The dynamic game between imported luxury car and domestic car and competition strategy comparison in Chinese car market

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Abstract: Chinese car market has became a differentiated market with the increasing income gap in China. According to the differentiation characteristic of Chinese car market, two models are built in this paper: One is output model and the other is price model. Then, the complex dynamic behaviors are analyzed by numerical simulations such as the local stability, three-dimensional dynamic evolutionary map, bifurcation, chaos, Lyapunov exponents, and initial conditions sensitivity analysis. Finally, two models are compared, and the complex dynamic behaviors are explained from the economic viewpoint. The comparison results have theoretical and practical significance for the development of Chinese car industry.

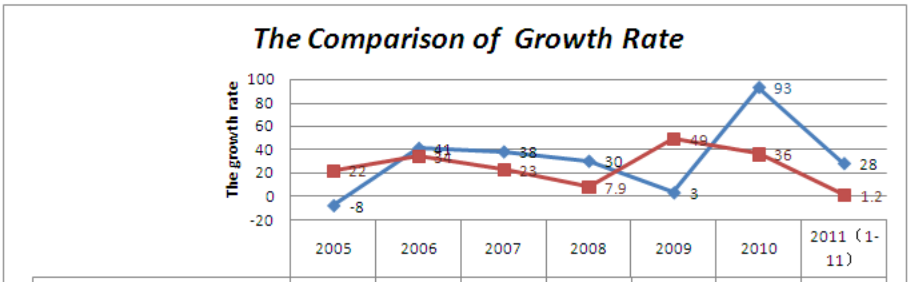
Key–Words: Repeated Game; Complex Dynamics; Output Game; Price Game; Product Differentiation; Chaos; Chinese Car Market

1 Introduction

In recent years, Chinese car market has made rapid development and has a significant characteristic of differentiation. China is a developing country with a middle and low average level of income. The domestic auto products are formed to meet the middle and low consumption, and it is the consumption demands of most people in China. The minority consumer demand of domestic high-income class is satisfied by importing top grade car from developed countries such as America and Germany. And we have to admit that the enlarging income gap between people increased the product differentiation of the car market.

The data used in our study are obtained from China Statistical Year Book. In Figure 1, especially from 2009, the imported car has an outstanding performance, while the increment speed of domestic car declined significantly. And the imported brands are mostly famous and luxury cars, such as Lexus, Benz, Audi, BMW, etc. Through the investigation to this phenomenon, we can conclude that the differentiation trend between imported luxury car and domestic car is more and more obvious.

The combination of chaos dynamics and repeated game theory is becoming the hot spot of research in the past few years. Cournot [1] put forward the Cournot model in which company competes on the output quantity in 1838. Bertrand [2] chose price rather than quantity as the strategic variable in 1883. Soon after, researchers have done a lot of works about complex dynamics of Cournot and Bertrand game models, such as Puu [3], Kopel [4][5], Agiza [6][7]. Based on the existing achievements, modern scholars pay more attention to the application in real economy field. Ji [8] built an electric power triopoly model based on heterogeneous expectations and studied its complicated dynamic behaviors. Xin et al. [9] studied the complex dynamics of an adnascent-type output game model. Guo et al. [10] established a collecting price game model for a close-loop supply chain system and analyzed its complex dynamic phenomena. Sun et al. [11] built a nonlinear Chinese cold rolled steel market model based on Bertrand game. Xu et al. [12] built a dynamic model of a duopoly price game with delay in insurance market and analyzed the existence and stability of the Nash equilibrium point of the dynamic system. Wang et al. [13] considered a multi-enterprise output game model under the circumstances of information asymmetry, and investigated its complexity. Ma et al. [14] bulit a new Cournot duopoly game model with delayed bounded rationality in the electricity market and analyzed its complexity. Zhang [15] built an nonlinear price game of insurance market in which one of the two competitors in the market made decision only with bounded rationality without delay, and the other competitor made the delayed decision with one period and two periods.



	2005	2006	2007	2008	2009	2010	2011 (1-11)
The growth rate of import product(%)	-8	41	38	30	3	93	28
The growth rate of domestic output(%)	22	34	23	7.9	49	36	1.2

Figure 1: The Comparison of Growth Rate.

However, little attention has been done on the comparison research of Cournot dynamic model (output decision) and Bertrand dynamic model (price decision) in the actual economic field .

This paper constructs an output and a price dynamical game models with bounded rationality according to the real situation of Chinese car industry. This research mainly contributes following points: (1) The comparison study between the two models supplies reference for policy decision of enterprises and government from economics angle. (2) The route to chaos of multi-enterprise is depicted by 3-d bifurcation diagram, 2-d bifurcation diagram and 1-d bifurcation diagram step by step, and it is clearer and more comprehensive than ever.

There are four parts in this paper: In section 2, the output model of Chinese car market is built. The Nash equilibrium, the local stability, and the complexity analysis are studied. In Section 3, the price model of Chinese car market is constructed, and the Nash equilibrium, the local stability and its complexity are analyzed. In Section 4, the comparison between output model and price model is given. The Section 5 is the conclusion.

