Robust mixed H_2/H_∞ output tracking control of uncertain discrete-time switched systems with state time-delay

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Abstract: This paper focuses on the problem of mixed H_2/H_∞ output tracking control for uncertain discrete-time switched systems with state time-delay. By using single Lyapunov function theory, a state feedback controller is presented to guarantee the closed-loop error system robust asymptoptically stable with mixed H_2/H_∞ performance is developed. The controller gains are obtained by a set of linear matrix inequalities(LMIs). The corresponding stabilizing switching rule is provided. A numerical example is given to demonstrate the effectiveness of the proposed approach.

Key–Words: discrete-time switched systems, time-delay systems, tacking control, mixed H_2/H_∞control, single Lyapunov function.

1 Introduction

Switched systems are special kinds of hybrid systems, which consist of some subsystems and a switching rule, it determines which system is activated at certain time interval [1, 2]. Many real-world processes and systems can be modeled as switched system, such as automotive engine control system, chemical process, computer controlled systems [2], and so on. In the last decades, many results have been reported on stability analysis and controller design for switched systems [2, 3, 4, 5, 6, 7, 8], which adopt methods including convex common Lyapunov function [2], switched Lyapunov function [5], dwell time and average dwell time [6], single Lyapunov function [2], multiple Lyapunov function [4], and so on.

For many practical control systems, uncertainties and time-delay are unavoidable and they often influence the control performance of the closed-loop system. Hence, there have been steadily growing interests in the stabilization and performance analysis of switched systems with delays [9, 10, 11, 12, 13, 14, 15, 16].

In order to obtain better control performance, some performance indexes should be considered in the controller design. For example, H_∞ disturbance attenuation property was discussed via linear matrix inequality(LMI) and eliminate element method in [17]. In [18], guaranteed cost control was studied based on the Lyapunov theory together with the LMI approach. In [19], the author considered the problem of observer-based mixed H_2/H_∞ controller design for linear systems with time-varying state, input and output delays via a convex optimization method. Several other papers, such as [20, 21, 22], were also devoted to studying H_∞ control or guaranteed cost control related problems for some kind of hybrid systems.

Output tracking control is an important topic in control fields, which has wide range of applications in dynamic processes, economics, biology and many other practical fields. The basic aim of output tracking control is to design a controller to reduce the tracking error between the system output and the reference model output [23]. The problem of robust output tracking control for an uncertain system with multiple delays has been studied in [24], where the reference signal is chosen as a known constant vector. The problem of H_2 output tracking control for wireless networked discrete-time systems has been considered in [25], where the reference signal is presented by external model.

Along with development of switched system theory, the tracking control problems for the switched systems have received more and more attention. In addition, switched control methods have been widely applied to flight control system [26, 27]. These applications make the research of tracking control for switched system more and more important. In [28, 29, 30], the authors have studied the tracking control problem for kinds of continuous switched system.

Up to date, few results are paid attention to the problem of multi-object tracking control for discrete-time switched systems.

In this paper, the problem of mixed H_2/H_∞ output tracking control for uncertain discrete-time switched systems with state time-delay is developed. By resorting to single Lyapunov function approach, a feasible condition for the problem of mixed H_2/H_∞ tracking control is presented in terms of LMI form. A numerical example is provided to demonstrate the effectiveness of the main result.

The remainder of this paper is organized as follows. In section 2, the problem and preliminaries are formulated. Section 3 states the main results. Section 4 presents a numerical example to demonstrate main results in section 3. Finally, the paper is concluded in section 5.

Notation. A symmetric matrix $P > 0$ denotes P being a positive definite matrix. I is used to denote an identity matrix with appropriate dimensions. l_2 refers to the space of square summable function on $[0, \infty)$ and $\|\omega\|_2 = (\sum_{k=0}^{\infty} \omega^T(k)\omega(k))^{1/2}$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2 Problem formulation and Preliminaries

Consider the following uncertain discrete switched system with state time-delay

$$\begin{aligned} x(k+1) &= (A_\sigma + \Delta A_\sigma(k))x(k) \\ &\quad +(A_{d\sigma} + \Delta A_{d\sigma})x(k-d) \\ &\quad +(B_\sigma + \Delta B_\sigma(k))u(k) \\ &\quad +(B_{1\sigma} + \Delta B_{1\sigma})\omega(k), \\ z(k) &= C_\sigma x(k) + F_\sigma u(k), \\ x(s) &= \phi(s),\ s \in [-d, 0]. \end{aligned} \quad (1)$$

where $x(k) \in R^n$ is the state, $u(k) \in R^m$ are the control inputs, $w(k) \in R^q$ is the disturbance input which belongs to l_2. $\phi(s)$ is the initial condition, $d > 0$ is fixed constant time-delay. $\sigma(k) \in \{1, 2, \cdots, m\} \triangleq M$ is switching signal, which specifies which subsystem will be activated at a certain discrete time instant. For any $i \in M$, A_i, A_{di}, B_i, B_{1i}, C_i, D_i are known matrices with appropriate dimensions, $\Delta A_i(k)$, $\Delta A_{di}(k)$, $\Delta B_i(k)$, $\Delta B_{1i}(k)$ are unknown time-varying parameter uncertainties of the form

$$\begin{aligned} &\begin{bmatrix} \Delta A_i(k) & \Delta A_{di}(k) & \Delta B_i(k) & \Delta B_{1i}(k) \end{bmatrix} \\ &= D_i\Gamma(k) \begin{bmatrix} E_{1i} & E_{di} & E_{2i} & E_{3i} \end{bmatrix}, \end{aligned} \quad (2)$$

where $\Gamma(k) \in R^{n\times n}$ is the uncertain matrix satisfying

$$\Gamma^T(k)\Gamma(k) \le I,\ \forall k. \quad (3)$$

D_i, E_{1i}, E_{di}, E_{2i}, E_{3i} are given constant matrices with appropriate dimensions.

