A Novel Inexact Smoothing Method for Second-Order Cone Complementarity Problems

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Abstract: A novel inexact smoothing method is presented for solving the second-order cone complementarity problems (SOCCP). Our method reformulates the SOCCP as an equivalent nonlinear system of equations by introducing a regularized Chen-Harker-Kanzow-Smale smoothing function. At each iteration, Newton's method is adopted to solve the system of equations approximately, which saves computation work compared to the calculations of exact search directions. Under rather weak assumptions, the algorithm is proved to possess global convergence and local quadratic convergence.

Key–Words: Second-order cone complementarity problem, Smoothing Newton method, Inexact search direction, Global convergence, Local quadratic convergence

1 Introduction

In this paper, we consider the following second-order cone complementarity problem (SOCCP) (see, e.g., [1]), which is to find vectors $(x, s, p) \in R^n \times R^n \times R^l$ such that

$$x \in K,\ s \in K,\ < x, s >= 0,\ F(x, s, p) = 0, \quad (1)$$

where $< \cdot, \cdot >$ represents the Euclidean inner product, $F : R^n \times R^n \times R^l \to R^{n+l}$ is a continuously differentiable function, and

$$K = K^{n_1} \times K^{n_2} \times \cdots \times K^{n_m}$$

with

$$n = n_1 + n_2 + \cdots + n_m$$

is the Cartesian product of second-order cones. The set $K^{n_i} (i = 1, \ldots, m)$ is the second-order cone (SOC) of dimension n_i defined by

$$K^{n_i} = \left\{ \begin{array}{l} x_i = (x_{i0}, x_{i1}) \in R \times R^{n_i - 1} : \\ x_{i0} - \|x_{i1}\| \geq 0 \end{array} \right\},$$

where $\| \cdot \|$ refers to the Euclidean norm. Then the interior of the SOC is

$$\mathrm{int}K^{n_i} = \left\{ \begin{array}{l} x_i = (x_{i0}, x_{i1}) \in R \times R^{n_i - 1} : \\ x_{i0} - \|x_{i1}\| > 0 \end{array} \right\},$$

and therefore

$$\mathrm{int}K = \mathrm{int}K^{n_1} \times \mathrm{int}K^{n_2} \times \cdots \times \mathrm{int}K^{n_m}.$$

It is easy to verify that the SOC K is self-dual, i.e.,

$$K = K^* := \{s \in R^n :\ s^T x \geq 0,\ \forall x \in K\}.$$

It should be noted that the SOCCP (1) considered in this paper is one of the most general expressions of the SOCCP. Actually, if $l = 0$ and

$$F(x, s, p) = f(x) - s$$

for some $f : R^n \to R^n$, then (1) becomes

$$x \in K,\ s \in K,\ < x, s >= 0,\ s = f(x),$$

which is the form considered by many researchers (see, e.g., [2, 3, 4]), and if $l = n$ and

$$F(x, s, p) = \begin{pmatrix} f(p) - x \\ g(p) - s \end{pmatrix}$$

for some $f : R^n \to R^n$ and $g : R^n \to R^n$, then (1) reduces to

$$x \in K,\ s \in K,\ < x, s >= 0,$$

$$x = f(p), \quad s = g(p),$$

which was studied by Chen and Tseng [5].

The SOCCPs have various important applications in many fields, such as management, control, and engineering (see, e.g., [6]). The SOCCPs have been utilized as a general framework for the linear complementarity problems, the nonlinear complementarity problems, the second-order cone programming problems and so on (see, e.g., [7]).

Recently great attention has been paid to smoothing methods (non–interior continuation methods) for solving the linear complementarity problems (see, e.g., [8]), the nonlinear complementarity problems and variational inequality problems (see, e.g., [9, 10]), partially due to their superior numerical performances and theoretical results. Unlike interior point methods, smoothing Newton methods do not require starting points and intermediate iteration points to stay in the sets of strict feasible solutions.

However, to obtain the global convergence and local superlinear (or quadratic) convergence, some algorithms available (see, e.g., [9]) strongly depend on uniform nonsingularity assumptions. Without uniform nonsingularity, most smoothing Newton methods (see, e.g., [8, 11]) need to solve two linear systems of equations and to perform two or three line searches at each iteration.

Moreover, most computation work in smoothing Newton methods is devoted to the computation of an exact search direction by solving a system of equations, especially when the problem is large. Even if a direct method is used to solve the system of equations, the solution may not satisfy the equations exactly due to rounding errors. These motivate the study of smoothing methods that use inexact search directions.

In [12], a smoothing inexact Newton method is proposed for the P_0 nonlinear complementarity problem, which is shown to possess global convergence and local superlinear convergence.

Motivated by the method in [12], we propose a novel inexact smoothing method to solve the SOCCP in this paper. At each iteration, our method allows the use of the search directions that are calculated from the system of equations with only moderate accuracy. Moreover, our method is shown to possess global convergence and local quadratic convergence under rather weak conditions.

The organization of this paper is as follows. In Section 2, we review some preliminaries, including the Euclidean Jordan algebra, semismoothness and the Cartesian mixed P_0-property, which will be used in the subsequent analysis. In Section 3, our inexact smoothing method is proposed for solving the SOCCP. Convergence of the method is analyzed in Section 4. Section 5 concludes this paper.

2 Some Preliminaries

In this section, we give a brief introduction to the Euclidean Jordan algebra (see, e.g., [7, 13]) associated with the SOC K^n, and the concepts of semismoothness and the Cartesian mixed P_0-property, which will be used in the subsequent analysis.

For any $x = (x_0, x_1)$ and $s = (s_0, s_1) \in R \times R^{n-1}$, the Jordan product is defined as

$$x \circ s = (x^T s, x_0 s_1 + s_0 x_1).$$

We will write x^2 to mean $x \circ x$ and $x + s$ to mean the usual componentwise addition of vectors x and s. Then, $\circ$, $+$, together with

$$e := (1, 0, \cdots, 0) \in R^n$$

have the following basic properties (see, e.g., [13]).

