A framework for faults in detectors within network monitoring systems

Peter J. Slater
University of Alabama in Huntsville
Mathematical Sciences Department and Computer Science Department
Huntsville, AL 35899, U.S.A
slaterp@math.uah.edu; pslater@cs.uah.edu

Abstract: Various different types of detectors can be used to identify malfunctioning elements in a network. The detectors themselves might not function properly. Four types of possible detector faults are identified, where detectors are located at vertices and are used to identify malfunctioning vertex locations. This leads to a sequence of four detector-failure parameters for various (domination-related) parameters.

Key–Words: intruder detection, locating-dominating sets, identifying codes, open-locating-dominating sets, detector faults

1 Introduction

Graphs and networks are frequently used to model situations in which elements are subject to failure or intrusion. In a multiprocessor network each vertex can represent one processor, each of which can be subject to a complete failure or might seem to operate but actually produce incorrect data. In a communication network a transmitter/receiver (represented by a vertex) can totally fail or might transmit and/or receive incorrect data, and a communication link (represented by an edge) might be subject to full or partial interference. In modeling a facility one might be interested in the (failure) location of a fire, a thief, or a saboteur.

Reliability measures are concerned with how well the system can be expected to operate given the probabilities with which individual elements (vertices and/or edges) will function properly. For example, what is the probability that all of the functioning vertices can still communicate? Or, what is the expected number of pairs of vertices that can communicate? Vulnerability measures are concerned with the minimum amount of element failures (perhaps induced by a knowledgeable opponent) before an overall failure is reached, such as disconnecting the network or resulting in accepting an incorrect result produced with one or more malfunctioning processors. Robustness is concerned with optimal operation of the network system in the presence of element failures.

A natural central concern is the determination that an element failure has occurred and the identification of which element is failing. Various types of detectors can be used for locating and reporting element failures. Some, like sonar devices, can be assumed to be able to detect failures anywhere in the system, while others, like heat sensors, might have a limited detection range.

For uniform terminology, the facility model will be used. The failure site will be termed an "intruder" location. One aspect of operation for a detector is the manner in which it determines where an intruder lies. For example, a camera gives an exact location and a heat sensor determines a region in which an intruder lies. A second aspect is the manner in which detectors communicate to a central responder. The central focus of this paper is the consideration of various possible faults in the use of detectors for determining failure locations and in the use of detectors for transmitting this information. Note that intruder locations (fire, thief, saboteur, malfunctioning processors, etc.) will be described as failures, while the terms fault/faulty will be applied to the detectors.

In Section 2 various detection modes will be described. In Section 3 a series of progressively more serious detector faults will be considered. The parameters associated with each detection mode/detection fault (many of which have yet to be studied) will be introduced. The framework described here is applicable much more widely.

2 Intruder detection models

Standard graph theory terminology will be used. In particular, for a graph $G = (V, E)$ the distance $d(u, v)$ is the minimum number of edges in a *uv* path for vertices *u* and *v* in $V(G)$. The (open) neighborhood $N(v)$ is the set of vertices at distance one from *v*, $N(v) =$

$\{w \in V(G) : vw \in E(G)\}$, and the closed neighborhood is $N[v] = N(v) \cup \{v\}$. Being concerned with determining an intruder location anywhere in the system, we will mainly be concerned with various types of dominating sets. Vertex set $D \subseteq V(G)$ is dominating if $\cup_{v \in D} N[v] = V(G)$, that is, every vertex w is in D or is adjacent to some $v \in D$. For a dominating set D of detectors, every vertex location is within distance one of at least one detector.