Assume that the reference signal is defined by the following system

$$\begin{aligned} x_r(k+1) &= A_r x_r(k) + B_r r(k), \\ z_r(k) &= C_r x_r(k), \end{aligned} \quad (4)$$

where $x_r(k)$ is reference states, $r(k)$ is reference input which belongs to l_2, A_r is Hurwitz matrix with appropriate dimensions, B_r and C_r are constant matrices with appropriate dimensions.

Here we are interesting designing a state feedback controller

$$u(k) = K_{1\sigma}x(k) + K_{2\sigma}x_r(k), \quad (5)$$

where $K_{1\sigma}$ and $K_{2\sigma}$ are controller gains. Substituting (5) into (1), we have

$$\begin{aligned} x(k+1) &= (\tilde{A}_\sigma + \Delta\tilde{A}_\sigma)x(k) \\ &\quad +(B_\sigma K_{2\sigma} + \Delta B_\sigma K_{2\sigma})x_r(k) \\ &\quad +(A_{d\sigma} + \Delta A_{d\sigma})x(k-d) \\ &\quad +(B_{1\sigma} + \Delta B_{1\sigma})\omega(k), \\ z(k) &= \tilde{C}_\sigma x(k) + F_\sigma K_{2\sigma}x_r(k), \\ x(s) &= \phi(s),\quad s \in [-d, 0], \end{aligned} \quad (6)$$

where $\tilde{A}_\sigma = A_\sigma + B_\sigma K_{1\sigma}$, $\Delta\tilde{A}_\sigma = \Delta A_\sigma + \Delta B_\sigma K_{1\sigma}$, $\tilde{C}_\sigma = C_\sigma + F_\sigma K_{1\sigma}$.

Letting $e(k) = z(k) - z_r(k)$, combining (1) and (6), we obtain

$$\begin{aligned} \xi(k+1) &= (\bar{A}_\sigma + \Delta\bar{A}_\sigma)\xi(k) \\ &\quad +(\bar{A}_{d\sigma} + \Delta\bar{A}_{d\sigma})\xi(k-d) \\ &\quad +(\bar{B}_{1\sigma} + \Delta\bar{B}_{1\sigma})\nu(k), \\ e(k) &= \bar{C}_\sigma\xi(k), \end{aligned} \quad (7)$$

where

$$\begin{aligned} \xi(k+1) &= \begin{bmatrix} x(k+1) \\ x_r(k+1) \end{bmatrix}, \\ \bar{A}_\sigma &= \begin{bmatrix} \tilde{A}_\sigma & B_\sigma K_{2\sigma} \\ 0 & A_r \end{bmatrix}, \\ \Delta\bar{A}_\sigma &= \begin{bmatrix} \Delta\tilde{A}_\sigma & \Delta B_\sigma K_{2\sigma} \\ 0 & 0 \end{bmatrix}, \\ \bar{A}_{d\sigma} &= \begin{bmatrix} 0 & A_{d\sigma} \\ 0 & 0 \end{bmatrix}, \\ \Delta\bar{A}_{d\sigma} &= \begin{bmatrix} 0 & \Delta A_{d\sigma} \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\bar{B}_{1\sigma} = \begin{bmatrix} B_{1\sigma} & 0 \\ 0 & B_r \end{bmatrix},$$
$$\Delta\bar{B}_{1\sigma} = \begin{bmatrix} \Delta B_{1\sigma} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\nu(k) = \begin{bmatrix} \omega(k) \\ r(k) \end{bmatrix}, \ \bar{C}_\sigma = [\tilde{C}_\sigma, F_\sigma K_{2\sigma} - C_r].$$

Controller can be expressed as

$$u(k) = \bar{K}_\sigma \xi(k), \bar{K}_\sigma = [K_{1\sigma}, \ \ K_{2\sigma}],$$

Remark 1 *The model-based output tracking control problem is studied in this paper. The reference input $r_(k)$ and the disturbance $\omega(k)$ of system (7) are consisted of a new variable and regarded as a disturbance vector $\nu(k) = [\omega^T(k) \ \ r^T(k)]^T$.*

Definition 2 *[32]. (i) The H_2 performance index of the system (1) is defined as*

$$J_1 = \sum_{k=0}^{\infty}(\xi^T(k)Q\xi(k) + u^T(k)Ru(k)),$$

where $\nu \equiv 0$, constant matrices $Q > 0$, $R > 0$ are given;
(ii) The H_∞ performance index of the system (1) is defined as

$$J_2 = \sum_{k=0}^{\infty}(e^T(k)e(k) - \gamma^2\nu^T(k)\nu(k)),$$

where the scalar $\gamma > 0$ is given;
(iii) The mixed H_2/H_∞ performance index of system (1) is defined as $J_1 \leq J_0$ and $J_2 < 0$.

