A Modified Power Spectrum Estimator for Minimizing the Prediction Error Energies

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Abstract: The power spectrum estimation plays a significant role in the marine monitoring and the quality of spectrum will seriously affect the results of marine monitoring. There have been various algorithms on power spectrum estimation, one of which is Burg algorithm. Though this method can effectively improve the resolution of spectral estimation, it will cause the shifting of peak frequency and the splitting of spectral lines accompanying spurious frequency components. In this paper, a novel method for minimizing the prediction error energies is presented, which is based on the optimum tapered Burg algorithm. The proposed method is implemented in two aspects. Firstly, the first order reflection coefficient is calculated by minimizing the averaged optimum tapered forward and backward prediction error energies of the third order filter. Then, the residual factor is introduced to estimate the model order. In the end of the paper, the experimental results demonstrate that in contrast to Burg algorithm and optimum tapered Burg algorithm, the proposed algorithm can not only effectively improve the spectrum resolution, lower the degree of peak frequency shifting and spectrum lines splitting, but also be able to alleviate the generation of spurious peak frequencies and the phenomenon of spectral leakage.

Key–Words: Burg algorithm, Spectrum estimation, Prediction error energy, Levinson-Durbin recursion.

1 Introduction

The power spectral analysis plays a significant role in the signal processing, and is the research focus of information science as well. In last decades, the power spectrum estimation has become popular in many application areas such as signal prediction, analysis of radar clutter, seismic signal processing, wave signal processing and marine monitoring. The fundamental theory of spectrum estimation is Fourier transformation, which is the main tool in signal processing and also the basis of other researching fields [1]. Generally speaking, there are two types of power spectral estimation: classical spectrum estimation and modern spectrum estimation. This paper mainly concentrates the attention on some algorithms of modern spectrum estimation.

The classical power spectrum estimation assumes the unknown data outside the observation areas to be zero, which is equivalent to impose a window function on the observed data, leading to lower resolution. The two most representative methods of classical spectrum estimation are autocorrelation function algorithm proposed by *Schuster* and periodogram algorithm proposed by *Blackman* and *Tukey*. However, both the two methods qualify the common limitations such as large variance, low resolution and relatively large errors caused by the calculation of correlation function [2]. Therefore, the modern power spectrum estimation is put forward and more widely spread in practical applications. Unlike the classical spectrum estimation, the modern power spectrum estimation first establishes a model which can describe the practical process according to priori knowledge of the process. Then, it estimates the model parameters using the observed data or correlation function, and it identifies the model finally. The advantage of modern power spectrum estimation lies in the maximum uncertainty for the unknown information of a process. As to the parameter model, some models such as the autoregressive (AR) model, moving average (MA) model and

autoregressive and moving average (ARMA) model have been proposed to approximate the practical process. However, AR model is commonly adopted and widely applied [3, 4]. The parameters of AR model can be simply obtained by solving a system of linear equations, besides, it is worth noting that AR model is suitable for extremely narrow spectrum. Moreover, it has the recursive characteristic during the parameters calculation with short length data. In short, the modern spectrum estimation presents two main problems: model parameters calculation and model order determination. For parameters estimation, a large number of algorithms have been put forward to calculate AR model parameters. *Levinson* proposed Yule-Walker algorithm which is a biased autocorrelation estimator based on moment estimation, but it makes the model severely biased. *Windrow* and *Hoff* raised the least squares algorithm which is simple to understand and of high efficiency, but it owns a relatively large variance and yields unstable models. *Burg* proposed Burg algorithm which is an algorithm calculating each AR parameters through minimizing the sum of forward and backward errors subject to the constraint that the optimal parameters satisfy Levinson recursion. It provides an increased spectral resolution over the conventional methods. Besides, Burg algorithm generates stable model with a smaller variance than Yule-walker algorithm and the least squares algorithm [4, 5]. Although Burg algorithm has the characteristics of low covariance and high resolution, it ignores the influence of the phenomena including line splitting, peak shifting and spurious frequency components. As to the limitations of Burg algorithm, *Kaveh* proposed the optimum tapered Burg algorithm which is more widely spread than other weighted Burg algorithms introduced in [6]-[9]. However, the optimum tapered Burg algorithm can just slightly lower the above phenomena, and particularly the peak shifting and spurious frequency components are still exist. For the problem of model order determination, which is the main influencing factor for spectral resolution representing the degree how two neighboring frequencies are resolved, a large number of researches have presented that the width of a spectral peak decreases and the resolution increases with an increasing signal power and model order. However, an excessive model order may lead to line splitting or spurious spectral peaks [10]. As to these conflicting factors, many methods have been proposed to determine the model order [11, 12]. *Akaike* proposed the Akaike information criterion (AIC) which is known to suffer from overfit [13]. The minimum description length (MDL) criterion or the Bayesian information criterion (BIC), proposed by *Rissanen*, is a consistent criterion, but unfortunately they are not suitable for complex signal. *Broersen* presented the combined information criterion (CIC) which depends on a penalty factor [14]. Moreover, relevant experiments have shown that the selected model order is too low or too high according to the above criterions.