2.1 Locating sets/metric bases

First, a case of location at arbitrary distances is considered. Assume that a detector at vertex x can determine the distance $d(x, w)$ to an intruder at w. With an (ordered) set $X = \{x_1, x_2, ..., x_k\}$ of vertices, for each $v \in V(G)$ we have a k-tuple $(d(x_1, v), d(x_2, v), ..., d(x_k, v))$. As introduced in Slater [11] and Harary and Melter [3], X is a locating-set or metric basis if all of these k-tuples are distinct. We can say that a vertex x resolves two vertices u and v if $d(x, u) \neq d(x, v)$. Then X is locating if for every two vertices u and v in $V(G)$ at least one $x \in X$ resolves u and v. The location number $LOC(G)$ is the minimum cardinality of a locating set $X \subseteq V(G)$. Because $d(x_i, x_i) = 0$, clearly each x_i in X resolves itself with any other vertex.

A branch of a tree T at vertex v is a subgraph induced by v and one component of $T \setminus \{v\}$. A branch B at vertex v is called a branch path if B is a path with degree $deg(v) \geq 3$, in which case v is called a stem of B. Tree T_{42} in Figure 1 has order $n = |V(T_{42})| = 42$, sixteen branch paths, and six stems: s_1, s_2, ..., s_6. Let L_1, L_2, ..., L_k be the components of the subtree induced by all of the branch paths, and for T_{42} we have $k = 6$.

Using the next result, one can see that $LOC(T_{42}) = 10$ and $\{1, 2, ..., 10\}$ is an $LOC(T_{42})$-set. For path P_n we clearly have $LOC(P_n) = 1$, using one endpoint as an $LOC(P_n)$-set.

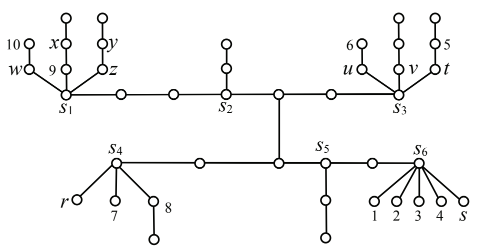

Figure 1: Tree T_{42} with $LOC(T_{42}) = 10$

Theorem 1 *(Slater [11]) Let T be a tree with maximum degree $\Delta(T) \geq 3$ (that is, with at least one stem). Vertex set S is locating if and only if for each vertex u there are vertices from S on at least $deg(u) - 1$ of the $deg(u)$ components of $T \setminus \{u\}$.*

Theorem 2 *(Slater [11]) Let T be a tree with set L of endpoints with $|L| \geq 3$, with components L_1, L_2, ..., L_k of the subtree induced by the set of all branch paths, and let e_i be the number of branch paths in L_i. Then $LOC(T) = |L| - k$, and S is an $LOC(T)$-set if and only if it consists of exactly one vertex from each of exactly $e_i - 1$ of the branch paths of L_i, for $1 \leq i \leq k$.*

As noted, various types of faults for a detector can be considered. One such is that a detector will become completely inoperable. Resolving set S is redundant-resolving if $S \setminus \{v\}$ is also resolving for each v in S. Let $RED{:}LOC(G)$ denote the minimum cardinality of a redundant-resolving set.

Theorem 3 *(Hernando, et al [4]) For a tree T with set L of endpoints with $|L| > 3$, with branch path components L_1, L_2, ..., L_k and e_i the number of endpoints of T in L_i, let E_1 be the set of endpoints corresponding to branch paths where e_i = 1. Then $RED{:}LOC(T) = |L \setminus E_1|$ and $L \setminus E_1$ is a $RED{:}LOC(T) - set$.*

For tree T_{42}, $\{1, 2, ..., 10, y, v, r, s\}$ is an $RED{:}LOC(T_{42}) - set$.

2.2 Domination-related models

Heat sensors and motion detectors have a limited range. Guards in a museum have a limited line of sight for the objects therein. If a processor A is testing the correct functioning of processor B, but communication between A and B must involve at least one intermediate processor C, then A can not conclude that B is malfunctioning because it might be C that is not functioning correctly. In brief, detectors might have a limited range.

