Global State-Feedback Stabilization of High-Order Nonholonomic Systems with Time-Varying Delays

DEHENG HOU
School of Software Engineering
Anyang Normal University
Anyang, 455000
CHINA
hdh@aynu.edu.cn

FANGZHENG GAO
School of Mathematics and Statistics
Anyang Normal University
Anyang, 455000
CHINA
gaofz@seu.edu.cn

Abstract: This paper investigates the problem of global stabilization by state-feedback for a class of high-order nonholonomic systems in power chained form with time-varying delays. By using input-state-scaling, and adding a power integrator techniques, and choosing an appropriate Lyapunov-Krasoviskii functional, a state-feedback controller is constructed. Based on switching strategy to eliminate the phenomenon of uncontrollability, the proposed controller could ensure that the closed-loop system is globally asymptotically regulated at origin. A simulation example is provided to illustrate the effectiveness of the proposed method.

Key–Words: High-order nonholonomic systems, Time-varying delays, Input-state-scaling, Adding a power integrator

1 Introduction

In the past decades, there has been a rapidly growing research interest in nonholonomic systems, which can be found frequently in the real world, such as mobile robots, car-like vehicle, under-actuated satellites, the knife-edge and so on. As pointed out in [1], such a class of nonlinear systems cannot be asymptotically stabilized at the origin by only using continuous state feedback control signal. In order to overcome this obstruction, several approaches have been proposed for the problem, such as discontinuous feedback [4, 8], time-varying feedback [2, 5, 9] and hybrid stabilization [3, 6, 7]. Using these valid approaches, the asymptotic stabilization or exponential regulation for nonholonomic systems has been extensively studied [10-16]. More meaningly, the high-order nonholonomic systems in power chained form, which can be viewed as the extension of the classical nonholonomic systems, have been achieved investigation[17-19].

However, the aforementioned results do not consider the effect of time delay. As a matter of fact, time-delay is actually widespread in state, input and output due to sensors, calculation, information processing or transport, and its emergence is often a significant cause of instability and serious deterioration in the system performance [20-23]. Therefore, how extending these methods to the systems with time delays is naturally regarded as an interesting research topic. Recently, [24] and [25] investigated the state-feedback stabilization problem for delayed nonholonomic systems with different structures. However, the control design for high-order nonholonomic systems with time delays is extremely challenging because on the one hand, some intrinsic features of high-order nonholonomic systems, such as its Jacobian linearization being neither controllable nor feedback linearizable, lead to the existing design tools being hardly applicable to this kind of systems, and on the other hand, the existence of time-delay effect will make the common assumption on the high-order systems nonlinearities infeasible and what conditions should be imposed on the systems remain unanswered. To the best of the authors' knowledge, there is no result for the high-order nonholonomic systems with time delays. It is precisely our intention of this paper.

In this paper, we introduce a class of high-order nonholonomic systems in power chained form with time-varying delays, and then study the problem of robust state-feedback stabilization for the concerned systems. The contribution of this paper is highlighted as follows. First, motivated by the work in [26] and flexibly using the methods of adding a power integrator, a recursive design procedure for the time-delay independent state-feedback controller is given. Then, by employed an appropriate Lyapunov- Krasovskii functional, we show that the controller designed guarantees global asymptotic regulation of the resulting closed-loop system.

The rest of this paper is organized as follows. In

Section 2, preliminary knowledge and the problem formulation are given. Section 3 presents the input-state-scaling technique and the recursive design procedure, while Section 4 provides the switching control strategy and the main result. Section 5 gives a simulation example to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 6.

2 Preliminaries and problem formulation

Consider the following high-order nonholonomic systems with time-varying delays

$$\begin{cases} \dot{x}_0(t) = d_0(t)u_0^{p_0}(t) + f_0(t, x_0(t)) \\ \dot{x}_i(t) = d_i(t)x_{i+1}^{p_i}(t)u_0^{q_i}(t) \\ \qquad\quad +f_i(t, x_0(t), x(t), x(t-d(t))) \\ \qquad\quad i = 1, \cdots, n-1 \\ \dot{x}_n(t) = d_n(t)u_1^{p_n}(t) \\ +f_n(t, x_0(t), x(t), x(t-d(t))) \end{cases} \tag{1}$$

where $x_0 \in R$ and $x = (x_1, \cdots, x_n)^T \in R^n$ are system states, u_0 and u_1 are control inputs, respectively; $p_i \geq 1, i = 0, 1, \cdots, n$ are odd integers, and $q_k \geq 1$, $k = 1, 2, \cdots, n-1$ are integers; $d_i(t)$ are disturbed virtual control coefficients; f_0 and f_i, $i = 1, \cdots, n$ are unknown continuous functions with $f_0(t, 0) = 0$ and $f_i(t, 0, 0, 0) = 0$; $d(t) : R_+ \to [0, d]$ is the time-varying delay satisfying $\dot{d}(t) \leq \eta < 1$ for a known constant η;

The control objective is to construct robust control laws of the form

$$u_0 = \mu_0(x_0), \quad u_1 = \mu(x_0, x) \tag{2}$$

such that all signals of the closed-loop system are bounded. Furthermore, global asymptotic regulation of the states are achieved, i.e.

$$\lim_{t\to\infty}(|x_0(t)| + |x(t)|) = 0.$$

In order to achieve the above control objective, throughout the paper, the following assumptions regarding system (1) are imposed.

Assumption 1. For $i = 0, 1, \cdots, n$, there are positive constants c_{i1} and c_{i2} such that

$$c_{i1} \leq d_i(t) \leq c_{i2}$$

Assumption 2. For f_0, there is a positive constant c_{03} such that

$$|f_0(t, x_0(t))| \leq c_{03}|x_0|$$

For $i = 1, \cdots, n$, there are $\mathcal{F}$ functions a_i such that

$$\begin{aligned} &|f_i(t, x_0(t), x(t), x(t-d(t)))| \\ &\leq a_i(x_0(t))\Big(|x_i(t)| + |x_i(t-d(t))|\Big) \\ &\quad +a_i(x_0(t))\sum_{j=1}^{i-1}\Big(|x_j(t)|^{\frac{1}{p_j\cdots p_{i-1}}} \\ &\quad +|x_j(t-d(t))|^{\frac{1}{p_j\cdots p_{i-1}}}\Big) \end{aligned}$$

To make the paper self-contained, we recall that a continuously differential function $f : R^m \to R$ is called a $\mathcal{F}$ function if it is nonnegative and monotone-nondecreasing on $[0, +\infty)$. It is worthwhile to point out that there exist many functions such as $f_1(x) \equiv c > 0$ and $f_2(x) = x^m$,where $m \geq 0$ being $\mathcal{F}$ functions. Furthermore, it can be showed that if f and g are $\mathcal{F}$ functions, then $f + g$, $f \cdot g$ and $f \circ g$ are also $\mathcal{F}$ functions.

