Global Exponential Stabilization of Uncertain Switched Nonlinear Systems with Time-Varying Delays

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Abstract: This paper is concerned with the problem of robust exponential stabilization for a class of uncertain hybrid systems with mixed time-varying delays in both the state and control. By using a Lyapunov-Krasovskii functional, a memoryless switching controller design is proposed to guarantee the global exponential stabilization. Based on matrix inequality technique, we establish some new delay-dependent exponential stabilization criteria for the system. Finally, some numerical examples are presented to illustrate the effectiveness of the theoretical results.

Key–Words: Uncertain switched system; Time-varying delays; Exponential stabilization; Switching controller design

1 Introduction

Time-delay phenomenon appears in many practical systems, such as AIDS epidemic, aircraft stabilization, chemical engineering system, inferred grinding model, manual control, neural network, nuclear reactor, population dynamical model, rolling mill, ship stabilization, and its existence is frequently a source of oscillation and instability [1,2]. In view of this, the stability issue of time-delay systems is a topic of theoretical and practical importance which has attracted a lot of interest over the decades [3-8].

Studies on dynamic systems with complicated switching law which are called switched systems have arisen in various disciplines of science and engineering in recent years [5-15]. Switched system usually consists of a family of subsystems of differential or difference equations and a rule that determines which subsystem is activated at a certain time interval. A different switching rule would produce different behavior of the system and hence lead to different system performances. Just owing to the complication of designing switching law for the systems, the stability analysis of switched systems becomes more difficult and attracts the interest of several scientists. To date, a number of works on the stability and stabilization for switched systems have appeared recently [5-11]. In the study on stability analysis for switched systems, multiple Lyapunov functions approach has been shown to be an effective tool [12-14]. Most recently, on the basis of dwell time analysis method, the stability and stabilization for switched systems have been further investigated [15-18]. The main approach for stability analysis relying on the use of Lyapunov-Krasovskii functional and linear matrix inequality (LMI) has been presented in [10, 19-24].

It is interesting to note that the stability for each subsystem cannot imply that of the overall system under arbitrary switching signal [9]. Another interesting fact is that the stability of a switched system can be achieved by choosing the switching signal even when each subsystem is unstable [6, 7, 9-11]. Although some important results have been obtained for linear/nonlinear switched systems, there are few results concerning the stability of switched nonlinear systems with time delay and uncertainties. In [25], the problem of stabilization via state feedback and/or state-based switching for switched linear systems with multiple time-varying delays without uncertainties was considered. It was proved in [25] that the switched linear delay system will be stabilizable via state feedback and/or switching if the corresponding system with zero delays has a Hurwitz stable convex combination and the delays less than an appropriate upper bound that satisfies a set of LMIs. On the other hand, it is worth noting that the existing stability conditions for time-delay systems must be solved upon a grid of the parameter space, which results in testing a nonlinear Riccati-type equation or a finite number of LMIs. In this case, the results using finite gridding points are unreliable and the numerical complexity of the tests grows rapidly. Therefore, finding new conditions for the robust exponential stability of uncertain switched time-delay systems are of interest.

In [26], the global exponential stability of BAM neural networks with inertial term and time delay was investigated. By chosen proper variable substitution the system was transformed to first order differential equation. Then some new sufficient conditions which ensured the globally exponential stability of the system were obtained by constructing suitable Lyapunov functional, using Halanay inequality and the fundamental solution matrix of coefficient matrix. In [27], by constructing suitable Lyapunov functional, using differential mean value theorem and homeomorphism, the global exponential stability of high-order bi-directional associative memory (BAM) neural networks with reaction-diffusion terms and S-type distributed delays was analyzed. Some sufficient theorems had been derived under different conditions to guarantee the global exponential stability of the networks.

In this paper, we study the problem of robust exponential stabilization for a class of uncertain nonlinear hybrid time-delay systems. Based on the Lyapunov-Krasovskii functional approach and the linear matrix inequality technique, the switching signal design method is proposed and delay-dependent stabilization conditions are provided, which guarantee that the uncertain switched nonlinear systems with time-varying delays in state and control are exponentially stablizable.

This paper is organized as follows. Section 2 presents notations, definitions and some technical lemmas required for the proof of the main results. Switching design for memoryless feedback controller for the exponential stabilization is presented in Section 3. Some numerical examples are presented in Section 4. Finally, some conclusion and remarks are drawn in section 5.

2 Preliminaries

The following notations will be used throughout this paper. R^+ denotes the set of all non-negative real numbers; R^n denotes the n-finite-dimensional space with the Euclidean norm $\|\cdot\|$ and scalar product x^Ty or $<x,y>$ of two vectors x,y; $\lambda_{max}(A)$ ($\lambda_{min}(A)$,respectively)denotes the maximal (the minimal, respectively) number of the real part of eigenvalues of A ; A^T denotes the transpose of the matrix A ; $R^{n\times m}$ denotes the set of all $(n\times m)$-matrices with the spectral norm defined by

$$\eta(A)=\sqrt{\lambda_{max}(AA^T)}.$$

$Q\geq 0(Q>0$,respectively)means Q is semi-positive definite (positive definite, respectively), $A\geq B$ means $A-B\geq 0, I$ denotes the identity matrix, $\bar{N}=\{1,2,\cdots,N\}$.

In the sequel, sometimes for the sake of brevity, we will omit the arguments of matrix functions, if it does not cause confusion.

