig. 9: represents the fourth region with y_4 that form the Rhombus.

We have the equation (8)

$$Rhjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe} \quad (8)$$

And we know from the figure 9 the following equalities:

$$\frac{b}{-a} = \frac{od_y}{-a - od_x} \quad (40)$$

$$\tan(\alpha) = \frac{od_y}{od_x} \quad (41)$$

By replacing the equation (41) in the equation (40) we obtain:

$$\frac{b}{-a} = \frac{\tan(\alpha)\cdot od_x}{-a - od_x}$$

$$\Rightarrow Rhjes(\alpha) = \frac{od_x}{a} = \frac{b}{a\cdot\tan(\alpha) - b} \quad (42)$$

So we obtain four equations for the Rh-Jes and we want to unify these equations into only one equation using the angular functions as following:

$$Rhjes(\alpha) = \frac{od_x}{a} = \frac{b}{a\cdot\tan(\alpha) + b} \quad (33)$$

$$Rhjes(\alpha) = \frac{od_x}{a} = \frac{-b}{a\cdot\tan(\alpha) - b} \quad (36)$$

$$Rhjes(\alpha) = \frac{od_x}{a} = \frac{-b}{a\cdot\tan(\alpha) + b} \quad (39)$$

$$Rhjes(\alpha) = \frac{od_x}{a} = \frac{b}{a\cdot\tan(\alpha) - b} \quad (42)$$

Therefore the final equation is written as below:

$$\Rightarrow Rhjes(\alpha) = \frac{b\cdot ang_y(\alpha)}{a\cdot\tan(\alpha) + ang_y(\alpha)\cdot ang_x(\alpha) b} \quad (43)$$

$$\Rightarrow Rhjes(x) = \frac{b\cdot ang_y(x)}{a\cdot\tan(x) + ang_y(x)\cdot ang_x(x) b} \quad (44)$$

• Expression of the Absolute Rhombus *Jes*:

$$\overline{Rhjes}_{i,b}(x) = \frac{b\cdot ang_y(x)}{a\cdot\tan(x) + ang_y(x)\cdot ang_x(x) b} \cdot \left(ang_x(x)\right)^i \quad (45)$$

The Absolute Rhombus *Jes* is a powerful function that can produce more than 14 different signals by varying only two parameters i and b.

• Multi form signals made by $\overline{Rhjes}_{i,b}(x)$:

Figures 10 and 11 represent multi form signals obtained by varying two parameters (i and b). For the figures 10.a to 10.f the value of $i = 2$, for the figures 11.a to 11.f the value of $i = 1$.