Remark 1 *Assumption 1 is common and similar to the ones usually imposed on the nonholonomic systems [10,11,15,19]. It is worth pointing out that Assumption 2 is somewhat stringent, and when $a_i(x_0(t)) = a$ (a is a positive constant) and $d(t) = 0$, it is the same as that in [26]. Particularly, $p_i = 1, i = 1, \cdots, n$ the assumption is equivalent to that in [20,27].*

The following two lemmas can be found in [27,28], which serve as the basis of the key tools for the adding a power integrator technique.

Lemma 2 *For $x \in R$, $y \in R$, and $p \geq 1$ is a constant, the following inequalities hold:*

$$|x + y|^p \leq 2^{p-1}|x^p + y^p|$$

$$(|x|+|y|)^{1/p} \leq |x|^{1/p}+|y|^{1/p} \leq 2^{(p-1)/p}(|x|+|y|)^{1/p}$$

If $p \geq 1$ is odd, then

$$|x - y|^p \leq 2^{p-1}|x^p - y^p|$$

$$|x^{1/p} - y^{1/p}| \leq 2^{(p-1)/p}|x - y|^{1/p}$$

Lemma 3 *Let c, d be positive real numbers and $\pi(x, y) > 0$ be a real-valued function. Then,*

$$|x|^c|y|^d \leq \frac{c\pi(x, y)|x|^{c+d}}{c + d} + \frac{d\pi^{-c/d}(x, y)|y|^{c+d}}{c + d}$$

3 Robust controller design

In this section, we focus on designing robust controller provided that $x_0(t_0) \neq 0$. The case where the initial condition $x_0(t_0) = 0$ will be treated in Section 4. The inherently triangular structure of system (1) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

3.1 Design u_0 for x_0-subsystem

For x_0-subsystem, the control u_0 can be chosen as

$$u_0^{p_0}(t) = -\lambda_0 x_0(t) \tag{3}$$

where λ_0 is a positive design constant and satisfies $\lambda_0 > 1 + (c_{03}/c_{01})$.

As a result, the following lemma can be established by considering the Lyapunov function candidate $V_0 = x_0^2/2$ and by applying directly the Gronwall-Bellman inequality [10].

Lemma 4 *For any initial $t_0 \geq 0$ and any initial condition $x_0(t_0) \in R$, the corresponding solution $x_0(t)$ exists for each $t \geq t_0$ and satisfies*

$$\begin{aligned} x_0(t_0) \geq 0 \Rightarrow x_0(t_0)e^{-(\lambda_0 c_{02}+c_{03})(t-t_0)} \\ \leq x_0(t) \leq x_0(t_0)e^{-(\lambda_0 c_{01}-c_{03})(t-t_0)} \end{aligned} \tag{4}$$

$$\begin{aligned} x_0(t_0) < 0 \Rightarrow x_0(t_0)e^{-(\lambda_0 c_{01}-c_{03})(t-t_0)} \\ \leq x_0(t) \leq x_0(t_0)e^{-(\lambda_0 c_{02}+c_{03})(t-t_0)} \end{aligned} \tag{5}$$

Remark 5 *From Lemma* **??**, *we can see that $x_0(t)$ can be zero only at $t = t_0$, when $x(t_0) = 0$ or $t = \infty$. Consequently, it is concluded that x_0 does not cross zero for all $t \in (t_0, \infty)$ provided that $x_0(t_0) \neq 0$. Furthermore, from (3), it follows that the u_0 exists, does not cross zero for all $t \in (t_0, \infty)$ independent of the x-subsystem and satisfies $\lim_{t\to\infty} u_0(t) = 0$ provided that $x_0(t_0) \neq 0$.*

3.2 Input-state-scaling transformation

From the above analysis, we can see the x_0-state in (1) can be globally exponentially regulated to zero via $u_0(t)$ in (3) as $t \to \infty$. It is troublesome in controlling the x-subsystem via the control input $u_1(t)$ because, in the limit (i.e. $u_0(t) = 0$), the x-subsystem is uncontrollable. This problem can be avoided by utilizing the following discontinuous input-state-scaling transformation

$$z_i(t) = \begin{cases} \dfrac{x_i(t)}{u_0^{r_i}(t)}, & t \geq t_0 \\ x_i(t), & t_0 - d \leq t < t_0 \end{cases} \quad i = 1, \cdots, n \tag{6}$$

where $r_i = q_i + p_i r_{i+1}, 1 \leq i \leq n-1$ and $r_n = 0$.

Remark 6 *Note that (6) is a modified version of input-state-scaling transformation [18,19], and is used to deal with time delay terms. Because of the particular choice (6), for $i = 1, \cdots, n$, $z_i(t - d(t))$ are well-defined.*

Under the new z-coordinates, the x-subsystem is transformed into

$$\begin{cases} \dot{z}_i(t) = d_i(t)z_{i+1}^{p_i}(t) \\ \qquad +g_i(t, x_0(t), x(t), x(t-d(t))) \\ \quad i = 1, \cdots, n-1 \\ \dot{z}_n(t) = d_n(t)u_1^{p_n}(t) \\ \qquad +g_n(t, x_0(t), x(t), x(t-d(t))) \end{cases} \tag{7}$$

where

$$\begin{aligned} & g_i(t, x_0(t), x(t), x(t-d(t))) \\ & \quad = \frac{f_i(t, x_0(t), x(t), x(t-d(t)))}{u_0^{r_i}(t)} \\ & \qquad + r_i z_i(t)\frac{u_0^{p_0}(t) + f_0(t, x_0(t))}{p_0 x_0(t)} \end{aligned} \tag{8}$$

In order to obtain the estimation for the nonlinear function g_i, the following lemma can be derived by the assumption before.