The basic limited range detector model is that of a dominating set. One can think of using a camera as a detector where a camera can be used to precisely determine any intruder location adjacent to it or at its own location. Then a set $D \subseteq V(G)$ of camera detector locations can identify any intruder location if and only if D is dominating, $\cup_{v \in D} N[v] = V(G)$. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set. When an intruder at v can interfere with the detection or reporting of an intruder at v, we need to consider open neighborhood domination.

Vertex set $D \subseteq V(G)$ is an open-dominating set (also called a total dominating set) if $\cup_{v \in D} N(v) = V(G)$.

Think of a locating set with range limited to distance one. A detector at *v* can determine if there is an intruder at v, or if there is an intruder in $N(v)$, but which element in $N(v)$ cannot be determined. These ideas of locating and dominating were merged in Slater [12, 13]. Dominating set $L \subseteq V(G)$ is a locating-dominating set if for any two vertices u and v in $V(G) - L$ one has $N(v) \cap L \neq N(u) \cap L$. Every graph has a locating-dominating set (namely $V(G)$) and $LD(G)$ is the minimum cardinality of such a set.

Identifying codes were introduced in Karpovsky, Chakrabarty and Levitin [6]. For this model a detection device at vertex *v* can determine if there is an intruder in $N[v]$, but which vertex location in $N[v]$ cannot be determined. Then $S \subseteq V(G)$ is an identifying code if for all *u* and *v* in $V(G)$ one has $N[v] \cap S \neq N[v] \cap S$. Graph *G* has an identifying code when no two vertices have the same closed neighborhood, and $IC(G)$ is the minimum cardinality of such a set.

When a detection device at vertex *v* can determine if an intruder is in $N[v]$ but will not/can not report if the intruder is at *v* itself, then we are interested in open-locating-dominating sets as introduced for the *k*-cubes Q_k by Honkala, Laihonen and Ranto [5] and for all graphs by Seo and Slater [9, 10]. Vertex set $S \subseteq V(G)$ is an open-locating-dominating set if for all *u* and *v* in $V(G)$ one has $N(u) \cap S \neq N(v) \cap S$. Graph *G* has an open-dominating set when no two vertices have the same open neighborhood, and $OLD(G)$ is the minimum cardinality of such as set.

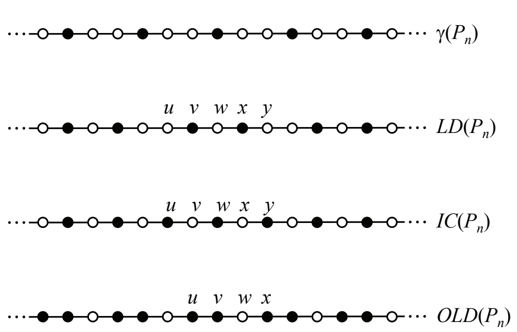

Figure 2: Distinguishing sets for paths

In general, a collection $C = \{S_1, S_2, ..., S_p\}$ of subsets of $V(G)$ is a distinguishing set for graph *G* if $\cup_{1 \leq i \leq p} S_i = V(G)$ and for every pair of distinct vertices *u* and *v* in V(G) some S_i contains exactly one of them. So, $L = \{w_1, w_2, ..., w_j\}$ is a locating-dominating set if $C_1 = \{\{w_1\}, N(w_1), \{w_2\}, N(w_2), ..., \{w_j\}, N(w_j)\}$ is distinguishing with $p = 2j$; $S = \{w_1, w_2, ..., w_j\}$ is an identifying code if $C_2 = \{N[w_1], N[w_2], ..., N[w_j]\}$ is distinguishing; and $S = \{w_1, w_2, ..., w_j\}$ is an open-locating-dominating set if $C_3 = \{N(w_1), N(w_2), ..., N(w_j)\}$ is distinguishing. Figure 2 illustrates how to achieve the values $\gamma(P_n) = \lceil (1/3)n \rceil, LD(P_n) = \lceil (2/5)n \rceil, IC(P_n) = \lceil n/2 \rceil$, and $OLD(P_n) = \lceil (2/3)n \rceil$. Note that for *LD*-sets, C_1 can be a multiset if, for example, $N(w_1) = N(w_2)$. For a dominating set *D* the associated collection C_4 has $I(D)$ singleton set entries where $I(D) = \Sigma_{v \in D}(1 + deg(v))$ is the influence of *D* as defined in Grinstead and Slater [1]. Each $\{v\}$ appears in C_4, in fact, $|N[v] \cap D|$ times. The redundance is important when we consider fault-tolerance.