Property 1 *([13])*

(i) $e \circ x = x \quad \forall x \in R^n$.
(ii) $x \circ s = s \circ x \quad \forall x, s \in R^n$.
(iii) $x \circ (x^2 \circ s) = x^2 \circ (x \circ s) \quad \forall x, s \in R^n$.
(iv) $(x + y) \circ s = x \circ s + y \circ s \quad \forall x, s, y \in R^n$.

Notice that the Jordan product "$\circ$", unlike scalar or matrix multiplication, is not associative in general, which is the main source on complication in the analysis of SOCCPs.

For any $x = (x_0, x_1) \in R \times R^{n-1}$, we define the symmetric matrix

$$L_x = \begin{pmatrix} x_0 & x_1^T \\ x_1 & x_0 I \end{pmatrix},$$

which can be viewed as a linear mapping with the following properties.

Property 2 *([13])*

(i) $L_x s = x \circ s$ *and* $L_{x+s} = L_x + L_s$ *for any* $x, s \in R^n$.
(ii) $x \in K^n \Leftrightarrow L_x$ *is positive semidefinite, and* $x \in \mathrm{int} K^n \Leftrightarrow L_x$ *is positive definite.*
(iii) L_x *is invertible whenever* $x \in \mathrm{int} K^n$ *with the inverse* L_x^{-1} *given by*

$$L_x^{-1} = \frac{1}{\det(x)} \begin{pmatrix} x_0 & -x_1^T \\ -x_1 & \frac{\det(x)}{x_0} I + \frac{x_1 x_1^T}{x_0} \end{pmatrix},$$

where $\det(x) := x_0^2 - \|x_1\|^2$ *denotes the determinant of* x.

We now introduce the spectral factorization of vectors in R^n associated with the SOC K^n, which is an important character of Jordan algebra. For any

$x = (x_0, x_1) \in R \times R^{n-1}$, its spectral factorization is defined as

$$x = \lambda_1 u^{(1)} + \lambda_2 u^{(2)}.$$

Here $\lambda_1,\ \lambda_2$ are the spectral values given by

$$\lambda_i = x_0 + (-1)^i \|x_1\|,\ i = 1, 2,$$

and $u^{(1)},\ u^{(2)}$ are the associated spectral vectors given by

$$u^{(i)} = \begin{cases} \frac{1}{2}(1, (-1)^i \frac{x_1}{\|x_1\|}) & \text{if } x_1 \neq 0, \\ \frac{1}{2}(1, (-1)^i \omega) & \text{otherwise}, \end{cases} \quad i = 1, 2,$$

with $\omega \in R^{n-1}$ being any vector satisfying $\|\omega\| = 1$. If $x_1 \neq 0$, the factorization is unique.

Some interesting properties of λ_1, λ_2 and $u^{(1)}, u^{(2)}$ are given as follows. Notice that the identity element e is uniquely identified by its two spectral values which are exactly equal to 1.

Property 3 *([13])*
For any $x = (x_0, x_1) \in R \times R^{n-1}$, the spectral values λ_1, λ_2 and spectral vectors $u^{(1)}, u^{(2)}$ have the following properties:
(i) $u^{(1)}$ and $u^{(2)}$ are orthogonal under the Jordan product and have length $1/\sqrt{2}$, i.e.,

$$u^{(1)} \circ u^{(2)} = 0, \quad \|u^{(1)}\| = \|u^{(2)}\| = 1/\sqrt{2}.$$

(ii) $u^{(1)}$ and $u^{(2)}$ are idempotent under the Jordan product, i.e.,

$$u^{(i)} \circ u^{(i)} = u^{(i)}, \quad i = 1, 2.$$

(iii) λ_1 and λ_2 are nonnegative (respectively, positive) if and only if $x \in K^n$ (respectively, $x \in \text{int}K^n$).
(iv) The determinant, the trace, and the Euclidean norm of x can be represented in terms of λ_1 and λ_2:

$$\det(x) := \lambda_1 \lambda_2,$$

$$\text{tr}(x) := \lambda_1 + \lambda_2,$$

$$2\|x\|^2 = \lambda_1^2 + \lambda_2^2.$$

By using the spectral factorization, we may extend scalar functions to SOC functions. For example, we define

$$x^2 = \lambda_1^2 u^{(1)} + \lambda_2^2 u^{(2)},\ \ \forall x \in R^n.$$

Since both λ_1 and λ_2 are nonnegative for any $x \in K^n$, we define

$$\sqrt{x} = \sqrt{\lambda_1} u^{(1)} + \sqrt{\lambda_2} u^{(2)},\ \ \forall x \in K^n.$$

Semismoothness is a generalization of the smoothness, which was originally introduced by Mifflin [14] for functionals and extended to vector-valued functions by Qi and Sun [15].

Definition 4 *([15]) Suppose that $G : R^m \to R^n$ is locally Lipschitz continuous around $x \in R^m$.*

(i) G is said to be <u>semismooth</u> at x if G is directionally differentiable at x and for any $V \in \partial G(x + \triangle x)$,

$$G(x + \triangle x) - G(x) - V(\triangle x) = o(\|\triangle x\|),$$

where ∂G stands for the generalized Jacobian of G in the sense of Clarke [16].

(ii) G is said to be <u>p-order $(0 < p < \infty)$ semismooth</u> at x if G is semismooth at x and

$$G(x + \triangle x) - G(x) - V(\triangle x) = O(\|\triangle x\|^{1+p}).$$

In particular, G is said to be <u>strongly semismooth</u> at x if G is said to be 1-order semismooth at x.

A function $G : R^m \to R^n$ is said to be a semismooth (respectively, p-order semismooth) function if it is semismooth (respectively, p-order semismooth) everywhere in R^m. Semismooth functions include smooth functions, piecewise smooth functions, and convex and concave functions. The composition of (strongly) semismooth functions is still a (strongly) semismooth function [14].

Now let us introduce the concept of the Cartesian mixed P_0-property.

Definition 5 *([1]) Define the matrix $Q = [A\ B\ C]$ where $A, B \in R^{(n+l)\times n}$ and $C \in R^{(n+l)\times l}$. The matrix Q is said to have the <u>Cartesian mixed P_0-property</u> iff C has full column rank and*

$$\left.\begin{array}{l} Au + Bv + Cw = 0, (u, v) \neq 0, w \in R^l, \\ u = (u_1, \cdots, u_m) \in R^{n_1} \times \cdots \times R^{n_m}, \\ v = (v_1, \cdots, v_m) \in R^{n_1} \times \cdots \times R^{n_m} \end{array}\right\} \Rightarrow$$

there exists an index i such that $(u_i, v_i) \neq 0$ and $\langle u_i, v_i \rangle \geq 0$.