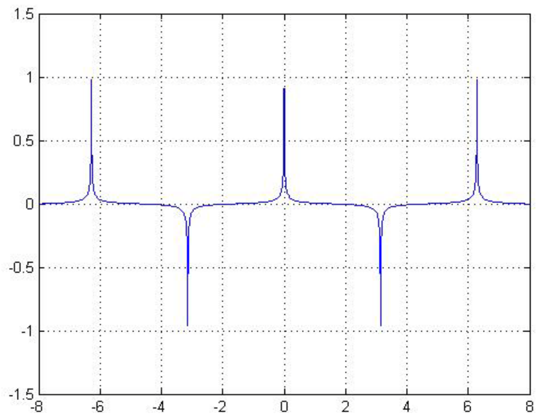


Fig 10.a: $i = 2, b = 0.01$

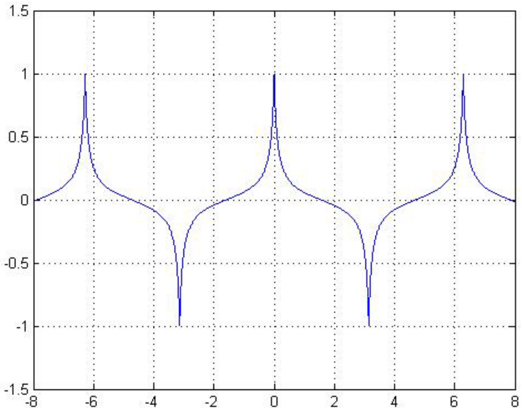


Fig 10.b: $i = 2, b = 0.1$

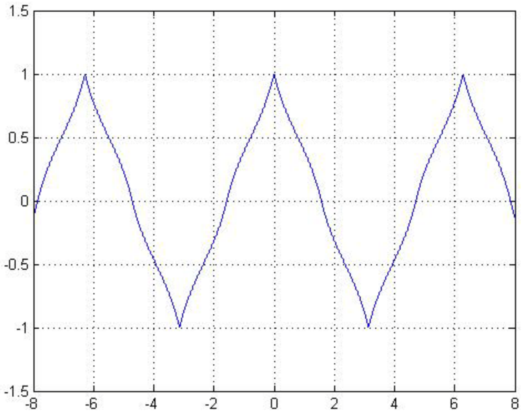


Fig 10.c: $i = 2, b = 1$

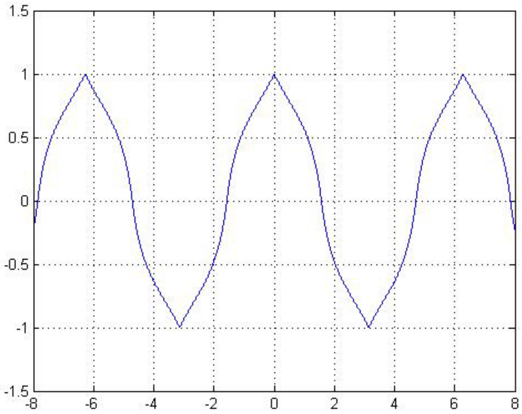


Fig 10.d: $i = 2, b = 2$

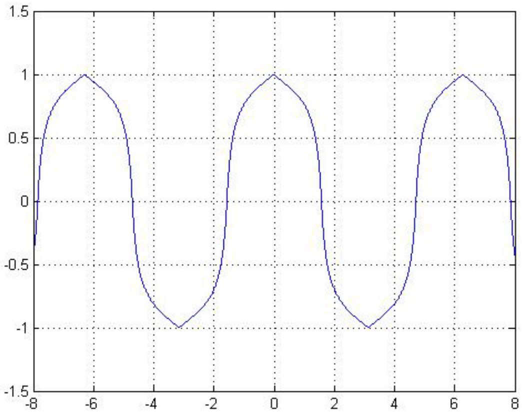


Fig 10.e: $i = 2, b = 5$

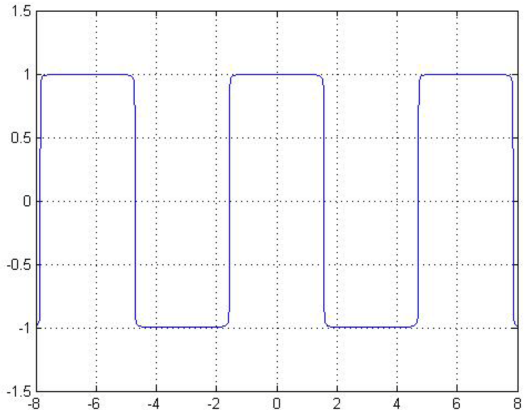


Fig 10.f: $i = 2, b = 500$

Fig. 10.a to 10.f: multi-form signals of the function $\overline{Rh}jes_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.