Consider the switched nonlinear system with time-varying delays:

$$\begin{aligned}\dot{x}(t)&=\bar{A}_\sigma x(t)+\bar{D}_\sigma x(t-\tau(t))+\bar{E}_\sigma\int_{t-\tau(t)}^t\\&\quad\times x(s)ds+\bar{B}_\sigma u(t)+\bar{C}_\sigma u(t-r(t))\\&\quad+\bar{F}_\sigma\int_{t-r(t)}^t u(s)ds,\\x(t)&=\phi(t),\ t\in[-h,0],\end{aligned}\tag{1}$$

where $h=\max\{\bar{\tau},\bar{r}\}$ and

$$\bar{A}_\sigma=A_\sigma+\Delta A_\sigma(t),\quad \bar{D}_\sigma=D_\sigma+\Delta D_\sigma(t),$$

$$\bar{E}_\sigma=E_\sigma+\Delta E_\sigma(t),\quad \bar{B}_\sigma=B_\sigma+\Delta B_\sigma(t),$$

$$\bar{C}_\sigma=C_\sigma+\Delta C_\sigma(t),\quad \bar{F}_\sigma=F_\sigma+\Delta F_\sigma(t),$$

$x(t)\in R^n$ is the state at time t, $u(t)\in R^p$ is the control, σ is a switching signal which is a piecewise constant function and depends on x, σ takes its values in finite set $\bar{N}$, moreover, $\sigma(x)=i$ implies that the ith subsystem of system (1) is active. The initial vector $\phi(t)\in C_0$, where C_0 is the set of continuous functions from $[-h,0]$ to R^n. The delay functions $\tau(t),r(t)$ are continuous functions satisfying

$$\begin{aligned}&0\leq\tau(t)\leq\bar{\tau},\quad \dot{\tau}(t)\leq\delta<1,\quad \forall t\geq 0,\\&0\leq r(t)\leq\bar{r},\quad \dot{r}(t)\leq\delta_1<1,\quad \forall t\geq 0.\end{aligned}$$

Matrices $A_i,B_i,C_i,D_i,E_i,F_i,i\in\bar{N}$, are real constant matrices with appropriate dimensions, $\Delta A_i(t),\Delta B_i(t),\Delta C_i(t),\Delta D_i(t),\Delta E_i(t),\Delta F_i(t)$, $i\in\bar{N}$, are some perturbed matrices and satisfy the following conditions

$$\begin{aligned}&[\Delta A_i(t)\ \Delta B_i(t)\ \Delta C_i(t)\ \Delta D_i(t)\ \Delta E_i(t)\ \Delta F_i(t)]\\&\quad=M_iJ_i(t)[N_{Ai}\ \ N_{Bi}\ \ N_{Ci}\ \ N_{Di}\ \ N_{Ei}\ \ N_{Fi}],\\&\forall i\in\bar{N},\ \forall t\geq 0,\end{aligned}\tag{2}$$

where $M_i,N_{Ai},N_{Bi},N_{Ci},N_{Di},N_{Ei}$ and $N_{Fi},i\in\bar{N}$, are some given constant matrices with appropriate dimensions. $J_i(t),i\in\bar{N}$, are unknown matrices representing the parameter perturbations which satisfy

$$J_i^T(t)J_i(t)\leq I,\ \forall i\in\bar{N},\ \forall t\geq 0.\tag{3}$$

Remark 1 *The conditions (2) and (3) are referred to as the admissible conditions. These conditions have been frequently used to describe parameter uncertainties for systems in many papers that deal with the stability analysis, see e.g [7,20,21,24,31].*

The stabilization problem for switched system (1) is to construct a switching rule and feedback control that make the closed-loop system is exponentially stable.

Definition 2 *[28] Given $\alpha > 0$. Switched control system (1) is exponentially stabilizable with rate $\alpha > 0$, if there exist a positive number q, switching rule $\sigma(\cdot)$ and feedback control $u(t) = K_\sigma x(t)$ such that every solution $x(t,\phi)$ of the closed-loop system satisfies the following condition:*

$$\| x(t,\phi) \| \le q \| \phi \| \mathrm{e}^{-\alpha t}, \qquad \forall t \ge 0,$$

where

$$\| \phi \| = \sup_{-h \le t \le 0} \| \phi(t) \| .$$

Definition 3 *[29] The system of matrices $\{L_i\}, i \in \bar{N}$, is said to be strictly complete if for every $x \in R^n \backslash \{0\}$ there exists $i \in \bar{N}$ such that $x^T L_i x < 0$.*

Let us define the

$$\xi_i = \{x \in R^n; x^T L_i x < 0\}, i \in \bar{N}.$$

It is easy to show that the system $\{L_i\}, i \in \bar{N}$, is strictly complete if and only if

$$\bigcup_{i=1}^{N} \xi_i = R^n \backslash \{0\}.$$

Remark 4 *In [29], it is shown that a sufficient condition for the strict completeness of the system $\{L_i\}$ is that there exist $\tau_i \ge 0$, $\sum_{i=1}^{N} \tau_i > 0$ such that*

$$\sum_{i=1}^{N} \tau_i L_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Lemma 5 *(Cauchy matrix inequality) [30] For any $0 < W \in R^{n\times n}, x, y \in R^n$, we have*

$$\pm 2x^T y \le x^T W x + y^T W^{-1} y.$$

Lemma 6 *[31] Let U, V, W and M be real matrices of appropriate dimensions with $M = M^T$, then*

$$M + UVW + W^T V^T U^T < 0,$$

for all $V^T V < I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + \varepsilon^{-1} UU^T + \varepsilon W^T W < 0.$$

Lemma 7 *[32] Given constant symmetric matrices S_1, S_2, S_3 and $S_1 = S_1^T < 0, S_3 = S_3^T > 0$, then $S_1 + S_2 S_3^{-1} S_2^T < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & -S_3 \end{bmatrix} < 0.$$

3 Main result

For given symmetric positive definite matrices $P, Q_i \in R^{n\times n}$, we set

$$L_i = A_i^T P + P A_i + (1+h)I + Q_i,$$

$$\Omega_i = \left\{ x \in R^n \mid \begin{array}{l} x^T(A_i^T P + P A_i + (1+h)I)x \\ < -x^T Q_i x \end{array} \right\},$$

and

$$\bar{\Omega}_1 = \Omega_1, \quad \bar{\Omega}_i = \Omega_i \backslash \bigcup_{j=1}^{i-1} \bar{\Omega}_j.$$

Now we present a delay-dependent condition for the global exponential stabilization of system (1).