In this paper, we provide a modified method to improve the spectrum estimation with short length signals. The main contributions of this paper lie in the first order reflection coefficient estimation and the model order determination. On one hand, we make some effort on estimating the AR parameters, especially the first order parameter, for decreasing errors and alleviating the phenomena of the peak shifting, line splitting and spurious frequency components. On the other hand, in order to guarantee the stability of spectrum estimation, we introduce the residual factor which notes the minimum difference of two adjacent averaged forward and backward prediction errors to determine the model order. Combining the calculation of residual factor, the first order reflection coefficient is solved by minimizing the averaged forward and backward prediction errors of the third order. Without loss of generality, we take the real signal as an example in this paper, and the algorithm can also be applied to complex signals. Some experimental results on real sinusoids in additive Gaussian white noise indicate that our approach improves the phenomena of line splitting, frequency shifting and spurious spectral peaks without degrading the resolution.

The remainder of the paper is organized as follows. Section 2 provides a description of an optimum tapered Burg algorithm. The improved AR model parameters calculation and the modified AR model order determination are presented in section 3 and section 4 respectively. We present some experimental results for real sinusoids in additive Gaussian white noise in section 5 and the paper ends with a conclusion in section 6.

2 The Optimum Tapered Burg Algorithm Power Spectrum Estimation

In this section, the specific process of the optimum tapered Burg algorithm is presented.

The basic linear AR model of ocean surface displacement is commonly applied in the maximum entropy spectrum estimation. p-order AR model is defined as:

$$X(t_n) = -a_1 X(t_{n-1}) - \cdots - a_p X(t_{n-p}) + W(t_n), \quad (1)$$

where $X(t)$ is a zero-mean stationary random process, $W(t)$ is a zero-mean white Gaussian noise with a constant variance σ_W^2.

By Fourier transform and time-shifting theorem for the above formula, we obtain the discrete power spectrum of signal $X(t)$ as follows [15]:

$$S_X(f_k) = \frac{2\sigma_W^2 \Delta t}{|1 + \sum_{j=1}^{p} a_j exp\{-i2\pi f_k j \Delta t\}|^2}, \quad (2)$$

the reason that the right side of the formula (2) generates 2 is that $S_X(f_k)$ is one-side spectrum. Δt is time slot, f_k is frequency, $a_1, a_2, \cdots, a_p$ are the model parameters which need to be estimated.

As to the model parameters estimation, Burg algorithm can not only get high accuracy, but also effectively improve the resolution of spectrum estimation. However, the phenomena of spectrum peak shifting and line splitting usually appear when we apply Burg algorithm to frequency spectral analysis. The estimation results are influenced by many factors, including data length, signal-to-noise ratio (SNR), model order, the accuracy of model parameters and the initial phase of signal data. Given that the optimum taper is insensitive to the initial phase, and some experiments have shown that the optimum tapered Burg algorithm can get better results than other weighted Burg algorithms, e.g., rectangular window, Hanning window, and Hamming window. Moreover, optimum tapered Burg algorithm is of low computational complexity degree[7, 8]. Therefore, our modified algorithm is based on the optimum tapered Burg algorithm. Before presenting optimum tapered Burg algorithm, we need to simply introduce the optimal window and give the theoretical supports in the sequel.

2.1 The optimal taper

Let the initial phase φ be a random variable which is uniformly distributed on $[-\pi, \pi)$. Then the mean value of the frequency error is zero. By minimizing the variance of averaged frequency error, we obtain the m-order optimal taper:

$$W_m(k) = \frac{6(k+1)(N-m-k+1)}{(N-m+1)(N-m+2)(N-m+3)}, \quad k = 0, 1, \cdots, N-m.$$

The specific derivation process of the optimal taper is discussed by *Kaveh* and *Lippert* [8].

From the above equation, this optimum taper is parabolic in form. It is even, positive and has a maximum at $k = (N-m+1)/2$. Furthermore, if $W_m(0)$ and $W_m(1)$ are known from the above equation, one can generate the remaining tapers recursively from:

$$W_m(k) = 2W_m(k-1) - W_m(k-2) - \lambda,$$

where $\lambda = \frac{12}{(N-m+1)(N-m+2)(N-m+3)}$.