Lemma 7 *For $i = 1, \cdots, n$, there exist $\mathcal{F}$ functions ϕ_i such that*

$$\begin{aligned} & |g_i(t, x_0(t), x(t), x(t-d(t)))| \\ & \leq \phi_i(x_0(t))\Big(|z_i(t)| + |z_i(t-d(t))|\Big) \\ & +\phi_i(x_0(t))\sum_{j=1}^{i-1}\Big(|z_j(t)|^{\frac{1}{p_j\cdots p_{i-1}}} \\ & +|z_j(t-d(t))|^{\frac{1}{p_j\cdots p_{i-1}}}\Big) \end{aligned} \tag{9}$$

Proof. In view of (3)-(6), (8) and Assumption 2, we get

$$\begin{aligned} & |g_i(t, x_0(t), x(t), x(t-d(t)))| \\ & \leq a_i\Big(\frac{|x_i(t)|}{|u_0^{r_i}(t)|} + \frac{|x_i(t-d(t))|}{|u_0^{r_i}(t)|}\Big) \\ & + \frac{r_i(\lambda_0 + c_{03})}{p_0}|z_i(t)| \\ & + a_i\sum_{j=1}^{i-1}\Big(\frac{|x_j(t)|^{\frac{1}{p_j\cdots p_{i-1}}}}{|u_0^{r_i}(t)|} + \frac{|x_j(t-d(t))|^{\frac{1}{p_j\cdots p_{i-1}}}}{|u_0^{r_i}(t)|}\Big) \\ & \leq \Big(a_i + \frac{r_i(\lambda_0 + c_{03})}{p_0}\Big)|z_i(t)| \\ & + \Big(a_i\frac{|u_0^{r_i}(t-d(t))|}{|u_0^{r_i}(t)|}\Big)|z_i(t-d(t))| \\ & + a_i\sum_{j=1}^{i-1}\Big(|z_j(t)|^{\frac{1}{p_j\cdots p_{i-1}}}|u_0(t)|^{\frac{r_j}{p_j\cdots p_{i-1}}-r_i} \\ & + |z_j(t-d(t))|^{\frac{1}{p_j\cdots p_{i-1}}}\frac{|u_0(t-d(t))|^{\frac{r_j}{p_j\cdots p_{i-1}}}}{|u_0^{r_i}(t)|}\Big) \end{aligned}$$

$$
\begin{aligned}
&\le b_{i1}|z_i(t)| + b_{i2}|z_i(t-d(t))| \\
&+b_{i3}\sum_{j=1}^{i-1}|z_j(t)|^{\frac{1}{p_j\cdots p_{i-1}}} \\
&+b_{i4}\sum_{j=1}^{i-1}|z_j(t-d(t))|^{\frac{1}{p_j\cdots p_{i-1}}} \\
&\le \phi_i\Big(|z_i(t)| + |z_i(t-d(t))|\Big) \\
&+\phi_i\sum_{j=1}^{i-1}\Big(|z_j(t)|^{\frac{1}{p_j\cdots p_{i-1}}} + |z_j(t-d(t))|^{\frac{1}{p_j\cdots p_{i-1}}}\Big)
\end{aligned} \tag{10}
$$

where $\phi_i = \max\{b_{i1}, b_{i2}, b_{i3}, b_{i4}\}$, $b_{i1} = a_i + (r_i(\lambda_0 + c_{03})/p_0)$, $b_{i2} = a_i exp\Big\{r_i d(\lambda_0 c_{01} - c_{03})/p_0\Big\}$, $b_{i3} = a_i|\lambda_0 x_0(t)|^{(r_1+\cdots+r_{i-1}-r_i)/p_0}$ and $b_{i4} = a_i|\lambda_0 x_0(t)|^{(r_1+\cdots+r_{i-1}-r_i)/p_0} \times \max\Big\{1, exp\{r_i d(r_1+\cdots+r_{i-1}-r_i)(\lambda_0 c_{01} - c_{03})/p_0\}\Big\}$ are $\mathcal{F}$ functions. Thus, the inequality (9) follows. □

3.3 Design u_1 for $x-$subsystem

In this subsection, we proceed to design the control input u_1 by using adding a power integrator technique. To simplify the deduction procedure, we sometimes denote $\chi(t)$ by χ, for any variable $\chi(t)$.

Step 1. Introduce the Lyapunov-Krasoviskii functional $V_1 = \frac{n}{2}x_0^2 + \frac{1}{2}z_1^2 + \frac{n}{1-\eta}\int_{t-d(t)}^{t} z_1^2(s)ds$. With the help of (7) and (9), it can be verified that

$$
\begin{aligned}
\dot V_1 \le& -nx_0^2 + d_1 z_1 z_2^{p_1} \\
&+\phi_1(x_0)|z_1|\Big(|z_1| + |z_1(t-d(t))|\Big) \\
&+\frac{n}{1-\eta}z_1^2 - \frac{n(1-\dot d(t))}{1-\eta}z_1^2(t-d(t)) \\
\le& -nx_0^2 + d_1 z_1 z_2^{p_1} \\
&+z_1^2\Big(\frac{n}{1-\eta} + \phi_1(x_0) + \frac{1}{4}\phi_1^2(x_0)\Big) \\
&-(n-1)z_1^2(t-d(t))
\end{aligned} \tag{11}
$$

Obviously, the first virtual controller

$$
\begin{aligned}
z_2^{*p_1} &= -\frac{1}{c_{01}}\Big(n + \frac{n}{1-\eta} + \phi_1(x_0) + \frac{1}{4}\phi_1^2(x_0)\Big)z_1 \\
&:= -\alpha_1(x_0)z_1
\end{aligned} \tag{12}
$$

leads to

$$
\begin{aligned}
\dot V_1 \le& -n(x_0^2 + z_1^2) - (n-1)z_1^2(t-d(t)) \\
&+d_1 z_1(z_2^{p_1} - z_2^{*p_1})
\end{aligned} \tag{13}
$$

Step i $(i = 2, \cdots, n)$**.** Suppose at step $i-1$, there is a positive-definite and proper Lyapunov functional V_{i-1}, and a set of virtual controllers $z_1^*, \cdots, z_i^*$ defined by defined by (19) shown at the top of the next page, with $\alpha_1(x_0) > 0$, $\cdots$, $\alpha_{i-1}(x_0) > 0$, being $\mathcal{F}$ function, such that

$$
\begin{aligned}
\dot V_{i-1} \le& -(n-i+2)\sum_{k=0}^{i-1}\xi_k^2 \\
&-(n-i+1)\sum_{k=1}^{i-1}\xi_k^2(t-d(t)) \\
&+d_{i-1}\xi_{i-1}^{2-\frac{1}{p_1\cdots p_{i-2}}}(z_i^{p_{i-1}} - z_i^{*p_{i-1}})
\end{aligned} \tag{14}
$$

here we let $\xi_0 = x_0$ for the simplicity of expression.

We intend to establish a similar property for $(z_1, \cdots, z_i)$-subsystem. Consider the following Lyapunov-Krasoviskii functional candidate

$$
V_i = V_{i-1} + W_i + \frac{n-i+1}{1-\eta}\int_{t-d(t)}^{t}\xi_i^2(s)ds \tag{15}
$$

where

$$
\begin{aligned}
&W_i(z_1, \cdots, z_i) \\
&\quad = \int_{z_i^*}^{z_i}(s^{p_1\cdots p_{i-1}} - z_i^{*p_1\cdots p_{i-1}})^{(2-\frac{1}{p_1\cdots p_{i-1}})}ds
\end{aligned} \tag{16}
$$

For W_i, some useful properties are given by the following proposition whose proof can be found in [27] and hence omitted here.

