A Novel Vague Set Based Score Function for Multi-Criteria Fuzzy Decision Making

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Abstract: - For handling multi-criteria fuzzy decision-making problems, the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria is often represented by a set of vague values. In multi-criteria fuzzy decision-making problems, vague set based score functions have become more and more popular for building models that concern the evaluation and comparison of alternatives in the decision-making process. However, several deficiencies remain evident when using these vague based score functions to handle multi-criteria decision-making problems. Therefore, the main objective of this study is propose a novel score function for a more effective and reasonable method for measuring the degree of suitability to which an alternative satisfied the decision maker's requirement. An illustrative example is provided, which shows that the proposed vague set based score function is more effective and reasonable than other existing score functions in handling multi-criteria decision-making problems.

Key-Words: - Multi-Criteria Fuzzy Decision Making, Score Function, Type-2 Fuzzy Sets, Vague Sets.

1 Introduction

Multi-criteria fuzzy decision-making problems deals with theoretical models, algorithms and practical applications in considering imprecise or uncertain human expertise and knowledge involving multiple criteria encountered in decision-making environments [6, 14, 17, 39].

In multi-criteria fuzzy decision-making problems, vague set based score functions have been defined and found to be applicable for building models that concern the evaluation and comparison of alternatives in the decision making process [3, 15, 19, 20, 21, 30, 32, 36]. In the past five years, many researchers have also presented some new methods supported by the aforementioned vague set based score functions for handling multi-criteria fuzzy decision making problems [4, 9, 10, 25, 29, 31]. However, several deficiencies remain evident when using these vague based score functions to handle multi-criteria decision-making problems. These include ignorance of the unknown part that can cause information loss, inefficient calculation of the score value and unreasonable comparison results for ranking the vague values. Therefore, there is an emerging demand for a more effective and reasonable score function.

In light of these deficiencies and limitations, the main objective of this study is to propose a novel score function for a more effective and reasonable method for measuring the degree of suitability to which an alternative satisfies the decision maker's requirement.

The rest of this paper is organized as follows. In section 2, relevant definitions and operations of vague sets are briefly reviewed. In Section 3, several familiar research works on existing score functions for transforming vague values into numerical values are reviewed and criticized. In Section 4, the proposed new score function is introduced and demonstrated to be suitable for multi-criteria decision-making problems under uncertain environments. An illustrative example is given to support the proposed score function. Finally, conclusions of this study are drawn in Section 5.

2 Preliminaries

In real-world applications, there may exist certain degree of hesitation regarding the belongingness of an element to a set. However, there are no means of expressing such degree of hesitation by using fuzzy set. Atanasov presented intuition fuzzy set theory [1].

Gau and Buehrer [12] proposed the concept of vague set, where the grade of membership is bounded to a subinterval $[t_A(x_i),\, 1 - f_A(x_i)]$ of $[0, 1]$. Instead of fuzzy sets, this paper used vague sets for handling multi-criteria decision-making problems. Bustince & Burillo showed that vague sets are indeed intuitionistic fuzzy sets and unified the vague sets and the intuition fuzzy sets [2]. Relevant definitions and operations of vague sets, which are in [12, 24, 30, 37], are briefly reviewed as follows.

Vague sets and vague values

A vague set, as well as being an intuitionistic fuzzy set, is a further generalization of fuzzy set [1, 2, 12]. Instead of using point-based membership as in fuzzy set, interval-based membership is used in vague set. Let X be a classical set of data objects, called the universe of discourse, $X = \{x_1,\, x_2, \ldots, x_n\}$, with a generic element of X denoted by x_i. A vague set A in the universe of discourse X is characterized by a truth membership function t_A, and a false membership function f_A, where t_A: $X \to [0, 1]$, f_A: $X \to [0, 1]$. $t_A(x_i)$ is a lower bound of the grade of membership of x_i derived from the "evidence for x_i", $f_A(x_i)$ is a lower bound on the negation of x_i derived from the "evidence against x_i", and $t_A(x_i)+f_A(x_i) \leq 1$. These lower bounds are used to create a subinterval on $[0, 1]$, namely $[t_A(x_i),\, 1 - f_A(x_i)]$, to generalize the $\mu_A(x_i)$ of fuzzy sets, where $t_A(x_i) < \mu_A(x_i) < 1 - f_A(x_i)$.

The vague set A is a set of ordered pairs, given by $A = \{(x_i, [t_A(x_i), 1 - f_A(x_i)])|x_i \in X\}$, where the grade of membership of x_i in the vague set is bounded to a subinterval $[t_A(x_i),\, 1 - f_A(x_i)]$ of $[0, 1]$. Here, the interval $[t_A(x_i),\, 1 - f_A(x_i)]$ is the vague membership (also called vague value) of the object x_i in vague set A. The vague value $[t_A(x_i), 1 - f_A(x_i)]$ indicates that the exact grade of membership $\mu_A(x_i)$ of x_i may be unknown but it is bounded by $t_A(x_i) \leq \mu_A(x_i) \leq 1 - f_A(x_i)$. For example, consider a universe X = {DOG, CAT, and RAT}. A vague set A of X could be $A = \{<$ DOG, [0.7, 0.2] $>, <$ CAT, [0.3, 0.5], $<$ RAT, [0.5, 0.9] $>\}$. For considering $[t_A(x_i), 1 - f_A(x_i)] = [0.5, 0.9]$, we can see that $t_A(x_i) = 0.5, 1 - f_A(x_i) = 0.9$ and $f_A(x_i) = 0.1$. The result can be explained as: x_i belong to vague set A and accept evidence is 0.5, decline evidence is 0.1 and hesitation is $\pi_A(x_i) = 1 - f_A(x_i) - t_A(x_i) = 0.4$. It can also be interpreted as: the vote for resolution is 5 in favor, 1 against, and 4 abstentions, or to say: the number of supporter is 5, the number of objector is 1, and the number of abstainer is 4.

Equality of two vague values

Let $x_A = [t_A(x_i),\, 1 - f_A(x_i)]$ be the vague value of x_i in the vague set A, and $x_B = [t_B(x_i),\, 1 - f_B(x_i)]$ be the vague value of x_i in the vague set B, where $t_A(x_i),\, t_B(x_i),\, f_A(x_i),\, f_B(x_i) \in [0, 1]$. *If* $t_A(x_i) = t_B(x_i)$ and $f_A(x_i) = f_B(x_i)$, then the vague values x_A and x_B are called equal (i.e., $[t_A(x_i),\, 1 - f_A(x_i)]$ = $[t_B(x_i),\, 1 - f_B(x_i)]$).

Maximum operation of two vague values

Let $x_A = [t_A(x_i), 1 - f_A(x_i)]$ be the vague value of x_i in the vague set A, and $x_B = [t_B(x_i), 1 - f_B(x_i)]$ be the vague value of x_i in the vague set B, where $t_A(x_i), t_B(x_i),\ f_A(x_i), f_B(x_i) \in [0, 1]$. The result of the maximum operation of the vague values x_A and x_B is a vague value x_C, written as $x_C = x_A \vee x_B = [t_C(x_i), 1 - f_C(x_i)] = [\max(t_A(x_i), t_B(x_i)), \max(1 - f_A(x_i), 1 - f_B(x_i))] = [\max(t_A(x_i), t_B(x_i)), 1 - \min(t_A(x_i), t_B(x_i))]$.