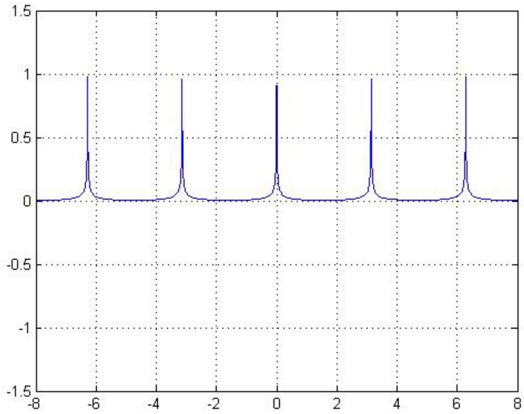


Fig 11.a: $i = 1, b = 0.01$

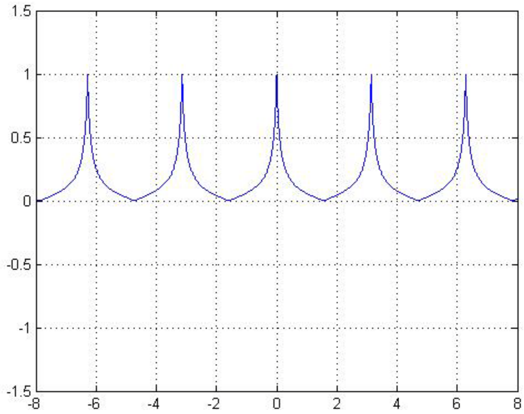


Fig 11.b: $i = 1, b = 0.1$

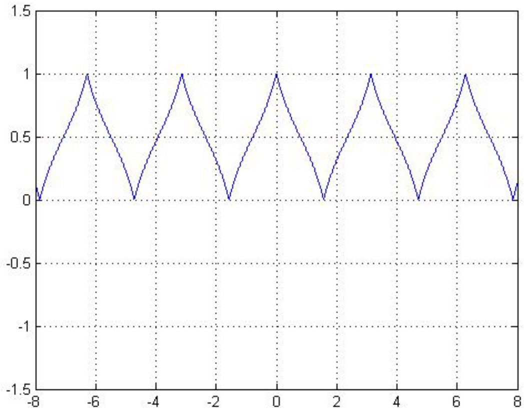


Fig 11.c: $i = 1, b = 1$

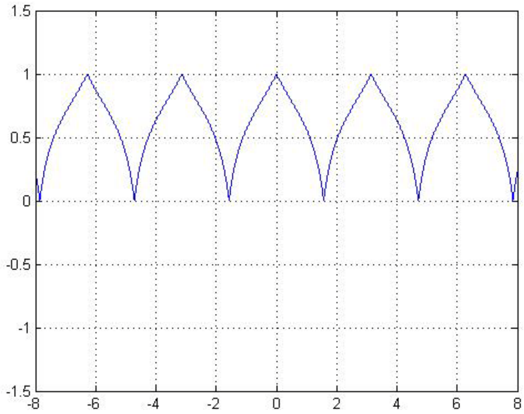


Fig 11.d: $i = 1, b = 2$

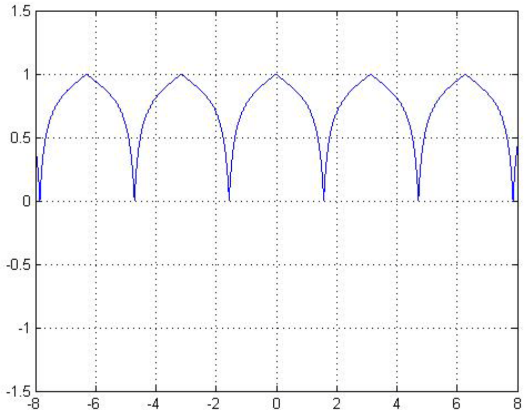


Fig 11.e: $i = 1, b = 5$

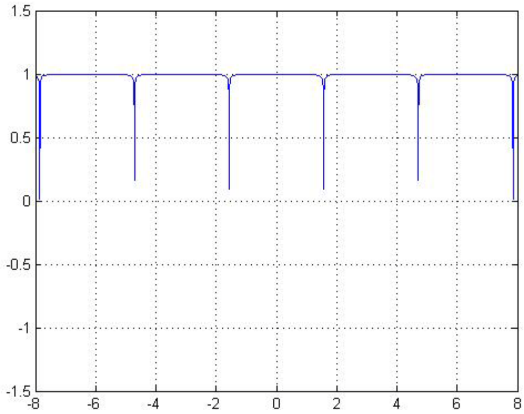


Fig 11.f: $i = 1, b = 500$

Fig. 11a to 11.f: multi-form signals of the function $\overline{Rhjes}_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Impulse train with positive and negative part, Rhombus deflated, quasi-triangular, quasi-sinusoidal, Rhombus swollen, square signal, rectangular signal, impulse train (positive part only), rectified Rhombus deflated, saw signal, rectified Rhombus swollen, continuous signal…

These types of signals are widely used in power electronics, electrical generator and in transmission of analog signals [18].

• **Example using the $Rhjes_b(x)$:**

In this part, an example is treated in a goal to form many shapes obtained by varying only one parameter. For this, consider the following equation:

$$y^2 = \left(k.rect_T(x - x_0).Rhjes_b(\alpha x)\right)^2 \quad (46)$$

With k is the amplitude of the signal,
'x' is a variable parameter,
α is the frequency of the $Ejes_b(\alpha x)$,
k, α and T are positive values(> 0).