Theorem 8 *Switched nonlinear control system (1) is globally exponentially stabilizable with rate $\alpha > 0$, if there exist symmetric positive definite matrices $P, Q_i \in R^{n\times n}$ and some positive constants $\varepsilon_i > 0, \tau_i > 0$ with $\sum_{i=1}^{N} \tau_i > 0$, such that the following LMIs hold:*

$$\sum_{i=1}^{N} \tau_i L_i < 0, \tag{4}$$

$$\begin{bmatrix} G_i(P) & W_i & S_i \\ W_i^T & -\varepsilon_i^{-1} I & 0 \\ S_i^T & 0 & -\varepsilon_i I \end{bmatrix} < 0, i \in \bar{N}, \tag{5}$$

where

$$\begin{aligned} W_i \ &= [PM_i \ \ \mu \mathrm{e}^{2\alpha h} PM_i \ \ h\mathrm{e}^{2\alpha h} PM_i \\ &\quad h\mathrm{e}^{2\alpha h} PM_i \ \ \mu_1 \mathrm{e}^{2\alpha h} PM_i \ \ PM_i], \end{aligned} \tag{6}$$

$$\begin{aligned} S_i \ &= [N_{Ai}^T \ \ D_i N_{Di}^T \ \ E_i N_{Ei}^T \ \ F_i N_{Fi}^T \\ &\quad C_i N_{Ci}^T \ \ (N_{Bi} B_i^T P)^T], \end{aligned} \tag{7}$$

$$\begin{aligned} G_i(P) \ &= 2\alpha P - Q_i + \lambda_0 \mathrm{e}^{2\alpha h}(\lambda_1 \mu + \lambda_2 h \\ &\quad + \lambda_3 \mu_1 + \lambda_4 h) P^2 + P(\mu \mathrm{e}^{2\alpha h} D_i D_i^T \\ &\quad + h\mathrm{e}^{2\alpha h} E_i E_i^T + \mu_1 \mathrm{e}^{2\alpha h} C_i C_i^T \\ &\quad + h\mathrm{e}^{2\alpha h} F_i F_i^T + (h+3) B_i B_i^T) P \end{aligned} \tag{8}$$

$$\begin{array}{ll} \mu=(1-\delta)^{-1}, & \mu_1=(1-\delta_1)^{-1}, \\ \lambda_0=\max_{i\in\bar{N}}\{\eta^2(M_i)\}, & \lambda_1=\max_{i\in\bar{N}}\{\eta^2(N_{Di})\}, \\ \lambda_2=\max_{i\in\bar{N}}\{\eta^2(N_{Ei})\}, & \lambda_3=\max_{i\in\bar{N}}\{\eta^2(N_{Ci})\}, \\ \lambda_4=\max_{i\in\bar{N}}\{\eta^2(N_{Fi})\}. & \end{array}$$

The switching rule is chosen as $\sigma(x(t))=i\in\bar{N}$ whenever $x(t)\in\bar{\Omega}_i$. The feedback control is

$$u(t)=B_\sigma^T Px(t),\ \ t\geq 0, \tag{9}$$

and the solution of the system satisfies

$$\| x(t,\phi)\| \leq q \,\|\phi\|\, \mathrm{e}^{-\alpha t},\ t\in R^+,$$

where

$$\begin{array}{l} q=\sqrt{\frac{m}{\lambda_{min}(P)}}, \quad \lambda_B=\max_{i\in\bar{N}}\{\eta^2(B_i)\}, \\ m=\lambda_{max}(P)+\bar{\tau}+\bar{r}\lambda_B\lambda_{max}^2(P)+\frac{1}{2}h^2 \\ \quad +\frac{1}{2}h^2\lambda_B\lambda_{max}^2(P). \end{array} \tag{10}$$

Proof: From (4), it follows that the system matrices $\{L_i\}$ is strictly complete and

$$\bigcup_{i=1}^{N}\Omega_i=R^n\backslash\{0\}.$$

Based on the set Ω_i, we construct the set $\bar{\Omega}_i$ and it is easily verified that

$$\textstyle\bigcup_{i=1}^N\bar{\Omega}_i=R^n\backslash\{0\}, \quad \bar{\Omega}_i\bigcap\bar{\Omega}_j=\emptyset,\ \ i\neq j. \tag{11}$$

The switching rule is chosen as $\sigma(x(t))=i$, whenever $x(t)\in\bar{\Omega}_i$ (this switching rule is well defined due to (11)). So when $x(t)\in\bar{\Omega}_i$, the ith subsystem is activated and then we have the following subsystem

$$\begin{array}{rl} \dot{x}(t) & =\bar{A}_ix(t)+\bar{D}_ix(t-\tau(t))+\bar{E}_i\int_{t-\tau(t)}^t \\ & \quad \times x(s)ds+\bar{B}_iu(t)+\bar{C}_iu(t-r(t)) \\ & \quad +\bar{F}_i\int_{t-r(t)}^t u(s)ds, \quad t\geq 0. \end{array} \tag{12}$$