Definition 3 *Consider the linear switched time-delay system (1). Given a prescribed level of disturbance attenuation $\gamma > 0$ and the admissible uncertainties satisfying (2) and (3), if there exists a mixed H_2/H_∞ controller $u(t)$ of the form (5), a scalar J_0 such that the following three conditions are satisfied:*

1. *the closed-loop error system (7) is robustly asymptotically stable when $\nu(k) \equiv 0$, under arbitrary switching rule;*

2. *the H_2 performance index guarantees $J_1 \leq J_0$, where the positive scalar J_0 is said to be a guaranteed cost;*

3. *under zero initial conditions and for all nonzero $\nu(k) \in l_2$, the H_∞ performance index $J_2 < 0$;*

then, the closed-loop error system (7) is said to be robustly asymptotically stable with a mixed H_2/H_∞ performance index.

To this end, we first introduce the following well-known lemmas.

Lemma 4 *(Schur complement)[33]. Given any symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0$, the following conditions are equivalent:*

$$(i) \qquad S_{11} < 0, \quad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0,$$

$$(ii) \qquad S_{22} < 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0.$$

Lemma 5 *[18]. Let A, E, F be matrices if $P > 0$ and constant $\eta > 0$, such that $\eta^{-1} - E^T PE > 0$, then*

$$\begin{aligned} &(A + E\Gamma F)^T P(A + E\Gamma F) \\ &\leq A^T(P^{-1} - \eta EE^T)^{-1}A + \eta^{-1}F^T F \quad (8) \end{aligned}$$

holds for the arbitrary norm-bounded time-varying uncertainly Γ with $\Gamma^T\Gamma \leq I$.

Lemma 6 *Given a $\gamma > 0$, chosen*

$$V(\xi(k)) = \xi^T(k)P\xi(k) + \sum_{l=k-d}^{k-1}\xi^T(l)S\xi(l), \quad (9)$$

if the closed-loop system (7) satisfies the following condition

$$\begin{aligned} &\Delta V(x(k)) + e^T(k)e(k) - \gamma^2\nu^T(k)\nu(k) \\ &+\xi^T(k)Q\xi(k) + u^T(k)Ru(k) < 0, \quad (10) \end{aligned}$$

then the switched systems (1) is said to be robustly asymptotically stable with a mixed H_2/H_∞ performance under any switching.

Proof. (1) assumption $\nu(k) \equiv 0$.

$$\text{(i)} \ \Delta V(\xi(k) \ < \ -e^T(k)e(k) - \xi^T(k)Q\xi(k) \\ -u^T(k)Ru(k) \leq 0,$$

then, the system (1) is robustly asymptotically stable with $\nu(k) \equiv 0$.

$$\begin{aligned} \text{(ii)} \ J_1 &= \sum_{k=0}^{\infty}(\xi^T(k)Q\xi(k) + u^T(k)Ru(k) \\ &< -\sum_{k=0}^{\infty}(\Delta V(\xi(k)) \\ &= \xi_0^T P\xi_0 + \sum_{l=-d}^{-1}\xi^T(l)S\xi(l) = J_0. \end{aligned}$$

(2) with zero-initial condition $\xi(0) = 0$,

$$\begin{aligned} J_2 &= \sum_{k=0}^{\infty}(e^T(k)e(k) - \gamma^2\nu^T(k)\nu(k)) \\ &< -\sum_{k=0}^{\infty}\Delta V(\xi(k)) = \sum_{k=0}^{\infty}(V(k) - V(k+1)) \\ &= \sum_{k=0}^{\infty}V(\xi(0)) = 0, \end{aligned}$$

with these and definition 2, the conclusion is correct.

3 Main Results

In this section, a sufficient condition will be develop to solve the mixed H_2/H_∞ tracking control problem formulated in the previous section.

Theorem 7 *For given constants matrices $Q > 0$, $R > 0$, scalars $d > 0$, $\gamma > 0$, $\eta_i > 0$ and $\alpha_i > 0$, $\sum_{i=1}^{m}\alpha_i = 1$, the closed-loop system (7) is robustly asymptotically stable with a mixed H_2/H_∞ performance index under designing switching rule and controller (5), if there exist matrices $P > 0$, $S > 0$, K_{1i}, and K_{2i}, $i \in M$ such that the following matrix inequality holds*

$$\Psi = \begin{bmatrix} -P & 0 & 0 & I & I & A^T & K^T & C^T & E_1^T \\ * & -S & 0 & 0 & 0 & A_d^T & 0 & 0 & E_d^T \\ * & * & -\gamma^2 I & 0 & 0 & B^T & 0 & 0 & E_3^T \\ * & * & * & -S^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & U & 0 & 0 & 0 \\ * & * & * & * & * & * & -\overline{R}^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -\overline{I} & 0 \\ * & * & * & * & * & * & * & * & -\eta\overline{I} \end{bmatrix} < 0, \quad (11)$$