Minimum operation of two vague values

Let $x_A = [t_A(x_i), 1 - f_A(x_i)]$ be the vague value of x_i in the vague set A, and $x_B = [t_B(x_i), 1 - f_B(x_i)]$ be the vague value of x_i in the vague set B, where $t_A(x_i), t_B(x_i),\, f_A(x_i), f_B(x_i) \in [0, 1]$. The result of the minimum operation of the vague values x_A and x_B is a vague value x_C, written as $x_C = x_A \wedge x_B = [t_C(x_i), 1 - f_C(x_i)] = [\min(t_A(x_i), t_B(x_i), \min(1 - f_A(x_i), 1 - f_B(x_i))] = [\min(t_A(x_i), t_B(x_i)), 1 - \max(t_A(x_i), t_B(x_i))]$.

Equality of two vague sets

Let A and B be two vague sets of the universe of discourse X. If $\forall i, 1 \leq i \leq n, t_A(x_i) = t_B(x_i)$, $f_A(x_i) = f_B(x_i)$, then the vague set A and B are called equal.

Intersection of two vague sets

The intersection of two vague sets A and B is a vague set C, written as $C = A \wedge B$, whose truth membership function and false-membership function are t_C and f_C, respectively, where $\forall x_i \in X$, $t_C(x_i) = \min(t_A(x_i), t_B(x_i)), 1 - f_C(x_i) = \min(1 - f_A(x_i), 1 - f_B(x_i))$. That is, $[t_C(x_i), 1 - f_C(x_i)] = [\min(t_A(x_i), t_B(x_i)), \min(1 - f_A(x_i), 1 - f_B(x_i))] = [\min(t_A(x_i), t_B(x_i)), 1 - \max(f_A(x_i), f_B(x_i))]$.

Union of two vague sets

The union of two vague sets A and B is a vague set Z, written as $Z = A \vee B$, whose truth membership function and false-membership function are t_Z and f_Z, respectively, where $\forall x_i \in X$, $t_Z(x_i) = \max(t_A(x_i), t_B(x_i))$, $1 - f_Z(x_i) = \max(1 - f_A(x_i), 1 - f_B(x_i))$. That is, $[t_Z(x_i), 1 - f_Z(x_i)] = [\max(t_A(x_i), t_B(x_i)), \max(1 - f_A(x_i), 1 - f_B(x_i))] = [\max(t_A(x_i), t_B(x_i)), 1 - \max(f_A(x_i), f_B(x_i))]$.

3 Existing Score Functions for M-CDM Based on Vague Set

A multi-criteria decision making problem is one that aims to find a desirable solution from a finite number of feasible alternatives, which are assessed on multiple criteria both quantitatively and qualitatively. Let A be a discrete set of m alternatives and let C be a set of n independent criteria, where $A = \{A_1, A_2, \dots, A_m\}$, $C = \{C_1, C_2, \dots, C_n\}$ respectively. An alternative satisfies a criterion should it meets the desired level of evaluation. The satisfaction is gradual and can be characterized by a vague value. A multi-criteria decision making problem formulated by vague sets can be concisely expressed in matrix format as in Table 1.

Table 1 The characteristics of the alternatives

	C_1	$\cdots$	C_j	$\cdots$	C_n
A_1	$[t_{11}, t^*_{11}]$	$\cdots$	$[t_{1j}, t^*_{1j}]$	$\cdots$	$[t_{1n}, t^*_{1n}]$
A_2	$[t_{21}, t^*_{21}]$	$\cdots$	$[t_{2j}, t^*_{2j}]$	$\cdots$	$[t_{2n}, t^*_{2n}]$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$
A_i	$[t_{i1}, t^*_{i1}]$	$\cdots$	$[t_{ij}, t^*_{ij}]$	$\cdots$	$[t_{in}, t^*_{in}]$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\cdots$	$\vdots$
A_m	$[t_{m1}, t^*_{m1}]$	$\cdots$	$[t_{mj}, t^*_{mj}]$	$\cdots$	$[t_{mn}, t^*_{mn}]$

Assume that the characteristics of the alternative A_i and the criterion C_j are represented by the vague set shown as follows:

$$A_i = \{(C_1, [t_{i1}, 1 - f_{i1}]), (C_2, [t_{i2}, 1 - f_{i2}]), \dots, (C_n, [t_{in}, 1 - f_{in}])\}, 1 \le j \le n, \text{ and } 1 \le i \le m.$$

With respect to criteria C_j, the alternative A_i performance is measured by the vague value A_{ij}, $A_{ij} = [t_{ij}, 1 - f_{ij}]$, where t_{ij} indicates the degree to which the alternative A_i satisfies criteria C_j, f_{ij} indicates the degree to which the alternative A_i does not satisfy criteria $C_j, t_{ij} \in [0, 1], f_{ij} \in [0, 1], t_{ij} + f_{ij} \le 1, 1 \le j \le n$, and $1 \le i \le m$. Let $1 - f_{ij} = t^*_{ij}$, where $1 \le j \le n, 1 \le i \le m$. Then A_i can be rewritten as

$$A_i = \{(C_1, [t_{1j}, t^*_{1j}]), (C_2, [t_{i2}, t^*_{i2}]), \dots, (C_n, [t_{in}, t^*_{in}])\}.$$

Then the characteristics of these alternatives may be represented by Table 1. Assume that there is a decision maker who wants to choose an alternative which satisfies the criteria $C_j, C_k, \dots$, and C_p or which satisfies the criteria C_s. This decision maker's requirements are represented by the following expression:

$$C_j \text{ AND } C_k \text{ AND }, \dots, \text{ AND } C_p \text{ OR } C_s.$$

Thus, the degrees that the alternative A_i satisfies and does not satisfy the decision maker's requirements can be measured by the evaluation function E as follows:

$$\begin{aligned} E(A_i) &= ([t_{ij}, t^*_{ij}] \wedge \dots \wedge [t_{ip}, t^*_{ip}]) \vee [t_{is}), t^*_{is}] \\ &= [(\min(t_{ij}, \dots, t_{ip})), \min(t^*_{ij}, \dots, t^*_{ip}))] \\ &\vee [t_{is}), t^*_{is}] \\ &= [\max(\min(t_{ij}, \dots, t_{ip}), t_{is}), \max(\min(t^*_{ij}, \\ &\dots, t^*_{ip}), t^*_{is})] = [t_{Ai}, t^*_{Ai}] = [t_{Ai}, 1 - f_{Ai}], \quad (1) \end{aligned}$$

where $\wedge$ denotes the minimum operator and $\vee$ stands for maximum operator of the vague values, and $E(A_i)$ is a vague value, and $1 \le i \le m$. The score of $E(A_i)$ can be measured by several familiar score function $S(E(A_i))$. The greater the value of $S(E(A_i))$, the higher the degree of appropriateness that alternatively satisfies some criteria.