For a symmetric taper, the averaged frequency error variance can be written as [8]:

$$\begin{aligned} & \langle var(\Delta f) \rangle \\ = & \frac{1}{16\pi^3}[\sum_{k=1}^{N-2} W_1(k)(W_1(k) - \\ & \frac{1}{2}W_1(k-1) - \frac{1}{2}W_1(k+1)) + \\ & 2W_1(0)(W_1(0) - \frac{1}{2}W_1(1))]. \end{aligned}$$

For a rectangular taper $W_1(k) = \frac{1}{N}$, we have

$$\begin{aligned} & \langle var(\Delta f) \rangle_R \\ = & \frac{1}{16\pi^3}[0 + \frac{2}{N}(\frac{1}{N} - \frac{1}{2N})] \\ = & \frac{1}{16\pi^3 N^2}. \end{aligned}$$

For a Hanning taper $W_1(k) = \frac{1}{2} + \frac{1}{2}cos(2\pi\frac{1}{N})$, we have

$$\begin{aligned} & \langle var(\Delta f) \rangle_H \\ = & \frac{1}{16\pi^3}\left(\frac{1}{2} + \frac{1}{2}cos(2\pi\frac{1}{N})\right)\left(\frac{1}{2} + \frac{1}{2}cos(2\pi\frac{1}{N})\right) \end{aligned}$$

As to the optimum taper, using the recursive form and the value of λ, we have

$$\begin{aligned} & \langle var(\Delta f) \rangle_O \\ = & \frac{1}{16\pi^3}[\sum_{k=1}^{N-2} W_1(k)\frac{6}{N(N+1)(N+2)} \\ & +2W_1(0)(W_1(0) - \frac{1}{2}W_1(1))] \\ = & \frac{6N^3 + 18N^2 + 12N}{16\pi^3 N^2(N+1)^2(N+2)^2} \\ = & \frac{6}{16\pi^3 N(N+1)(N+2)}. \end{aligned}$$

Therefore,

$$\begin{aligned} & \frac{\langle var(\Delta f) \rangle_O}{\langle var(\Delta f) \rangle_R} \\ = & \frac{6N}{(N+1)(N+2)}. \end{aligned}$$

$$\frac{\langle var(\Delta f) \rangle_O}{\langle var(\Delta f) \rangle_H}$$

$$= \frac{6}{N(N+1)(N+2)(0.5+0.5cos(2\pi\frac{1}{N}))^2}$$
$$\leq \frac{24}{N(N+1)(N+2)} \leq \frac{1}{5}, \quad (N \geq 4).$$

The above equations imply that $\langle var(\Delta f)\rangle_O$ is always less than or equal to $\langle var(\Delta f)\rangle_R$. The former is less than one half the latter for $N \geq 9$ in the case of real sinusoid. However, $\langle var(\Delta f)\rangle_O$ is less than one fifth of $\langle var(\Delta f)\rangle_H$ for $N \geq 4$. Similarly, we can prove that the optimum taper has less averaged frequency error variance than the Hamming window.

2.2 The optimum tapered Burg algorithm

Burg algorithm applies Levinson-Durbin recursion formula to calculate AR model parameters from low to high order. The optimum tapered Burg (TBO) algorithm shares the same basic principle with Burg algorithm. However, the difference between Burg algorithm and TBO algorithm lies in the later imposing an optimal taper on the forward and backward prediction errors for degrading the phenomena of spectral peak shifting and line splitting. The specific process is as follows.

Suppose a signal $x_n(n = 1, 2, \cdots, N)$ is given. Then the forward and backward prediction error energies of order M ($M = 1, 2, \cdots, p$) are defined as:

$$e^f_{M,j} = x_j + \sum_{m=1}^{M} a_M(m)x_{j-m} = \sum_{m=0}^{M} a_M(m)x_{j-m},$$

$$e^b_{M,j} = x_{j-M} + \sum_{m=1}^{M} a_M(m)x_{j+m-M} = \sum_{m=0}^{M} a_M(m)x_{j+m-M}, \quad j = M+1, \cdots, N,$$

where $a_M(0) = 1$.

At each order M, the mean of the forward and backward prediction errors (FBPE) is defined as ρ_M:

$$\rho_M = \frac{1}{2(N-M)} \sum_{j=M+1}^{N} \{(e^f_{M,j})^2 + (e^b_{M,j})^2\}.$$

Define the mean of optimum tapered FBPE ρ_M as

$$\rho_M = \frac{1}{2(N-M)} \sum_{j=M+1}^{N} W_M(j)\{(e^f_{M,j})^2 + (e^b_{M,j})^2\}, \quad (3)$$

where $W_M(j)$ is the M-order optimal window, and $\rho_0 = \sum_{j=1}^{N} W_0(j)x_j^2$.