where

$$\begin{aligned} A^T &= \left[\ \sqrt{\alpha_1}\bar{A}_1^T \ \cdots \ \sqrt{\alpha_m}\bar{A}_m^T\ \right], \\ A_d^T &= \left[\ \sqrt{\alpha_1}\bar{A}_{d1}^T \ \cdots \ \sqrt{\alpha_m}\bar{A}_{dm}^T\ \right], \\ E_1^T &= \left[\ \sqrt{\alpha_1}\bar{E}_{11}^T \ \cdots \ \sqrt{\alpha_m}\bar{E}_{1m}^T\ \right], \\ E_d^T &= \left[\ \sqrt{\alpha_1}\bar{E}_{d1}^T \ \cdots \ \sqrt{\alpha_m}\bar{E}_{dm}^T\ \right], \\ B^T &= \left[\ \sqrt{\alpha_1}\bar{B}_{11}^T \ \cdots \ \sqrt{\alpha_m}\bar{B}_{1m}^T\ \right], \\ \overline{R} &= diag\{\underbrace{R, \cdots, R}_{m}\},\ \overline{P} = diag\{\underbrace{P, \cdots, P}_{m}\}, \\ K^T &= \left[\ \sqrt{\alpha_1}\bar{K}_1^T \ \cdots \ \sqrt{\alpha_m}\bar{K}_m^T\ \right], \\ \sum_{i=1}^{m}\alpha_i &= 1,\ U = -\overline{P}^{-1} + \eta\hat{D}\hat{D}^T, \\ \overline{I} &= diag\{\underbrace{I, \cdots, I}_{m}\},\ \eta = diag\{\eta_1, \cdots, \eta_m\}, \\ C^T &= \left[\ \sqrt{\alpha_1}\bar{C}_1^T \ \cdots \ \sqrt{\alpha_m}\bar{C}_m^T\ \right], \\ E_3^T &= \left[\ \sqrt{\alpha_1}\bar{E}_{31}^T \ \cdots \ \sqrt{\alpha_m}\bar{E}_{3m}^T\ \right], \\ \hat{D} &= diag\{\bar{D}_1, \cdots, \bar{D}_m\}, \\ \bar{D}_i &= \begin{bmatrix} D_i & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{E}_{1i} &= \begin{bmatrix} E_{1i} + E_{2i}K_{1i} & E_{2i}K_{2i} \\ 0 & 0 \end{bmatrix}, \\ \bar{E}_{di} &= \begin{bmatrix} 0 & E_{di} \\ 0 & 0 \end{bmatrix},\ \bar{E}_{3i} = \begin{bmatrix} E_{3i} & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

Moreover, the corresponding cost function value is

$$J_1 < \xi_0^T P\xi_0 + \sum_{l=-d}^{-1}\xi^T(l)S\xi(l). \quad (12)$$

Proof. Substituting (12) into (11) and applying Lemma 4, we obtain

$$\Psi < 0 \Leftrightarrow \sum_{i=1}^{m}\alpha_i\theta_i < 0,$$

where

$$\begin{aligned} \theta_i &= \begin{bmatrix} -P + S + Q & 0 & 0 \\ * & -S & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \\ &+ \begin{bmatrix} \bar{A}_i^T \\ \bar{A}_{di}^T \\ \bar{B}_{1i}^T \end{bmatrix}(P^{-1} - \eta_i\bar{D}_i\bar{D}_i^T)^{-1}\begin{bmatrix} \bar{A}_i^T \\ \bar{A}_{di}^T \\ \bar{B}_{1i}^T \end{bmatrix}^T \\ &+ \begin{bmatrix} \bar{K}_i^T \\ 0 \\ 0 \end{bmatrix} R \begin{bmatrix} \bar{K}_i^T \\ 0 \\ 0 \end{bmatrix}^T \\ &+ \begin{bmatrix} \bar{C}_i^T \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \bar{C}_i^T \\ 0 \\ 0 \end{bmatrix}^T \\ &+ \eta_i^{-1}\begin{bmatrix} \bar{E}_{1i}^T \\ \bar{E}_{di}^T \\ \bar{E}_{3i}^T \end{bmatrix}\begin{bmatrix} \bar{E}_{1i}^T \\ \bar{E}_{di}^T \\ \bar{E}_{3i}^T \end{bmatrix}^T. \end{aligned} \quad (13)$$

Let

$$\Omega_i = \{\bar{\xi}(k)|\bar{\xi}^T(k)\theta_i\bar{\xi}(k) < 0, \ \ i = 1, 2, \cdots, m\},$$

where $\bar{\xi}^T(k) = \begin{bmatrix} \xi^T(k) & \xi^T(k-d) & \nu^T(k) \end{bmatrix}$. Then

$$\bigcup_{i=1}^{m} \Omega_i = R^l \backslash \{0\}.$$

Set

$$\Delta_1 = \Omega_1, \ \ \Delta_i = \Omega_i - \bigcup_{j=1}^{i-1} \Omega_j,$$

obviously

$$\bigcup_{i=1}^{m} \Delta_i = R^l \backslash \{0\}, \ \Delta_i \bigcap \Delta_j = \varnothing, \ i, j \in M, \ i \neq j.$$

Design the following switching rule

$$\sigma(k) = i, \ \ \bar{\xi}(k) \in \Delta_i, \ \ i \in M, \tag{14}$$

then we have

$$\sum_{i=1}^{m} \alpha_i \bar{\xi}^T(k)\theta_i\bar{\xi}(k) < 0,$$

According to the switching rule as (14) and the candidate Lyapunov function as (9), assume $\bar{\xi}(k) \in \Delta_i$, we have