2 The Model of Chinese Car Market Based on Output Decision

2.1 A Real Word Example

In China, there are usually three kinds of cars: the imported luxury car, the domestic self-owned brand car and the joint venture car. Such as the level of A00: the Smart Fortwo of Mercedes-Benz is an imported luxury car, the QQ of Chery is self-owned brand car and the Spark of Chevrolet is a joint venture car. The self-owned brand car QQ and the joint venture car Spark meet domestic common demand, and the imported luxury car Smart Fortwo meets domestic high-grade demand of minority. From the data in Figure 1, we can see that the product differentiation between imported luxury car and domestic car (including the self-owned brand and the joint brand) is getting more and more obvious from 2009. This new tendency is mainly analyzed in this paper.

2.2 The Notations

Notation 2.2.1. For a certain class cars, enterprises 1, 2 and 3 produce imported luxury car, self-owned brand car and the joint venture car, respectively, with outputs q_1, q_2, q_3 in period t. And it is assumed that the three enterprises are in oligopoly positions and do not cooperate.

Notation 2.2.2. The prices of the imported luxury car, self-owned car and the joint venture car are respectively:

$$\begin{aligned} p_1 &= m - n(q_1 + e_{12}q_2 + e_{13}q_3), \\ p_2 &= m - n(q_2 + e_{12}q_1 + e_{23}q_3), \\ p_3 &= m - n(q_3 + e_{13}q_1 + e_{23}q_2). m, n > 0 \end{aligned} \tag{1}$$

where e_{12},e_{13}, and e_{23} are substitution factors. According to the differentiation characteristic of Chinese car market, $1-e_{12}, 1-e_{13} \gg 1-e_{23}$, namely the differentiation degree between imported luxury car and domestic car (including the self-owned brand and the joint brand) is far greater than the differentiation degree among domestic brands. In order to analysis the main characteristic of the market, we should seize the major features of the problem. So, for convenient research, we set $e_{12} = e_{13} = e$. Products are independent when substitution factor equals to 0, and are complete substitution when substitution factor equals to 1. The nonlinear inverse demand functions are as below:

$$\begin{aligned} p_1 &= m - n(q_1 + eq_2 + eq_3), \\ p_2 &= m - n(q_2 + eq_1 + e_{23}q_3), \\ p_3 &= m - n(q_3 + eq_1 + e_{23}q_2).m, n > 0 \end{aligned} \tag{2}$$

Notation 2.2.3. The cost functions are

$$C_i = c_i q_i, i = 1, 2, 3. \tag{3}$$

Notation 2.2.4. The profits are

$$\Pi_i = p_i q_i - c_i q_i, i = 1, 2, 3. \tag{4}$$

2.3 The Model

According to above notations, the profits of the three enterprises can be obtained:

$$\begin{cases} \Pi_1 = mq_1 - nq_1^2 - neq_1q_2 - neq_1q_3 - c_1q_1 \\ \Pi_2 = mq_2 - nq_2^2 - neq_1q_2 - ne_{23}q_2q_3 - c_2q_2 \\ \Pi_3 = mq_3 - ne_{23}q_3^2 - nq_2q_3 - neq_1q_3 - c_3q_3 \end{cases} \tag{5}$$

The first order derivatives of the profits (the marginal profits) are respectively,

$$\begin{cases} \frac{\partial \Pi_1}{\partial q_1} = -2nq_1 + m - neq_2 - neq_3 - c_1 \\ \frac{\partial \Pi_2}{\partial q_2} = -2nq_2 + m - neq_1 - ne_{23}q_3 - c_2 \\ \frac{\partial \Pi_3}{\partial q_3} = -2ne_{23}q_3 + m - nq_2 - neq_1 - c_3 \end{cases} \tag{6}$$

The bounded rationality expectation is used in the system. The enterprise decides its output according to the marginal profit of the last period. When the marginal profit is positive, the output will be increased in the next period. On the contrary, when the marginal profit is negative, the output will be decreased in the next period. So the dynamic evolution process of Chinese car market is as follows:

$$\begin{cases} q_1(t+1) = q_1 + \alpha q_1 \dfrac{\partial \Pi_1}{\partial q_1} \\ \quad = q_1 + \alpha q_1(-2nq_1 + m - neq_2 - neq_3 - c_1), \\ \qquad 0 \le \alpha \le 1 \\ q_2(t+1) = q_2 + \beta q_2 \dfrac{\partial \Pi_2}{\partial q_2} \\ \quad = q_2 + \beta q_2(-2nq_2 + m - neq_1 - ne_{23}q_3 - c_2), \\ \qquad 0 \le \beta \le 1 \\ q_3(t+1) = q_3 + \gamma q_3 \dfrac{\partial \Pi_3}{\partial q_3} \\ \quad = q_3 + \gamma q_3(-2ne_{23}q_3 + m - nq_2 - neq_1 - c_3), \\ \qquad 0 \le \gamma \le 1 \end{cases} \tag{7}$$

where α, β, γ are output adjustment speed parameters.