In fact, the rectangular function $rect_T(x - x_0)$ which is illustrated in figure 12 is defined as

$$rect_T(x - x_0) = \begin{cases} 1 \ \ for \ \ x_0 - \frac{T}{2} \leq x \leq x_0 + \frac{T}{2} \\ 0 \qquad otherwise \end{cases} \quad (47)$$

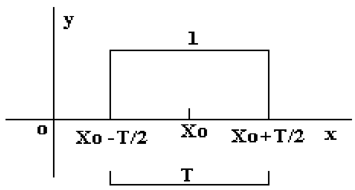


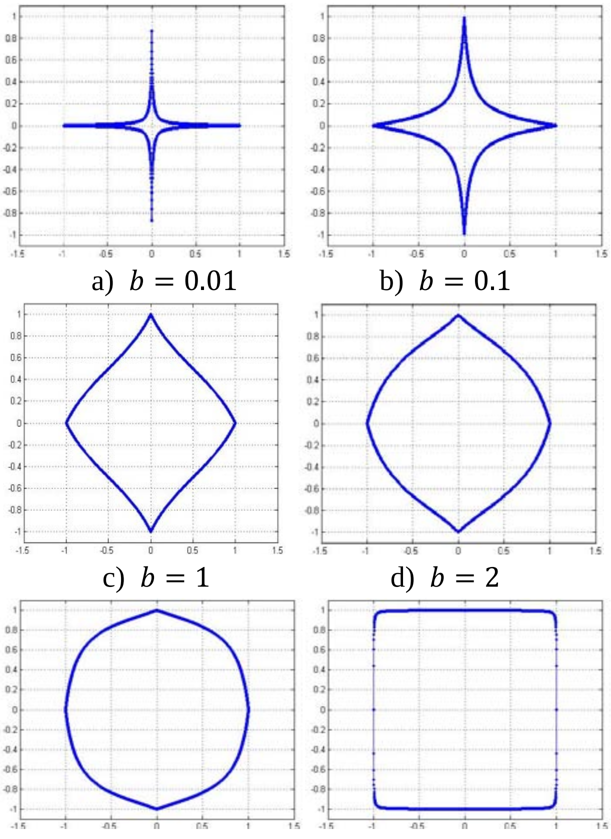
Fig. 12: $rect_T(x - x_0)$ wave form

Then $y = \pm\left(k.rect_T(x - x_0).Rhjes_b(\alpha x)\right)$ (48)

The period of $Rhjes_b(\alpha x)$ is equal to $T_1 = 2\pi/\alpha$

For a particular case, let's take $T = 2; \alpha = \pi/2$; $x_0 = 0$ and $k = 1$.

By varying the parameter b in (48), the following remarkable shapes are illustrated in figures 13.a to 13.f.



a) $b = 0.01$ b) $b = 0.1$ c) $b = 1$ d) $b = 2$ e) $b = 5$ f) $b = 500$

Fig. 13: different shapes of the function (48) for different values of b (presented in blue color).

Important shapes obtained respectively using this expression:

Plus form, star form, Quasi-rhombus, eye form, quasi-circular form, square form and rectangle form. More forms can be obtained by varying more than one parameter.

4.2 The Rhombus *Jes-x* function

The Rhombus Jes-x form can be obtained from equations (9) and equation (44)