We consider the following Lyapunov-Krasovskii functional

$$V(x(t))=V_1(\cdot)+V_2(\cdot)+V_3(\cdot)+V_4(\cdot)+V_5(\cdot),$$

where

$$V_1(\cdot)=x^T(t)Px(t),$$

$$V_2(\cdot)=\int_{t-\tau(t)}^t \mathrm{e}^{2\alpha(s-t)}\ \|x(s)\|^2\ ds,$$

$$V_3(\cdot)=\int_{-h}^0\int_{t+s}^t \mathrm{e}^{2\alpha(v-t)}\ \|x(v)\|^2\ dvds,$$

$$V_4(\cdot)=\int_{t-r(t)}^t \mathrm{e}^{2\alpha(s-t)}\ \|u(s)\|^2\ ds,$$

$$V_5(\cdot)=\int_{-h}^0\int_{t+s}^t \mathrm{e}^{2\alpha(v-t)}\ \|u(v)\|^2\ dvds.$$

Taking derivative of $V_1(\cdot)$ along the trajectory of any subsystem ith, we have

$$\begin{array}{rl} \dot{V}_1= & x^T(t)(\bar{A}_i^TP+P\bar{A}_i)x(t)+2x^T(t)P\bar{D}_i \\ & \times x(t-\tau(t))+2x^TP\bar{E}_i\int_{t-\tau(t)}^t x(s)ds \\ & +x^T(t)P\bar{B}_iu(t)+u^T(t)\bar{B}_i^TPx(t) \\ & +2x^T(t)P\bar{C}_iu(t-r(t)) \\ & +2x^T(t)P\bar{F}_i\int_{t-r(t)}^t u(s)ds. \end{array} \tag{13}$$

Applying Lemma 5 gives

$$\begin{array}{l} 2x^T(t)P\bar{D}_ix(t-\tau(t)) \\ \quad \leq (1-\delta)\mathrm{e}^{-2\alpha h}\ \|x(t-\tau(t))\|^2 \\ \qquad +(1-\delta)^{-1}\mathrm{e}^{2\alpha h}x^T(t)P\bar{D}_i\bar{D}_i^TPx(t), \end{array} \tag{14}$$

$$\begin{array}{l} 2x^P(t)P\bar{E}_i\int_{t-\tau(t)}^t x(s)ds \\ \quad =\int_{t-\tau(t)}^t 2x^T(t)P\bar{E}_ix(s)ds \\ \quad \leq \int_{t-\tau(t)}^t \mathrm{e}^{2\alpha h}x^T(t)P\bar{E}_i\bar{E}_i^TPx(t)ds \\ \qquad +\int_{t-\tau(t)}^t \mathrm{e}^{-2\alpha h}\ \|x(s)\|^2\ ds \\ \quad \leq h\mathrm{e}^{2\alpha h}x^T(t)P\bar{E}_i\bar{E}_i^TPx(t)+\mathrm{e}^{-2\alpha h} \\ \qquad \times\int_{-h}^0\ \|x(t+s)\|^2\ ds, \end{array} \tag{15}$$

$$\begin{array}{l} 2x^T(t)P\bar{C}_iu(t-r(t)) \\ \quad \leq (1-\delta_1)\mathrm{e}^{-2\alpha h}\ \|u(t-r(t))\|^2 \\ \qquad +(1-\delta_1)^{-1}\mathrm{e}^{2\alpha h}x^T(t)P\bar{C}_i\bar{C}_i^TPx(t), \end{array} \tag{16}$$

$$\begin{array}{l} 2x^T(t)P\bar{F}_i\int_{t-r(t)}^t u(s)ds \\ \quad =\int_{t-r(t)}^t 2x^T(t)P\bar{F}_iu(s)ds \\ \quad \leq \int_{t-r(t)}^t \mathrm{e}^{2\alpha h}x^T(t)P\bar{F}_i\bar{F}_i^TPx(t)ds \\ \qquad +\int_{t-r(t)}^t \mathrm{e}^{-2\alpha h}\ \|u(s)\|^2\ ds \\ \quad \leq h\mathrm{e}^{2\alpha h}x^T(t)P\bar{F}_i\bar{F}_i^TPx(t) \\ \qquad +\mathrm{e}^{-2\alpha h}\int_{-h}^0\ \|u(t+s)\|^2\ ds. \end{array} \tag{17}$$

Therefore, let $\mu=(1-\delta)^{-1},\mu_1=(1-\delta_1)^{-1}$, from (13) to (17) we have

$$\begin{array}{rl} \dot{V}_1 & \leq x^T(t)[\bar{A}_iP+P\bar{A}_i \\ & \quad +P(\bar{B}_iB_i^T+B_i\bar{B}_i^T)P]x(t) \\ & \quad +(1-\delta)\mathrm{e}^{-2\alpha h}\ \|x(t-\tau(t))\|^2 \\ & \quad +\mathrm{e}^{-2\alpha h}\int_{-h}^0\ \|x(t+s)\|^2\ ds \\ & \quad +(1-\delta_1)\mathrm{e}^{-2\alpha h}\ \|u(t-r(t))\|^2 \\ & \quad +\mathrm{e}^{-2\alpha h}\int_{-h}^0\ \|u(t+s)\|^2\ ds \\ & \quad +\mathrm{e}^{2\alpha h}x^T(t)P(\mu\bar{D}_i\bar{D}_i^T+h\bar{E}_i\bar{E}_i^T \\ & \quad +\mu_1\bar{C}_i\bar{C}_i^T+h\bar{F}_i\bar{F}_i^T)Px(t). \end{array} \tag{18}$$

Next, taking derivative of $V_i, i = 2,3,4,5$, respectively, along the system trajectories yield

$$\begin{aligned}\dot V_2 \le\ & \| x(t) \|^2 -(1-\delta)\mathrm{e}^{-2\alpha h} \| x(t-\tau(t)) \|^2 \\ & -2\alpha V_2,\\ \dot V_3 \le\ & h \| x(t) \|^2 -\mathrm{e}^{-2\alpha h}\int_{-h}^0 \| x(t+s) \|^2\, ds \\ & -2\alpha V_3,\\ \dot V_4 \le\ & \| u(t) \|^2 -(1-\delta_1)\mathrm{e}^{-2\alpha h} \| u(t-r(t)) \|^2 \\ & -2\alpha V_4,\\ \dot V_5 \le\ & h \| u(t) \|^2 -\mathrm{e}^{-2\alpha h}\int_{-h}^0 \| u(t+s) \|^2\, ds \\ & -2\alpha V_5,\end{aligned} \tag{19}$$