Then setting the derivative of ρ_M with respect to $a_M(M)$ equals to zero yields M-order reflection coefficient $a_M(M)$:

$$a_M(M) = \frac{-2\sum_{j=M+1}^{N} W_M(j)e^f_{M-1,j}e^b_{M-1,j-1}}{\sum_{j=M+1}^{N} W_M(j)\{(e^f_{M-1,j})^2 + (e^b_{M-1,j-1})^2\}}. \quad (4)$$

Obviously, the absolute value of $a_M(M)$ is always smaller than unity. Therefore, the stability of the estimated AR model is guaranteed. Based on the above fact, each order reflection coefficient can be calculated from equation (4) and other parameters can be obtained via the Levinson-Durbin recursion:

$$a_M(i) = a_{M-1}(i) + a_M(M)a_{M-1}(M-i), \quad 1 \leq i \leq M-1. \quad (5)$$

Finally, the power spectrum is obtained by substituting the parameters in equation (2) with the estimated p-order coefficients and the variance of white Gaussian noise σ^2_W. In practical experiments, σ^2_W is usually taken as the $(p+1)$-order averaged FBPE.

3 The Improved Optimum Tapered Burg Algorithm

In this section, the modified calculation method of AR parameters is presented.

According to the equation (5), if the parameters of order $m-1$ are known, the m-order parameters can be solved under the condition that $a_m(m)$ is known. Obviously, in order to estimate the parameters more accurately, the key problem is how to calculate each order reflection coefficient. Therefore, $a_1(1)$, whose value influences the accuracy of each parameters, takes a significant role in all the reflection coefficients. The modified idea is to calculate $a_1(1)$ not by the equation (4) directly but by minimizing the averaged optimum tapered FBPE of each order. *Ibrahlm* [7] has proposed to calculate the first order reflection coefficient by minimizing the optimum tapered FBPE of the second order filter with respect to $a_2(1)$ and $a_2(2)$, but he fails to consider the determination criterion of model order.

Taking the determination criterion of model order into consideration, the paper modifies the first order reflection coefficient by minimizing the third order

averaged optimum tapered FBPE for further improving the accuracy of $a_2(1)$ and $a_2(2)$. Then $a_1(1)$ is obtained by Levinson-Durbin recursion formula from high to low order. In fact, compared with the second order and third order, we find that the latter will not impose much complexity on the parameters calculation (just increasing $7N + 27$ steps addition and $6N + 20$ steps multiplication). The simulations will show that the optimal performance is minimizing the p-order averaged optimum tapered FBPE firstly in section 5. There is no denying that the calculation process is extremely complicated. Based on the above evidence, we calculate the first order reflection coefficient by minimizing the averaged optimum tapered FBPE of the third order, which will make preparations for AR model order determination. The specific process is presented in the sequel.

The forward and backward prediction errors of order 3 are defined as

$$\begin{aligned} & e^f_{3,j} \\ = \; & x_j + \sum_{m=1}^{3} a_3(m) x_{j-m} \\ = \; & x_j + a_3(1)x_{j-1} + a_3(2)x_{j-2} + a_3(3)x_{j-3} \, , \end{aligned}$$

$$\begin{aligned} & e^b_{3,j} \\ = \; & x_{j-3} + \sum_{m=1}^{3} a_3(m) x_{j+m-3} \\ = \; & x_{j-3} + a_3(1)x_{j-2} + a_3(2)x_{j-1} + a_3 3 x_j \, , \\ & j = 4, 5, \cdots, N. \end{aligned}$$

The averaged optimum tapered FBPE of the third-order is written as

$$\begin{aligned} & \rho_3 \\ = \; & \frac{1}{2(N-3)} \sum_{j=4}^{N} W_3(j)[(x_j + a_3(1)x_{j-1} \\ & + a_3(2)x_{j-2} + a_3(3)x_{j-3})^2 + (x_{j-3} \\ & + a_3(1)x_{j-2} + a_3(2)x_{j-1} + a_3(3)x_j)^2], \end{aligned}$$

where $W_3(j)$ is 3-order optimal taper. In order to minimize the averaged optimum tapered FBPE of the third order, we set the derivative of ρ_3 with respect to $a_3(1)$, $a_3(2)$ and $a_3(3)$ yielding the following linear equations:

$$\begin{cases} \dfrac{\partial \rho_3}{\partial a_3(1)} = 0, \\ \dfrac{\partial \rho_3}{\partial a_3(2)} = 0, \\ \dfrac{\partial \rho_3}{\partial a_3(3)} = 0. \end{cases}$$

Let

$$b_{mn} = \sum_{j=4}^{N} W_3(j)(x_{j+m-3}x_{j+n-3} + x_{j-m}x_{j-n}),$$

$$c_{mn} = \sum_{j=4}^{N} W_3(j)(x_{j+m-3}x_{j-n} + x_{j-m}x_{j+n-3}),$$

where $b = \{b_{mn}\}$ and $c = \{c_{mn}\}$ are both three dimensional symmetric matrixes, and satisfy

$$\begin{cases} b_{12} = b_{21} = c_{11} = c_{22}, \\ b_{11} = b_{22} = c_{12} = c_{21}, \\ b_{23} = b_{32} = c_{13} = c_{31}, \\ b_{13} = b_{31} = c_{23} = c_{32}. \end{cases}$$