3 Fault tolerant detection models

There are two aspects to using a detector in deciding where an intruder is located. The detector must be able to determine the presence of the intruder and to transmit this information to a central point *P* where this information can be used. For now we assume there is at most one faulty detector. For the redundant-locating sets in Section 2.1, a detector becomes completely inoperable and does not transmit any information. This type of detector fault can be determined if we, for example, synchronize transmission times. In this case, a detector's failure is indicated by the fact that it has not reported, and the failure is known.

Contrast this with a case in which the detector's ability to actually detect an intruder is lost, but the detector itself can not determine its own failure. It will transmit the information that no intruder is present to a point *P*. At *P* this failure might not be apparent. Locating-dominating set *L* is redundant-locating-dominating if $L \backslash \{v\}$ is also locating-dominating for each *v* in *L*. Let $RED{:}LD(G)$ denote the minimum cardinality of a redundant-locating-dominating set. For this framework, let $DET{:}LD(G)$ denote the minimum cardinality of a locating-dominating set *L* with the property that if any one detector in *L* inaccurately reports that no intruder is present, then the intruder location can still be determined. Such sets *L* will be called detector-redundant. Consider $L = \{v_1, v_2, v_3, v_5, v_6, v_7\}$ in cycle C_8, as in Figure 3.

Theorem 4 *(Slater [14]) If $L \subseteq V(G)$ is a detection-redundant set and $v \in L$, then $L - v$ is a locating-dominating set. In particular, $RED{:}LD(G) \leq DET{:}LD(G)$.*

Proposition 5 *For cycle C_8, $RED{:}LD(C_8) = 6 < DET{:}LD(C_8) = 7$.*

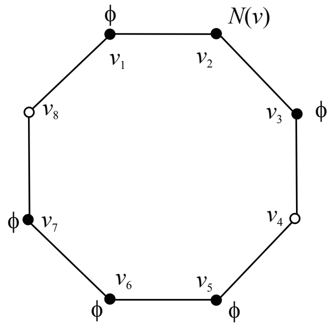

Figure 3: $RED{:}LD(C_8) < DET{:}LD(C_8)$

Proof: $L = \{v_1, v_2, v_3, v_5, v_6, v_7\}$ is a redundant-locating-dominating set, so $RED{:}LD(C_8) \le 6$. If any vertex is dominated only once by a dominating set *D*, then a single *RED*-fault can leave that intruder location unreported. Hence, any $RED{:}LD(C_8)$-set *D* must dominate every vertex twice. It follows that the influence of D must satisfy $I(D) \ge 2 \cdot |V(C_8)| = 16$. But $|D| \le 5$ implies that $I(D) \le 15$. So, $RED{:}LD(C_8) = 6$.

Likewise, each detection-redundant set $D \subseteq V(C_8)$ must dominate every vertex twice. Thus, for example, $V(C_8)\backslash\{v_1, v_2\}$ and $V(C_8)\backslash\{v_1, v_3\}$ are not detection-redundant. As observed above, $V(C_8)\backslash\{v_1, v_5\}$ is not detection -redundant. Finally, consider $D = V(C_8) - \{v_1, v_4\}$, and assume v_2 reports an intruder in $N(v_2) = \{v_1, v_3\}$ while all other detectors report no intruder. The intruder can be at *v*, with detector v_8 being faulty or at v_3 with detector v_3 faulty. Thus, $DET{:}LD(C_8) \ge 7$. Let $L = V(C_8)\backslash\{v1\}$. For $3 \le i \le 7$, because at most one detector is faulty, either an intruder at v_i is reported by v_i as being at v_i or by both v_{i-1} and v_{i+1} as being in $N(v_{i-1}) \cap N(v_{i+1}) = \{v_i\}$. Assume an intruder is at v_2 or v_8, say v_2. If v_2 does not report itself, then v_3 reports $N(v_3)$. Because v_4 and v_5 report no intruder, the intruder in $N(v_3)$ must be at v_2. If the intruder is at v_1, we can assume that v_2 reports $N(v_2)$. Because both v_3 and v_4 report no intruder, the intruder in $N(v_2)$ must be at v_1. Thus $V(C_8)\backslash\{v_1\}$ is a $DET{:}LD(C_8)$-set. □