3.1 Criticism of existing score functions for MCDM based on vague set

A score function can be adopted for ranking and selection in decision-making process based on vague sets. It can then be used to measure the degree of suitability of each alternative, with respect to a set of criteria presented by vague values. Suppose E is the evaluation function of alternative A_i, and $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ is the evaluated vague value for the alternative, where $t_{Ai} \in [0, 1]$, $f_{Ai} \in [0, 1]$, and $t_{Ai} + f_{Ai} \le 1$. In this section, several familiar research works on score functions are reviewed and criticised

Chen and Tan's score function [3]
Chen and Tan defined a score function S_{CT} and the score value of $E(A_i)$ can be defined as: $S_{CT}(E(A_i)) = t_{Ai} - f_{Ai}$, where S_{CT} $((E(A_i))$ $\in [-1, 1]$. A larger score value implies a higher degree of suitability that the alternative A_i satisfies the decision maker's requirement.

In [9, 25], the authors used Chen and Tan's score function for evaluating scores of intuitionistic fuzzy values. In Deng and Wibowo's study [5], the authors used Chen and Tan's score function and presented a multi-criteria group decision-making approach for effectively evaluating the performance of e-waste recycling programs under uncertainty in an organization. Zhu et al. [40] proposed a vague set based adjustment model using Chen and Tan's score for regional blood supply and demand balance adjustment.

Example 1. *Suppose there are two alternatives A_1 and A_2, and the evaluated vague values for the two alternatives are $E(A_1) = [0, 1]$ and $E(A_2) = [0.5, 0.5]$. By applying Chen and Tan's score function, $S_{CT}(E(A_i)) = t_{Ai} - f_{Ai}$, we obtain $S_{CT}(E(A_1)) =$*

$0-(1-1)=0$, $S_{CT}(E(A_2))=0.5-(1-0.5)=0$. *We cannot know which alternative is better.*

It is intuitively appealing that if $S_{CT}(E(A_1)) > S_{CT}(E(A_2))$ then A_1 should be better than A_2, but if $S_{CT}(E(A_1)) = S_{CT}(E(A_2))$ this does not always mean that A_1 is equal to A_2. That is, it cannot differentiate many vague values even though they are obviously different. For this reason, Hong and Choi [15] added an accuracy function to measure the degree of accuracy in the grades of membership of each alternative with respect to a set of criteria.

Hong and Choi's accuracy function [15]
Hong and Choi defined the accuracy function H and the accuracy value of $E(A_i)$ can be defined as: $H(E(A_i)) = t_{Ai} + f_{Ai}$, where $H(E(A_i)) \in [0,1]$. A larger value of $H(E(A_i))$ implies a higher degree of accuracy in the grades of membership of the alternative A_i.

This measure provides additional useful information to efficiently help the decision-maker to make his decisions. They also showed that the relationship between the score function S_{CT} and the accuracy function H is similar to the relationship between mean and variance in statistics. A conservative person might choose the alternative with high accuracy value, but an aggressive person may choose the alternative with low accuracy value.

Example 2. *For the two vague values of the two alternatives presented earlier,* $E(A_1) = [0,1]$ *and* $E(A_2) = [0.5, 0.5]$, *we get* $S(E(A_1)) = S(E(A_2)) = 0$ *and we still do not know which one is better. By using the accuracy function, we get* $H(E(A_1)) = 0+1-1 = 0$, $H(E(A_2)) = 0.5 + 1 - 0.5 = 1$. *In this case, it remains unclear which alternative is better. The choice of alternative may depend on the decision maker's preferences: a conservative person might choose the alternative with high accuracy value* $H(E(A_2))$, *but an aggressive person may choose the alternative with low accuracy value* $H(E(A_1))$.

Li and Rao [19] also analyzed the inadequacy of Chen and Tan's score function and defined the following two score functions S_1^{Ai} and S_2^{Ai}, so as to jointly measure the degree of suitability to which alternative A_i satisfies the decision maker's requirement.

Li and Rao's score function [19]
Li and Rao defined two score functions S_1^{Ai} and S_2^{Ai} to jointly measure the degree of suitability to which the alternative A_i satisfies the decision maker's requirement:

Method I: $S_1^{Ai} = t_{Ai}$, $S_2^{Ai} = 1 - f_{Ai}$, or
Method II: $S_1^{Ai} = t_{Ai} - f_{Ai}$, $S_2^{Ai} = 1 - f_{Ai}$.

A larger score value implies a higher degree of suitability. The larger the score value of S_1^{Ai}, the higher degree of suitability to which the alternative A_i satisfies the decision maker's requirement. If the score values of S_1^{A1} and S_1^{A2} for the two alternatives A_1 and A_2 are equal, the score values of S_2^{A1} *and* S_2^{A2} are further calculated and compared to decide the ranking order for the two alternatives.

Example 3. *If two evaluated vague values for two alternatives are* $E(A_1) = [0.1, 0.9]$ *and* $E(A_2) = [0.5, 0.5]$. *If we use method I, then we obtain* $S_1(E(A_1)) = 0.1$ *and* $S_1(E(A_2)) = 0.5$. *The alternative* A_2 *is better than alternative* A_1 *in terms of values of* S_1; *If we use method II, then we obtain* $S_1\ (E(A_1)) = S_1(E(A_2)) = 0$, *we further obtain* $S_2(E(A_1)) = 0.9$, $S_2(E(A_2)) = 0.5$. *The alternative* A_1 *is better than the alternative* A_2 *in terms of values of* S_2. *The two results contradict to each other. Obviously, we know that Li and Rao's score function cannot maintain consistency.*

In fact, Chen and Tan's score function, Hong and Choi's accuracy function and Li's score functions do not give sufficient information about vague values to compare with alternatives. They consider only the truth membership part (i.e., t_{Ai}) and the false membership part (i.e., f_{Ai}), but ignore the unknown part (i.e., $1 - t_{Ai} - f_{Ai}$). This ignorance may ultimately lead to information loss. Thus, Li et al. [20] proposed two different transforming methods for comparing vague values.

Li et al.'s transforming function [20]
Let $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ be a vague value for alternative A_i, where $t_{Ai} \in [0,1]$, $f_{Ai} \in [0,1]$, and $t_{Ai} + f_{Ai} \le 1$. The score of $E(A_i)$ can be evaluated by the transforming function defined as: $S_{LLC}(E(A_i)) = t_{Ai} + (1 - t_{Ai} - f_{Ai})/2 = (t_{Ai} + 1 - f_{Ai})/2$, where $S_{LLC}(E(A_i)) \in [-1, 1]$.

This is also called as the median membership value of the vague set. The larger the value of $S_{LLC}(E(A_i))$, the greater the vague value $E(A_i)$ *is* and the higher degree of suitability that the alternative A_i satisfies the decision maker's requirement.