$$Rhjes_x(\alpha) = \frac{od_x}{oe_x} = \frac{Rhjes(\alpha)}{Cjes(\alpha)}$$

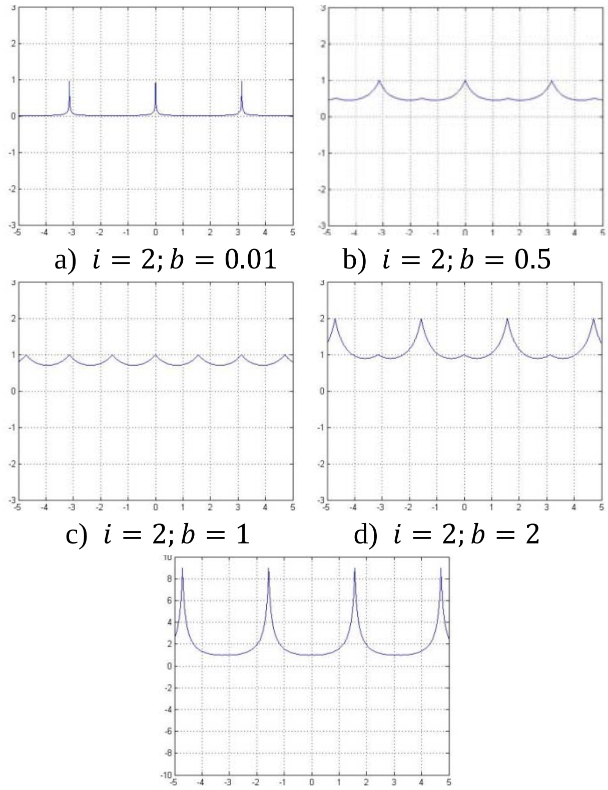
$$\Rightarrow Rhjes_{x_b}(x) = \frac{b \cdot ang_y(x)}{Cjes(\alpha)(a \cdot \tan(x) + ang_y(x) \cdot ang_x(x) b)} \quad (49)$$

• Expression of the Absolute Rhombus *Jes-x*

$$\overline{Rhjes}_{x_b^i}(x) = Rhjes_{x_b}(x) \cdot \left(ang_x(x-\gamma)\right)^i \quad (50)$$

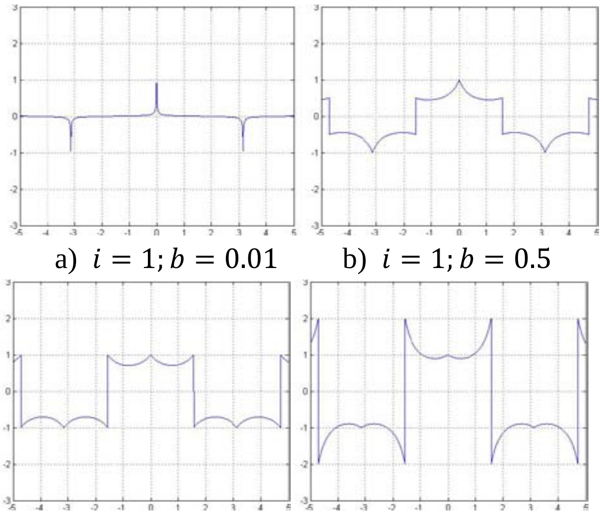
• Multi-form signals made by $\overline{Rhjes}_{x_b^i}(x)$:

Taking $\gamma = 0$ for this example, figures 14 and 15 represent multi form signals obtained by varying two parameters (i and b). For the figures 14.a to 14.e the value of $i = 2$, for the figures 15.a to 15.f the value of $i = 1$.

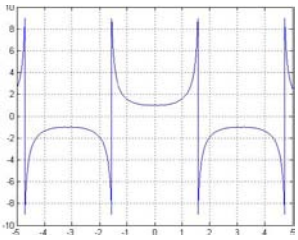


a) $i = 2; b = 0.01$ b) $i = 2; b = 0.5$ c) $i = 2; b = 1$ d) $i = 2; b = 2$ e) $i = 2; b = 9$

Fig. 14: multi-form signals of the function $\overline{Rhjes}_{x_b^i}(x)$ for $i = 2$ and for different values of $b > 0$.



a) $i = 1; b = 0.01$ b) $i = 1; b = 0.5$ c) $i = 1; b = 1$ d) $i = 1; b = 2$



e) $i = 2; b = 9$

Fig.15: multi-form signals of the function $\overline{Rh}jes_{x_b^i}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Impulse train with positive part only, sea waves, amplified sea waves, impulse train with positive and negative part, U signal …

4.3 The Rhombus *Mar* function

The study of the Rhombus *Mar* function is similar to the Rhombus *Jes* function in the previous sub-section 4.1. The formed equations for the Rhombus *Mar* function are as following:

$$- Rhmar(\alpha) = \frac{od_y}{ob} = \frac{a \cdot tan(\alpha)}{b + a \cdot tan(\alpha)} \quad (51)$$

$$- Rhmar(\alpha) = \frac{-a \cdot tan(\alpha)}{-b + a \cdot tan(\alpha)} \quad (52)$$

$$- Rhmar(\alpha) = \frac{-a \cdot tan(\alpha)}{b + a \cdot tan(\alpha)} \quad (53)$$

$$- Rhmar(\alpha) = \frac{a \cdot tan(\alpha)}{-b + a \cdot tan(\alpha)} \quad (54)$$

