Uniform Persistence and Global Attractivity of A Predator-Prey Model with Mutual Interference and Delays

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Abstract: In this paper, a predator-prey model with mutual interference and delays is studied by utilizing the comparison theorem and Lyapunov second method. Some sufficient conditions for uniform persistence and global attractivity of positive periodic solution of the model are obtained. It is very interesting that the results obtained are related to delays, especially based on the mutual interference constant. Furthermore, it is the first time that such a model is considered. An example is illustrated to verify the feasibility of the results in the last part.

Key–Words: Mutual interference, Delays, Uniform persistence, Lyapunov second method, Global attractivity

1 Introduction

Recently, the following predator-prey(PP for short) model with mutual interference,

$$\begin{cases} \dot{x} = x(a_1(t) - b_1(t)x) - \varphi_1(t,x)y^m, \\ \dot{y} = y\left(-a_2(t) - b_2(t)y\right) + \varphi_2(t,x)y^m, \end{cases}$$

where $m \in (0,1]$ is the mutual interference constant, which was introduced by Hassell in 1971 (see [1–3] for more details), has been studied by some authors, such as Wang (2008-2010), Lin (2009), Wang (2010), Chen (2009) and Wu (2010), see [4–10] for more details. They investigated the existence and stability of periodic or almost periodic solutions of some special cases of the above model, and the models they considered are all ordinary differential systems or difference systems. It was pointed by Kuang (1993) [11] that any model of species dynamics without delays is an approximation at best, more detailed arguments on the importance and usefulness of time-delays in realistic models may also be found in the classical books of Macdonald (1989) [12] and Gopalsamy (1992) [13]. There are many papers on study of dynamics of delayed population models, see[18,19,22-26] and the references cited therein for more details. But there are few literatures considering time-delays in PP model with mutual interference.

Motivated by the above reasons, in this paper we investigate a PP model with mutual interference and delays in the form:

$$\begin{cases} \dot{x}(t) = x(t)(a_1(t) - b_1(t)x(t-\tau_1(t)) \\ \qquad - c_1(t)y^m(t-\tau_2(t))), \\ \dot{y}(t) = y(t)(-a_2(t) - b_2(t)y(t-\tau_3(t)) \\ \qquad + c_2(t)x(t)y^{m-1}(t)), \end{cases} \tag{1}$$

where $x(t)$, $y(t)$ denote the size of prey and predator at time t, respectively; $m \in (0,1]$ is mutual interference constant; $a_i(t),\ b_i(t)$ and $c_i(t)(i=1,2)$ are continuous and bounded above and below by positive constants on $[0,+\infty)$, $\tau_i(t)(i=1,2,3)$ are nonnegative, bounded and continuous functions on $[0,+\infty)$. If set $\tau = \sup\{\tau_i(t) : t \in [0,+\infty), i = 1,2,3\}$, then we get $\tau \in [0,+\infty)$.

Considering the application of model (1) to population dynamics, we assume that all positive solutions of model (1) satisfy the following initial conditions,

$$\begin{cases} x(\theta) = \varphi(\theta),\ \theta \in [-\tau,0],\ \varphi(0) = \varphi_0 > 0, \\ y(\theta) = \psi(\theta),\ \psi \in [-\tau,0],\ \psi(0) = \psi_0 > 0, \end{cases} \tag{2}$$

where φ and ψ are given nonnegative and bounded continuous functions on $[-\tau,0]$.

Remark 1 *If $m = 1$, i.e., there is no mutual interference between preys and predators, then model (1) transforms to the classical PP model:*

$$\begin{cases} \dot{x}(t) = x(t)(a_1(t) - b_1(t)x(t-\tau_1(t)) \\ \qquad - c_1(t)y(t-\tau_2(t))), \\ \dot{y}(t) = y(t)(-a_2(t) - b_2(t)y(t-\tau_3(t)) \\ \qquad + c_2(t)x(t)), \end{cases}$$

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which has been extensively studied with or without delays, such as existence of periodic solutions, permanence, stability of equilibria and so on, see [14–24] and the references cited therein.

Remark 2 *If we neglect the influence of delays in model (1), i.e., $\tau_i(t) \equiv 0, i = 1,2,3$, then model (1) reduces to the following ordinary differential population system:*

$$\begin{cases} \dot{x}(t) = x(t)(a_1(t) - b_1(t)x(t) - c_1(t)y^m(t)), \\ \dot{y}(t) = y(t)(-a_2(t) - b_2(t)y(t) + c_2(t)x(t)y^{m-1}). \end{cases} \quad (3)$$

In paper [5], we studied the existence and global asymptotic stability of positive periodic solution of model (3) with $c_2(t) = kc_1(t)$, and in paper [6], we investigated the permanence, and global asymptotic stability of the positive solutions of model (3).