$$\begin{aligned}
& \Delta V(\xi(k)) + e^T e - \gamma^2 \nu^T \nu \\
+ \ & \xi(k)^T Q \xi(k) + u^T R u \\
= \ & \bar{\xi}^T(k) \begin{bmatrix} -P + S + Q & 0 & 0 \\ * & -S & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \bar{\xi}(k) \\
+ \ & \bar{\xi}^T(k) \begin{bmatrix} (\bar{A}_i + \Delta\bar{A}_i)^T \\ (\bar{A}_{di} + \Delta\bar{A}_{di})^T \\ (\bar{B}_{1i} + \Delta\bar{B}_{1i})^T \end{bmatrix} P \qquad (15) \\
\times \ & \begin{bmatrix} (\bar{A}_i + \Delta\bar{A}_i)^T \\ (\bar{A}_{di} + \Delta\bar{A}_{di})^T \\ (\bar{B}_{1i} + \Delta\bar{B}_{1i})^T \end{bmatrix}^T \bar{\xi}(k) \\
+ \ & \bar{\xi}^T(k) \begin{bmatrix} \bar{K}_i^T \\ 0 \\ 0 \end{bmatrix} R \begin{bmatrix} \bar{K}_i^T \\ 0 \\ 0 \end{bmatrix}^T \bar{\xi}(k) \\
+ \ & \bar{\xi}^T(k) \begin{bmatrix} \bar{C}_i^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \bar{C}_i^T \\ 0 \\ 0 \end{bmatrix}^T \bar{\xi}(k). \qquad (16)
\end{aligned}$$

By Lemma 5 and (2), (3), we derive

$$\begin{aligned}
& \begin{bmatrix} (\bar{A}_i + \Delta\bar{A}_i)^T \\ (\bar{A}_{di} + \Delta\bar{A}_{di})^T \\ (\bar{B}_{1i} + \Delta\bar{B}_{1i})^T \end{bmatrix} P \begin{bmatrix} (\bar{A}_i + \Delta\bar{A}_i)^T \\ (\bar{A}_{di} + \Delta\bar{A}_{di})^T \\ (\bar{B}_{1i} + \Delta\bar{B}_{1i})^T \end{bmatrix}^T \\
\leq \ & \begin{bmatrix} \bar{A}_i^T \\ \bar{A}_{di}^T \\ \bar{B}_{1i}^T \end{bmatrix} (P^{-1} - \eta_i \bar{D}_i \bar{D}_i^T)^{-1} \begin{bmatrix} \bar{A}_i^T \\ \bar{A}_{di}^T \\ \bar{B}_{1i}^T \end{bmatrix}^T \qquad (17) \\
& + \eta_i^{-1} \begin{bmatrix} \bar{E}_{1i}^T \\ \bar{E}_{di}^T \\ \bar{E}_{3i}^T \end{bmatrix} \begin{bmatrix} \bar{E}_{1i}^T \\ \bar{E}_{di}^T \\ \bar{E}_{3i}^T \end{bmatrix}^T.
\end{aligned}$$

Combining (15) and (17) give rise to

$$\begin{aligned}
& \Delta V(\xi(k)) + e^T(k)e(k) - \gamma^2 \nu^T(k)\nu(k) \\
+ \ & \xi(k)^T Q \xi(k) + u^T(k) R u(k) \\
\leq \ & \bar{\xi}^T(k)\theta_i\bar{\xi}(k) < 0. \qquad (18)
\end{aligned}$$

Hence, we have

$$\begin{aligned}
& \Delta V(\xi(k)) + e^T(k)e(k) - \gamma^2 \nu^T(k)\nu(k) \\
+ \ & \xi(k)^T Q \xi(k) + u^T(k) R u(k) \\
\leq \ & \xi^T(k)\theta_i\xi(k) < 0, \ \ i \in M. \qquad (19)
\end{aligned}$$

Therefore, on the basis of Lemma 6, under switching rule (14), we obtain the conclusion.

Remark 8 *It is obvious that the condition (11) is not an LMI. In order to solve out tracking controller gains in (5), the condition (11) is converted into an LMI.*

Defining $F = diag\{P^{-1}, S^{-1}, I, I, I, \bar{I}, \bar{I}, \bar{I}, \bar{I}\} = diag\{X, T, I, I, I, \bar{I}, \bar{I}, \bar{I}, \bar{I}\}$, pre-multiplying F^T and post-multiplying F to (11), and letting $W_{1i} = K_{1i}X_1$, $W_{2i} = K_{2i}X_2$, we have the following Theorem.

Theorem 9 *For given constants matrices $Q > 0$, $R > 0$, scalars $d > 0$, $\gamma > 0$, $\eta_i > 0$ and $\alpha_i > 0$, $\sum_{i=1}^{m} \alpha_i = 1$, the closed-loop system (7) is robustly asymptotically stable under designing switching rule and controller (4), if there exist matrices $X > 0$, $T > 0$, W_{1i} and W_{2i}, $i \in M$, such that the following LMI holds*

$$\begin{bmatrix}
-X & 0 & 0 & X & X \\
* & -T & 0 & 0 & 0 \\
* & * & -\gamma^2 I & 0 & 0 \\
* & * & * & -T & 0 \\
* & * & * & * & -Q^{-1} \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & *
\end{bmatrix}$$

$$\begin{bmatrix} \cdots & \hat{A}^T & W^T & \hat{C}^T & \hat{E}_1^T \\ & \hat{A}_d^T & 0 & 0 & \hat{E}_d^T \\ & B^T & 0 & 0 & E_3^T \\ & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & \overline{U} & 0 & 0 & 0 \\ & * & -\overline{R}^{-1} & 0 & 0 \\ & * & * & -\overline{I} & 0 \\ & * & * & * & -\eta\overline{I} \end{bmatrix} < 0, \qquad (20)$$