Clearly, when $m = n$ and $n_1 = \cdots = n_m = 1$, the matrix Q having the Cartesian mixed P_0-property coincides with Q having the mixed P_0-property [17]. Therefore, $F'(x, s, p)$ having the Cartesian mixed P_0-property, which will be adopted in this paper, is a weaker assumption than the monotonicity assumption usually used in SOCCPs (see, e.g., [4]).

3 Inexact Smoothing Method

In this section, a novel inexact smoothing method is proposed for the SOCCP and is shown to be well defined. Our method reformulates (1) as an equivalent nonlinear system of equations, and then applies Newton's method to solving the system of equations approximately.

We firstly recall the following smoothing function $\phi : R \times R^n \times R^n \to R^n$ defined by [18]:

$$\begin{aligned}\phi(\mu, x, s) &= (1+\mu)(x+s) \\ &-\sqrt{(1-\mu)^2(x-s)^2+\mu^2 e},\end{aligned} \tag{2}$$

which is a regularized version of Chen-Harker-Kanzow-Smale smoothing function [2]. Notice that

$$\phi(0, x, s) = 0 \Leftrightarrow x \in K, s \in K. \tag{3}$$

Let $z := (\mu, x, s, p) \in R \times R^n \times R^n \times R^l$. Based on the regularized Chen-Harker-Kanzow-Smale smoothing function (2), we define

$$G(z) = \begin{pmatrix} e^{\mu} - 1 \\ \Phi(z) \end{pmatrix}, \tag{4}$$

$$\Phi(z) = \begin{pmatrix} \phi(\mu, x, s) \\ F(x, s, p) \end{pmatrix}, \tag{5}$$

and

$$\Psi(z) = \|G(z)\|^2.$$

By (1), (2), (3), (4) and (5), $z^* := (0, x^*, s^*, p^*)$ is a root of the system of equations $G(z) = 0$ if and only if (x^*, s^*, p^*) is the optimal solution of the SOCCP (1).

By Theorem 3.2 in [19], we obtain the following properties of $G(z)$.

Lemma 6 *Let $G : R_+ \times R^{2n+l} \to R_+ \times R^{2n+l}$ be defined in (5). Then the following results hold.*

(i) G is locally Lipschitz continuous and semismooth everywhere in R^{1+2n+l}. Moreover, if F' is locally Lipschitzian, then G is strongly semismooth on R^{1+2n+l}.

(ii) G is continuously differentiable at any point $z := (\mu, x, s, p) \in R_{++} \times R^{2n+l}$ with the Jacobian matrix

$$G'(z) = \begin{pmatrix} e^{\mu} & 0 & 0 & 0 \\ \phi'_{\mu}(z) & \phi'_x(z) & \phi'_s(z) & 0 \\ 0 & F'_x(x,s,p) & F'_s(x,s,p) & F'_p(x,s,p) \end{pmatrix},$$

where

$$\phi'_{\mu}(z) = x + s - L_w^{-1}[-(1-\mu)v^2 + 4\mu e],$$

$$\phi'_x(z) = (1+\mu)I - (1-\mu)^2 L_w^{-1} L_v,$$
$$\phi'_s(z) = (1+\mu)I + (1-\mu)^2 L_w^{-1} L_v,$$

$$v := x - s, \quad w := \sqrt{(1-\mu)^2 v^2 + 4\mu^2 e}.$$

(iii) If $F'(x, s, p)$ has the Cartesian mixed P_0−property at $z := (\mu, x, s, p) \in R_{++} \times R^{2n+l}$, i.e., $F'(x, s, p)$ satisfies

$$\text{rank} F'_p(x, s, p) = l, \tag{6}$$

and

$$\left.\begin{array}{l} F'(x,s,p)(\xi,\eta,\varphi) = 0,\ (\xi,\eta) \neq 0,\ \varphi \in R^l, \\ \xi = (\xi_1, \cdots, \xi_m) \in R^{n_1} \times \cdots \times R^{n_m}, \\ \eta = (\eta_1, \cdots, \eta_m) \in R^{n_1} \times \cdots \times R^{n_m} \end{array}\right\} \Rightarrow$$

there exists an index i such that $(\xi_i, \eta_i) \neq 0$ and

$$\langle \xi_i, \eta_i \rangle \geq 0, \tag{7}$$

then the matrix $G'(z)$ is nonsingular.

Proof. It follows from Theorem 3.2 in [19], Lemma 3.1 in [18] and Definition 4 that (i) and (ii) hold. Now we prove (iii). Let $z := (\mu, x, s, p) \in R_{++} \times R^{2n+l}$ be any point satisfying (6) and (7). It is easy to see that the nonsingularity of $F'(x, s, p)$ is equivalent to the nonsingularity of the following matrix

$$J(z) = \begin{pmatrix} \phi'_x(z) & \phi'_s(z) & 0 \\ F'_x(x,s,p) & F'_s(x,s,p) & F'_p(x,s,p) \end{pmatrix}. \tag{8}$$

Now let us show that the matrix $J(z)$ is nonsingular. Suppose that

$$J(z)(\xi, \eta, \varphi) = 0.$$

It suffices to prove $(\xi, \eta, \varphi) = 0$. From (8), we have

$$\begin{aligned}&[(1+\mu)I - (1-\mu)^2 L_w^{-1} L_v]\xi + [(1+\mu)I + \\ &(1-\mu)^2 L_w^{-1} L_v]\eta = 0,\end{aligned} \tag{9}$$

and

$$F'(x, s, p)(\xi, \eta, \varphi) = 0, \tag{10}$$

where

$$\xi = (\xi_1, \cdots, \xi_m) \in R^{n_1} \times \cdots \times R^{n_m},$$

$$\eta = (\eta_1, \cdots, \eta_m) \in R^{n_1} \times \cdots \times R^{n_m}.$$

Multiplying both sides of (9) by L_w from the left yields

$$\begin{aligned}&[(1+\mu)L_w - (1-\mu)^2 L_v]\xi + [(1+\mu)L_w + \\ &(1-\mu)^2 L_v]\eta = 0.\end{aligned}$$