From(18) and (19), it yields that

$$\begin{aligned}\dot V(x(t)) + 2\alpha V(x(t)) \le\ & x^T(t)[\bar A_i^T P + P\bar A_i \\ & +P(\bar B_i B_i^T + B_i\bar B_i^T)P + 2\alpha P]x(t) \\ & +(1+h) \| x(t) \|^2 +(1+h) \| u(t) \|^2 \\ & +\mathrm{e}^{2\alpha h}x^T(t)P(\mu\bar D_i\bar D_i^T + h\bar E_i\bar E_i^T \\ & +\mu_1\bar C_i\bar C_i^T + h\bar F_i\bar F_i^T)Px(t).\end{aligned} \tag{20}$$

From (2), we have

$$\begin{aligned} x^T(t)P\bar D_i\bar D_i^T Px(t) =\ & x^T(t)P(D_iD_i^T + M_iJ_i \\ & \times(D_iN_{Di}^T)^T + D_iN_{Di}^TJ_i^TM_i^T)Px(t) \\ & +x^T(t)PM_iJ_iN_{Di}N_{Di}^TJ_i^TM_i^TPx(t) \\ \le\ & x^T(t)P(D_iD_i^T + M_iJ_i(D_iN_{Di}^T)^T \\ & +D_iN_{Di}^TJ_i^TM_i^T)Px(t) + \lambda_0\lambda_1x^T(t)P^2x(t) \\ =\ & x^TP(D_iD_i^T + M_iJ_i(D_iN_{Di}^T)^T \\ & +D_iN_{Di}^TJ_i^TM_i^T + \lambda_0\lambda_1 I)Px(t),\end{aligned} \tag{21}$$

where

$$\lambda_0 = \max_{i\in\bar N}\{\eta^2(M_i)\},$$

$$\lambda_1 = \max_{i\in\bar N}\{\eta^2(N_{Di})\}.$$

Similarly, for

$$x^T(t)P\bar E_i\bar E_i^TPx(t), \qquad x^T(t)P\bar C_i\bar C_i^TPx(t)$$

and $x^T(t)P\bar F_i\bar F_i^TPx(t)$, we have

$$\begin{aligned} x^T(t)P\bar E_i\bar E_i^TPx(t) \le\ & x^TP(E_iE_i^T + M_iJ_i \\ & \times(E_iN_{Ei}^T)^T + E_iN_{Ei}^TJ_i^TM_i^T + \lambda_0\lambda_2 I)Px(t),\end{aligned} \tag{22}$$

$$\begin{aligned} x^T(t)P\bar C_i\bar C_i^TPx(t) \le\ & x^TP(C_iC_i^T + M_iJ_i \\ & \times(C_iN_{Ci}^T)^T + C_iN_{Ci}^TJ_i^TM_i^T + \lambda_0\lambda_3 I)Px(t),\end{aligned} \tag{23}$$

$$\begin{aligned} x^T(t)P\bar F_i\bar F_i^TPx(t) \le\ & x^TP(F_iF_i^T + M_iJ_i \\ & \times(F_iN_{Fi}^T)^T + F_iN_{Fi}^TJ_i^TM_i^T + \lambda_0\lambda_4 I)Px(t),\end{aligned} \tag{24}$$

where

$$\lambda_2 = \max_{i\in\bar N}\{\eta^2(N_{Ei})\},$$

$$\lambda_3 = \max_{i\in\bar N}\{\eta^2(N_{Ci})\},$$

$$\lambda_4 = \max_{i\in\bar N}\{\eta^2(N_{Fi})\}.$$

From (9), we have

$$\| u(t) \|^2 = x^T(t)PB_iB_i^TPx(t). \tag{25}$$

And from $x(t) \in \bar\Omega_i$ and the definition of $\bar\Omega_i$, it follows that

$$x^T(t)(A_i^TP + PA_i + (1+h)I)x(t) < -x^T(t)Q_ix(t),$$

then we obtain

$$\begin{aligned}&\dot V(x(t)) + 2\alpha V(x(t)) \\ &\quad \le x^T(t)(2\alpha P - Q_i + \lambda_0\mathrm{e}^{2\alpha h}(\lambda_1\mu + \lambda_2 h \\ &\quad + \lambda_3\mu_1 + \lambda_4 h)P^2 + P(\mu\mathrm{e}^{2\alpha h}D_iD_i^T + h \\ &\quad \times \mathrm{e}^{2\alpha h}E_iE_i^T + \mu_1\mathrm{e}^{2\alpha h}C_iC_i^T + h\mathrm{e}^{2\alpha h}F_iF_i^T \\ &\quad + (h+3)B_iB_i^T)P + [PM_i\ \ \mu\mathrm{e}^{2\alpha h}PM_i \\ &\quad h\mathrm{e}^{2\alpha h}PM_i\ \ h\mathrm{e}^{2\alpha h}PM_i\ \ \mu_1\mathrm{e}^{2\alpha h}PM_i\ \ PM_i]J_i \\ &\quad \times [N_{Ai}^T\ \ D_iN_{Di}^T\ \ E_iN_{Ei}^T\ \ F_iN_{Fi}^T\ \ C_iN_{Ci}^T \\ &\quad (N_{Bi}B_i^TP)^T]^T + [N_{Ai}^T\ \ D_iN_{Di}^T\ \ E_iN_{Ei}^T \\ &\quad F_iN_{Fi}^T\ \ C_iN_{Ci}^T\ \ (N_{Bi}B_i^TP)^T]J_i \\ &\quad \times [PM_i\ \ \mu\mathrm{e}^{2\alpha h}PM_i\ \ h\mathrm{e}^{2\alpha h}PM_i\ \ h\mathrm{e}^{2\alpha h}PM_i \\ &\quad \mu_1\mathrm{e}^{2\alpha h}PM_i\ \ PM_i]^T)x(t).\end{aligned}$$