Proposition 8 *$W_i(z_1, \cdots, z_i)$ is C^1. Moreover*

$$
\begin{aligned}
\frac{\partial W_i}{\partial z_i} &= \xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}} \\
\frac{\partial W_i}{\partial y} &= -\Big(2 - \frac{1}{p_1\cdots p_{i-1}}\Big)\frac{\partial z_i^{*p_1\cdots p_{i-1}}}{\partial y} \\
&\quad\times\int_{z_i^*}^{z_i}(s^{p_1\cdots p_{i-1}} - z_i^{*p_1\cdots p_{i-1}})^{(1-\frac{1}{p_1\cdots p_{i-1}})}ds
\end{aligned} \tag{17}
$$

where y is an argument of W_i except z_i.

Proposition 9 *There is a positive constant m such that*

$$
\Big|\frac{\partial W_i}{\partial y}\Big| \le m|\xi_i|\Big|\frac{\partial z_i^{*p_1\cdots p_{i-1}}}{\partial y}\Big| \tag{18}
$$

Using Proposition 8, it is deduced from (16) that

$$
\begin{array}{ll}
z_1^* = 0 & \xi_1 = z_1 - z_1^* \\
z_2^{*p_1} = -\alpha_1(x_0)\xi_1 & \xi_2 = z_2^{p_1} - z_2^{*p_1} \\
\cdots & \cdots \\
z_i^{*p_1\cdots p_{i-1}} = -\alpha_{i-1}(x_0)\xi_{i-1} & \xi_i = z_i^{p_1\cdots p_{i-1}} - z_i^{*p_1\cdots p_{i-1}}
\end{array} \tag{19}
$$

$$\begin{aligned}\dot{V}_i \leq & -(n-i+2)\sum_{k=0}^{i-1}\xi_k^2 \\ & -(n-i+1)\sum_{k=1}^{i-1}\xi_k^2(t-d(t)) \\ & +d_{i-1}\xi_{i-1}^{2-\frac{1}{p_1\cdots p_{i-2}}}(z_i^{p_{i-1}}-z_i^{*p_{i-1}}) \\ & +d_i\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}(z_{i+1}^{p_i}-z_{i+1}^{*p_i}) \\ & +d_i\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}z_{i+1}^{*p_i}+\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}g_i \\ & +\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial z_j}(d_jz_{j+1}^{p_j}+g_j) \\ & +\frac{\partial W_i}{\partial x_0}(d_0u_0^{p_0}+f_0)+\frac{n-i+1}{1-\eta}\xi_i^2 \\ & -\frac{(n-i+1)(1-\dot{d}(t))}{1-\eta}\xi_i^2(t-d(t))\end{aligned} \tag{20}$$

Similarly, to give the explicit form of z_{i+1}^*, we should estimate each term on the righthand side of (20). First, from (14), Assumption 1 and Lemma 3, it follows that

$$\begin{aligned}&\Big|d_{i-1}\xi_{i-1}^{2-\frac{1}{p_1\cdots p_{i-2}}}(z_i^{p_{i-1}}-z_i^{*p_{i-1}})\Big| \\ &\leq c_{i-1,2}\Big|\xi_{i-1}\Big|^{2-\frac{1}{p_1\cdots p_{i-2}}} \\ &\times\Big|(z_i^{p_1\cdots p_{i-1}})^{\frac{1}{p_1\cdots p_{i-2}}}-(z_i^{*p_1\cdots p_{i-1}})^{\frac{1}{p_1\cdots p_{i-2}}}\Big| \\ &\leq c_{i-1,2}\Big|\xi_{i-1}\Big|^{2-\frac{1}{p_1\cdots p_{i-2}}}2^{1-\frac{1}{p_1\cdots p_{i-2}}} \\ &\times\Big|z_i^{p_1\cdots p_{i-1}}-z_i^{*p_1\cdots p_{i-1}}\Big|^{\frac{1}{p_1\cdots p_{i-2}}} \\ &=2^{1-\frac{1}{p_1\cdots p_{i-2}}}c_{i-1,2}\Big|\xi_{i-1}\Big|^{2-\frac{1}{p_1\cdots p_{i-2}}}\Big|\xi_i\Big|^{\frac{1}{p_1\cdots p_{i-2}}} \\ &\leq\frac{1}{4}\xi_{i-1}^2+l_{i1}\xi_i^2\end{aligned} \tag{21}$$

where l_{i1} is a positive constant.

From (9) and (14), there are $\mathcal{F}$ functions $\rho_k(x_0)$, $k=1,\cdots,i$ such that

$$\begin{aligned}|g_k| \leq & \phi_k\Big(|z_k(t)|+|z_k(t-d(t))|\Big) \\ & +\phi_k\sum_{j=1}^{k-1}\Big(|z_j(t)|^{\frac{1}{p_j\cdots p_{k-1}}} \\ & +|z_j(t-d(t))|^{\frac{1}{p_j\cdots p_{k-1}}}\Big) \\ & \leq\phi_k\Big(|\xi_k(t)|^{\frac{1}{p_1\cdots p_{k-1}}}+|\xi_k(t-d(t))|^{\frac{1}{p_1\cdots p_{k-1}}}\Big) \\ & +\phi_k\sum_{j=1}^{k-1}\Big(1+a_j^{\frac{1}{p_1\cdots p_{k-1}}}\Big)|\xi_j(t)|^{\frac{1}{p_1\cdots p_{k-1}}} \\ & +\phi_k\sum_{j=1}^{k-1}\Big(1+a_j^{\frac{1}{p_1\cdots p_{k-1}}}\Big)|\xi_j(t-d(t))|^{\frac{1}{p_1\cdots p_{k-1}}} \\ & \leq\rho_k\sum_{j=1}^{k}\Big(|\xi_j(t)|^{\frac{1}{p_1\cdots p_{k-1}}} \\ & +|\xi_j(t-d(t))|^{\frac{1}{p_1\cdots p_{k-1}}}\Big)\end{aligned} \tag{22}$$

and hence by Lemma 3

$$\begin{aligned}&\Big|\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}g_i\Big| \\ &\leq\Big|\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}\Big|\rho_j\sum_{j=1}^{i}\Big(|\xi_j(t)|^{\frac{1}{p_1\cdots p_{i-1}}} \\ &+|\xi_j(t-d(t))|^{\frac{1}{p_1\cdots p_{i-1}}}\Big) \\ &\leq\frac{1}{4}\sum_{j=1}^{i-1}\xi_j^2+\frac{1}{2}\sum_{j=1}^{i}\xi_j^2(t-d(t))+l_{i2}\xi_i^2\end{aligned} \tag{23}$$

where $l_{i2}(x_0)$ is a $\mathcal{F}$ function.