2.4 The Nash Equilibrium and the Local Stability

Through calculating $q_i(t+1) = q_i(t), i = 1, 2, 3$, eight solutions can be obtained:

$$\begin{aligned} &E_1(0,0,0), E_2(R,0,0), E_3(0,P,0), E_4(0,0,Q), \\ &E_5(R,P,0), E_6(0,P,Q), E_7(R,0,Q), E_8(R,P,Q) \end{aligned} \tag{8}$$

where

$$\begin{aligned} R &= \frac{-\frac{1}{2}(-m + ney + nez + c_1)}{n} \\ P &= \frac{-\frac{1}{2}(-m + nex + ne_{23}z + c_2)}{n} \\ Q &= \frac{-\frac{1}{2}(-m + ny + nex + c_3)}{ne_{23}} \end{aligned} \tag{9}$$

Obviously, $E_0, E_1, E_2, E_3, E_4, E_5, E_6, E_7$ are all bounded equilibrium points [16] and unstable [15]. It is difficult to analyze the stability of the formula $E_8(R, P, Q)$, so the values of parameters are set as $c_1 = 0.5, c_2 = 0.2, c_3 = 0.35, e = 0.03, e_{23} = 0.7, m = 5.1, n = 1.6$. Here, we only consider the stability of Nash equilibrium point $E_8(1.4017, 1.0381, 1.349)$.

The Jacobian matrix of E_8 is:

$$J(E_8) = \begin{bmatrix} 1-4.4855\alpha & -0.0673\alpha & -0.0673\alpha \\ -0.0498\beta & 1-3.3220\beta & -1.1627\beta \\ -0.0648\gamma & -2.1584\gamma & 1-3.0218\gamma \end{bmatrix} \tag{10}$$

And its characteristic equation is

$$f(\lambda) = \lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 \tag{11}$$

According to Routh-Hurwitz's [17] stability test, the local stable conditions are as follows:

$$\begin{cases} f(1) = A_1 + A_2 + A_3 + 1 > 0 \\ -f(-1) = -A_1 + A_2 - A_3 + 1 > 0 \\ A_3^2 - 1 < 0 \\ (1 - A_3^2)^2 - (A_2 - A_1A_3)^2 > 0 \end{cases} \tag{12}$$

In Eqs.(12),

$$\begin{aligned} A_1 &= -3 + 3.0218\gamma + 3.3220\beta + 4.4855\alpha \\ A_2 &= 13.5496\alpha\gamma + 7.5288\beta\gamma + 3 + 14.8974\alpha\beta \\ &\quad - 6.6440\beta - 8.9709\alpha - 6.0435\gamma \\ A_3 &= -1 + 4.4855\alpha + 3.0218\gamma + 3.3220\beta + \\ &\quad + 33.7332\alpha\beta\gamma - 14.8974\alpha\beta - 13.5496\alpha\gamma - 7.5288\beta\gamma \end{aligned} \tag{13}$$

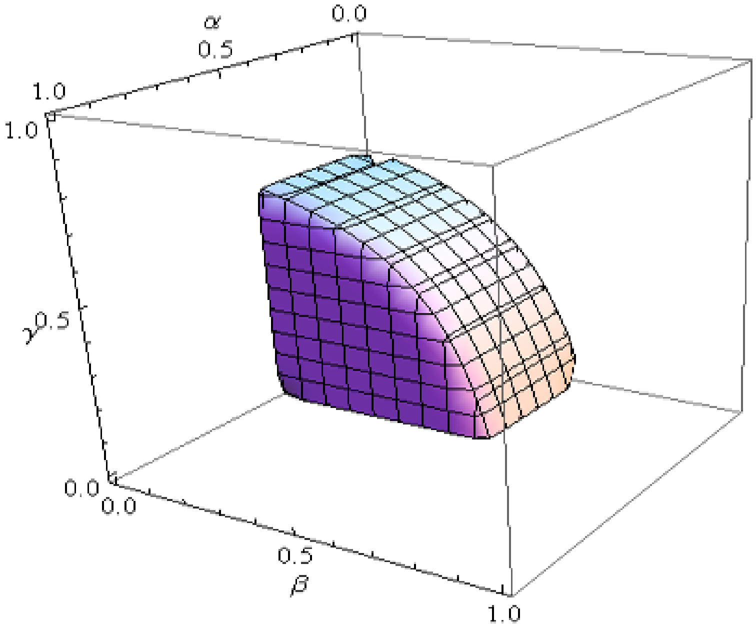


Figure 2: The stable region of the Nash equilibrium point $E_8(1.4017, 1.0381, 1.349)$.

By calculating inequalities (12), the local stable region of $E_8(1.4017, 1.0381, 1.349)$ can be got as Figure 2 shows. We can see the stable region only depends on α, β and γ. In stable region, the outputs of the three enterprises will eventually achieve Nash equilibrium point $E_8(1.4017, 1.0381, 1.349)$, whatever the initial outputs are. If one of enterprises accelerates output adjustment speed for some reasons and make (α, β, γ) escape the region, the market will fall into unstable state.

2.5 The Complexity and the Numerical Simulations

We will use 3-d evolutionary map (also called 3-d bifurcation diagram), 2-d evolutionary map (2-d bifurcation diagram) and 1-d bifurcation diagram to reflect the complex dynamics characters of system (7), step by step and layer by layer.

2.5.1 The Evolution Map of Complicated Dynamic

In order to analyze the evolution history of system (7), we further draw 3-d evolutionary process map with the initial value (1,1,1) as shown in Figure 3. It shows bifurcation, chaos and stable region of the system with different colors.