For detection-redundant intruder reporting, any detector actually reporting that there is an intruder is accurate ("telling the truth"). That is, for *RED* and *DET* a detector problem is in the inability to correctly report an intruder location. Following are two models involving a single fault that involves inaccurately reporting the actual presence of an intruder. Consider the simple domination model in which each detector at a vertex *v* is camera-like and can precisely identify at which vertex in $N[v]$ an intruder is located. For any situation in which a detector can fail to report or can inaccurately report that there is no intruder at a location where one actually exists, clearly no intruder location can be covered by only one detector. That is, every vertex must be double dominated. As defined by Harary and Haynes[27], vertex set $D \subseteq V(G)$ is a *k*-tuple dominating set if $|N[x] \cap D| \ge k$ for every $x \in V(G)$, and the minimum cardinality of a *k*-tuple dominating set is denoted by $\gamma_{xk}(G)$. The double domination number of *G* is $\gamma_{x2}(G)$. Note that if *D* is double dominated, then any intruder at a vertex *x* will be accurately reported at least once even with one *RED* or *DET* fault. Thus we have the following.

Theorem 6 *For any graph G,* $\gamma_{x2}(G) = RED{:}\gamma(G) = DET{:}\gamma(G)$

As introduced in Slater [15] and Roden and Slater [7]. For a "liar's dominating set" it is assumed that any one detector *v* in the neighborhood of an intruder vertex *x* might (either deliberately or through a transmission error) misreport the location of *x* by indicating that there is no intruder or by misreporting another location $y \in N(v)$ as the intruder location.

A dominating set $D \subseteq V(G)$ is *a liar's dominating set* if for any designated (intruder) vertex *x*, if all or all but one of the vertices in $N[x] \cap D$ report *x* as the intruder location, and at most one vertex *v* in $N[x] \cap D$ either reports a vertex $y \in N[v]$ or fails to report any vertex, then the vertex *x* can be correctly identified as the intruder location. That is, if an intruder is at $x \in V(G)$ then the detectors outside $N[x]$ are assumed to indicate there is no intruder (thus, we have no "false alarm"), one vertex *v* in $N[x] \cap D$ can report no intruder exists or report any vertex in $N[v]$ as the intruder location, every other detector in $N[x] \cap D$ correctly reports location *x*, and *x* will be identifiable. The *liar's domination number* $\gamma_{LR}(G)$ or simply $LR(G)$ is the minimum cardinality of a liar's dominating set. It is being assumed that all detectors can, in fact, detect correctly and there will be at most one fault in the reporting to central location *P*.

Theorem 7 *(Slater[15]) For every connected graph G of order* $n \ge 3$ *we have* $\gamma_{x2}(G) \le LR(G)$*, and, if G has minimum degree* $\delta(G) \ge 2$*, then* $\gamma_{x2}(G) \le LR(G) \le \gamma_{x3}(G)$.

Theorem 8 *(Slater[15]) Vertex set* $D \subseteq V(G)$ *is a liar's dominating set if and only if (1) D double dominates every* $v \in V(G)$ *and (2) for every pair of distinct vertices u,v we have* $|(N[u] \cup N[v]) \cap D| \ge 3$.