Thus, we get

$$\begin{aligned}\dot V(x(t)) + 2\alpha V(x(t)) \ \le\ & x^T(t)[G_i(P) + W_iJ_iS_i^T \\ & + S_iJ_i^TW_i^T]x(t),\end{aligned}$$

where $G_i(P), W_i, S_i$ are defined by (8), (6), (7), respectively. Applying Lemma 7, matrix inequality (5) implies

$$G_i(P) + \varepsilon_i^{-1}W_iW_i^T + \varepsilon_iS_iS_i^T < 0. \tag{26}$$

Using Lemma 6, we then have

$$G_i(P) + W_iJ_iS_i^T + S_iJ_i^TW_i^T < 0.$$

Hence,

$$\dot V(x(t)) + 2\alpha V(x(t)) \le 0, \quad t \ge 0. \tag{27}$$

Using the expression of $V(x(t))$ and estimation (27), we get

$$\lambda_{min}(P) \| x(t) \|^2 \le V(x(t)) \le V(x(0))\mathrm{e}^{-2\alpha t},$$

where the estimate of $V(x(0))$ is easily verified by

$$\begin{aligned}V(x(0)) \ \le\ & [\lambda_{max}(P) + \bar\tau + \tfrac{1}{2}h^2 + \bar r\lambda_B\lambda_{max}^2(P) \\ & + \tfrac{1}{2}h^2\lambda_B\lambda_{max}^2(P)] \| \phi \|^2 .\end{aligned}$$

Therefore,

$$\| x(t) \| \leq q\mathrm{e}^{-\alpha t} \| \phi \|, \forall t \geq 0,$$

where q is defined by(10). This completes the proof. □

Remark 9 *In [7], the problem of exponential stabilization for a class of linear systems with time-varying delay is studied, and this system contains only a time delay in the state. But, in this paper we deal with the global exponential stabilization for a class of uncertain switched nonlinear systems, and our system contains with time-varying delay in both the state and control. So, our results have a greater range of application.*

If

$$M_i = 0,\ N_{Ai} = 0,\ N_{Bi} = 0,\ N_{Ci} = 0,$$

$$N_{Di} = 0,\ N_{Ei} = 0,\ N_{Fi} = 0,$$

then the systems (1) can be written as

$$\begin{aligned} \dot{x}(t) &= A_\sigma x(t) + D_\sigma x(t-\tau(t)) + E_\sigma \int_{t-\tau(t)}^{t} \\ &\quad \times x(s)ds + B_\sigma u(t) + C_\sigma u(t-r(t)) \\ &\quad + F_\sigma \int_{t-r(t)}^{t} u(s)ds, \\ x(t) &= \phi(t), \quad t \in [-h, 0]. \end{aligned} \tag{28}$$

Corollary 10 *If for some constant $\alpha > 0$, there exist some numbers $\tau_i \geq 0$, with*

$$\sum_{i=1}^{N} \tau_i > 0,$$

and some positive definite matrices $P, Q_i \in R^{n\times n}$ such that the following LMIs hold:

$$\begin{aligned} &\sum_{i=1}^{N} \tau_i L_i < 0, \\ &\bar{G}_i(P) < 0, \ \ i \in \bar{N}, \end{aligned} \tag{29}$$

where

$$\begin{aligned} \bar{G}_i(P) &= 2\alpha P - Q_i + P(\mu \mathrm{e}^{2\alpha h} D_i D_i^T + h\mathrm{e}^{2\alpha h} \\ &\quad \times E_i E_i^T + \mu_1 \mathrm{e}^{2\alpha h} C_i C_i^T + h\mathrm{e}^{2\alpha h} F_i F_i^T \\ &\quad + (h+3) B_i B_i^T) P, \end{aligned}$$

then the switched nonlinear control system (28) is globally exponentially stabilizable with convergence rate $\alpha > 0$. The switching rule is chosen as $\sigma(x(t)) = i \in \bar{N}$ whenever $x(t) \in \bar{\Omega}_i$. The feedback control is

$$u(t) = B_\sigma^T P x(t),\ t \geq 0,$$

and the solution $x(t,\phi)$ of the system satisfies

$$\| x(t,\phi) \| \leq q \| \phi \| \mathrm{e}^{-\alpha t}, t \in R^+,$$

where

$$q = \sqrt{\frac{m}{\lambda_{min}(P)}},$$

$$\begin{aligned} m &= \lambda_{max}(P) + \bar{\tau} + \bar{r}\lambda_B \lambda_{max}^2(P) \\ &\quad + \tfrac{1}{2}h^2 + \tfrac{1}{2}h^2 \lambda_B \lambda_{max}^2(P), \\ &\lambda_B = \max_{i\in\bar{N}} \{\eta^2(B_i)\}. \end{aligned}$$

Proof: Similar to the proof of Theorem 8, Corollary 10 can be proved. □

Moreover, if $E_i = 0, B_i = 0, C_i = 0, F_i = 0$, then the system (28) can be written as

$$\begin{aligned} &\dot{x}(t) = A_\sigma x(t) + D_\sigma x(t-\tau(t)), \\ &x(t) = \phi(t),\ \ t \in [-h, 0]. \end{aligned} \tag{30}$$

We can get the following corollary.