A section is extracted from Figure 3, as shown in Figure 4. Figure 4 shows bifurcation, chaos and stable region of the system when $\gamma = 0.33$. The straight line $\beta = 0.35$ reflects the complex dynamic process of the system (7) with changing α and fixed β, γ , and the line experiences stable region, period bifurcation and enters into chaos in turn. The dynamic process of the line also is described by the bifurcation Figure 5 (a).

2.5.2 The Bifurcation and the Largest Lyapunov Exponents

Figure 5 (a) shows the bifurcation and the largest Lyapunov exponents (LLEs) with changing the output adjustment speed of the imported luxury cars. When $\alpha \in (0, 0.437)$, the outputs of the three kinds of products are fixed at $E_8(1.4017, 1.0381, 1.349)$. When $\alpha \in (0.437, 0.66)$, the system enters into period-doubling bifurcation and chaos in turn. The positive LLEs mean chaos. The LLEs also express the same evolution with the bifurcation diagram. In chaotic region, there are a few dots of LLEs less than zero, which means periodic windows. Similarly, increasing β or γ can also push the market into chaos.

Figure 5 (b) is the bifurcation with changing product substitution parameter e . From Figure 5 (b), the market experiences bifurcation and enters into chaos with decreasing e (namely with increasing product differentiation $1 - e$). The imported luxury car enters chaos firstly because of its bigger output adjustment speed ($\alpha > \beta, \gamma$). So, the Chinese car market may enter into unstable state with increasing product differentiation between the imported luxury car and the domestic mainstream cars.

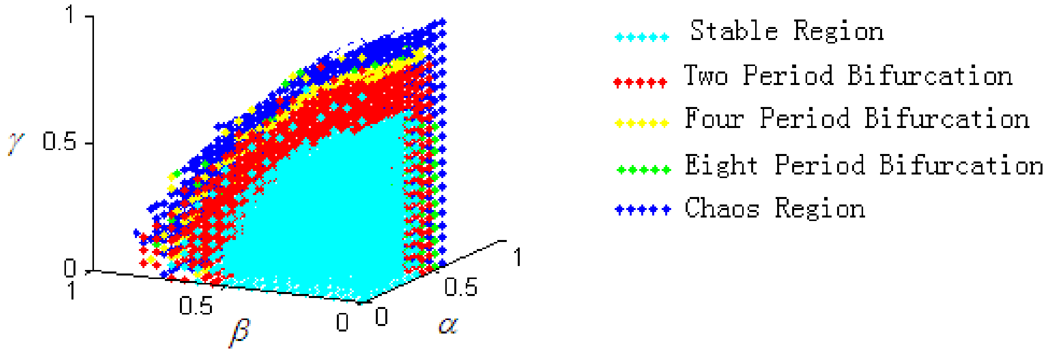


Figure 3: The three-dimensional dynamic evolutionary map of output model.

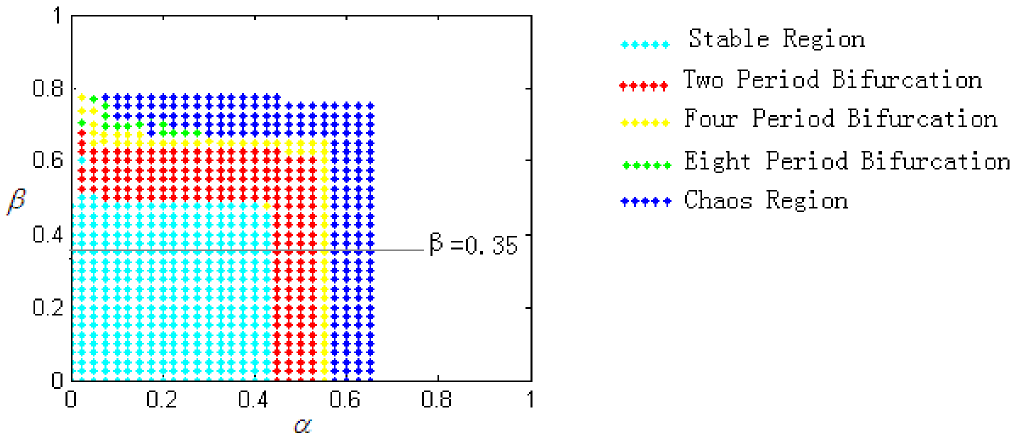


Figure 4: The two-dimensional evolutionary map of complicated dynamic with $\gamma = 0.33$.

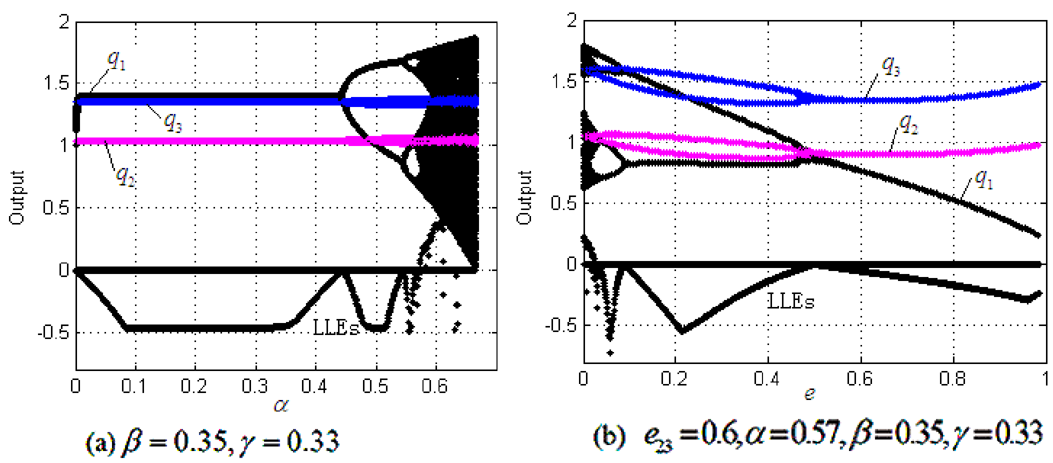


Figure 5: The bifurcation and the largest Lyapunov exponents.