Thus, we have for $i = 1, 2, \ldots, m$ that

$$[(1+\mu)L_{w_i} - (1-\mu)^2 L_{v_i}]\xi_i + [(1+\mu)L_{w_i} + (1-\mu)^2 L_{v_i}]\eta_i = 0, \quad (11)$$

with

$$v = (v_1, \cdots, v_m) \in R^{n_1} \times \cdots \times R^{n_m},$$

$$w = (w_1, \cdots, w_m) \in R^{n_1} \times \cdots \times R^{n_m}.$$

Let

$$\overline{L}_i := (1+\mu)L_{w_i} - (1-\mu)^2 L_{v_i},$$

$$\underline{L}_i := (1+\mu)L_{w_i} + (1-\mu)^2 L_{v_i},$$

for $i = 1, 2, \cdots, m$. Since

$$\begin{aligned} &[(1+\mu)w_i]^2 - [(1-\mu)^2(\pm v_i)]^2 \\ &= 4\mu(1-\mu)^2 v_i^2 + 4\mu^2(1+\mu)^2 e_i \\ &\in \mathrm{int}K^{n_i}, \end{aligned} \quad (12)$$

Lemma 3.5 in [2] shows that both $\overline{L}_i$ and $\underline{L}_i$ are nonsingular. It follows from (12) and Proposition 3.4 in [2] that the symmetric part of $\overline{L}_i\underline{L}_i$ is positive definite. Multiplying both sides of (11) by $\xi_i^T \underline{L}_i^{-1}$ from the left yields

$$\xi_i^T \underline{L}_i^{-1} \overline{L}_i \xi_i + \langle \xi_i, \eta_i \rangle = 0, \ i = 1, 2, ..., m,$$

or equivalently

$$\bar{\xi}_i^T \overline{L}_i \underline{L}_i \bar{\xi}_i + \langle \xi_i, \eta_i \rangle = 0, \ i = 1, 2, ..., m, \quad (13)$$

with $\bar{\xi}_i := \underline{L}_i^{-1}\xi_i$. Now let us assume $(\xi, \eta) \neq 0$. Then by (10) and assumption (7), there exists an index i such that $(\xi_i, \eta_i) \neq 0$ and $\langle \xi_i, \eta_i \rangle \geq 0$. But since the symmetric part of $\overline{L}_i\underline{L}_i$ is positive definite, relation (13) implies

$$\bar{\xi}_i^T \overline{L}_i \underline{L}_i \bar{\xi}_i = 0$$

and therefore $\bar{\xi}_i = 0$. Then $\xi_i = 0$ and since $\underline{L}_i$ is nonsingular, relation (11) implies $\eta_i = 0$. This contradicts $(\xi_i, \eta_i) \neq 0$. Thus we must have $(\xi, \eta) = 0$. Since by assumption (6) the matrix $F_p'(x, s, p)$ has full column rank l, relation (10) implies $\varphi = 0$. Therefore, the matrix $J(z)$ is nonsingular. □

By Lemma 6 , $G(z)$ is continuously differentiable at any point $z := (\mu, x, s, p) \in R_{++} \times R^{2n+l}$. Thus, for any $\mu > 0$, we can apply Newton's method to solving the smooth system of equations $G(z) = 0$ approximately at each iteration, and make $\Psi(z) \downarrow 0$ so that the solutions of the SOCCP (1) can be found.

Let $\gamma \in (0, 1)$ and $\mu_0 > 0$. Define the function $\beta : R_+ \times R^{2n+l} \to R_+$ by

$$\beta(z) := e^{\mu}\gamma \min\{1, \Psi(z)\}. \quad (14)$$

Algorithm 7 *(A novel inexact smoothing method)*

Step 0 *Choose constants $\delta, \sigma \in (0, 1)$, and $\mu_0 > 0$. Let $\bar{z} := (\mu_0, 0, 0, 0) \in R_{++} \times R^n \times R^n \times R^l$, and $z_0 := (\mu_0, x_0, s_0, p_0) \in R_{++} \times R^{2n+l}$ be an arbitrary point. Choose $\gamma \in (0, 1)$ such that $\gamma\mu_0 < 1/4$, and choose a sequence $\{\eta_k\}$ such that $\eta_k \in [0, \theta)$, where $\theta \in [0, (1 - 4\gamma\mu_0)/2]$. Set $k := 0$.*

Step 1 *If $\Psi(z_k) = 0$, stop.*

Step 2 *Compute a solution $\Delta z_k = (\Delta\mu_k, \Delta x_k, \Delta s_k, \Delta p_k) \in R \times R^{2n+l}$ of the linear system of equations*

$$G(z_k) + G'(z_k)\Delta z_k = \beta_k \bar{z} + \mathbf{r}_k, \quad (15)$$

where the residual $\mathbf{r}_k = \begin{pmatrix} 0 \\ r_{k1} \end{pmatrix} \in R \times R^{2n+l}$ satisfies $\|\mathbf{r}_k\| \leq \eta_k \min\{1, \Psi(z_k)\}$.

Step 3 *Let $\lambda_k = \max\{\delta^l | = 0, 1, 2, \ldots\}$ such that*

$$\begin{aligned} &\Psi(z_k + \lambda_k \Delta z_k) \\ &\leq [1 - \sigma(1 - 4\gamma\mu_0 - 2\eta_k)\lambda_k]\Psi(z_k). \end{aligned} \quad (16)$$

Step 4 *Set $z_{k+1} = z_k + \lambda_k \Delta z_k$, and $k := k + 1$. Go to Step 1.*

By Lemma 6, we could show the well-definedness of Algorithm 7.