Example 4. *If two vague values for two alternatives are* $E(A_1) = [0.6, 0.8]$ *and* $E(A_2) = [0.5, 0.9]$. *Utilizing Li et al.'s transforming function, we can obtain:* $S_{LLC}(E(A_1)) = t_{A1} + (1 - t_{A1} - f_{A1})/2 = (t_{A1} + 1 - f_{A1})/2 = 0.7$, *and* $S_{LLC}(E(A_2)) = t_{A2} + (1 - t_{A2} - f_{A2})/2 = (t_{A2} + 1 - f_{A2})/2 = 0.7$. *In this case, we do not know which alternative is better.*

Li et al.'s defuzzification function [20]
Let $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ be a evaluated vague value of alternative A_i. The defuzzification function to

measure the numerical value of the vague value is defined as: $S_D(E(A_i)) = t_{Ai}/(t_{Ai} + f_{Ai})$, where $S_D(E(A_i)) \in [0, 1]$.

Example 5. *If three evaluated vague values for three alternatives are $E(A_1) = [0, 0.1]$, $E(A_2) = [0, 0.9]$ and $E(A_3) = [0, 1]$. Utilizing Li et al.'s defuzzification function, we can obtain $S_D(E(A_1)) = 0/0.9 = 0, S_D(E(A_2)) = 0/0.1 = 0$ and $S_D(E(A_3)) = 0/0$. The vague value $E(A_2)$ is equal to the vague value $E(A_1)$, even though they are obviously different. In this scenario, the vague values are not only indistinguishable, but the result is also unreasonable. For defuzzification of the vague value $E(A_3) = [0, 1]$, we obtain $S_D(E(A_3)) = 0/0$; thus, it would be either mathematically meaningless operations or undefined results in this case.*

Lin et al. [21] defined an improved function to provide a more useful way than those of Chen and Tan's score function, Hong and Choi's accuracy function and Li and Rao's score functions. The improved score function S_{LXW} is written as follows.

Lin et al.'s score function[21]
Let $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ be a evaluated vague value for alternative A_i, where $t_{Ai} \in [0, 1]$, $f_{Ai} \in [0, 1]$, and $t_{Ai} + f_{Ai} \leq 1$. Then, the score of the vague value can be calculated by Lin et al.'s score function defined as: $S_{LXW}(E(A_i)) = t_{Ai} - m_{Ai} = t_{Ai} - (1 - t_{Ai} - f_{Ai}) = 2t_{Ai} + f_{Ai} - 1 = 2t_{Ai} - t^*_{Ai}$, where $S_{LXW}(E(A_i)) \in [-1, +1]$ and m_{Ai} is defined by $m_{Ai} = 1 - t_{Ai} - f_{Ai}, 0 \leq m_{Ai} \leq 1$. That is, m_{Ai} stands for the unknown degree, indefinite degree or hesitancy degree of the vague value.

Although Lin et al. improved Chen and Tan's score function, Hong and Choi's accuracy function and Li and Rao's score functions, there are still some problems. Now, we will illustrate to describe the unsolvable problems as follows.

Example 6. *For the two evaluated vague values of two alternatives $E(A_1) = [0.7, 0.8]$ and $E(A_2) = [0.6, 0.6]$, by applying Lin et al.'s score function, we can get $S_{LXW}(E(A_1)) = 2 \times 0.7 - 0.8 = 0.6$ and $S_{LXW}(E(A_2)) = 2 \times 0.6 - 0.6 = 0.6$. The two vague values are identical; therefore, it is impossible to know which one is the better choice.*

Here is another example to show its deficiency.

Example 7. *For the two evaluated vague values of two alternatives, $E(A_1) = [0.4, 0.9]$ and $E(A_2) = [0.4, 1]$, by applying Lin et al.'s accuracy function, we can get $S_{LXW}(E(A_1)) = 2 \times 0.4 - 0.9 = -0.1$ and $S_{LXW}(E(A_2)) = 2 \times 0.4 - 1 = -0.2$. It seems that the alternative with vague value $E(A_1)$ is better than the alternative with vague value $E(A_2)$ in terms of the values of Lin et al.'s score function. However, the result is not reasonable if explained in voting model. For example, alternative A_1 has 4 votes in favor and 1 vote against it. Alternative A_2 has 4 votes in favor and no vote against it. Most people may choose the alternative with vague value $E(A_2)$ as the better choice, which is conflict with the result drawn by Lin et al.'s score function.*

Wang et al. [30] proposed a new score function S_{WZL} to measure the score of the vague values. The proposed score function place simultaneous emphasis on three parts, i.e., t_{Ai}, f_{Ai} and $1 - t_{Ai} - f_{Ai}$. Therefore, it provides a more sufficient way than those of Chen and Tan's score function S_{CT}, Hong's accuracy function H and Lin et al.'s score function S_{LXW} to measure the score of the vague values and to discriminate two vague values.

Wang et al.'s score function [30]
Let $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ be a evaluate vague value for alternative A_i, where $t_{Ai} \in [0, 1]$, $f_{Ai} \in [0, 1]$, and $t_{Ai} + f_{Ai} \leq 1$. The score value of E(A_i) can be evaluated by the score function defined as: $S_{WZL}(E(A_i)) = t_{Ai} - f_{Ai} - (1 - t_{Ai} - f_{Ai})/2 = (3t_{Ai} - f_{Ai} - 1)/2$, where $S_{WZL}(E(A_i)) \in [-1, 1]$. The larger the score value of $S_{WZL}(E(A_i))$, the greater the vague value $E(A_i)$ *is* and the higher degree of suitability that the alternative A_i satisfies the decision maker's requirement.

Example 8. *For the two evaluated vague values of two alternatives, $E(A_1) = [0.2, 0.6]$ and $E(A_2) = [0.1, 0.9]$. Utilizing the score function S_{WZL}, we obtain $S_{WZL}(E(A_1)) = (3 \times 0.2 - 0.4 - 1)/2 = -0.4$, and $S_{WZL}(E(A_2)) = (3 \times 0.1 - 0.1 - 1)/2 = -0.4$. Therefore, they are still indistinguishable. In this case, we still do not know whether the vague value $E(A_1)$ is superior or inferior with regard to the vague value $E(A_2)$.*

The relationship between Chen and Tan's score function S_{CT} and Hong and Choi's accuracy function H is similar to the relationship between mean and variance in statistics. Based on the concepts of score function and accuracy function, Xu used functions S_{CT} and H to develop a method for comparing two vague values [32, 33, 34].