For the seventh term, noting that

$$\begin{aligned}z_i^{*p_1\cdots p_{i-1}}&=-\alpha_{i-1}\xi_{i-1} \\ &=-\sum_{k=1}^{i-1}\Big(\prod_{h=k}^{i-1}\alpha_h\Big)z_k^{p_1\cdots p_{k-1}}\end{aligned} \tag{24}$$

By this, (14), Proposition 9 and Lemma 3, we obtain

$$\begin{aligned}&\Big|\sum_{j=1}^{i-1}\frac{\partial W_i}{\partial z_j}(d_jz_{j+1}^{p_j}+g_j)\Big| \\ &\leq\sum_{j=1}^{i-1}m|\xi_i|\Big|\frac{\partial z_i^{*p_1\cdots p_{i-1}}}{\partial z_j}\Big|(c_{j2}|z_{j+1}^{p_j}|+|g_j|) \\ &\leq\sum_{j=1}^{i-1}mp_1\cdots p_{j-1}|\xi_i|\Big(\prod_{h=j}^{i-1}\alpha_j\Big)|z_j^{p_1\cdots p_{j-1}-1}| \\ &\times(c_{j2}|z_{j+1}^{p_j}|+|g_j|) \\ &\leq\sum_{j=1}^{i-1}mp_1\cdots p_{j-1}|\xi_i|\Big(\prod_{h=j}^{i-1}\alpha_j\Big) \\ &\times|\xi_j+\alpha_{j-1}\xi_{j-1}|^{1-\frac{1}{p_1\cdots p_{j-1}}} \\ &\times\Big[c_{j2}\Big|\xi_{j+1}+\alpha_j\xi_j\Big|^{\frac{1}{p_1\cdots p_{j-1}}} \\ &+\rho_j\sum_{k=1}^{j}\Big(|\xi_k(t)|^{\frac{1}{p_1\cdots p_{j-1}}} \\ &+|\xi_k(t-d(t))|^{\frac{1}{p_1\cdots p_{j-1}}}\Big)\Big]\end{aligned}$$

$$\leq \frac{1}{4}\sum_{j=1}^{i-1}\xi_j^2 + \frac{1}{2}\sum_{j=1}^{i}\xi_j^2(t-d(t)) + l_{i3}\xi_i^2 \tag{25}$$

where $l_{i3}(x_0)$ is a $\mathcal{F}$ function.

According to Proposition 9 and Lemma 3, we easily get

$$\begin{aligned}&\Big|\frac{\partial W_i}{\partial x_0}(d_0u_0^{p_0}+f_0)\Big|\\&\leq m|\xi_i|\Big|\frac{\partial z_i^{*p_1\cdots p_{i-1}}}{\partial x_0}\Big|(c_{02}\lambda_0+c_{03})|x_0|\\&\leq x_0^2+l_{i4}\xi_i^2\end{aligned} \tag{26}$$

where $l_{i4}(x_0)$ is a $\mathcal{F}$ function.

Substituting (21), (23), (25)and (26) into (20) yields

$$\begin{aligned}\dot{V}_i &\leq -(n-i+1)\sum_{k=0}^{i-1}\xi_k^2-(n-i)\sum_{k=1}^{i}\xi_k^2(t-d(t))\\&+d_i\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}z_{i+1}^{*p_i}+\Big(\sum_{j=1}^{4}l_{ij}+\frac{n-i+1}{1-\eta}\Big)\xi_i^2\\&+d_i\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}(z_{i+1}^{p_i}-z_{i+1}^{*p_i})\end{aligned} \tag{27}$$

Now, it easy to see that the virtual controller

$$\begin{aligned}z_{i+1}^{*p_1\cdots p_i}&=-\Big(n-i+1+\sum_{j=1}^{4}l_{ij}+\frac{n-i+1}{1-\eta}\Big)\xi_i\\&:=-\alpha_i(x_0)\xi_i\end{aligned} \tag{28}$$

renders

$$\begin{aligned}\dot{V}_i &\leq -(n-i+1)\sum_{k=0}^{i}\xi_k^2-(n-i)\sum_{k=1}^{i}\xi_k^2(t-d(t))\\&+d_i\xi_i^{2-\frac{1}{p_1\cdots p_{i-1}}}(z_{i+1}^{p_i}-z_{i+1}^{*p_i})\end{aligned} \tag{29}$$

As $i=n$, the last step, we can construct explicitly a change of coordinates $(\xi_1,\cdots,\xi_n)$, a positive-definite and proper Lyapunov-Krasoviskii functional $V_n(\xi_1,\cdots,\xi_n)$ and a state feedback controller z_{n+1}^* of form (28) such that

$$\dot{V}_n \leq -\sum_{k=0}^{n}\xi_k^2 + d_n\xi_n^{2-\frac{1}{p_1\cdots p_{n-1}}}(u_1^{p_n}-z_{n+1}^{*p_n}) \tag{30}$$

Therefore, by choosing the actual control u_1 as

$$u_1 = z_{n+1}^* = -\Big(\alpha_n(x_0)\xi_n\Big)^{\frac{1}{p_1\cdots p_n}} \tag{31}$$

we get

$$\dot{V}_n \leq -\sum_{k=0}^{n}\xi_k^2 \tag{32}$$

Thus far the controller design procedure for $x_0(t_0)\neq 0$ has been completed.

4 Switching controller and main result

In the preceding section, we have given controller design for $x_0(t_0)\neq 0$. Now, we discuss how to select the control laws u_0 and u_1 when the initial $x_0(t_0)=0$. Without loss of generality, we can assume that $t_0=0$. In the absence of the disturbances, most of the commonly used control strategies use constant control $u_0=u_0^*\neq 0$ in time interval $[0,t_s)$. In this paper, we also use this method when $x_0(0)=0$, with u_0 chosen as follows:

$$u_0 = u_0^*, \;\; u_0^*>0. \tag{33}$$

Since $f_0(t,x_0(t))$ in this paper satisfies the linear growth condition, the x_0-state does not escape and $x_0(t_s)\neq 0$, for any given finite time $t_s>0$. Thus, input-state-scaling for the control design can be carried out.

During the time period $[0,t_s)$, using u_0 defined in (33), new control law $u_1=u_1^*(x_0,x)$ can be obtained by the control procedure described above to the original $x-$subsystem in (1). Then we can conclude that the $x-$state of (1) cannot blow up during the time period $[0,t_s)$. Since $x(t_s)\neq 0$ at t_s, we can switch the control inputs u_0 and u_1 to (3) and (31), respectively.