Theorem 8 *Suppose that $F'(x, s, p)$ has the Cartesian mixed P_0-property at any point $z := (\mu, x, s, p) \in R_{++} \times R^{2n+l}$. Then for any $k \geq 0$, Algorithm 7 is well-defined and generates an infinite sequence $\{z_k := (\mu_k, x_k, s_k, p_k)\}$. Moreover, we have $\mu_k \in R_{++}$ and $z_k \in \Omega$ for any $k \geq 0$, where*

$$\Omega = \left\{ \begin{array}{l} z = (\mu, x, s, p) \in R_{++} \times R^{2n+l} : \\ \mu \geq \gamma \min\{1, \Psi(z)\}\mu_0 \end{array} \right\}. \quad (17)$$

Proof. We divide the proof into four steps.

(i) It is obvious that $\mu_0 > 0$. Suppose that $\mu_k > 0$. Thus by (15), we get

$$\Delta\mu_k = \frac{1 - e^{\mu_k}}{e^{\mu_k}} + \frac{\beta_k \mu_0}{e^{\mu_k}}.$$

Then for any $\alpha \in (0, 1]$, we obtain

$$\begin{aligned} &\mu_{k+1} = \mu_k + \alpha\Delta\mu_k \\ &= \mu_k + \alpha\left(\tfrac{1-e^{\mu_k}}{e^{\mu_k}} + \tfrac{\beta_k\mu_0}{e^{\mu_k}}\right) \\ &\geq \mu_k + \alpha\left(-\mu_k + \gamma \min\{1, \Psi(z_k)\}\mu_0\right) \\ &= (1 - \alpha)\mu_k + \alpha\gamma\mu_0 \min\{1, \Psi(z_k)\} \\ &> 0. \end{aligned}$$

By mathematical induction on k, we have that $\mu_k > 0$ for any $k \geq 0$.

(ii) It follows from Lemma 6 that the matrix $G'(z_k)$ is nonsingular for any $\mu_k > 0$, since $F'(x, s, p)$ has the Cartesian mixed P_0-property at any point $z := (\mu, x, s, p) \in R_{++} \times R^{2n+l}$. Therefore, Step 2 is well-defined.

(iii) By the Taylor expansion and (15), we have

$$\begin{aligned} & e^{\mu_k+\alpha\Delta\mu_k} - 1 \\ &= e^{\mu_k}\left[1 + \alpha\Delta\mu_k + O(\alpha^2)\right] - 1 \\ &= e^{\mu_k} - 1 + \alpha e^{\mu_k}\Delta\mu_k + O(\alpha^2) \\ &= e^{\mu_k} - 1 + \alpha\left(1 - e^{\mu_k} + \beta_k\mu_0\right) + O(\alpha^2) \\ &= (1-\alpha)(e^{\mu_k} - 1) + \alpha\beta_k\mu_0 + O(\alpha^2). \end{aligned} \tag{18}$$

Since $e^{\mu_k} - 1 \leq \|G(z_k)\|$ and

$$\begin{aligned} & (\|G(z_k)\| + 1)\min\{1, \Psi(z_k)\} \\ &= (\|G(z_k)\| + 1)\min\{1, \|G(z_k)\|^2\} \\ &\leq 2\|G(z_k)\|, \end{aligned}$$

we obtain from (18) that

$$\begin{aligned} & (e^{\mu_k+\alpha\Delta\mu_k} - 1)^2 \\ &= [(1-\alpha)(e^{\mu_k} - 1) + \alpha\beta_k\mu_0 + O(\alpha^2)]^2 \\ &= (1-\alpha)^2(e^{\mu_k} - 1)^2 + \alpha^2\beta_k^2\mu_0^2 \\ &\quad +2\alpha(1-\alpha)\beta_k\mu_0(e^{\mu_k} - 1) + O(\alpha^2) \\ &\leq (1-\alpha)(e^{\mu_k} - 1)^2 + 2\alpha\beta_k\mu_0(e^{\mu_k} - 1) \\ &\quad +O(\alpha^2) \\ &= (1-\alpha)(e^{\mu_k} - 1)^2 + O(\alpha^2) \\ &\quad +2\alpha\gamma\mu_0 e^{\mu_k}(e^{\mu_k} - 1)\min\{1, \Psi(z_k)\} \\ &\leq (1-\alpha)(e^{\mu_k} - 1)^2 + 2\alpha\gamma\mu_0(\|G(z_k)\| + 1) \\ &\quad \cdot\|G(z_k)\| \cdot \min\{1, \Psi(z_k)\} + O(\alpha^2) \\ &\leq (1-\alpha)(e^{\mu_k} - 1)^2 + 4\alpha\gamma\mu_0\|G(z_k)\|^2 \\ &\quad +O(\alpha^2) \\ &= (1-\alpha)(e^{\mu_k} - 1)^2 + 4\alpha\gamma\mu_0\Psi(z_k) + o(\alpha). \end{aligned} \tag{19}$$

From (15), we obtain

$$\Phi(z_k) + \Phi'(z_k)\Delta z_k = r_{k1},$$

and therefore

$$\begin{aligned} & \|\Phi(z_k + \alpha\Delta z_k)\|^2 \\ &= \|\Phi(z_k) + \alpha\Phi'(z_k)\Delta z_k + O(\alpha^2)\|^2 \\ &= \|(1-\alpha)\Phi(z_k) + \alpha r_{k1} + O(\alpha^2)\|^2 \\ &\leq (1-\alpha)^2\|\Phi(z_k)\|^2 + 2\alpha(1-\alpha)\|\Phi(z_k)\| \\ &\quad \cdot\|r_{k1}\| + O(\alpha^2) \\ &\leq (1-\alpha)\|\Phi(z_k)\|^2 + 2\alpha\eta_k\min\{1, \Psi(z_k)\} \\ &\quad \cdot\|\Phi(z_k)\| + o(\alpha) \\ &\leq (1-\alpha)\|\Phi(z_k)\|^2 + 2\alpha\eta_k\min\{1, \|G(z_k)\|^2\} \\ &\quad \cdot\|G(z_k)\| + o(\alpha) \\ &\leq (1-\alpha)\|\Phi(z_k)\|^2 + 2\alpha\eta_k\|G(z_k)\|^2 + o(\alpha) \\ &= (1-\alpha)\|\Phi(z_k)\|^2 + 2\alpha\eta_k\Psi(z_k) + o(\alpha). \end{aligned} \tag{20}$$