Xu's order relation method [32, 33, 34].
Suppose $E(A_1) = [t_{A1}, 1 - f_{A1}]$, $E(A_2) = [t_{A2}, 1 - f_{A2}]$ are two evaluated vague values for two alternatives. Let $S_{CT}(E(A_1))$ and $S_{CT}(E(A_2))$ be the score values of $E(A_1)$ and $E(A_2)$. Let $H(E(A_1))$ and $H(E(A_2))$ be the accuracy values of $E(A_1)$ and $E(A_2)$. Then, an order relation between the two

vague values is given according to the following principles:

1. If $S_{CT}(E(A_1)) < S_{CT}(E(A_2))$, then $E(A_1)$ is smaller than $E(A_2)$, denoted by $E(A_1) < E(A_2)$;

2. If $S_{CT}(E(A_1)) = S_{CT}(E(A_2))$, then:

(1) If $S_{CT}(E(A_1)) = S_{CT}(E(A_2))$ and $H(E(A_1)) = H(E(A_2))$, then $E(A_1) = E(A_2)$;

(2) If $S_{CT}(E(A_1)) = S_{CT}(E(A_2))$ and $H(E(A_1)) < H(E(A_2))$, then $E(A_1)$ is smaller than $E(A_2)$.

In recent years, Xu's method has been actively researched for extending new decision-making models. Some real-world case studies have also been carried out to deal with multi-criteria fuzzy decision making problems. In the studies of [11, 16, 29], the authors used Xu's method to compare intuitionistic fuzzy values. Das et al.'s case study [4] applied Xu's method for the comparison of intuitionistic multi-fuzzy values in a real-life case study related to a heart disease diagnosis problem. Goyal et al. used Xu's method to handle the uncertainty of students' knowledge on domain concepts in an E-learning system [13].

Example 9. *For the two evaluated vague values of two alternatives presented in [10],* $E(A_1) = [0.4, 0.8001]$ *and* $E(A_2) = [0.4, 0.8]$*, we get* $S(E(A_1)) = 0.2001$ *and* $S(E(A_2)) = 0.2$*. Since* $S(E(A_1)) - S(E(A_2)) = 0.0001$*, we get* $S(E(A_1)) > S(E(A_2))$*, which is the evidence for* $A_1 > A_2$*. Further, by the accuracy function,* $H(E(A_1)) = 0.4 + 1 - 0.8001 = 0.5999$*,* $H(E(A_2)) = 0.5 + 1 - 0.7 = 0.8$*. Since* $H(E(A_1)) - H(E(A_2)) = 0.2001$*, we get* $H(E(A_1)) > H(E(A_3))$*, which is the evidence for* $A_1 > A_2$*. However, by Xu's order relation method, we are forced to conclude that* $A_1 > A_2$ *even though the difference* $S(E(A_1)) - S(E(A_2)) = 0.0001$ *is negligible in comparison to the difference* $H(E(A_1)) - H(E(A_2)) = 0.2001$.

Xu's order relation method provides additional useful information to help the decision-makers make their decisions. Like Hong and Choi's accuracy function, the limitation of this method is that the score and the accuracy degree are not taken into account simultaneously [10]. Ye [36] defined an modified score function to improve Hong and Choi's accuracy function as follows.

Ye's modified score function [36]
Let $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ be the vague value for alternative A_i, where $t_{Ai} \in [0, 1]$, $f_{Ai} \in [0, 1]$, and $t_{Ai} + f_{Ai} \leq 1$. Then the score of $E(A_i)$ can be evaluated by the modified score function J, as follows: $J(E(A_i)) = t_{Ai} - f_{Ai} + \mu m_{Ai} = t_{Ai}(1 - \mu_{Ai}) + t_{Ai}(1 + \mu_{Ai}) - 1$, where $J(E(A_i)) \in [-1, +1], \mu \in [-1, 1]$, and m_{Ai} is defined by $m_{Ai} = 1 - t_{Ai} - f_{Ai}, 0 \leq m_{Ai} \leq 1$. The parameter μ is introduced to indicate that there is a trend of unanimous modification by taking into account the effect of the unknown degree m_{Ai} on the score of $E(A_i)$, and can be chosen according to actual cases:

1. If $\mu > 0$, the value of the third term μm_{Ai} of $J(E(A_i))$ is positive, and it tends to increase the score of $E(A_i)$ due to its addition in the first term t_{Ai};

2. If $\mu < 0$, the value of the third term μm_{Ai} is negative, then it tends to decrease the score of x due to its addition to the second item f_{Ai};

3. If $\mu = 0$, the improved score function $J(E(A_i))$ is the same score function as $S_{CT}(E(A_i))$ proposed by Chen and Tan [3].

In the renewed vote for resolution, μm_{Ai} can also be interpreted as: if $\mu > 0$, most abstentions may be in favor of; if $\mu < 0$, most of them for; or if $\mu = 0$, they still keep abstentions with.

Recently, Dou used Ye's score function to handle multi-criteria QoS routing decision-making problems [7]. Although Ye's accuracy function seems to better reflect the differences between truth membership and false membership, there are still some problems.

Example 10. *For the two vague values of two alternatives presented earlier,* $E(A_1) = [0, 1]$ *and* $E(A_2) = [0.5, 0.5]$*, by applying Ye's modified score function [33], when parameter* $\mu = 0$*, then we get:* $J(E(A_1)) = 0 \times (1 - 0) + 1 \times (1 + 0) - 1 = 0$ *and* $J(E(A_2)) = 0.5 \times (1 - 0) + 0.5 \times (1 + 0) - 1 = 0$*. We still do not know which one is the better choice. Although Ye's modified score function seems to better reflect the differences between truth membership and false membership, this method is still invalid in some cases.*

In order to overcome the above problem, in the following definition, Ye [37] proposed an improved score function which provides additional useful information to efficiently evaluate the degree of suitability of each alternative for decision making.

Ye's improved score function [37]
Suppose there is a vague value $E(A_i) = [t_{Ai}, 1 - f_{Ai}]$ for alternative A_i, where $t_{Ai} \in [0, 1]$, $f_{Ai} \in [0, 1]$, and $t_{Ai} + f_{Ai} \leq 1$. The score of $E(A_i)$ can be evaluated by the improved score function defined as: $J(E(A_i)) = t_{Ai}(1 + m_{Ai}) - m_{Ai}^2$, where $J(E(A_i)) \in [0, 1]$.

Example 11. *For two vague values,* $E(A_1) =$

$[0.7, 0.9]$ *and* $E(A_2) = [0.67, 0.7]$*, by applying Ye's improved score function, we can obtain* $J(E(A_1)) = t_{A1}(1+m_{A1}) - m_{A1}^2 = 0.7(1+0.1) - 0.1^2 = 0.76$ *and* $J(E(A_2)) = t_{A2}(1+m_{A2}) - m_{A2}^2 = 0.67(1+0.3) - 0.3^2 = 0.78$*. It seems that the alternative with vague value* $E(A_1)$ *is inferior to the alternative with vague value* $E(A_2)$*. However, the result is not reasonable if explained in terms of a voting model: alternative* A_1 *has 70% in favour and 10% against it; alternative* A_2 *has 67% in favour and 30% against it. In this scenario, Ye's improved accuracy function will produce the opposite result, which would be in conflict with the decision makers' desired outcomes.*

In addition to the aforementioned main score functions, many other researchers have also proposed different definitions of score functions, such as Liu, Wang, and Lin [23], Priya and Supriya [26], Solairaju et al. [28], Wang and Li [31], Dymova et al. [10]. However, these score functions have the same problematic deficiencies that the above familiar works have.