For the fourth fault tolerant detection model considered here, we have an error correcting code problem. Detectors might be able to detect the intruder location, but there can be an error in any one detector's transmission to central point *P*, including the possibility of a false alarm. *ER* will be used to indicate the possibility of such a case.

4 Fault tolerant detection parameters

We have a sequence of four fault tolerant parameters associated with each resolving or distinguishing parameter. For *RED* we need redundancy in that one of our detectors might fail to transmit; for *DET* every detector transmits but one of the detectors transmitting a "no intruder" message can be incorrect; for *LR* one no intruder message can be incorrect, or one detector *v* can misreport an intruder location in its range as being anywhere in *v*'s detection range; and for *ER* any one detector can misreport whether or not there is an actual intruder in its range.

Depending on the basic parameter Ψ, not all four fault tolerant parameters need be distinct. As noted, for example, $RED{:}\gamma(G) = DET{:}\gamma(G)$ for all graphs *G*. Long cycles C_n will usually suffice to demonstrate when the parameters are different.

4.1 $\gamma(G) \leq RED{:}\gamma(G) = DET{:}\gamma(G) \leq LR{:}\gamma(G) \leq ER{:}\gamma(G)$

From Theorems 6 we have $RED{:}\gamma(G) = DET{:}\gamma(G) = \gamma_{x2}(G)$. By theorem 3.5, $L \subseteq V(G)$ is a liar's dominating set if and only if L double dominates each vertex and triple dominates each pair of vertices. As in Slater[15] and Roden and Slater[7], such a set is an $X(2,3)(G)$-set. In general, given a sequence of nonnegative integers$(c_1, c_2, ..., c_t)$ with $0 \leq c_1 \leq c_2 \leq ... \leq c_t)$ and $c_t \geq 1$, a set $D \subseteq V(G)$ is a $X(c_1, c_2, ..., c_t)(G)$-dominating set if for $1 \leq i \leq t$ every $S \subseteq V(G)$ with $|S| = i$ has $|N[S] \cap D| \geq c_i$. The minimum cardinality of a $X(c_1, c_2, ..., c_t)(G)$-dominating set is the $X(c_1, c_2, ..., c_t)(G)$-domination number, denoted by $\gamma_{X(c_1,c_2,...,c_t)}(G)$. Rephrasing Theorem 3.5, we have $LR(G) = \gamma_{x(2,3)}(G)$.

For $ER{:}\gamma(G)$ and detector set $S \subseteq V(G)$, any $x \in S$ can report an intruder location in $N[x]$, even when no intruder is actually there. Assume that $N[w] \cap S = \{x, y\}$. If *x* reports an intruder at *w* and all other detectors report no intruder, there can be an intruder at *w* with an error from *y* or no intruder with an error from *x*. Note that if any two detectors in $N[w]$ agree that there is or is not an intruder at *w*, at least one (and hence both) will be correct. It follows that any triple dominating set allows for all possible single intruder error correction.

Theorem 9 $\gamma(G) \leq RED{:}\gamma(G) = DET{:}\gamma(G) = \gamma_{x2}(G) \leq LR{:}\gamma(G) = \gamma_{x(2,3)} \leq ER{:}\gamma(G) = \gamma_{x3}(G)$.

One can verify that for cycle C_n we have $\gamma(C_n) = \lceil n/3 \rceil$, $RED{:}\gamma(C_n) = DET{:}\gamma(C_n) = \gamma_{x2}(C_n) = \lceil 2n/3 \rceil$, $LR{:}\gamma(C_n) = \lceil 3n/4 \rceil$, and $ER{:}\gamma(C_n) = n$.

4.2 $LOC(G) \leq RED{:}LOC(G) = DET{:}LOC(G) \leq LR{:}LOC(G) = ER{:}LOC(G)$

For locating sets the existence of an intruder anywhere in the graph can be detected by any detection-functioning detector. This implies that $RED{:}LOC(G) = DET{:}LOC(G)$, as follows. One can identify a detector with an *RED*-fault because it fails to report. For a *DET*-fault a detector can report that there is no intruder within the range of that detector, in this case all of $V(G)$. However, any operating detector will indicate the presence of an intruder, enabling us to determine that a detector reporting no intruder is the faulty one.