where

$$\begin{aligned} \hat{A}_i &= \begin{bmatrix} A_iX_1 + B_iW_{1i} & B_iW_{2i} \\ 0 & A_rX_2 \end{bmatrix}, \\ \hat{E}_{1i} &= \begin{bmatrix} E_{1i}X_1 + E_{2i}W_{1i} & E_{2i}W_{2i} \\ 0 & 0 \end{bmatrix}, \\ \hat{A}^T &= \begin{bmatrix} \sqrt{\alpha_1}\hat{A}_1^T & \sqrt{\alpha_2}\hat{A}_2^T & \cdots & \sqrt{\alpha_m}\hat{A}_3^T \end{bmatrix}, \\ \hat{A}_d{}^T &= \begin{bmatrix} \sqrt{\alpha_1}(\bar{A}_{d1}T)^T & \cdots & \sqrt{\alpha_m}(\bar{A}_{dm}T)^T \end{bmatrix}, \\ \hat{E}_1^T &= \begin{bmatrix} \sqrt{\alpha_1}\hat{E}_{11}^T & \cdots & \sqrt{\alpha_m}\hat{E}_{1m}^T \end{bmatrix}, \\ \hat{E}_d{}^T &= \begin{bmatrix} \sqrt{\alpha_1}(\bar{E}_{d1}T)^T & \cdots & \sqrt{\alpha_m}(\bar{E}_{dm}T)^T \end{bmatrix}, \\ W^T &= \begin{bmatrix} \sqrt{\alpha_1}\bar{W}_1^T & \cdots & \sqrt{\alpha_m}\bar{W}_m^T \end{bmatrix}, \\ \bar{W}_i &= [W_{1i} \;\; W_{2i}], \\ \hat{C}_i &= [C_iX_1 + F_iW_{1i} \;\; F_iW_{2i} - C_rX_2], \\ \hat{C}^T &= \begin{bmatrix} \sqrt{\alpha_1}\hat{C}_1^T & \cdots & \sqrt{\alpha_m}\hat{C}_m^T \end{bmatrix}, \end{aligned}$$

$\overline{U} = \overline{X} + \eta\hat{D}\hat{D}^T$, $\overline{X} = diag\{\underbrace{X, \cdots, X}_{m}\}$, B, E_3, $\hat{D}$, $\overline{R}$, $\overline{I}$, α_i, η *definite as (12).*

Moreover, the corresponding cost function value is defined as (12).

4 A numerical example

Now, we provide an example to show the effectiveness of the main result in this paper.
Consider the system (1) with the parameter $m = 2$. The system matrices are given by

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.2 & -1.25 \\ 0.6 & -0.075 \end{bmatrix}, B_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \\ A_{d1} &= \begin{bmatrix} 0.02 & 0 \\ 0.01 & 0.01 \end{bmatrix}, \; B_{11} = \begin{bmatrix} -0.1 \\ 0.05 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -0.01 & -0.5 \\ -0.16 & -0.175 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.06 \end{bmatrix}, \\ A_{d2} &= \begin{bmatrix} 0.01 & -0.01 \\ 0.2 & -0.05 \end{bmatrix}, \; B_{12} = \begin{bmatrix} 0.05 \\ -0.15 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.03 & -0.02 \end{bmatrix}, \; F_1 = \begin{bmatrix} 2 & 1 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} -0.02 & 0 \end{bmatrix}, \; F_2 = \begin{bmatrix} 1 & 2 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \; D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ E_{11} &= \begin{bmatrix} 0.5 & 0.1 \\ 0 & 0.5 \end{bmatrix}, \; E_{d1} = \begin{bmatrix} 0.1 & 0.01 \\ 0 & 0.15 \end{bmatrix}, \\ E_{21} &= \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \; E_{31} = \begin{bmatrix} 0.03 \\ 0.01 \end{bmatrix}, \\ A_r &= \begin{bmatrix} 0.3 & 0 \\ -1 & 0.6 \end{bmatrix}, \; B_r = \begin{bmatrix} 1.5 & 0 \\ 0 & 3.5 \end{bmatrix}, \\ C_r &= \begin{bmatrix} 0.4 & 0.8 \end{bmatrix}, E_{32} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \\ \Gamma_1 &= \Gamma_2 = \begin{bmatrix} 0.1\sin(k) & 0 \\ 0 & 0.5\sin(k) \end{bmatrix}. \end{aligned}$$

Choose $d = 1$, $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, $\gamma = 4.4721$, $R = 0.02I$, $Q = 0.05I$.
Solving LMI (20), we get a set of feasible solutions as follows.

$$\begin{aligned} P &= \begin{bmatrix} 0.0978 & 0.0085 & -0.0012 & -0.0011 \\ 0.0085 & 0.1619 & -0.0007 & 0.0006 \\ -0.0012 & -0.0007 & 0.4737 & -0.1292 \\ -0.0011 & 0.0006 & -0.1292 & 0.1812 \end{bmatrix}, \\ S &= \begin{bmatrix} 0.0007 & 0 & 0 & 0 \\ 0 & 0.0007 & 0 & 0 \\ 0 & 0 & 0.0108 & -0.0031 \\ 0 & 0 & -0.0031 & 0.0019 \end{bmatrix}, \\ K_{11} &= \begin{bmatrix} -0.0100 & 0.0085 \\ -0.0070 & -0.0005 \end{bmatrix}, \\ K_{21} &= \begin{bmatrix} 0.1333 & 0.2665 \\ 0.0665 & 0.1330 \end{bmatrix}, \\ K_{12} &= \begin{bmatrix} -0.0458 & -0.0240 \\ 0.0334 & 0.0131 \end{bmatrix}, \\ K_{22} &= \begin{bmatrix} 0.0839 & 0.1698 \\ 0.1590 & 0.3135 \end{bmatrix}. \end{aligned}$$

In addition, the initial values of system (1) is chosen $x(s) = [-1 \exp(s) \; 2\exp(s)]^T$, and the initial condition of reference model (4) is $x_r(0) = [1 \; -2]^T$.