Parameters $a_3(1)$, $a_3(2)$ and $a_3(3)$ satisfy the following equations:

$$\begin{cases} c_{13} + b_{11}a_3(1) + b_{12}a_3(2) + b_{13}a_3(3) = 0, \\ c_{23} + b_{21}a_3(1) + b_{22}a_3(2) + b_{23}a_3(3) = 0, \\ c_{33} + b_{31}a_3 1 + b_{32}a_3(2) + b_{33}a_3(3) = 0. \end{cases} \quad (6)$$

According to the Levinson-Durbin recursion formula (5), the prediction error coefficients of the second and third order satisfy

$$\begin{cases} a_3(1) = a_2(1) + a_3(3)a_2(2), \\ a_3(2) = a_2(2) + a_3(3)a_2(1), \\ a_2(1) = a_1(1) + a_2(2)a_1(1). \end{cases} \quad (7)$$

Then $a_1(1)$ can be solved:

$$a_1(1) = \frac{BB_1 - B_2B_3}{B^2 - B_3^2 + BB_2 - B_1B_3}, \quad (8)$$

where B represents the determinant of symmetric matrix $\{b_{mn}\}$, B_1, B_2 and B_3 are the determinant of matrix $\{b_{mn}\}$ whose first row, second row and third row are replaced by $-c_{11}$, $-c_{12}$ and $-c_{13}$, respectively.

Based on the above calculation process, the first three order AR model parameters are obtained. Conversely, if we put the first three order AR model parameters in FBPE respectively, the values of FBPE will be smaller than the results got by TBO algorithm. Other model parameters are still calculated by the Levinson-Durbin recursion formula.

4 The Determination of AR Model Order

In this section, a new criterion for the selection of model order will be proposed.

Generally speaking, the optimal selection of model order is unknown to a priori, and it needs to select a model order in practical experiments. Suppose the optimal model order is p, where the model error is the lowest. A large number of experiments have proved that the spectrum seems to be too smooth when the model order $m < p$, which will lower the resolution of the power spectrum. While it will generate spurious frequency components and cause dramatic changes and shocks under the condition that the selected order is larger than the optimal model order [16]. Therefore, the determination of model order directly influences the quality of spectrum estimation.

The prediction error energy decreases or remains unchanged with the increased model order, and arrives the minimum at the optimal model order. In other words, when the selected order $m \geq n$, the m-order averaged FBPE ρ_m equals to or less than the n-order averaged FBPE ρ_n. Considering that the first order reflection coefficient is calculated by minimizing the averaged optimum tapered FBPE of the third order, this paper introduces a residual factor to determine the model order from the view of limit convergence. We regard the residual factor ε as an arbitrary positive number less than unity.

If

$$\begin{cases} |\rho_{m-2} - \rho_{m-3}| < \varepsilon \\ |\rho_{m-1} - \rho_{m-2}| < \varepsilon \\ |\rho_m - \rho_{m-1}| < \varepsilon \end{cases}$$

always holds, we consider the optimal order as $p = m$, where the value of ε is close to zero (we take it as 10^{-4} in practical experiment).

Summarily, the steps of model order determination are as follows:

(1) Calculate $a_3(1)$, $a_3(2)$, $a_3(3)$, $a_2(2)$, $a_2(1)$ and $a_1(1)$ by equations (6)–(8);

(2) Judge the order of AR model. Take ε as 10^{-4}, if $|\rho_{m-2} - \rho_{m-3}| < \varepsilon$, $|\rho_{m-1} - \rho_{m-2}| < \varepsilon$, $|\rho_m - \rho_{m-1}| < \varepsilon$ always hold, we assume $p = m$, then the recursion finish; otherwise, continue the next step;

(3) If $m = 3$, judge the model order directly by residual factor; If $m \geq 4$, calculate the $m-$order mean of optimum tapered forward and backward prediction error energies and reflection coefficients by equation (3)–(4);

(4) Let $m = m + 1$, iterate the steps of (2)–(3) until we get the appropriate model order.

5 MATLAB Simulation

In this section, we check the performance of the proposed method and compare the results with that of Burg algorithm and TBO algorithm.

In order to investigate the abilities of the proposed method, we define the output signal $x(t) = s(t)+\mu w(t)$, where $s(t)$ represents a sinusoidal signal with non-zero initial phase (generally the initial phase is defined odd multiple of $45°$), $w(t)$ is a zero-mean white Gaussian noise. The parameter μ is computed such that [5]

$$SNR = 10 \log \frac{P_s}{\mu^2 P_w},$$

$$P_s = \frac{1}{N} \sum_{n=1}^{N} s(n)^2,$$

$$P_w = \frac{1}{N} \sum_{n=1}^{N} w(n)^2,$$

where N is the length of the $s(n)$ and $w(n)$. SNR represents the Signal-to-Noise Ratio.