As mentioned above, when using these vague based score functions to address multi-criteria decision-making problems, the problematic deficiencies of existing score functions can be summarized as follows. Firstly, for the vague values, insufficient information of the unknown part may cause information loss during score value transformation. In addition to that, for some researchers' works, over-complicated calculations are inefficient for transferring the vague values into comparable score values. Furthermore, for some cases, undesirable or unreasonable results, which are conflict with the decision makers' desires, may be obtained. Finally, for some cases, mathematically meaningless operation or undefined results during vague values transformation could arise.

To address these problematic deficiencies, there is an emerging demand for proposing a novel score function to transform the uncertain and imprecise vague values into comparable numerical values.

4 The Proposed Novel Score Function for MCDM Based on Vague Set

In order to overcome the aforementioned problems of score functions for decision making, we proposed a novel vague set based score function inspired by type-2 fuzzy sets theory to provide another useful way to assist the decision making problems. The concept of type-2 fuzzy sets was initially proposed by Zadeh [38] as an extension of ordinary fuzzy sets. In the following, notions of Type-2 fuzzy sets are briefly recalled, and the proposed vague based score function which is fundamental to measure the degree of suitability of each alternative is defined.

Definition 1. Type-2 fuzzy set *[8, 27, 38]*
A type-2 fuzzy set A *in* X *is characterized by a three dimensional type-2 membership function* $\mu_A(x, u)$: $X \times [0, 1] \to [0, 1]$, $A = \{(x, u), \mu_A(x, u)) \,|\forall x \in X \wedge \forall u \in J_x \subseteq U = [0, 1]\}$*, where* x *is a point in the primary domain* X*, and* J_x *is called the primary membership function of* x*;* u *is a point in the secondary domain* U*, and* $\mu_A(x, u)$ *is called the secondary membership function whose domain is the primary membership of* x*. When all* $\mu_A(x, u) = 1$, $\forall u \in J_x \subseteq U = [0, 1]$*, then the type-2 membership function is an interval type-2 membership function and the type-2 fuzzy set* A *is an interval type-2 fuzzy set. The interval secondary membership function reflects a uniform uncertainty at the primary memberships of* x*.*

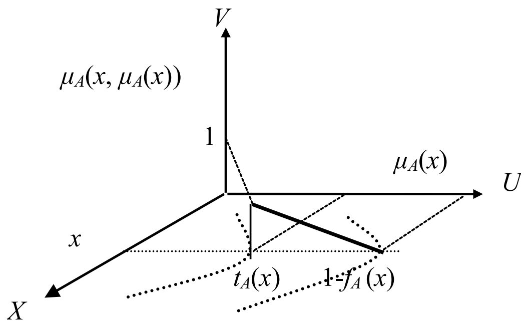

Figure 1 Secondary membership function of vague value

Fig. 1 shows a 3D representation of a vague value $[t_A(x), 1 - f_A(x)]$, where vague set A in the universe of discourse X, $A = \{(x, [t_A(x), 1 - f_A(x)])|x \in X\}$, and where the interval $[t_A(x), 1 - f_A(x)]$ of $[0, 1]$ is a vague value to the object x in vague set A. In the third dimension, a corresponding second membership function $\mu_A(x, \mu_A(x))$ maps the membership degree of the elements in the interval $[t_A(x), 1 - f_A(x)]$. The value $\mu_A(x, \mu_A(x))$ is a random value from the interval $[0, 1]$. It means that the second membership function $\mu_A(x, \mu_A(x))$ indicates to what degree of support an element on the interval $[t_A(x), 1 - f_A(x)]$ falls under "the concept x_i is true".

In the interval $[t_A(x), 1 - f_A(x)]$, if an element has a grade of second membership function $\mu_A(x, \mu_A(x))$ equal to 1, this reflects a complete fitness between the element and "the concept x is true"; if an element has a grade of support membership function $\mu_A(x, \mu_A(x))$ equal to 0, then the element does not belong to that "the concept x is true". For the element $t_A(x)$, the property of "being true" is fully satisfied. Hence the membership degree under "being true" is equal to 1. For the element $(1 - f_A(x))$, the property of "being true" is equal to zero. For the elements $\mu_A(x)$ less than $t_A(x)$ or more than $(1-f_A(x))$, the property of "being true" is completely excluded from this set. For the element $\mu_A(x)$ between $t_A(x)$ and $(1 - f_A(x))$, the property of "being true" is partially satisfied.

The second membership value of each element in the interval $[t_A(x), 1 - f_A(x)] = [p, q]$ can be read as follows: the second membership function $\mu_A(x, \mu_A(x))$ takes numerical values "Equal to 1 and is continuous and strictly decreasing to 0 as the $\mu_A(x)$ value increases between $t_A(x)$ and $1-f_A(x)$". Therefore, the second membership function $\mu_A(x, \mu_A(x))$ are plausibly to be strictly decreasing on the interval $[t_A(x), 1 - f_A(x)]$. Using this function, the second membership value $\mu_A(x, \mu_A(x))$ on the interval $[t_A(x), 1 - f_A(x)]$ is linearly mapped to a value in range $[1, 0]$.

Definition 2. Vague value set and its secondary membership function
If X is a collection of objects denoted generically by x, then A is defined to be a vague set of the universe of discourse X, written as $A = \{(x, [t_A(x), 1 - f_A(x)]) | x \in X\}$. The vague value $[t_A(x), 1 - f_A(x)]$ indicates that the exact grade of membership $\mu_A(x)$ of x may be unknown but it is bounded by $t_A(x) \leq \mu_A(x) \leq 1 - f_A(x)$. Therefore, the vague set A can also be generalized as a type-2 fuzzy set, whose secondary membership function can be presented as $\mu_A(x, \mu_A(x))$. This implies that the value of the primary membership of x, $\mu_A(x)$, is also referred to as the secondary domain of the secondary membership function $\mu_A(x, \mu_A(x))$.

In this case, X is referred to as the primary domain; U is referred to as the secondary domain, as well as the value of the primary membership of x; V is referred to as the secondary membership value of x. As shown in Fig.1, $\mu_A(x)$ of $[0, 1]$ is the primary membership function, whose primary domain is the universe of discourse X; $\mu_A(x, \mu_A(x)))$ of $[0, 1]$ is the secondary membership function, whose secondary domain is the vague value U.

At x, the secondary membership value corresponding to each primary membership value in the closed interval $[t_A(x), 1 - f_A(x)] = [t_A(x), t_A(x)^*]$ can be represented as follows.

$$\mu_A(x, \mu_A(x)) = \begin{cases} 0, & \textit{for } \mu_A(x) < t_A(x); \\ (1 - f_A(x) - \mu_A(x))/(1 - f_A(x) - t_A(x)), & \\ \quad \textit{for } t_A(x) \leq \mu_A(x) \leq 1 - f_A(x); & \\ 0, & \textit{for } 1 - f_A(x) < \mu_A(x), \end{cases}$$

where $\mu_A(x, \mu_A(x))$: $X \times [0, 1] \to [0, 1]$.