We are now ready to state the main theorem of this paper.

Theorem 10 *Under Assumptions 1-2, if the proposed control design procedure together with the above switching control strategy is applied to system (1), then, for any initial conditions in the state space* $(x_0,x)\in R^{n+1}$ *, the closed-loop system is globally asymptotically regulated at origin.*

Proof. According to the above analysis, it suffices to prove the statement in the case where $x_0(0)\neq 0$.

Since we have already proven that x_0 can be globally exponentially regulated to zero as $t\to\infty$ in Section 3.1, we just need to show that $\lim_{t\to\infty}x(t)=0$. In this case, choose the Lyapunov functional

$$\begin{aligned}V=V_n&=\frac{n}{2}x_0^2+\frac{1}{2}\sum_{k=1}^{n}\xi_i^2\\&+\sum_{k=1}^{n}\frac{n-i+1}{1-\eta}\int_{t-d(t)}^{t}\xi_i^2(s)ds\end{aligned}$$

from (32), we obtain

$$\dot{V}\leq -(x_0^2+\xi_1^2+\cdots+\xi_n^2)$$

Then by Lyapunov-Krasovskii stability theorem [20], we have $\lim_{t\to\infty}\xi(t) = 0$. This together with the definitions of z_i^*'s and the input-state-scaling transformation(6) directly concludes that $\lim_{t\to\infty} x(t) = 0$. This completes the proof of Theorem 10. □

Remark 11 *From the above design procedure, we can see that the upper bound of the change rate of time delays have important impact on the control effort. To keep the control effort within the certain range, the upper bound of the change rate of time delays cannot be arbitrarily close to* 1*, which should be considered in practical engineering design.*

Remark 12 *It should be mentioned that the control law* u_1 *may exhibit extremely large value when* $x_0(t_0) \neq 0$ *is sufficiently small. This is unacceptable from a practical point of view. It is therefore recommended to apply (33) in order to enlarge the initial value of* x_0 *before we appeal to the converging controllers (3) and (31).*

5 Simulation example

To verify the proposed controller, we consider the following low-dimensional system

$$\begin{cases} \dot{x}_0(t) = (1.5 + 0.5\cos t)u_0(t) \\ \dot{x}_1(t) = x_2^3(t)u_0(t) + \sin x_0(t)x_1(t - d(t)) \\ \dot{x}_2(t) = u_1^3(t) \end{cases} \tag{34}$$

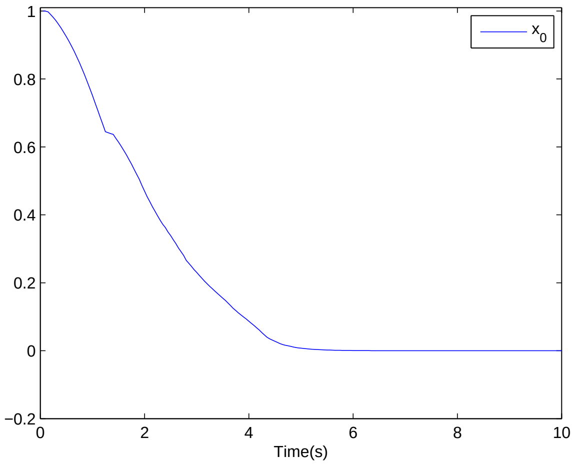
where $d(t) = \frac{1}{2}(1 + \sin(t))$. It is very easy to verify that Assumptions 1-2 holds. Hence the controller proposed in this paper is applicable.

If $x(0) = 0$, controls u_0 and u_1 are set as in Section 4 in interval $[0, t_s)$, such that $x(t_s) \neq 0$, then we can adopt the controls developed below. Therefore, without loss of generality, we assume that $x(0) \neq 0$. Noting that $\dot{d}(t) = \frac{1}{2}\cos t \leq \frac{1}{2} < 1$, we define the control law $u_0(t) = -\lambda_0 x_0(t)$ and introduce the state scaling transformation

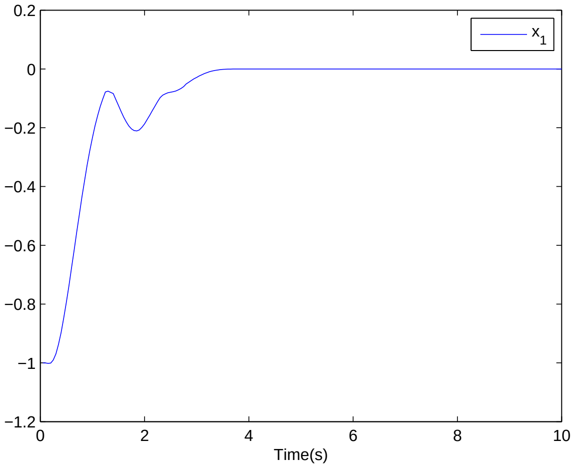
$$z_1(t) = \begin{cases} \frac{x_1(t)}{u_0(t)}, & t \geq 0 \\ x_1(t), & -1 \leq t < 0 \end{cases}, \quad z_2(t) = x_2(t) \tag{35}$$

In new $z-$coordinates, the $(x_1, x_2)-$ subsystem of (34) is rewritten as

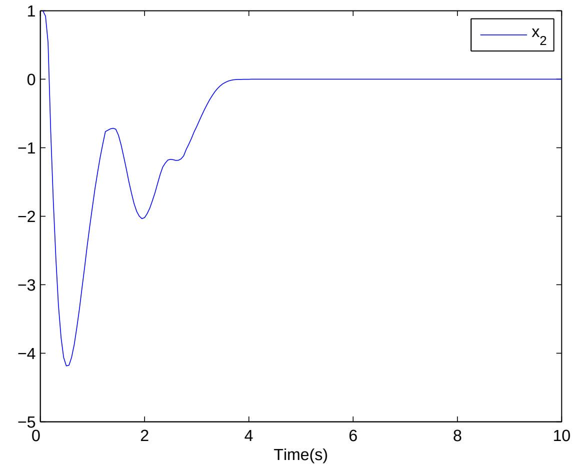
$$\begin{cases} \dot{z}_i(t) = z_{i+1}^3(t) \\ \qquad +g_1(t, x_0(t), x(t), x(t - d(t))) \\ \dot{z}_2(t) = u_1^3(t) \\ \qquad +g_2(t, x_0(t), x(t), x(t - d(t))) \end{cases} \tag{36}$$



(a) x_0



(b) x_1



(c) x_2

Figure 1: The responses of system states.