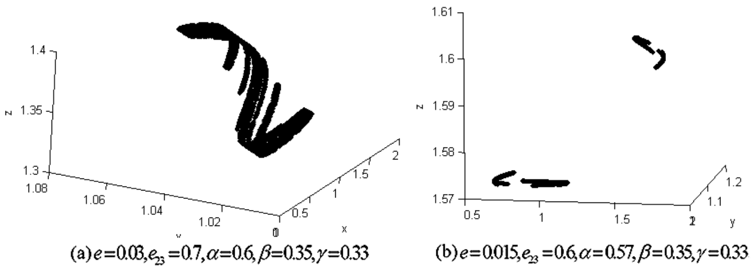


Figure 6: The chaotic attractors.

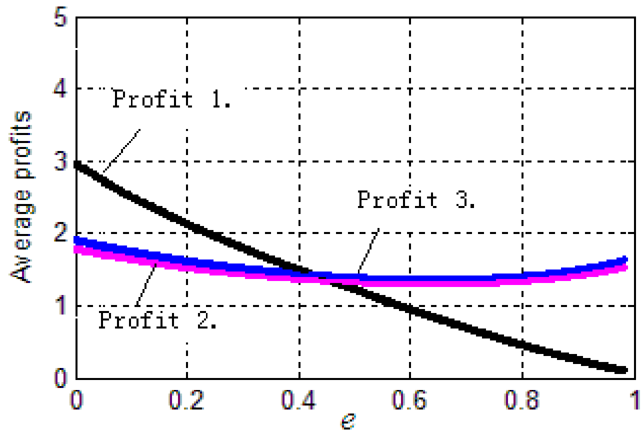


Figure 7: The average profits: $e_{23} = 0.6, \alpha = 0.57, \beta = 0.35, \gamma = 0.33$.

2.5.3 The Chaotic Attractor

The Figure 6 (a) and (b) are the chaotic attractors with different parameters. The chaotic attractor has complicated structure with definite boundary. In the range of chaos attractor, the chaotic motion is stochastic.

2.5.4 The Average Profit

Figure 7 is the average profits of Nash equilibrium and chaotic trajectory with changing product substitution parameter e . The average profits of chaotic trajectory can be got by (14) ,

$$\Pi_i = (1/T) \sum_{t=0}^{T-1} \Pi_i(x_t, y_t, z_t), \quad i = X, Y, Z \quad (14)$$

(see, [18]) Taking $T = 5.99 \times 10^5$.

With increasing product substitution e, the average profit of imported luxury brand decreased obviously, when $e > 0.45$, the average profits of domestic mainstream brand is bigger than imported luxury brand. So reducing product differentiation $1 - e$ (namely increasing e) is beneficial to the development of the domestic mainstream brand in output decision model.

3 The Model of Chinese Car Market Based on Price Decision

3.1 The Notations

Notation 3.1.1. For a certain class cars, the prices of the imported luxury car, self-owned brand car and the joint venture car are respectively p_1, p_2, p_3 in period t . And it is assumed that the three enterprises occupy the oligopolistic statuses in Chinese car market and do not cooperate.

Notation 3.1.2. The demand functions are as follows:

$$\begin{aligned} q_1 &= a - p_1 + dp_2 + dp_3, \\ q_2 &= a - p_2 + dp_1 + d_{23}p_3, \\ q_3 &= a - p_3 + dp_1 + d_{23}p_2 \\ a &> 0, 0 < d < 1. \end{aligned} \quad (15)$$

In (15), the product substitution parameter between the imported luxury car and the domestic mainstream brands (including the self-owned brand and the joint brand) is d . And the product substitution parameter among the domestic mainstream brands is d_{23} . According to actual situation of Chinese car market, the substitution parameter $d \ll d_{23}$ (the product differentiation $1-d \gg 1-d_{23}$).