Therefore the final equation is written as below:

$$Rhmar_b(\alpha) = \frac{a \cdot tan(\alpha) \cdot ang_y(x)}{ang_x(x) ang_y(x) b + a \cdot tan(\alpha)} \quad (55)$$

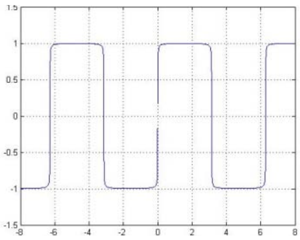
• Expression of the Absolute Rhombus *Mar*:

$$\overline{Rh}mar_{i,b}(x) = Rhmar_b(x) \cdot \left(ang_y(x)\right)^i \quad (56)$$

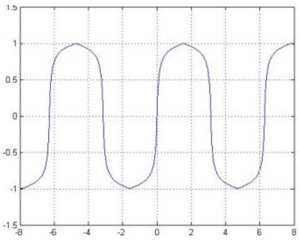
Similar to the Absolute Rhombus *Jes*, the Absolute Rhombus *Mar* is a powerful function that can produce more than 14 different signals by varying only two parameters i and b.

• Multi-form signals made by $\overline{Rh}mar_{i,b}(x)$:

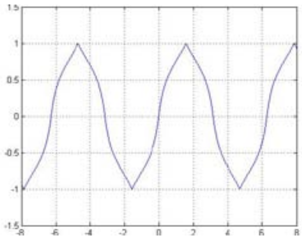
Figures 16 and 17 represent multi form signals obtained by varying two parameters (i and b). For the figures 16.a to 16.f the value of $i = 2$, for the figures 17.a to 17.f the value of $i = 1$.



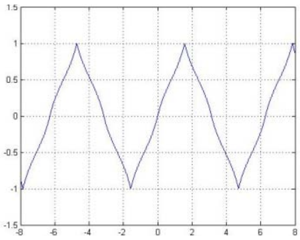
a) $i = 2; b = 0.005$



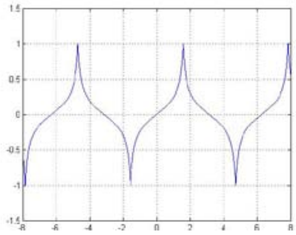
b) $i = 2; b = 0.1$



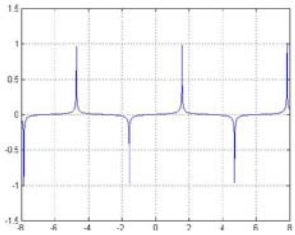
c) $i = 2; b = 0.5$



d) $i = 2; b = 1$

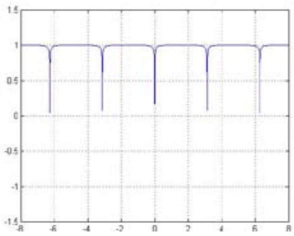


e) $i = 2; b = 5$

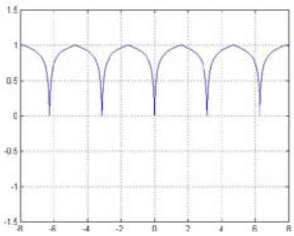


f) $i = 2; b = 100$

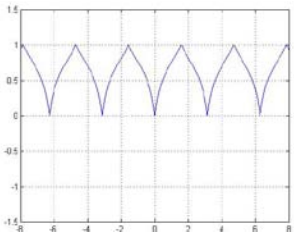
Fig. 16.a to 16.f: multi-form signals of the function $\overline{Rh}mar_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