In the sequel, two kinds of reference input $r(k)$, step disturbances and sinusoidal disturbances, are employed to demonstrate the effectiveness of the proposed method.

Case I: Step reference input
Let

$$r(k) = \begin{cases} \begin{bmatrix} 10 & 10 \end{bmatrix}^T, & 40 \le k < 80; \\ \begin{bmatrix} 0 & 0 \end{bmatrix}^T, & \text{other } k; \end{cases}$$

$$\omega(k) = \begin{cases} \frac{2}{k+1}, & 30 \le k < 80; \\ 0, & \text{other } k. \end{cases}$$

Curves of system output $z(k)$ and reference model output $z_r(k)$ are presented in Fig.1. From Fig. 1, one can see that the proposed controller can guarantee system output have good tracking performance. It powerfully proves the effectiveness of the proposed method. Response curves of system states depict in Fig. 3. It can be seen that system states are asymptotically stable when external input signals turn into zeros.

Fig. 3 and Fig. 4 show the curve of control inputs and switching signal.

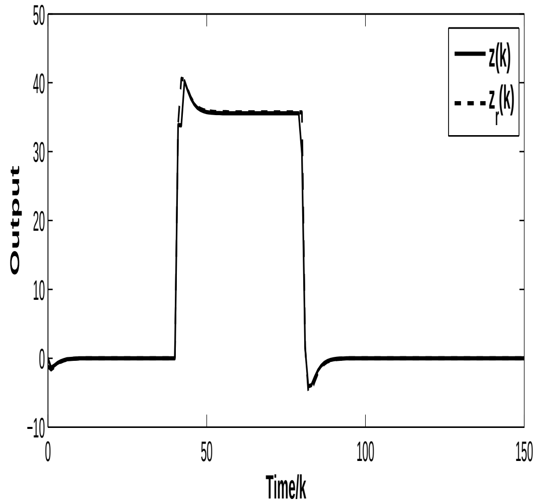

Fig. 1. Curves of system output $z(k)$ and reference model output $z_r(k)$ under Case I.

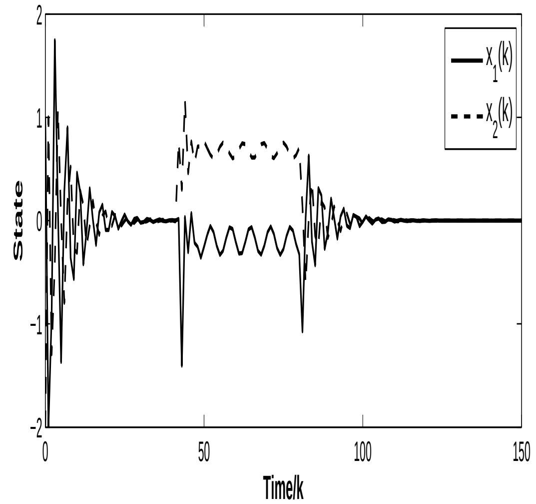

Fig. 2. Response curves of system state $x(k)$ under Case I.

Case II: Sinusoidal reference input

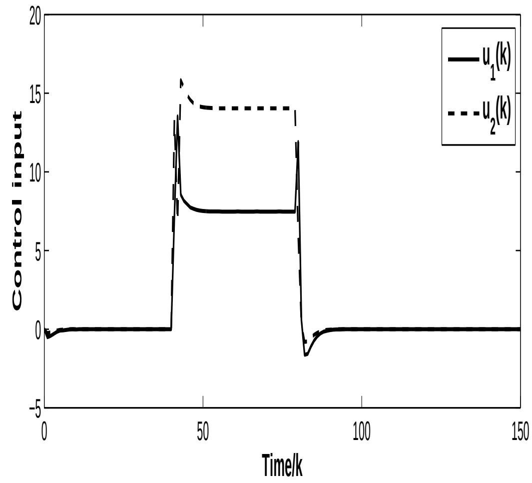

Fig. 3. Curve of control input $u(t)$ under Case I.

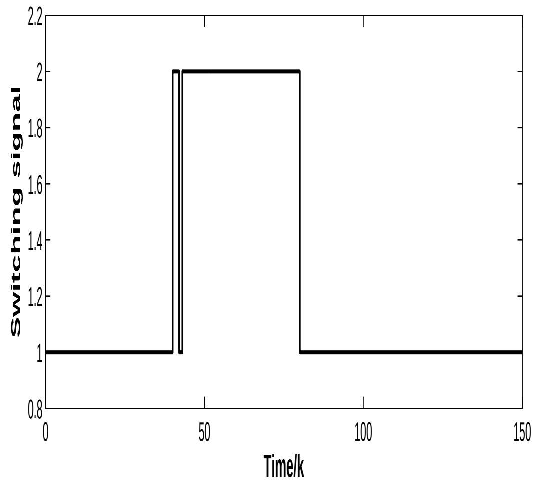

Fig. 4. Curve of switching signal under Case I.