Notation 3.1.3. The cost functions are

$$C_i = c_i q_i, \quad i = 1,2,3. \tag{16}$$

Notation 3.1.4. The profits are

$$\Pi_i = p_i q_i - c_i q_i, \quad i = 1,2,3. \tag{17}$$

3.2 The Model

According to above notations, the profits of the three enterprises can be obtained,

$$\begin{cases} \Pi_1 & = ap_1 - p_1^2 + dp_1p_2 + dp_1p_3 - ac_1 + c_1p_1 \\ & -dc_1p_2 - dc_1p_3 \\ \Pi_2 & = ap_2 - p_2^2 + dp_1p_2 + d_{23}p_2p_3 - ac_2 + c_2p_2 \\ & -dc_2p_1 - d_{23}c_2p_3 \\ \Pi_3 & = ap_3 - p_3^2 + dp_1p_3 + d_{23}p_2p_3 - ac_3 + c_3p_3 \\ & -c_3dp_1 - c_3d_{23}p_2 \end{cases} \tag{18}$$

The marginal profits are respectively,

$$\begin{cases} \frac{\partial \Pi_1}{\partial p_1} = a - 2p_1 + dp_2 + dp_3 + c_1 \\ \frac{\partial \Pi_2}{\partial p_2} = a - 2p_2 + dp_1 + d_{23}p_3 + c_2 \\ \frac{\partial \Pi_3}{\partial p_3} = a - 2p_3 + dp_1 + d_{23}p_2 + c_3 \end{cases} \tag{19}$$

The expression of the complex dynamic with bounded rationality expectation is as follows:

$$\begin{cases} p_1(t+1) = p_1 + \mu p_1 \dfrac{\partial \Pi_1}{\partial p_1} \\ \quad = p_1 + \mu p_1(a - 2p_1 + dp_2 + dp_3 + c_1), \\ \qquad\qquad 0 \le \mu \le 1 \\ p_2(t+1) = p_2 + \sigma p_2 \dfrac{\partial \Pi_2}{\partial p_2} \\ \quad = p_2 + \sigma p_2(a - 2p_2 + dp_1 + d_{23}p_3 + c_2), \\ \qquad\qquad 0 \le \sigma \le 1 \\ p_3(t+1) = p_3 + \omega p_3 \dfrac{\partial \Pi_3}{\partial p_3} \\ \quad = p_3 + \omega p_3(a - 2p_3 + dp_1 + d_{23}p_2 + c_3), \\ \qquad\qquad 0 \le \omega \le 1 \end{cases} \tag{20}$$

where μ, σ, ω are price adjustment speed.

3.3 The Nash Equilibrium and the Local Stability

By calculating $p_i(t+1) = p_i, i = 1,2,3$, the only Nash equilibrium point can be obtained:

$$E^* = (\frac{a_1 + dp_2 + dp_3 + c_1}{2}, \frac{a_2 + dp_1 + d_{23}p_3 + c_2}{2}, \frac{a_3 + dp_1 + d_{23}p_2 + c_3}{2}) \tag{21}$$

Because it is hard to study the stability of formula (21), parameters are set as $a_1 = 3.5, a_2 = 2.5, a_3 = 3.5, c_1 = 0.1, c_2 = 0.04, c_3 = 0.05, d = 0.1, d_{23} = 0.7$, so $E^* = (2.05, 2.313, 2.687)$. The Jacobian matrix of Nash equilibrium point is

$$J(E^*) = \begin{bmatrix} 1 - 4.1\mu & 0.205\mu & 0.205\mu \\ 0.2313\sigma & 1 - 3.6259\sigma & 1.6191\sigma \\ 0.2687\omega & 1.8809\omega & 1 - 5.3741\omega \end{bmatrix} \tag{22}$$

And the characteristic equation is:

$$f(\lambda) = \lambda^3 + B_1\lambda^2 + B_1\lambda + B_3 \tag{23}$$

Based on stability conditions of Routh-Hurwitz's [14], follow inequalities are obtained:

$$\begin{cases} f(1) = B_1 + B_2 + B_3 + 1 > 0 \\ -f(-1) = -B_1 + B_2 - B_3 + 1 > 0 \\ B_3^2 - 1 < 0 \\ \\ (1 - B_3^2)^2 - (B_2 - B_1B_3)^2 > 0 \end{cases} \tag{24}$$

In (24),

$$\begin{aligned} B_1 &= -3 + 5.3741\omega + 3.6259\sigma + 4.1\mu \\ B_2 &= 14.8189\mu\sigma + 16.4406\sigma\omega + 21.9786\mu\omega + 3 \\ &- 7.2519\sigma - 8.2\mu - 10.7481\omega \\ B_3 &= -1 + 67.1304\mu\sigma\omega - 14.8189\mu\sigma + 4.1\mu + \\ &5.3741\omega + 3.6259\sigma - 21.9786\mu\omega - 16.4406\sigma\omega \end{aligned} \tag{25}$$

By calculating inequalities (24), the stable region can be got as Figure 8 shows.

3.4 The Complexity and the Numerical Simulations

The 3-d evolution map (3-d bifurcation diagram), 2-d evolution map (2-d bifurcation diagram) and the 1-d bifurcation diagram are used to describe the dynamic characteristics progressively.