Hence, we have from (19) and (20) that

$$\begin{aligned} & \|G(z_k + \alpha\Delta z_k)\|^2 \\ &= \left(e^{\mu_k+\alpha\Delta\mu_k} - 1\right)^2 + \|\Phi(z_k + \alpha\Delta z_k)\|^2 \\ &\leq (1-\alpha)\left(e^{\mu_k} - 1\right)^2 + (1-\alpha)\|\Phi(z_k)\|^2 \\ &\quad +4\alpha\gamma\mu_0\Psi(z_k) + 2\alpha\eta_k\Psi(z_k) + o(\alpha) \\ &= [1 - (1 - 4\gamma\mu_0 - 2\eta_k)\alpha]\Psi(z_k) + o(\alpha). \end{aligned}$$

Then, there exists $\overline{\alpha} \in (0, 1]$ such that for all $\alpha \in (0, \overline{\alpha}]$ and $\sigma \in (0, 1)$,

$$\Psi(z_k + \alpha\Delta z_k) \leq [1 - \sigma(1 - 4\gamma\mu_0 - 2\eta_k)\alpha]\Psi(z_k),$$

which implies that there exists some λ_k such that (16) holds. Therefore, Step 3 is well-defined.

(iv) By Theorem 8(i), we have $\mu_k \in R_{++}$ for any $k \geq 0$. Now, we prove

$$\mu_k \geq \gamma\min\{1, \Psi(z_k)\}\mu_0$$

for any $k \geq 0$ by mathematical induction on k. It is obvious that

$$\mu_0 \geq \gamma\min\{1, \Psi(z_0)\}\mu_0,$$

since

$$\gamma\min\{1, \Psi(z)\} \leq \gamma < 1.$$

Suppose that $z_k \in \Omega$. Then it follows from (15) and (17) that

$$\begin{aligned} \mu_{k+1} &= \mu_k + \lambda_k\Delta\mu_k \\ &= \mu_k + \lambda_k\left(\tfrac{1-e^{\mu_k}}{e^{\mu_k}} + \tfrac{\beta_k\mu_0}{e^{\mu_k}}\right) \\ &\geq \mu_k + \lambda_k\left(-\mu_k + \gamma\min\{1, \Psi(z_k)\}\mu_0\right) \\ &= (1-\lambda_k)\mu_k + \lambda_k\gamma\mu_0\min\{1, \Psi(z_k)\} \\ &\geq \gamma\min\{1, \Psi(z_k)\}\mu_0 \\ &\geq \gamma\min\{1, \Psi(z_{k+1})\}\mu_0, \end{aligned}$$

where the last inequality is due to the fact that $\{\Psi(z_k)\}$ is monotonically decreasing. By induction on k, we have that $z_k \in \Omega$ for any $k \geq 0$. This completes the proof. □

4 Convergence Analysis

According to Theorem 8, Algorithm 7 generates an infinite sequence $\{z_k\} := \{(\mu_k, x_k, s_k, p_k)\}$ under suitable assumptions. Let $z^* := (\mu^*, x^*, s^*, p^*)$ be an accumulation point of the iteration sequence $\{z_k\}$ generated by the novel inexact smoothing method. In this section, we establish the global convergence and local quadratic convergence of Algorithm 7.

For any $\mu > 0$ and $c > 0$, let

$$L_\mu(c) = \left\{(x, s) \in R^{2n} : \|\Phi(z)\| \leq c\right\},$$

and for any $0 < \underline{\mu} \leq \overline{\mu}$ and $c > 0$, let

$$L(c) = \bigcup_{\underline{\mu}\leq\mu\leq\overline{\mu}} L_\mu(c).$$

Lemma 9 *Suppose that $F'(x,s,p)$ has the Cartesian mixed P_0-property at any point $z := (\mu,x,s,p) \in R_{++} \times R^{2n+l}$, and for any $\mu > 0$, $0 < \underline{\mu} \leq \overline{\mu}$ and $c > 0$, the set*

$$L(c) = \bigcup_{\underline{\mu}\leq\mu\leq\overline{\mu}} L_\mu(c)$$

is bounded. Then the sequence $\{\Psi(z^k)\}$ generated by Algorithm 7 is convergent. If it does not converge to zero, the sequence $\{z^k\}$ is bounded.

Proof. Since $F'(x,s,p)$ has the Cartesian mixed P_0-property at any point $z := (\mu,x,s,p) \in R_{++} \times R^{2n+l}$, it follows from Theorem 8 that Algorithm 7 is well-defined.

From (16), the sequence $\{\Psi(z^k)\}$ is monotonically decreasing. Then $\{\Psi(z^k)\}$ is convergent, i.e., there exists $\Psi^* \geq 0$ such that $\Psi(z^k) \to \Psi^*$ as $k \to \infty$. If $\{\Psi(z^k)\}$ does not converge to zero, then $\Psi^* > 0$. By (4) and (16), we have

$$\mu_k \leq e^{\mu_k} - 1 \leq \sqrt{\Psi(z_k)} \leq \sqrt{\Psi(z_0)},$$

and thus $\{\mu_k\}$ is bounded. Then there exist $\underline{\mu}, \overline{\mu} > 0$ such that

$$0 < \underline{\mu} \leq \mu_k \leq \overline{\mu}$$

for all $k \geq 0$. Let $c_0 := \sqrt{\Psi(z_0)}$ and

$$L(c_0) = \bigcup_{\underline{\mu}\leq\mu_k\leq\overline{\mu}} L_{\mu_k}(c_0).$$

It is not difficult to see that $(x_k, s_k, p_k) \in L(c_0)$, because of $(x_k, s_k, p_k) \in L_{\mu_k}(c_0)$. It follows from the assumption that the set $L(c_0)$ is bounded and therefore $\{(x_k, s_k, p_k)\}$ is bounded. Hence, the sequence $\{z_k\} := \{(\mu_k, x_k, s_k, p_k)\}$ is bounded. □

To discuss the convergence property of Algorithm 7, we make the following assumptions.