where

$$
\begin{aligned}
&g_i(t, x_0(t), x(t), x(t-d(t))) \\
&\quad = \frac{f_i(t, x_0(t), x(t), x(t-d(t)))}{u_0^{r_i}(t)} \\
&\quad\quad + r_i z_i(t) \frac{u_0^{p_0}(t) + f_0(t, x_0(t))}{p_0 x_0(t)}
\end{aligned} \tag{37}
$$

Similar to (10), it is very easy to verify that Lemma 7 is satisfied with $\phi_1 = 1 + e^{\lambda_0}$ and $\phi_2 = 1$

Obviously, the $(z_1, z_2)-$ subsystem of (36) with nonlinear parameter ε. Define $\gamma_1 = -1.5 g_0$, $\Theta = 1 + \varepsilon^2$, this subsystem satisfies Lemma 7, i.e. $|g_0(1 - 0.5\varepsilon^2) z_1| \leq |z_1| \gamma_1 \Theta$. Now consider $V_1 = x_0^2 + \frac{1}{2} z_1^2 + 4 \int_{t-d(t)}^{t} z_1^2(s) ds$. A simple calculation yields

$$
\begin{aligned}
\dot{V}_1 \leq& -2x_0^2 + z_1 z_2^3 \\
&+ z_1^2 \Big(4 + \phi_1 + \frac{1}{4}\phi_1^2\Big) - z_1^2(t - d(t))
\end{aligned} \tag{38}
$$

Hence, the virtual controller

$$
\begin{aligned}
z_2^{*3} &= -\Big(6 + \phi_1 + \frac{1}{4}\phi_1^2\Big) z_1 \\
&:= -\alpha_1 z_1
\end{aligned}
$$

renders

$$
\dot{V}_1 \leq -(x_0^2 + z_1^2) - z_1^2(t - d(t)) + z_1(z_2^3 - z_2^{*3})
$$

Next, define $\xi_2 = z_2^3 - z_2^{*3}$ and construct Lyapunov-Krasoviskii functional

$$
V_2 = V_2 + W_2 + 2 \int_{t-d(t)}^{t} \xi_i^2(s) ds \tag{39}
$$

where

$$
W_2 = \int_{z_2^*}^{z_2} (s^3 - z_2^{*3})^{\frac{5}{3}} ds \tag{40}
$$

Clearly

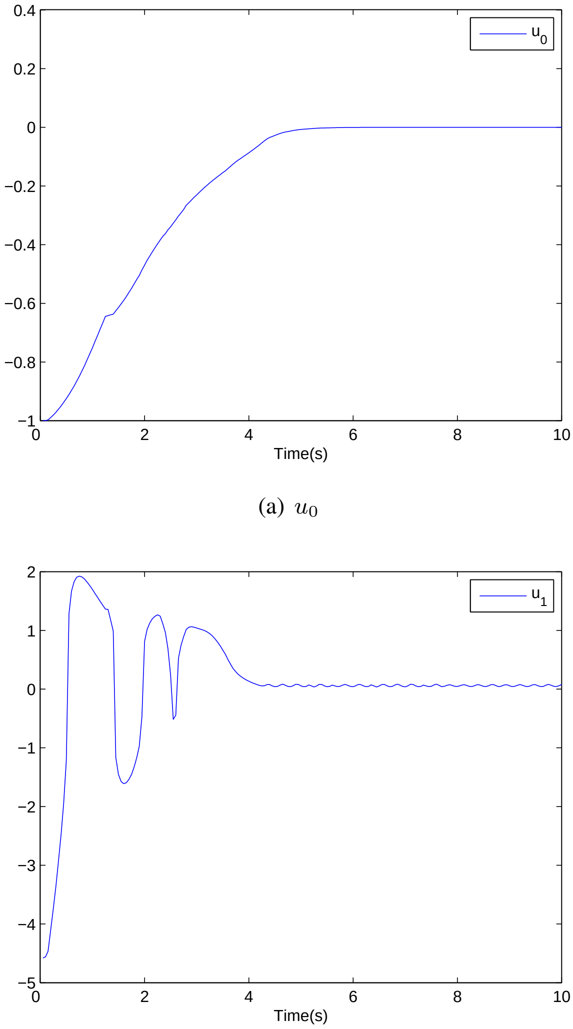
$$
\begin{aligned}
\dot{V}_2 \leq& -(x_0^2 + z_1^2) - z_1^2(t - d(t)) \\
&+ z_1(z_2^3 - z_2^{*3}) + \xi_2^{\frac{5}{3}} u_1^3 + \xi_2^{\frac{5}{3}} g_2 \\
&+ \frac{\partial W_2}{\partial z_1}(z_2^3 + g_1) + \frac{\partial W_2}{\partial x_0} d_0 u_0 \\
&+ 2\xi_1^2 - \xi_1^2(t - d(t))
\end{aligned} \tag{41}
$$

By Lemma 3, we have

$$
z_1(z_2^3 - z_2^{*3}) \leq \frac{1}{4} z_1^2 + l_{21}\xi_2^2
$$

$$
\xi_2^{\frac{5}{3}} g_2 \leq \frac{1}{4} z_1^2 + l_{22}\xi_2^2
$$

$$
\frac{\partial W_2}{\partial z_1}(z_2^3 + g_1) \leq \frac{1}{4} z_1^2 + \xi_1^2(t - d(t)) + l_{23}\xi_2^2
$$



(a) u_0

(b) u_1

Figure 2: The responses of control inputs.

$$
\frac{\partial W_2}{\partial x_0} d_0 u_0 \leq_1^2 + l_{24}\xi_2^2
$$

where l_{2j}, $j = 1, 2, 3, 4$ are known $\mathcal{F}$ functions

It is easy to verify that the controller

$$
u_1(t) = -\Big(3 + \sum_{j=1}^{4} l_{ij}\Big)^{\frac{1}{9}}
$$

renders

$$
\dot{V}_2 \leq -x_0^2 - z_1^2 - \xi_2^2
$$

thus achieving global stability with asymptotic state regulation.

In the simulation, by choosing design parameter as $\lambda_0 = 1$, the responses of the closed-loop system for initial conditions $(x_0(0), x_1(0), x_2(0)) = (1, -1, 1)$ are shown in Figures 1 and 2. From the figures, we can see that under the constructed controller, the solution process of the closed-loop system asymptotically converges to zero.