Corollary 11 *If for some constant $\alpha > 0$, there exist some numbers $\tau_i \geq 0$, with*

$$\sum_{i=1}^{N} \tau_i > 0,$$

and some positive definite matrices $P, Q_i \in R^{n\times n}$ such that the following LMIs hold:

$$\begin{aligned} &\sum_{i=1}^{N} \tau_i L_i < 0, \\ &\tilde{G}_i(P) = \begin{bmatrix} \Gamma_i & PD_i \\ * & -(1-\delta)e^{-2\alpha\bar{\tau}} I \end{bmatrix} < 0,\ i \in \bar{N}, \end{aligned} \tag{31}$$

where

$$\Gamma_i = 2\alpha P - Q_i - \bar{\tau} I,$$

then the switched system (30) is globally exponentially stable with convergence rate $\alpha > 0$. The switching rule is chosen as $\sigma(x(t)) = i \in \bar{N}$ whenever $x(t) \in \bar{\Omega}_i$, and the solution $x(t,\phi)$ of the system satisfies

$$\| x(t,\phi) \| \leq q \| \phi \| \mathrm{e}^{-\alpha t}, t \in R^+,$$

where

$$\begin{aligned} &q = \sqrt{\tfrac{m_1}{\lambda_{min}(P)}}, \\ &m_1 = \lambda_{max}(P) + \bar{\tau}. \end{aligned}$$

Proof: We consider the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(x(t)) \ &= \ \int_{t-\tau(t)}^{t} \mathrm{e}^{2\alpha(s-t)} \| x(s) \|^2 ds \\ &\quad + x^T(t) P x(t). \end{aligned}$$

The proof is similar to that for Theorem 8 and is omitted here.

4 Numerical examples

Example 12 *Consider the following switching system*

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + D_i x(t-\tau(t)) + E_i \textstyle\int_{t-\tau(t)}^{t} \\ &\quad \times x(s)ds + B_i u(t) + C_i u(t-r(t)) \\ &\quad + F_i \textstyle\int_{t-r(t)}^{t} u(s)ds,\end{aligned} \tag{32}$$

where

$$i = 1, 2, \tau(t) = 0.5\sin^2 t, r(t) = 0.5\cos^2 t,$$

and

$$A_1 = \begin{bmatrix} 3 & 1 \\ 0 & -22 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -15 & 0 \\ -1 & 1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

It is easy to check that both A_i and $A_i + D_i, i = 1, 2$, are unstable matrices. Moreover, neither system $(A_i, B_i), i = 1, 2$, nor $(A_i + D_i, B_i), i = 1, 2$, are controllable systems. However, for $\alpha = 0.5$ and by fixing $\tau_1 = 0.4, \tau_2 = 0.6$, Corollary 10 is feasible using LMI toolbox of Matlab. Therefore, the switched system (32) is exponentially stabilizable with the rate $\alpha = 0.5$. The LMIs (29) in Corollary 10 are satisfied with

$$P = \begin{bmatrix} 0.2 & -0.02 \\ -0.02 & 0.1885 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 1.1185 & -0.0544 \\ -0.0544 & 1.1063 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 1.0557 & -0.0766 \\ -0.0766 & 1.0409 \end{bmatrix}.$$

It is easy to check that the system of matrices $\{L_1, L_2\}$, where

$$L_1 = \begin{bmatrix} 3.8187 & 0.5265 \\ 0.5265 & -5.5946 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -3.4053 & 0.0186 \\ 0.0186 & 2.9118 \end{bmatrix},$$

are strictly complete. The set Ω_1 and Ω_2 are defined as

$$\begin{aligned}\Omega_1 &= \{(x_1, x_2) \in R^2 : \ 3.8187x_1^2 \\ &\qquad +1.053x_1x_2 - 5.5946x_2^2 < 0\}, \\ \Omega_2 &= \{(x_1, x_2) \in R^2 : \ -3.4053x_1^2 \\ &\qquad +0.0372x_1x_2 + 2.9118x_2^2 < 0\},\end{aligned}$$

which can be represented in Fig.1.

It is seen that

$$\Omega_1 \cup \Omega_2 = R^2 \backslash \{0\}.$$

Therefore, the switching regions are given as

$$\begin{aligned}\bar{\Omega}_1 &= \{(x_1, x_2) \in R^2 : \ 3.8187x_1^2 \\ &\qquad + 1.053x_1x_2 - 5.5946x_2^2 < 0\}, \\ \bar{\Omega}_2 &= \{(x_1, x_2) \in R^2 : \ 3.8187x_1^2 + 1.053x_1x_2 \\ &\qquad -5.5946x_2^2 \geq 0, \ (x, y) \neq (0, 0)\}.\end{aligned}$$

We have that $\bar{\Omega}_1 \cup \bar{\Omega}_2 = R^2 \backslash \{0\}$ and $\bar{\Omega}_1 \bigcap \bar{\Omega}_2 = \emptyset$.

The switching rule is chosen as $\sigma(x(t)) = i$ whenever $x(t) \in \bar{\Omega}_i, i = 1, 2$, and the state feedback controller

$$u(t) = K_i x(t), t \geq 0,$$

where

$$K_1 = [0.2 \quad -0.02],$$
$$K_2 = [-0.02 \quad 0.1885],$$

the system (32) is 0.5-exponentially stabilizable. By computation we find that every solution $x(t, \phi)$ of the closed-loop system satisfies

$$\parallel x(t, \phi) \parallel \leq 2.2499\mathrm{e}^{-0.5t} \parallel \phi \parallel, \quad t \geq 0.$$