Let

$$
\begin{aligned}
r(k) &= \begin{cases} \begin{bmatrix} 10\sin(0.3k) \\ 10\sin(0.3k) \end{bmatrix}, k \leq 80; \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \text{other } k; \end{cases} \\
\omega(k) &= \begin{cases} \frac{2}{k+1}, k \leq 80; \\ 0, \text{other } k. \end{cases}
\end{aligned}
$$

In Fig. 5, curves of system output $z(k)$ and reference model output are shown. It can be seen that system output effectively tracks the reference mode output. A conclusion is obtained that the proposed control method is efficient. Fig. 6 is shown the response curves of system states. Curves of control inputs and switching signal are presented in Fig. 7 and Fig. 8, respectively.

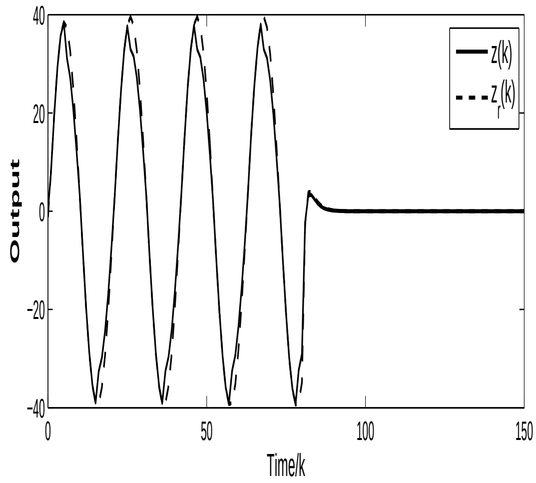

Fig. 5. Curves of system output $z(k)$ and reference model output $z_r(k)$ under Case II.

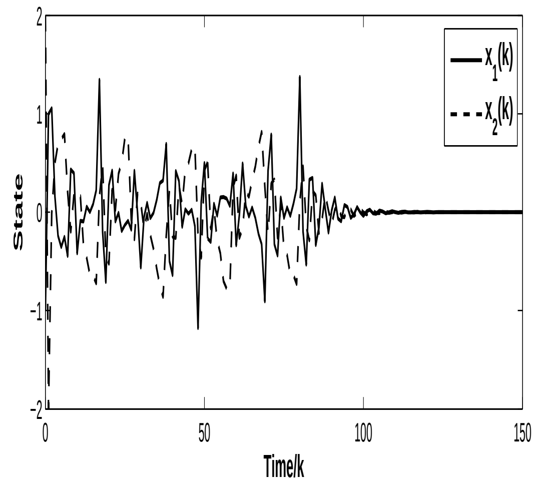

Fig. 6. Response curves of system state $x(k)$ under Case II.

5 Conclusions

This paper has considered the problem of mixed H_2/H_∞ output tracking control for uncertain discrete-time switched systems with state time-delay. By using single Lyapunov function theory, a tracking controller has developed to guarantee the closed-loop error system robust asymptotically stable with mixed H_2/H_∞ performance. A sufficient condition for the existence of the desired controller gains has been proposed in terms of linear matrix inequalities(LMIs). The corresponding stabilizing switching rule has been provided. A numerical example has been given to demonstrate the applicability of the proposed approach.

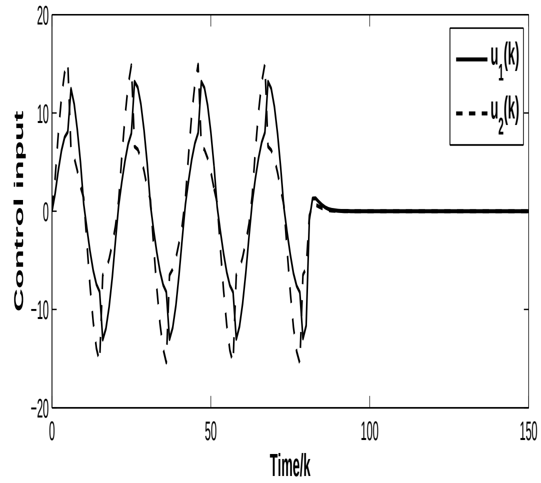

Fig. 7. Curve of control input $u(t)$ under Case II.

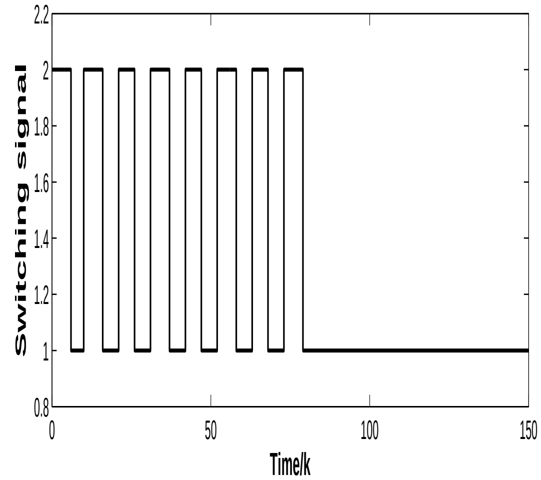

Fig. 8. Curve of switching signal under Case II.

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