For liar's-locating and error-locating faults, consider the following. If we have only two detectors, then the one can report an incorrect distance and we clearly cannot decide which is misreporting. That is, $LR{:}LOC(G) \geq 3$. For an error-locating fault, as noted, an error based on reporting no intruder can be identified. Because the range of a detector is all of $V(G)$, there can be no false alarms, and we have $3 \leq LR{:}LOC(G) = ER{:}LOC(G)$. For path P_n, $n \geq 4$, we have $LOC(P_n) = 1$, $RED{:}LOC(P_n) = DET{:}LOC(P_n) = 2$, and $LR{:}LOC(P_n) = ER{:}LOC(P_n) = 4$.

5 Summary

For various domination-related parameters involving locating vertices that function as places from which detectors can determine information about the location of an "intruder", four types of possible detector faults have been identified. For each parameter Ψ this leads to four fault-related parameters, and we have $\Psi(G) \leq RED{:}\Psi(G) \leq DET{:}(G) \leq LR{:}\Psi(G) \leq ER{:}\Psi(G)$, with some of these pairs of parameters turning out to be identical for specific examples. Most of the parameters described here (for example, the *IC*-parameters and *OLD*-parameters) are only now being studied. There are general results applying to general

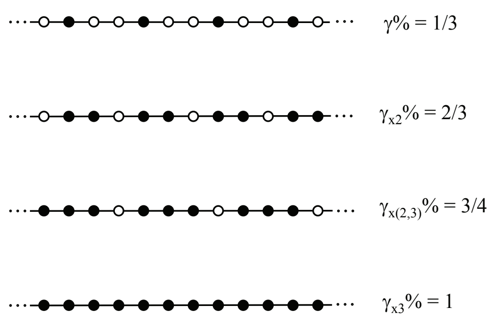

Figure 4: Domination fault parameter for cycles and (infinite) paths

distinguishing sets. Collection $S = \{s_1, s_2, ..., s_t\}$ is distinguishing for graph *G* if $\cup_{1 \le i \le t} S_i = V(G)$ and for each pair *u*, *v* of vertices there is some S_i containing exactly one of them. For $v \in V(G)$, let $S(v) = \{i | v \in S_i\}$ and $A \Delta B$ is the symmetric difference of sets *A* and *B*. Then *S* is distinguishing if $S(v) \neq \emptyset$ for all $v \in V(G)$ and $|S(u) \Delta S(v)| \ge 1$ for all pairs *u*,*v*. As examples of general results, we have the following. *S* is redundant-distinguishing if $S - S_i$ is distinguishing for $1 \le i \le t$.

Theorem 10 *S is redundant-distinguishing if each* $|S(u)| \ge 2$ *and* $|S(u) \Delta S(v)| \ge 2$.

S is detection-distinguishing if *S* can distinguish under the condition that one S_i can falsely report "no intruder" in S_i.

Theorem 11 *S is detection-distinguishing if each* $|S(u)| \ge 2$ *and for each pair u,v we have either* $|S(u) \setminus S(v)| \ge 2$ *or* $|S(v) \setminus S(u)| \ge 2$.

All of these problems can be extended to consider multiple intruders and/or multiple faults. For example, in Roden and Slater[8] we consider liars' domination, where for $LR(i, j)(G)$ we allow up to *i* intruders and *j* liars.

Finally, note that these ideas formed the basis of a talk at the Bordeaux Workshop on Identifying Codes (Bordeaux, 2011). Copies of the slides used are available at the website. However, there *FT* was used for what is here denoted by *DET*. In Slater[14] $FTLD(G)$ was used for what is here $DET{:}LD(G)$. The latter now seems preferable so as to identify the specific type of detector fault.

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