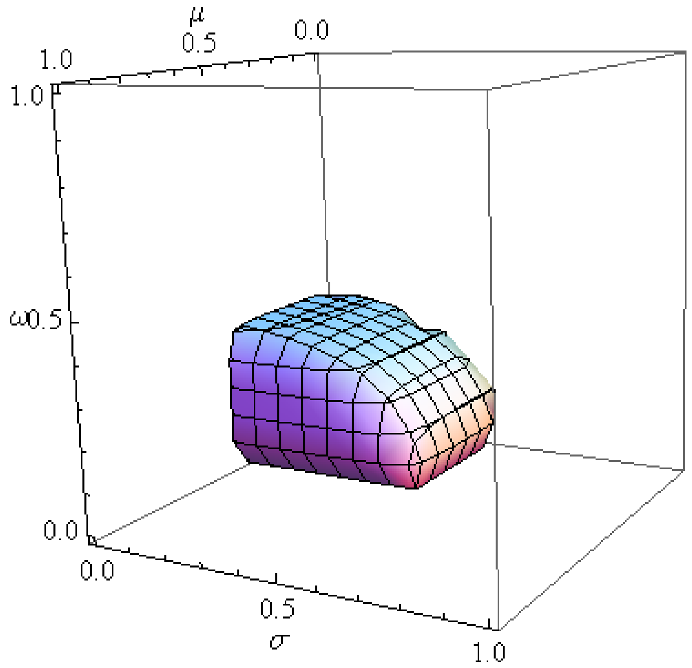


Figure 8: The local stable region of the Nash equilibrium point $E^*(2.05, 2.313, 2.687)$.

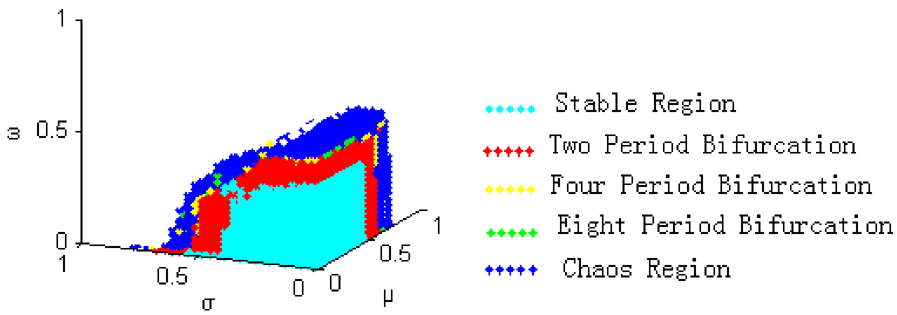


Figure 9: The three dimensional dynamic evolutionary map of price model.

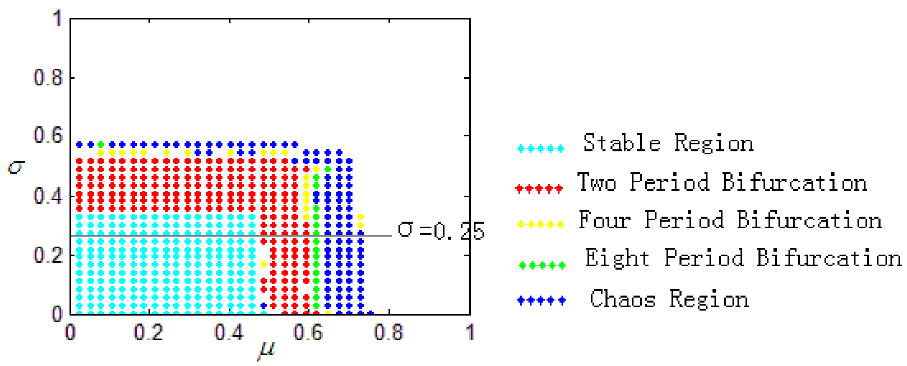


Figure 10: The two dimensional evolutionary map of complicated dynamic with $\omega = 0.25$.

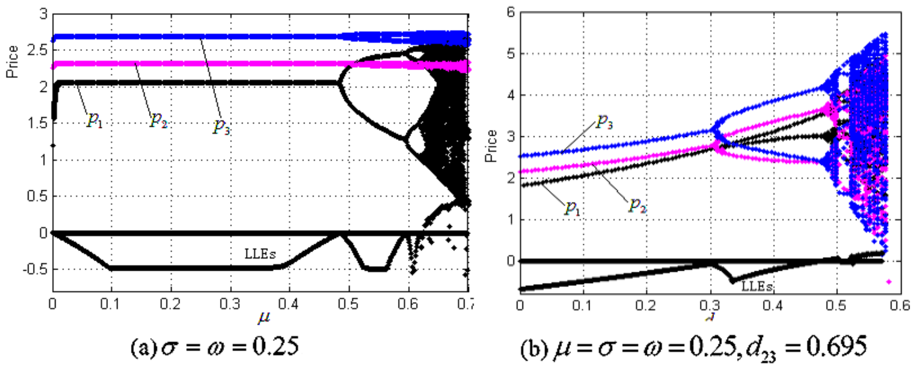


Figure 11: The bifurcation and the largest Lyapunov exponents(LLEs).

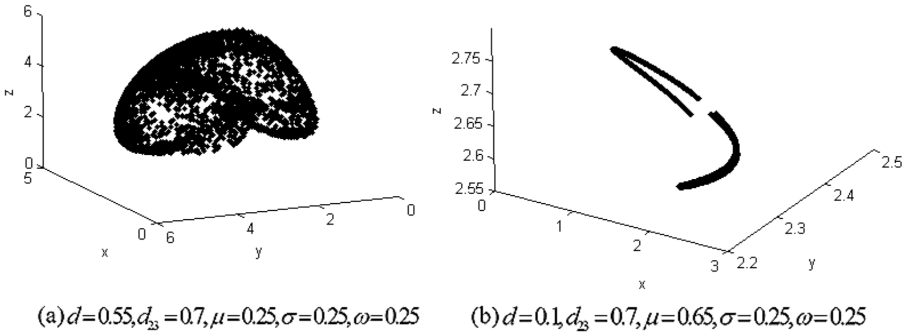


Figure 12: The chaotic attractors.