Clearly, the interval $[t_A(x),\ 1 - f_A(x)]$ and the secondary membership value, which is both "normal" and "convex", define a right triangular fuzzy number denoted as TFN($t_A(x)$, $t_A(x)$, $1 - f_A(x)$). Each data object in the interval $[t_A(x),\ 1 - f_A(x)]$ represents an element of the "the concept x is true" associated with a degree of secondary membership function $\mu_A(x, \mu_A(x))$, linearly decreasing from 1 to 0.

For example, at x, the primary membership values are in the vague interval $\mu_A(x) = [t_A(x),\ 1 - f_A(x)] =$ [0.6, 0.90] and their associated secondary membership values are as follows:

$\mu_A(x, \mu_A(x)) = 0$, for all $\mu_A(x) < 0.6$;

$\mu_A(x, \mu_A(x)) = 1$, for $\mu_A(x) = 0.6$;

$\mu_A(x, \mu_A(x))$ is strictly decrease from 1 to 0, for $\mu_A(x)$ from 0.6 to 0.90;

$\mu_A(x, \mu_A(x)) = 0$, for all $\mu_A(x) > 0.90$.

Yager's centroid method [35]
Yager proposed a centroid index ranking method for calculating the value u_N^* for a fuzzy number N : $u_N^* = \int_a^b w(u)\mu_N(u)du / \int_a^b \mu_N(u)du$, where a and b are the lower bound and the upper bound of the fuzzy number; w is a weighing function and $w(u)$ denotes the weighing value of the weighing function measuring the importance of the value u; μ_N is a membership function and $\mu_N(u)$ indicates the membership value of the element u in the fuzzy number N. When $w(u) = u$ (i.e. linear weight), the value u_N^* becomes the geometric center of gravity (CoG) shown as follows: $u_N^* = \int_a^b u \cdot \mu_N(u)du / \int_a^b \mu_N(u)du$. This method is used as a transforming method to transform the aforementioned right triangular fuzzy number TFN($t_A(x), t_A(x), 1 - f_A(x)$) into a single comparable numerical score. The larger the value of μ_N, the larger the expected truth value and the better the ranking of the fuzzy number N.

4.1 Proposed new score function

Suppose the evaluation function of an alternative can be expressed by the vague value $E(A) = [t_A(x),\ 1 - f_A(x)]$, where 0≤ $t_A(x) \leq 1 - f_A(x) \leq 1$. By the above definition, the elements in the vague value $[t(x),\ 1 - f(x)]$ can be generalized as a type-2 fuzzy set, whose secondary membership function can be presented as $\mu(x, \mu_A(x))$ to define a right triangular fuzzy number TFN($t_A(x), t_A(x), 1 - f_A(x)$).

As shown in Figure 2, by applying Yager's centroid method, the transformed numerical score of the defined triangular fuzzy number TFN($t_A(x)$, $t_A(x), 1 - f_A(x)$) can be regarded as the centroid index of the right triangular fuzzy number TFN($t_A(x), t_A(x), 1 - f_A(x)$). Thus, the numerical

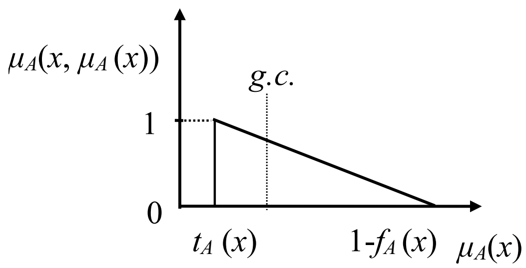

Figure 2 Membership function of a vague value $V(x) = [t_A(x), 1 - f_A(x)]$

score of the vague value $S_L(E(A))$ can be calculated by the following transforming score function:

$$S_L(E(A)) = \int_{t_A(x)}^{1-f_A(x)} \frac{x \cdot (1 - f_A(x) - t_A(x))}{1 - f_A(x) - t_A(x)} dx \Big/ \int_{t_A(x)}^{1-f_A(x)} \frac{(1 - f_A(x) - t_A(x))}{1 - f_A(x) - t_A(x)} dx$$
$$= t_A(x) + (1 - f_A(x) - t_A(x))/3$$
$$= \frac{2}{3} t_A(x) + \frac{1}{3}(1 - f_A(x)) \quad (2)$$

The secondary membership function $\mu(x, \mu_A(x))$ maps each element in the primary membership value $\mu_A(x) = [t_A(x), 1 - f_A(x)]$ to a secondary membership value between 0 and 1.

This method is used as an effective and efficient transforming method to transform the vague membership values of the elements in the vague interval into a single comparable crisp value.

To illustrate, at x, the primary membership values are in the vague interval $[t_A(x), 1 - f_A(x)] =$ [0.6, 0.9]. The primary membership function $\mu_A(x)$ and the secondary membership function $\mu_A(x, \mu_A(x))$ define a right triangular fuzzy number TFN$(t(x), t_A(x), 1 - f_A(x))$ = TFN(0.6, 0.6, 0.9) and can be transformed as follows:

$$S_L(E(A)) = \frac{2}{3} \times 0.6 + \frac{1}{3} \times 0.9 = 0.7$$

4.2 Illustrative example

In the above section, a novel score function for measuring the degree of suitability that an alternative satisfies the decision maker's requirement is shown. the proposed score function differ from the previous ones in that the proposed score function is a linear function to transform the vague values rather than non-linear functions proposed by other authors.

In this section, we took a case application as an illustration using the data set which is slightly modified from an example presented earlier in [30]. The proposed new score function is demonstrated as follows to draw comparisons of ranking orders with some commonly used score functions.

Example 12. *For a multi-criteria decision making problem based on vague set, let* $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$ *be a set of alternatives and let* $C = \{C_1, C_2, C_3, C_4\}$ *be a set of criteria. The characteristics of the alternatives are represented by the vague sets shown as follows:*

$A_1 = \{(C_1, [0.2, 0.4]), (C_2, [0.5, 0.8]), (C_3, [0.0, 0.6]), (C_4, [0.0, 1.0])\}$
$A_2 = \{(C_1, [0.8, 1.0]), (C_2, [0.6, 0.8]), (C_3, [0.5, 0.5]), (C_4, [0.3, 0.5])\}$
$A_3 = \{(C_1, [0.4, 0.6]), (C_2, [0.1, 0.9]), (C_3, [0.5, 0.8]), (C_4, [0.1, 0.4])\}$
$A_4 = \{(C_1, [0.4, 0.6]), (C_2, [0.5, 0.7]), (C_3, [0.6, 0.6]), (C_4, [0.1, 0.9])\}$
$A_5 = \{(C_1, [0.8, 0.9]), (C_2, [0.2, 0.5]), (C_3, [0.4, 0.6]), (C_4, [0.6, 1.0])\}$
$A_6 = \{(C_1, [0.3, 1.0]), (C_2, [0.4, 0.9]), (C_3, [0.2, 0.85]), (C_4, [0.65, 0.7])\}$
$A_7 = \{(C_1, [0.5, 0.8]), (C_2, [0.0, 0.9]), (C_3, [0.3, 0.6]), (C_4, [0, 0.7])\}$

Assume that the decision maker want to rank the alternatives A_1 through A_7 which satisfy the criteria C_1 and C_2 and C_3 or which satisfy the criteria C_4.