5.1 Spectral peak shifting and resolution of estimation problems

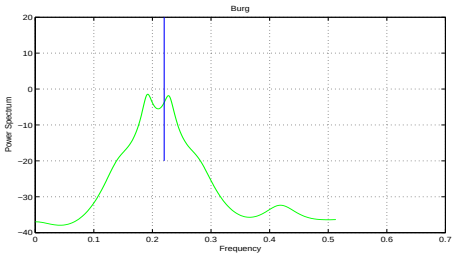
The problem of spectral peak shifting means that the estimated peak frequency apparently departs from the theoretic frequency value. The problem of resolution means that the degree how the closely-spaced frequencies can be distinguished. Take the signal $x(t) = \sin(2\pi \times 0.2t + \frac{\pi}{4}) + \sin(2\pi \times 0.22t + \frac{\pi}{4}) + \mu w(t)$ as an example. The sampling frequency and the length of the signal $x(n)$ are taken as 1000 Hz and 51 respectively when carrying out the simulation. By simulation experiments, for the second order averaged FBPE, TBO algorithm is 0.0132, while the improved TBO algorithm is 0.0008, which is much smaller than the second order averaged FBPE of TBO algorithm. Each order for the averaged FBPE of Burg algorithm is shown in Table 1.

Table 1 The averaged FBPE of Burg algorithm

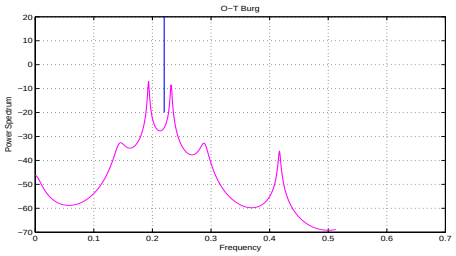
Order	1	2	3	4
FBPE	1.0400	0.9763	0.0890	0.0878
5	6	7	8	9
0.0507	0.0420	0.0417	0.0410	0.0401
10	11	12	13	14
0.0399	0.0395	0.0392	0.0380	0.0380

As is shown in Table 1, there is no significant change in prediction error energy under the condition that the model order is larger than 8. While AR model order reaches $p = 12$, the prediction error energy is little changed with the increase of model order. Taking the order as $p = 12$, a distinct difference in performance among Burg algorithm, TBO algorithm and

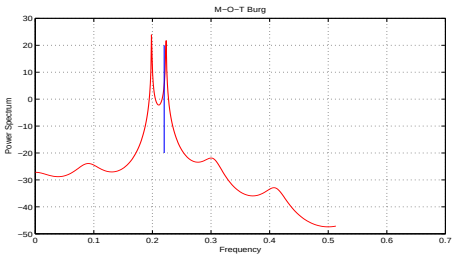
the improved TBO method is illustrated in Figure 1. It shows that the proposed algorithm is of highest resolution and least peak shifting compared with the Burg and TBO algorithm. However, the original Burg algorithm can distinguish the peak frequency reluctantly accompanying apparent peak shifting phenomenon. Although TBO algorithm differentiates the two close-spaced peak frequencies, it accompanies obvious peak shifting problem especially at $f_2 = 0.22Hz$. Besides, the three algorithms imply a common phenomenon of spurious frequency components. Now we take the order as $p = 8$ and $p = 6$, its corresponding simulations are presented in Figure 2 and Figure 3 respectively.



(a)



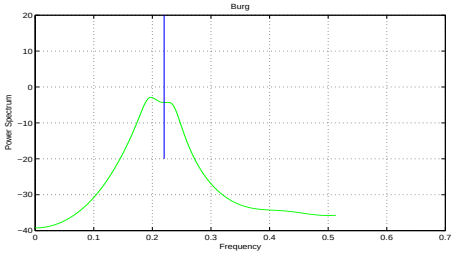
(b)



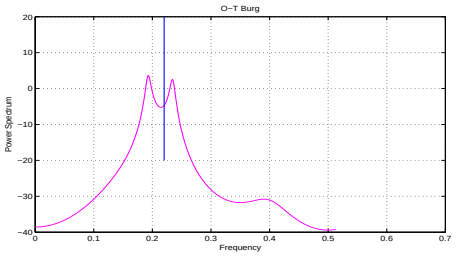
(c)

Figure 1 $f_1 = 0.2$, $f_2 = 0.22$, $SNR = 30$, $p = 12$
(a) Burg algorithm
(b) Optimal Burg algorithm
(c) Modified optimal Burg algorithm

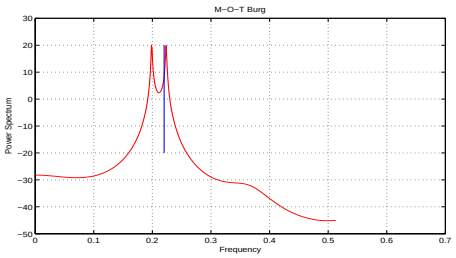
Compared with the three figures, we find that the improved TBO algorithm always owns the properties of highest resolution and slightest spectral peak shifting. The lager model order is, the higher resolution will be. Moreover, the phenomenon of spurious frequency components severely appears in Figure 1 while no spurious frequency component occurs in Figure 3. That is to say, the higher model order is, the easier spurious frequency component appears. The three figures indicate that model order is a significant factor for spurious frequency, and the following subsection will describe that SNR is also a key factor which can not be ignored.