6 Conclusion

In this paper, a state-feedback stabilization controller independent of time-delays is presented for a class of high-order nonholonomic systems with time-varying delays. It should be mentioned that the stabilization approaches in literature may fail be applied for the existence of time delays. In order to overcome the difficulty, a novel Lyapunov-Krasovskii functional is introduced to deal with time delays. The controller design is developed by using input-state-scaling and adding a power integrator techniques. Based on switching control strategy, global asymptotic regulation of the closed-loop system is achieved. It should be noted that the proposed controller can only work well when the whole state vector is measurable. Therefore, a natural and more interesting problem is how to design output feedback stabilization controller for the systems studied in the paper if only partial state vector are measurable, which are now under our further investigation.

Acknowledgements: The authors thank the editor and the anonymous reviewers for their constructive comments and suggestions for improving the quality of the paper. This work has been supported in part by National Nature Science Foundation of China under Grants 61073065, 61403003 and the Key Program of Science Technology Research of Education Department of Henan Province under Grants 13A120016, 14A520003.

References:

[1] R. W. Brockett, Asymptotic stability and feedback stabilization, in: R.W. Brockett, R.S. Millman, H.J. Sussmann (Eds.), *Differential Geometric Control Theory*, 1983, pp. 2961–2963.

[2] R. R. Murray, S. S. Sastry, Nonholonomic motion planning: steering using sinusoids, *IEEE Transactions on Automatic Control*, Vol.38, No.5, 1993, pp. 700–716.

[3] G. C. Walsh, L. G. Bushnell, Stabilization of multiple input chained form control systems, *Systems and Control Letters*, Vol.25, 1995, pp. 227–234.

[4] A. Astolfi, Discontinuous control of nonholonomic systems, *Systems and Control Letters*, Vol.27, No.1, 1996, pp. 37–45.

[5] Z. P. Jiang, Iterative design of time-varying stabilizers for multi-input systems in chained form, *Systems and Control Letters*, Vol.28, No.5, 1996, pp. 255–262.

[6] C. de Wit Canudas, B. Siciliano, G. Bastin (Eds.), *Theory of Robot Control*, Springer, London, 1996.

[7] J. P. Hespanha, S. Liberzon, A. S. Morse, Towards the supervisory control of uncertain nonholonomic systems, in *Proceedings of 1999 American Control Conference*, San Diego, CA, USA, June 1999, pp.3520–3524.

[8] W. L. Xu, W. Huo, Variable structure exponential stabilization of chained systems based on the extended nonholonomic integrator, *Systems and Control Letters*, Vol.41, No.1, 2000, pp. 225–235.

[9] Z. D. Sun, S. S. Ge, W. Huo, T. H. Lee, Stabilization of nonholonomic chained systems via nonregular feedback linerization, *Systems and Control Letters*, Vol.44, No.4, 2001, pp.279–289.

[10] Z. P. Jiang, Robust exponential regulation of nonholonomic systems with uncertainties, *Automatica*, Vol.36, No.2, 2000, pp.189–209.

[11] S. S. Ge, Z. P. Wang, T. H. Lee, Adaptive stabilization of uncertain nonholonomic systems by state and output feedback, *Automatica*, Vol.39, No.8, 2003, pp.1451–1460.

[12] Y. G. Liu, J. F. Zhang, Output feedback adaptive stabilization control design for nonholonomic systems with strong nonlinear drifts, *International Journal of Control*, Vol.78, No.7, 2005, pp.474–490.

[13] Z. R. Xi, G. Feng, Z. P. Jiang, D. Z. Cheng, Output feedback exponential stabilization of uncertain chained systems, *Journal of the Franklin Institute*, Vol.344, No.1, 2007, pp.36–57.

[14] X. Y. Zheng, Y. Q. Wu, Adaptive output feedback stabilization for nonholonomic systems with strong nonlinear drifts, *Nonlinear Analysis: Theory, Methods and Applications*, Vol.70, No.2, 2009, pp. 904–920.

[15] F. Z. Gao, F. S. Yuan, H. J. Yao, Robust adaptive control for nonholonomic systems with nonlinear parameterization, *Nonlinear Analysis: Real World Applications*, Vol.11, No.4, 2010, pp. 3242–3250.

[16] F. Z. Gao, F. S. Yuan, Y. Q. Wu, Semi-global finite-time stabilization of uncertain nonholonomic systems via output feedback, *Transactions of the Institute of Measurement and Control*, Vol.37, No.1, 2015, pp. 122-130.

[17] X. Y. Zheng, Y. Q. Wu, Controller design of high order nonholonomic system with nonlinear drifts, *International Journal of Automation and Computing*, Vol.6, No.3, 2009, pp. 240–244.

[18] F. Z. Gao, F. S. Yuan, H. J. Yao, X. W. Mu, Adaptive stabilization of high order nonholonomic systems with strong nonlinear drifts, *Applied Mathematical Modelling*, Vol.35, No.9, 2011, pp.4222–4233.

[19] F. Z. Gao, F. S. Yuan, Adaptive finite-time stabilization for a class of uncertain high order nonholonomic systems, *ISA Transactions*, Vol.54, No.1, pp.75–82.

[20] K. Gu, V. L. Kharitonov, J. Chen (Eds.), *Stability of time-delay systems*, Berlin: Birkhauser, 2003.

[21] F. Y. Zhou, Global exponential synchronization of a class of BAM neural networks with time-varying delays, *WSEAS Transactions Mathematics*, Vol.12, No.2, 2013, pp. 138–148.

[22] F. W. Yang, Y. L. Dong, Global exponential stabilization of uncertain switched nonlinear systems with time-varying delays, *WSEAS Transactions Mathematics*, Vol.12, No.3, pp. 256–265.

[23] C. J. Xu, Q. M. Zhang, Qualitative analysis for a Lotka-Volterra model with time delays, *WSEAS Transactions Mathematics*, Vol.13, 2014, pp. 603-614.

[24] Y. Y. Wu, Y. Q. Wu, Robust stabilization of delayed nonholonomic systems with strong nonlinear drifts, *Nonlinear Analysis: Real World Applications*, Vol.11, No.5, 2010, pp.3620–3627.

[25] Y. Y. Wu, Y. Q. Wu, Robust stabilization of nonholonomic systems with unknown time delays, *Journal of Control Theory and Applications*, Vol.9, 2011, pp.256–260.

[26] Z. Y. Sun, Y. G. Liu, State-feedback stabilization control design for a class of time-delay high-order nonlinear systems, in *Proceedings of 30th Chinese Control Conference*, Yantai, China, 2011, pp.476–480.

[27] C. J. Qian, W. Lin, A continuous feedback approach to global strong stabilization of nonlinear systems, *IEEE Transactions on Automatic Control*, Vol.46, No.7, 2001, pp.1061–1079.

[28] X. Huang, W. Lin, B. Yang, Global finite-time stabilization of a class of uncertain nonlinear systems, *Automaica*, Vol.41, No.5, 2005, pp. 881–888.