Assumption 10 *$F'(x,s,p)$ has the Cartesian mixed P_0-property in the sense that $F'(x,s,p)$ satisfies (6) and (7) at any point $z := (\mu,x,s,p) \in R_{++} \times R^{2n+l}$.*

Assumption 11 *For any $\mu > 0$, $0 < \underline{\mu} \leq \overline{\mu}$ and $c > 0$, the set*

$$L(c) = \bigcup_{\underline{\mu}\leq\mu\leq\overline{\mu}} L_\mu(c)$$

is bounded.

Remark 12 *(i) From Theorem 8(ii), if Assumption 10 holds, the matrix $G'(z)$ is nonsingular for any $z := (\mu,x,s,p) \in R_{++} \times R^{2n+l}$. Obviously Assumption 10 is a weaker assumption than the monotonicity assumption usually used in SOCCPs.*

(ii) From Lemma 9, if Assumption 11 holds, the sequence $\{z_k\} := \{(\mu_k, x_k, s_k, p_k)\}$ generated by Algorithm 7 is bounded.

The following Lemma shows that if $l = 0$ and $F(x,s,p) := f(x) - s$ where f is a continuously differentiable monotone function, then Assumption 10 and Assumption 11 hold, and therefore the sequence $\{z_k\} := \{(\mu_k, x_k, s_k, p_k)\}$ generated by Algorithm 7 is bounded.

Lemma 13 *Suppose that $l = 0$ and $F(x,s,p) := f(x) - s$ where f is a continuously differentiable monotone function. Then for any $\mu > 0$, $0 < \underline{\mu} \leq \overline{\mu}$ and $c > 0$, the set*

$$L(c) = \bigcup_{\underline{\mu}\leq\mu\leq\overline{\mu}} L_\mu(c)$$

is bounded.

Proof. Since $l = 0$ and $F(x,s,p) := f(x) - s$ where f is a continuously differentiable monotone function, it follows from Definition 5 that $F'(x,s,p)$ has the Cartesian mixed P_0-property at any point $z := (\mu,x,s,p) \in R_{++} \times R^{2n+l}$. Then from Theorem 8, we obtain that Algorithm 7 is well-defined.

On the contrary, we assume that $L(c)$ is unbounded. Then for some fixed $c > 0$, there exists a sequence $\{z_k\} := \{(\mu_k, x_k, s_k, p_k)\}$ such that $0 < \underline{\mu} \leq \mu_k \leq \overline{\mu}$, $\|\Phi(z_k)\| \leq c$, but $\|(x_k, s_k)\| \to \infty$. By (5), we have

$$\begin{aligned}&\|\Phi(z_k)\|^2 \\ &= \|f(x_k) - s_k\|^2 + \|\phi(\mu_k, x_k, s_k)\|^2 \\ &\leq c.\end{aligned}$$

Then from the proof of Lemma 5.3 in [18], we can obtain

$$\lim_{\|(x_k,s_k)\|\to\infty} \|\phi(\mu_k, x_k, s_k)\| = +\infty,$$

which contradicts the boundedness of $\{\phi(\mu_k, x_k, s_k)\}$. Therefore, we obtain the desired result. □

Under Assumption 10 and Assumption 11, we obtain the global convergence of Algorithm 7.

Theorem 14 *Suppose that Assumption 10 and Assumption 11 hold. Then any accumulation point of the sequence $\{z_k\}$ generated by Algorithm 7 is a solution of $G(z) = 0$.*

Proof. By Assumption 10 and Theorem 8, Algorithm 7 is well-defined and generates an infinite sequence $\{z_k\} := \{(\mu_k, x_k, s_k, p_k)\}$. From (16), we obtain that the sequence $\{\Psi(z^k)\}$ is monotonically decreasing, denoting its limit by Ψ^*. If $\Psi^* = 0$, we obtain the desired result. On the contrary, suppose $\Psi^* > 0$. Then it follows from Assumption 11 and Lemma 9 that the sequence $\{z_k\}$ is bounded. By taking a subsequence if necessary, suppose that $\{z_k\}$ converges to $z^* := (\mu^*, x^*, s^*, p^*)$ as $k \to \infty$. Then from the continuity of $G(\cdot)$ and the definition of $\beta(\cdot)$, we obtain that

$$\lim_{k\to\infty} \Psi(z_k) = \Psi(z^*) = \|H(z^*)\|^2,$$

$$\lim_{k\to\infty} \beta_k = \beta^* := e^{\mu^*}\gamma \min\{1, \Psi(z^*)\}.$$

By Theorem 8, we get

$$0 < \gamma \min\{1, \Psi(z^*)\}\mu_0 \le \mu^*.$$

It follows from Lemma 6 that $G(\cdot)$ is continuously differentiable at z^* and thus $G'(z^*)$ exists. By (16), we have

$$\lim_{k\to\infty} \lambda_k = 0.$$

From Algorithm 7, we obtain that the steplength $\bar\lambda_k := \lambda_k/\delta$ does not satisfy Step 3, i.e.,

$$\Psi(z_k+\bar\lambda_k\Delta z_k) > \left[1 - \sigma(1 - 4\gamma\mu_0 - 2\eta_k)\bar\lambda_k\right] \Psi(z_k),$$

which implies

$$\begin{aligned}&[\Psi(z_k + \bar\lambda_k\Delta z_k) - \Psi(z_k)]/\bar\lambda_k \\ &> -\sigma(1 - 4\gamma\mu_0 - 2\eta_k)\Psi(z_k).\end{aligned} \tag{21}$$

On the other hand, we have by (14) for any $k \ge 0$ that

$$\begin{aligned}\beta_k\mu_0 &= e^{\mu_k}\gamma \min\{1, \Psi(z_k)\}\mu_0 \\ &\le |e^{\mu_k} - 1|\gamma \min\{1, \Psi(z_k)\}\mu_0 \\ &\quad +\gamma \min\{1, \|G(z_k)\|^2\}\mu_0 \\ &\le \|G(z_k)\|\gamma\mu_0 + \gamma\mu_0\|G(z_k)\| \\ &\le 2\gamma\mu_0\|G(z_k)\|,\end{aligned}$$

and therefore

$$\beta^*\mu_0 \le 2\gamma\mu_0\|G(z^*)\|. \tag{22}$$

From the definition of $\mathbf{r}_k$, we obtain for any $k \ge 0$ that

$$\begin{aligned}\|\mathbf{r}_k\| &\le \eta_k \min\{1, \Psi(z_k)\} \\ &\le \eta_k\|G(z_k)\|,\end{aligned}$$

and therefore as $k \to \infty$,

$$\|\mathbf{r}_k\| \le \eta_k\|G(z^*)\|. \tag{23}$$

Note that

$$\begin{aligned}&\lim_{k\to\infty} [\Psi(z_k + \bar\lambda_k\Delta z_k) - \Psi(z_k)]/\bar\lambda_k \\ &= 2G(z^*)^T G'(z^*)\Delta z^*.\end{aligned} \tag{24}$$