(a)

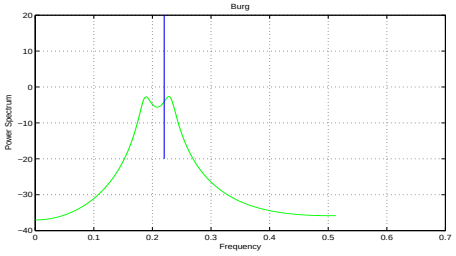


(b)

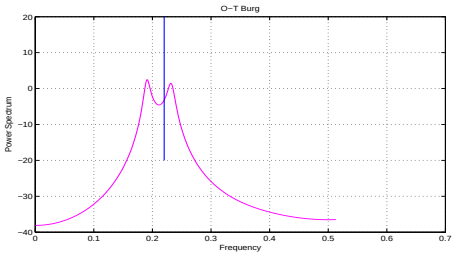


(c)

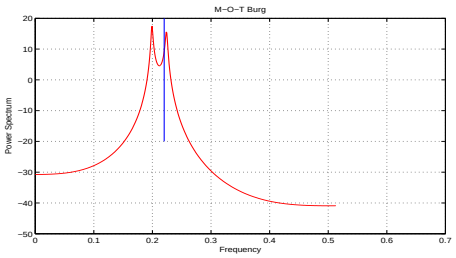
Figure 2 $f_1 = 0.2$, $f_2 = 0.22$, $SNR = 30$, $p = 8$
(a) Burg algorithm
(b) Optimal Burg algorithm
(c) Modified optimal Burg algorithm



(a)



(b)



(c)

Figure 3 $f_1 = 0.2$, $f_2 = 0.22$, $SNR = 30$, $p = 6$
(a) Burg algorithm
(b) Optimal Burg algorithm
(c) Modified optimal Burg algorithm

5.2 Spurious frequency components and spectral line splitting problems

In the simulation of spurious frequency components and line splitting problems, 45 points real sample sequence is generated by a unit amplitude sinusoid with $\varphi = \pi/4$ initial phase and frequency $f = 0.2Hz$. The sample rate is 1000 Hz, and AR model order of 5 is selected. Let $SNR = 20$, $SNR = 35$, $SNR = 50$, respectively. The differences affected by SNR are presented in Figure 4–6. As can be seen from the three figures, the improved TBO algorithm well avoids the appearance of spurious frequency components. We can hardly find the phenomenon of line splitting. In fact, TBO algorithm has well overcome the spectral line splitting problem, but it fails to control the spurious frequency components problem especially in the case of high SNR. As is described in Figure 4-6, in addition to the improved TBO algorithm, the higher SNR is, the easier spurious frequency components problem generates. What is the most distinctive by Burg algorithm is that it can hardly represent the inherent spectral shape (variance instability) while the improved TBO algorithm can well keep the stability of its spectral shape (variance stability) within a certain range of SNR. Through continuous experiments, it turns out that only when the SNR is more than or equal to 120dB, can the problems of spectral line splitting and spurious frequency components appear by the improved TBO algorithm. Besides, the stability of improved TBO algorithm can be evaluated by the variance of power spectrum. In order to make unified comparison, the variance for the logarithm of power spectrum is illustrated in Table 2. The improved TBO algorithm performs smallest fluctuations in variance as SNR varies.

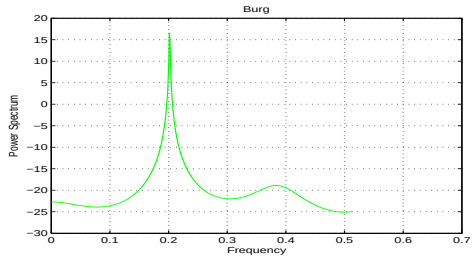
Table 2 The variance of logarithmic power spectrum

Variance	SNR=20	SNR=35	SNR=50
Burg	47.156	120.7314	159.8413
TBO	37.3861	33.8982	50.9992
NEW	35.8307	36.0245	36.0730

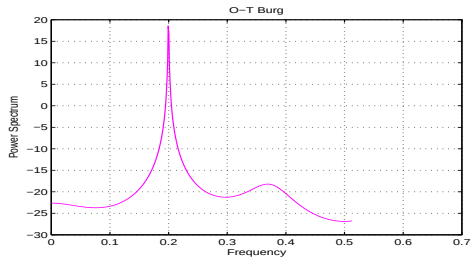
Summarily, spectral lines splitting and spurious frequency components are most likely to occur when the SNR and model order are both high. Besides, they occur under the condition that the initial phase of sinusoidal components is some odd multiple of 45° and the number of AR parameters estimated is a large percentage of the number of data values used for the estimation [17].