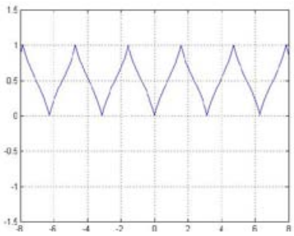
a) $i = 1; b = 0.005$



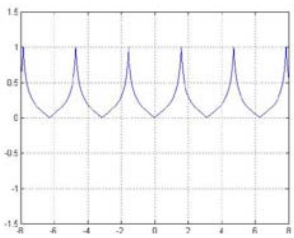
b) $i = 1; b = 0.1$



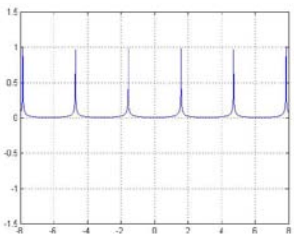
c) $i = 1; b = 0.5$



d) $i = 1; b = 1$



e) $i = 1; b = 5$



f) $i = 1; b = 100$

Fig. 17.a to 17.f: multi-form signals of the function $\overline{Rhmar}_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Impulse train with positive and negative part, Rhombus deflated, quasi-triangular, quasi-sinusoidal, Rhombus swollen, square signal, rectangular signal, impulse train (positive part only), rectified Rhombus deflated, saw signal, rectified Rhombus swollen, continuous signal…
These types of signals are widely used in power electronics, electrical generator and in transmission of analog signals [18].

4.4 The Rhombus *Rit* function

The Rhombus *Rit* function can be obtained from the equations (17) and (55). In fact:

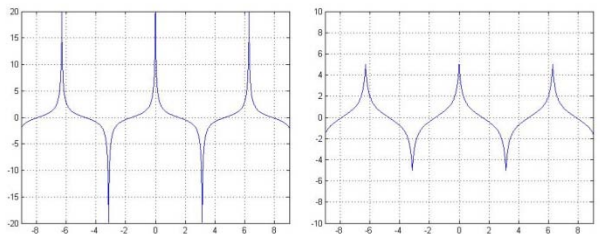
$$Rhrit_b(x) = \frac{od_x}{ob} = \frac{od_x}{oc} = \frac{Rhmar(x)}{\text{Cter}\ (x)} \quad (17)$$

$$\Rightarrow Rhrit_b(x) = \frac{a\cdot tan(\alpha)\cdot ang_y(x)}{\text{Cter}(x)(ang_x(x)ang_y(x)b + a\cdot tan(\alpha))} \quad (57)$$

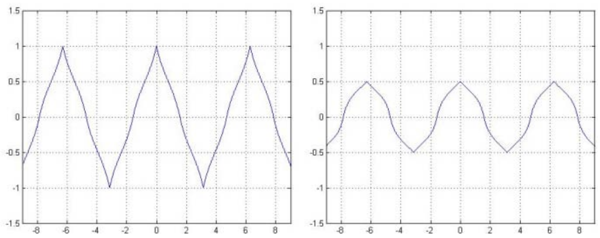
• Expression of the Absolute Rhombus *Rit*:

$$\overline{Rhrit}_{i,b}(x) = Rhrit_b(x) \cdot \left(ang_x(x)\right)^i \quad (58)$$

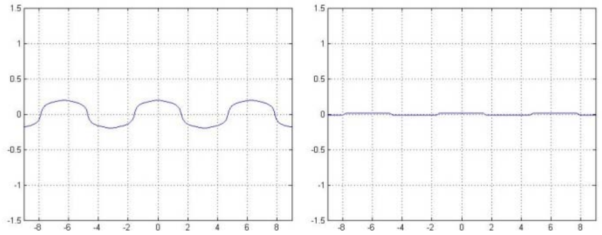
• Multi form signals made by $\overline{Rhrit}_{i,b}(x)$:
Figures 18 and 19 represent multi form signals obtained by varying two parameters (i and b). For the figures 18.a to 18.e the value of $i = 2$, for the figures 19.a to 19.e the value of $i = 1$.