Taking the limits on both sides of (21) and combining (15), (22), (23) and (24) yield

$$\begin{aligned}&-\sigma(1 - 4\gamma\mu_0 - 2\eta_k)\Psi(z^*) \\ &\le -2\|G(z^*)\|^2 + 2\|G(z^*)\|\beta^*\mu_0 + \|G(z^*)\| \cdot \|\mathbf{r}_k\| \\ &\le (-2 + 4\gamma\mu_0 + \eta_k)\Psi(z^*) \\ &\le (-1 + 4\gamma\mu_0 + 2\eta_k)\Psi(z^*),\end{aligned}$$

which implies

$$-\sigma(1 - 4\gamma\mu_0 - 2\eta_k) \le (-1 + 4\gamma\mu_0 + 2\eta_k).$$

Since $1 - 4\gamma\mu_0 - 2\eta_k > 0$, we obtain $\sigma \ge 1$, which contradicts $\sigma < 1$. Thus we have $\Psi(z^*) = 0$. □

The following results show that our method is locally quadratically convergent without strict complementarity assumption.

Theorem 15 *Suppose that Assumption 10 and Assumption 11 hold, and F' is locally Lipschitzian. Assume that all $V \in \partial G(z^*)$ are nonsingular. Then the sequence $\{z_k\}$ generated by Algorithm 7 converges to z^* quadratically, i.e.,*

$$\|z_{k+1} - z^*\| = O(\|z_k - z^*\|^2);$$

moreover,

$$\mu_{k+1} = O(\mu_k^2).$$

Proof. From Theorem 14, z^* is a solution of $G(z) = 0$. Since $V \in \partial G(z^*)$ are nonsingular, it follows from Proposition 3.1 in [15] that for all z_k sufficiently close to z^*,

$$\|[G'(z_k)]^{-1}\| = O(1). \tag{25}$$

It follows from Lemma 6 that $G(\cdot)$ is locally Lipschitz continuous and strongly semismooth at z^*, since F' is locally Lipschitzian. Then we have for all z_k sufficiently close to z^*,

$$\|G(z_k) - G(z^*)\| = O(\|z_k - z^*\|), \tag{26}$$

$$\begin{aligned}&\|G(z_k) - G(z^*) - G'(z_k)(z_k - z^*)\| \\ &= O(\|z_k - z^*\|^2).\end{aligned} \tag{27}$$

Hence, by (14), (15), (25), (26) and (27), we have for all z_k sufficiently close to z^* that

$$\begin{aligned}&\|z_k + \Delta z_k - z^*\| \\ &= \|z_k + [G'(z_k)]^{-1}[-G(z_k) + \beta_k\bar z \\ &\quad +\mathbf{r}_k] - z^*\| \\ &\le \|[G'(z_k)]^{-1}\|[\|G(z_k) - G(z^*) \\ &\quad -G'(z_k)(z_k - z^*)\| + \beta_k\mu_0 + \|\mathbf{r}_k\|] \\ &= O(\|z_k - z^*\|^2).\end{aligned} \tag{28}$$

By the proof of Theorem 3.1 in [20], for all z_k sufficiently close to z^*, we have

$$\|z_k - z^*\| = O(\|G(z_k) - G(z^*)\|). \tag{29}$$

By Lemma 6(i), $G(\cdot)$ is locally Lipschitz continuous. Hence, we have from (28) and (29) that

$$\begin{aligned} \Psi(z_k) &= \|G(z_k + \Delta z_k)\|^2 \\ &= O(\|z_k + \Delta z_k - z^*\|^2) \\ &= O(\|z_k - z^*\|^4) \\ &= O(\|G(z_k) - G(z^*)\|^4) \\ &= O(\Psi(z_k)^2). \end{aligned} \tag{30}$$

Then, it follows from (16) and (30) that for all z_k sufficiently close to z^*,

$$z_{k+1} = z_k + \Delta z_k, \tag{31}$$

Therefore, we obtain from (28) and (31) that for all z_k sufficiently close to z^*,

$$\|z_{k+1} - z^*\| = O(\|z_k - z^*\|^2). \tag{32}$$

By (14), (15) and (32), we have for all z_k sufficiently close to z^* that

$$\begin{aligned} \mu_{k+1} &= \mu_k + \Delta\mu_k = \beta_k\mu_0 \\ &= e^{\mu_k}\gamma\mu_0\|G(z_k)\|^2. \end{aligned}$$

Then from (26), (31) and (32), it follows that

$$\begin{aligned} \mu_{k+1} &= O(\|G(z_k)\|^2) \\ &= O(\|G(z_k) - G(z^*)\|^2) \\ &= O(\|z_k - z^*\|^2) \\ &= O(\|z_{k-1} - z^*\|^4) \\ &= O(\|G(z_{k-1}) - G(z^*)\|^4) \\ &= O(\mu_k^2). \end{aligned}$$

□

5 Conclusion

In this paper, we reformulate the SOCCP (1) as an equivalent system of equations by a regularized Chen-Harker-Kanzow-Smale smoothing function. Then we propose an inexact smoothing Newton method to solve the system of equations. Our method is shown to possess the following good properties.

- Our method does not have any restrictions regarding its starting points;
- The method is well-defined, if $F'(x, s, p)$ has the Cartesian mixed P_0−property, which is a weaker assumption than the monotonicity assumption usually used in the SOCCP;
- At each iteration, Newton's method is adopted to solve the system approximately, which saves computation work compared to the calculations of exact search directions;
- The method is shown to possess global convergence and local quadratic convergence without the strict complementarity assumption.

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