6 Conclusion

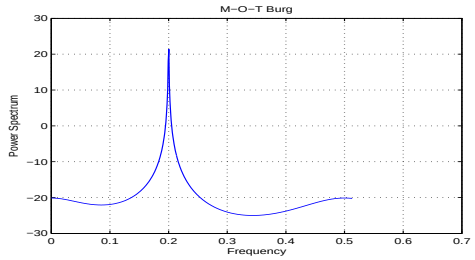
The limitations of Burg algorithm for the AR power spectrum estimation are the frequency bias, line splitting and spurious frequency components in processing the sinusoidal signals in noise. However, its attractive features are high resolution spectral estimates with short signal records, an efficient recursive implementation and guaranteed stable models. In this paper, a new method based on Burg algorithm and TBO algorithm has been proposed for estimating closely-spaced frequencies of the short length signals in the noisy environment. The simulation results have shown that improvements including reducing sensitivity to initial phase, reducing the bias of peak frequency, lessening the shift of spectral line and avoiding the generation



(a)

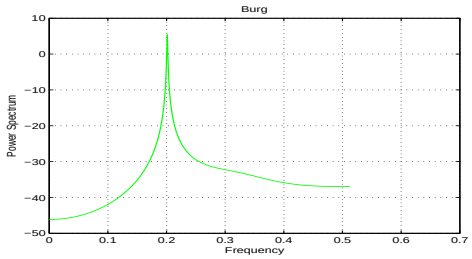


(b)

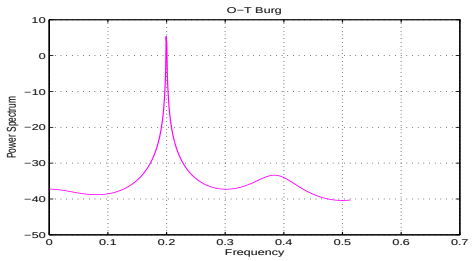


(c)

Figure 4 $f = 0.2$, $SNR = 20$, $p = 5$
(a) Burg algorithm
(b) Optimal Burg algorithm
(c) Modified optimal Burg algorithm



(a)



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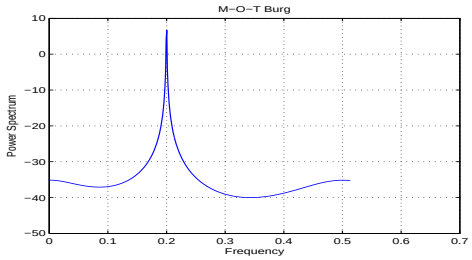

(c)

Figure 5 $f = 0.2$, $SNR = 35$, $p = 5$
(a) Burg algorithm
(b) Optimal Burg algorithm
(c) Modified optimal Burg algorithm

of spurious frequency components are achieved. All these improvements suggest that the improved algorithm is an attractive alternative to the Burg algorithm for AR spectral estimation.

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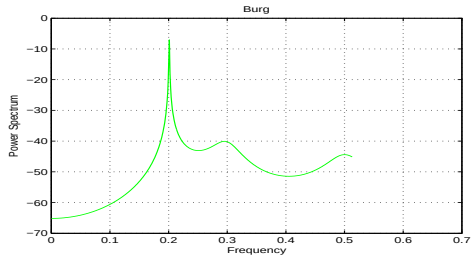
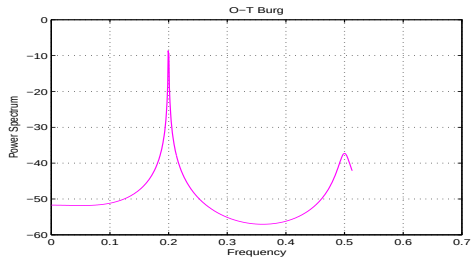
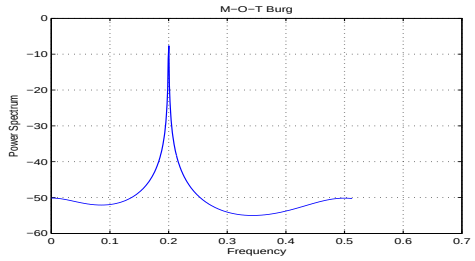


(a)



(b)



(c)

Figure 6 $f = 0.2, SNR = 50, p = 5$
(a) Burg algorithm
(b) Optimal Burg algorithm
(c) Modified optimal Burg algorithm

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