Example 13 *Consider the following switching system*

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + D_i x(t-\tau(t)) + B_i u(t) \\ &\quad + C_i u(t-r(t)),\end{aligned} \tag{33}$$

where

$$i = 1, 2, \quad \tau(t) \equiv 0.3, \quad r(t) \equiv 0.5$$

and

$$A_1 = \begin{bmatrix} 2 & 1 \\ 0 & -20 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -10 & 0 \\ -1 & 2 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1 & -1 \\ 0 & 10 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 5 & 0 \\ 1 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

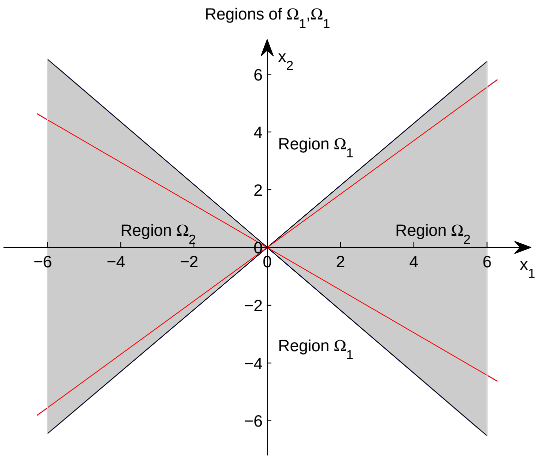

Figure 1: Regions of Ω_1,Ω_2.

For $\alpha = 0.6$ and by fixing $\tau_1 = 0.4, \tau_2 = 0.5$, Corollary 10 is feasible using LMI toolbox of Matlab. Therefore, the switched system (33) is exponentially stabilizable with the rate $\alpha = 0.6$. The LMIs (29) in Corollary 10 are satisfied with

$$P = \begin{bmatrix} 0.2063 & -0.0037 \\ -0.0037 & 0.1277 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.1973 & -0.3325 \\ -0.3325 & 1.9796 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 1.4261 & 0.1661 \\ 0.1661 & 0.1010 \end{bmatrix}.$$

It is easy to check that the system of matrices $\{L_1, L_2\}$, where

$$L_1 = \begin{bmatrix} 7.5 & 3 \\ 3 & -36.5 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -15.5 & 2 \\ 2 & 8.5 \end{bmatrix},$$

are strictly complete. The set Ω_1, Ω_2 are defined as

$$\begin{aligned} \Omega_1 &= \{(x_1, x_2) \in R^2 : \ 7.5x_1^2 \\ &\quad + 6x_1x_2 - 36.5x_2^2 < 0\}, \end{aligned}$$

$$\begin{aligned} \Omega_2 &= \{(x_1, x_2) \in R^2 : \ -15.5x_1^2 \\ &\quad + 4x_1x_2 + 8.5x_2^2 < 0\}. \end{aligned}$$

We see that

$$\Omega_1 \cup \Omega_2 = R^2 \backslash \{0\}.$$

Table 1: Comparing the previous results in [31] with this paper.

Results	$\delta = 0$	$\delta = 0.1$	$\delta = 0.5$	$\delta = 0.9$
$\bar{\tau}$([31])	0.0307	0.0271	0.0197	0.0185
$\bar{\tau}$(Our results)	0.0413	0.0354	0.0254	0.0196

Therefore, the switching regions are given as

$$\begin{aligned} \bar{\Omega}_1 &= \{(x_1, x_2) \in R^2 : 7.5x_1^2 \\ &\quad + 6x_1x_2 - 36.5x_2^2 < 0\}, \end{aligned}$$

$$\begin{aligned} \bar{\Omega}_2 &= \{(x_1, x_2) \in R^2 : 7.5x_1^2 + 6x_1x_2 \\ &\quad -36.5x_2^2 \geq 0, \ (x, y) \neq (0, 0)\}. \end{aligned}$$

We have that

$$\bar{\Omega}_1 \cup \bar{\Omega}_2 = R^2 \backslash \{0\}.$$

With the switching rule $\sigma(x(t)) = i$ whenever $x(t) \in \bar{\Omega}_i, i = 1, 2$, and the state feedback controller

$$u(t) = K_i x(t), t \geq 0,$$

where

$$K_1 = [0.2026 \quad 0.1240],$$

$$K_2 = [-0.0073 \quad 0.2554],$$

the system (33) is 0.6-exponentially stabilizable. By computation we find that every solution $x(t, \phi)$ of the closed-loop system satisfies

$$||x(t, \phi)|| \leq 0.7380\mathrm{e}^{-0.6t} \parallel \phi \parallel, \quad t \geq 0.$$

Example 14 *Consider system (30) with the following parameters: (Example 3 of [31])*

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & 2 \\ -20 & -2 \end{bmatrix}, \ A_2 = \begin{bmatrix} -2 & 10 \\ -4 & -2 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} -1 & -7 \\ 23 & 6 \end{bmatrix}, \ D_2 = \begin{bmatrix} 4 & -5 \\ 1 & -8 \end{bmatrix}. \end{aligned} \tag{34}$$

Set $\alpha = 0, \ \tau_1 = 0.4, \ \tau_2 = 0.5$. By Corollary 11, some comparisons for system (30) with (34) are made. Table 1 shows that the results of this paper provide a larger allowable upper bound for time delay to guarantee the global asymptotic stability of system (30) with (34) by the switching rule $\sigma(x(t)) = i \in \bar{N}$. So our results are provided to have the less conservativeness.

5 Conclusion

In this paper, the switching signal design for global exponential stabilization of uncertain switched nonlinear systems with time-varying delays in state and control has been considered. Switching laws design and techniques to deal with delay systems are two important issues about switched delay systems. Based on the Lyapunov-Krasovskii functional, the delay-dependent exponential stabilization conditions are derived in terms of linear matrix inequalities. If there is a feasible solution for the proposed LMI conditions under some given upper bounds of delays, the switching law can be designed and the exponential stabilization of systems can be achieved. The obtained results are show to be less conservative than previous one via the numerical examples.

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