a) $i = 2; b = 0.05$ b) $i = 2; b = 0.2$

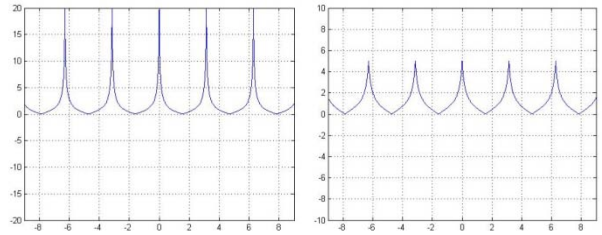


c) $i = 2; b = 1$ d) $i = 2; b = 2$

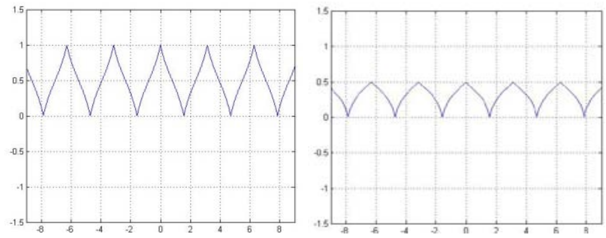


e) $i = 2; b = 5$ f) $i = 2; b = 100$

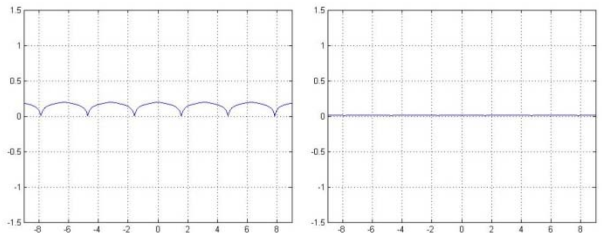
Fig. 18.a to 18.f: multi-form signals of the function $\overline{Rhrit}_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



a) $i = 1; b = 0.05$ b) $i = 1; b = 0.2$



c) $i = 1; b = 1$ d) $i = 2; b = 2$



e) $i = 1; b = 5$ f) $i = 1; b = 100$

Fig. 19.a to 19.f: multi-form signals of the function $\overline{Rhrit}_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Rhombus swollen compressed, quasi-sinusoidal, quasi-triangular, Rhombus deflated amplified, impulse train (positive and negative part) with controlled amplitude, impulse train (positive part only) with controlled amplitude, saw signal, continuous signal, rectified quasi-sinusoidal …

5 Survey on the application of the Rhombus trigonometry in engineering domain

As we saw in the previous sections, the main goal of the Rhombus Trigonometry is to produce a huge number of multi-form signals using a single function and by varying some parameters of this function. For this reason, one can imagine the importance of this trigonometry in all domains especially in telecommunication, signal theory, signal processing, electrical and electronic engineering.

Particularly in electrical engineering: motor drives, robotics, and other electronic applications need many controlled circuits that produce different type of signals. The purpose is to increase the efficiency, by using this trigonometry. One can develop a circuit that produces multiple forms of signals that respond to requirements.

The functions of this trigonometry are easily programmed and simulated with software as Matlab and Labview, and many circuits can be formed to describe these functions.

For a particular case, the Rhombus *Mar* function can be obtained as an output signal by using a simple circuit as shown in the figure 17.

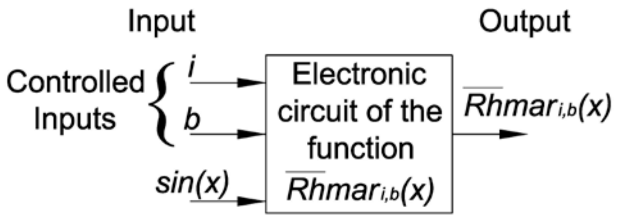


Fig. 20: Electronic circuit with its inputs and output.

Thus, this simple circuit can produce more than 14 different signals by varying only the two parameters (i and b). These output signals have an extreme importance in the power electronics in which the regulation of electronic systems will become simpler and more efficient. In another hand circuits will be more reduced and costless.

6 Conclusion

In this paper, an original study in trigonometry is introduced. The Rhombus unit and its trigonometric functions are presented and analyzed. In fact the proposed Rhombus Trigonometry is a new form of trigonometry that permits to produce multiple forms of signals by varying some parameters; it can be used in numerous scientific domains and particularly in mathematics and in engineering. For the case treated in this paper, 32 Rhombus trigonometric functions are defined; only four functions are simulated using software as Matlab with a brief study. For each Rhombus function, many periodic signals are produced by varying some parameters.

The Rhombus trigonometric functions will be widely used in electronic domain especially in power electronics. Thus, several studies will be improved and developed after introducing the new functions of this trigonometry. Some mathematical expressions and electronic circuits will be replaced by simplified expressions and reduced circuits.

In order to illustrate the importance of this trigonometry, some functions are defined briefly and some examples are treated.

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