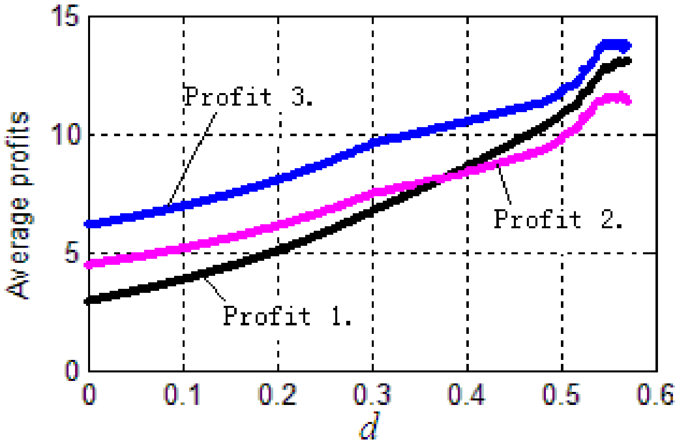


Figure 13: The average profits: $d = 0.1, d_{23} = 0.7, \mu = 0.65, \sigma = 0.25, \omega = 0.25$.

	Output model	Price model
The change of complexity.	The chaos appears with decreasing product substitution parameter e.	The chaos appears with increasing product substitution parameter d.
The dynamic tendency of Nash equilibrium point.	The Nash equilibrium point is invariant with changing output adjustment speed α, β, γ. The equilibrium value of imported car is descent with increasing product substitution parameter e.	The Nash equilibrium point is invariant with changing price adjustment speed μ, σ, ω. The equilibrium values are ascend with increasing product substitution degree d.
The average profit.	The average profits of the three enterprises show declining trend with increasing product substitution degree e, and the imported car is especially obvious.	The average profits of the three enterprises show uptrend with increasing product substitution degree d.

Table 1: The comparison between the output model and the price model.

3.4.1 The Evolution Map of Complicated Dynamic

Three-dimensional dynamic evolution map is drawn with the initial value (1.2,0.8,1), as shown in Figure 9. From Figure 9, we can examine the complex dynamic process of (20) comprehensively. We take a section $\omega = 0.25$ from it as shown in Figure 10.

Figure 10 shows the complex dynamic process with $\omega = 0.25$. In Figure 10, the straight line $\sigma = 0.25$ reflects the dynamic change of the system with changing μ, and the corresponding dynamic process also is decribed by Figure 11 (a).

3.4.2 The Bifurcation and the Largest Lyapunov Exponents

Figure 11 (a) shows the bifurcation and the LLEs with changing price adjustment speed of imported cars. After finite games, the system enters into stable state $E^*(2.05, 2.313, 2.687)$ with $\mu \in (0, 0.478)$, whatever the initial values are. When $\mu > 0.478$, the market falls into bifurcation and chaos in turn.

Figure 11 (b) is the bifurcation and LLEs with changing product substitution factor d . When $d \in (0, 0.3)$, the system is in equilibrium state. The emphasis should be pointed out that the equilibrium values of the three kinds of products are increasing. When $d \in (0.3, 0.576)$, the system experiences bifurcation and enters into chaos.

3.4.3 The Chaotic Attractor

Figure 12 is the chaotic attractors with different parameters.

3.4.4 The Average Profit

Figure 13 is the average profits with changing product substitution factor d . The average profits of all players show uptrend with increasing d (namely decreasing product differentiation $1 - d$). Thus, increasing product substitution is beneficial to the whole market in price decision model.

4 The Comparison between Output Model and Price Model

Table 1 compares output and price model from three aspects: the change of complexity, the dynamic tendency of Nash equilibrium point and the average profit. We can get some thoughts from economics perspectives:

(i) Increasing product differentiation between imported luxury brand and domestic brands can lead to complex dynamic in output model. On the contrary, decreasing product differentiation can lead to complex dynamics in price model. That is, decreasing product differentiation $(1 - e)$ can help to stabilize domestic car market in output competition, and increasing product differentiation $(1 - d)$ can help to stabilize domestic car market in price competition.

(ii) Whether in output model or price model, increasing the output or price adjustment speed can not change the values of Nash equilibrium point. However, the change of product differentiation can cause the change of Nash equilibrium in the both models. In output model, the equilibrium value of imported cars shows a decline trend with decreasing product differentiation. In price model, the equilibrium values of the three kind of cars appear obviously uptrend with decreasing product differentiation.

(iii) In output model, increasing product differentiation is beneficial to the profit of imported cars obviously. So enhancing product quality and brand, shortening product differentiation are important to develop Chinese car industry. In price model, increasing product differentiation can decrease profits of the three kind of cars. So differentiation competitive strategy is unsuitable for price decision model in the long run. In other words, price decision can gain competitive advantage much more easily for domestic car industry at present.

5 Conclusion

In this paper, the output decision model and price decision model of Chinese car market are built. The complexities of the two models are studied and the differences between them are summarized. The compared results have much theoretic values and actual meanings.

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