By applying the minimum operation and maximum operation in Eq. (1), we can yield the evaluation values of alternative:

$E(A_1) = ([0.2, 0.4] \wedge [0.5, 0.8] \wedge [0.0, 0.6]) \vee [0.0, 1.0] = [0.0, 1.0]$
$E(A_2) = ([0.8, 1.0] \wedge [0.6, 0.8] \wedge [0.5, 0.5]) \vee [0.3, 0.5] = [0.5, 0.5]$
$E(A_3) = ([0.4, 0.6] \wedge [0.1, 0.9] \wedge [0.5, 0.8]) \vee [0.1, 0.4] = [0.1, 0.6]$
$E(A_4) = ([0.4, 0.6] \wedge [0.5, 0.7] \wedge [0.6, 0.6]) \vee [0.1, 0.9] = [0.4, 0.9]$
$E(A_5) = ([0.8, 0.9] \wedge [0.2, 0.5] \wedge [0.4, 0.6]) \vee [0.6, 1.0] = [0.6, 1.0]$
$E(A_6) = ([0.3, 1.0] \wedge [0.4, 0.9] \wedge [0.2, 0.85]) \vee [0.65, 0.7] = [0.65, 0.85]$
$E(A_7) = ([0.5, 0.8] \wedge [0.0, 0.9] \wedge [0.3, 0.6]) \vee [0, 0.7] = [0, 0.7]$.

4.3 Results and discussions

By using the proposed new score function in Eq. (2), the degree of suitability to which the alternative A_i satisfies the decision maker's requirements can be calculated as follows:

$$S_L(E(A_1)) = \frac{2}{3} \times 0 + \frac{1}{3} \times 1.0 = 0.33$$

Table 2 Score function and ranking order

score function	ranking order
Chen and Tan's score function $S_{CT}(x)$ [3]	$A_5 \succ A_6 \succ A_4 \succ A_2 = A_1 \succ A_3 = A_7$
Li et al.'s transforming function [20]	$A_5 \succ A_6 \succ A_4 \succ A_2 = A_1 \succ A_3 = A_7$
Li et al.'s defuzzification function [20]	$A_5 \succ A_6 \succ A_4 \succ A_2 \succ A_3 \succ A_7, A_1$ is meaningless
Wang et al.'s score function, $S_{WZL}(x)$ [30]	$A_5 = A_6 \succ A_4 \succ A_2 \succ A_1 \succ A_3 \succ A_7$
Xu's order relation method [32, 33, 34]	$A_5 \succ A_6 \succ A_4 \succ A_2 \succ A_1 \succ A_7 \succ A_3$
Proposed score function in this study, $S_L(x)$	$A_5 \succ A_6 \succ A_4 \succ A_2 \succ A_1 \succ A_3 \succ A_7$

$$S_L(E(A_2)) = \frac{2}{3} \times 0.5 + \frac{1}{3} \times 0.5 = 0.5$$

$$S_L(E(A_3)) = \frac{2}{3} \times 0.1 + \frac{1}{3} \times 0.6 = 0.27$$

$$S_L(E(A_4)) = \frac{2}{3} \times 0.4 + \frac{1}{3} \times 0.9 = 0.57$$

$$S_L(E(A_5)) = \frac{2}{3} \times 0.6 + \frac{1}{3} \times 1.0 = 0.73$$

$$S_L(E(A_6)) = \frac{2}{3} \times 0.65 + \frac{1}{3} \times 0.85 = 0.7$$

$$S_L(E(A_7)) = \frac{2}{3} \times 0 + \frac{1}{3} \times 0.7 = 0.23$$

The ranking order of the scores for the seven vague values is given as follows:

$$S_L(E(A_5)) \succ S_L(E(A_6)) \succ S_L(E(A_4)) \succ S_L(E(A_2)) \succ S_L(E(A_1)) \succ S_L(E(A_3)) \succ S_L(E(A_7))$$

Consequently, the ranking order of the seven alternatives is given as follows:

$$A_5 \succ A_6 \succ A_4 \succ A_2 \succ A_1 \succ A_3 \succ A_7$$

Table 2 shows the comparison results of the ranking orders obtained from using familiar score functions and from using the proposed new score function. By using Chen and Tan's score function, Li et al.'s transforming function, and Wang et al.'s score function, alternatives are still indistinguishable. By using Li et al.'s defuzzification function, for the vague value $E(A_1) = [0, 1]$, we obtain $S_D(E(A_1)) = 0/0$. Thus, it would be either mathematically meaningless or undefined to calculate the score $S_D(E(A_1))$ in this case. By using Xu's order relation method, the alternative with evaluated vague value $E(A_3)$ is inferior to the alternative with vague value $E(A_7)$. It can be easily seen that the unreasonable result may be in conflict with decision makers' desired outcome in this scenario.

These analytical results demonstrate that by using existing score functions for comparing vague values, some comparison are indistinguishable, some operation are mathematically undefined and some comparison results are unreasonable. However, these deficiencies can be resolved by using the proposed score function. These results suggest that the novel score function provides a more distinguishable, easily computable and reasonable way than other score functions for discriminating vague values. Therefore, we can rank all alternatives through this novel score function.

5 Conclusions and Further Research

After reviewing and examining existing vague set based score functions in the literature, our analysis of these works revealed that there exist several deficiencies when using these score functions in multi-criteria fuzzy decision-making problems. To address these problematic deficiencies, this paper has presented a novel score function to measure the degree of suitability of each alternative with respect to a set of criteria to be represented by vague values.

The proposed novel score function is a linear score function which emphasizes three parts, namely, t_x, f_x and $1 - t_x - f_x$, simultaneously. As expected, during vague score transformation, the aforementioned problematic deficiencies of existing score functions can be overcome: First, during vague value transformation, the information loss caused by the insufficient information of the unknown part can be overcome; Second, the proposed score function is easily computable with higher precision and consistency than other existing score functions; Third, undesirable or unreasonable transformed results, which are in conflict with the decision makers' desires, can be avoided; Fourth, mathematically meaningless operations or undefined results during vague values transformation can be prevented.

An illustrative example has also been given to allow a comparison of the proposed score function with the existing main score functions. The analytical results demonstrate that the proposed vague set based score function is more effective and reasonable than other existing score functions in handling multi-criteria decision-making problems.

In the future, further work focused on the development on new multi-criteria fuzzy decision making models based on the proposed score function will be carried out. Another area warranting attention can be concentrated on conducting real-word case studies to tackle multi-criteria fuzzy decision making problems by using the developed decision-making models.

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