The aim of this paper is to establish sufficient conditions for the uniform persistence of model (1) with $m \in (0,1)$, and present sufficient conditions for the global attractivity of model (1) with $m \in (0,1)$.

The structure of the paper is: In section 2, some useful lemmas and definitions are given. In section 3, some main results on the uniform persistence and the existence of positive periodic solutions for model (1) are established. In section 4, an example is given to verify the feasibility of our results by simulation. The significance is that we verify that the mutual interference constant m has intrinsic effect on the uniform persistence, the existence of positive periodic solutions and the global attractivity of model (1).

2 Lemmas and definitions

In this section, we give some important lemmas and definitions which will be used in next sections.

For the sake of convenience, we let $f^L = \inf_{t\in E} f(t)$, $f^U = \inf_{t\in E} f(t)$, where f is a continuously bounded function defined on interval E, and denote by $\varphi_i^{-1}(t)$ the inverse function of $\varphi_i(t) = t - \tau_i(t), i = 1,2,3$, respectively.

Lemma 3 *(See [23]) If $a > 0, b > 0$ and $\dot{z}(t) \geqslant (\leqslant)\, z(t)(b - a\,z(t))$ for $t \geqslant 0$, $z(0) > 0$, then the following inequality hold:*

$$z(t) \geqslant (\leqslant) \frac{b}{a}\left[1 + \left(\frac{b}{a\,z(0)} - 1\right)\exp\{-b\,t\}\right]^{-1}.$$

Lemma 4 *If $a > 0$, $b > 0$, $\dot{z}(t) \geqslant z^m(t)(b - az^{1-m}(t))$ with $0 < m < 1$ and $z(0) > 0$, then for any small constant $\varepsilon > 0$ we have*

$$z(t) \geqslant (b/a)^{1/(1-m)} - \varepsilon \ \text{for}\ \ t \geqslant T,$$

where T is a large enough positive constant.

Proof: It follows from

$$\dot{z}(t) \geqslant z^m(t)(b - az^{1-m}(t))$$

that

$$d(z^{1-m})(t)/dt \geqslant (1-m)(b - az^{1-m}(t)).$$

From Lemma 3 we get

$$z^{1-m}(t) \geqslant b/a + \left(z^{1-m}(0) - b/a\right)e^{-a(1-m)t}$$

for $t \geqslant 0$ i.e.,

$$z(t) \geqslant \left[b/a + \left(z^{1-m}(0) - b/a\right)e^{-a(1-m)t}\right]^{1/(1-m)}$$

for $t \geqslant 0$. Then for any small positive constant ε there exists a positive constant T such that

$$z(t) \geqslant (b/a)^{1/(1-m)} - \varepsilon \ \text{ for } t \geqslant T.$$

□

Definition 5 *Model (1) is said to be uniformly persistent if there exists a compact region $D \subseteq Int\, R^2$ such that every solution $(x(t), y(t))^\top$ of model (1) with initial condition (2) eventually enters and remains in region D.*

Definition 6 *Model (1) is called global attractivity if for any positive solutions $(x(t), y(t))$, $(x_0(t), y_0(t))$, $\lim\limits_{t\to+\infty}(|x(t) - x_0(t)| + |y(t) - y_0(t)|) = 0$.*

3 Main results

In this section, we will present some sufficient conditions on uniform persistence, existence of positive periodic solutions and global attractivity of model (1), respectively.

3.1 Uniform persistence and positive periodic solutions

Theorem 7 *If the following conditions hold:*
(1) $K_1 := (a_1 - c_1 M_2^m)^L > 0$,
(2) $K_2 := \left(c_2\, M_3\, M_2^{m-1} - a_2\right)^L > 0$ or $K_3 := \left(c_2\, M_3 - b_2\, M_2^{2-m}\right)^L > 0$.
Then model (1) is uniformly persistent.

Proof: It follows from model (1)-(2) that

$$\begin{cases} \frac{x(t)}{x(0)} = e^{\int_0^t (a_1(s) - b_1(s)x(s-\tau_1(s)) - c_1(s)y^m(s-\tau_2(s)))ds}, \\ \frac{y(t)}{y(0)} = e^{\int_0^t (-a_2(s) - b_2(s)y(s-\tau_3(s)) + c_2(s)x(s)y^{m-1}(s))ds}, \end{cases}$$

which together with the positivity of $(x(0), y(0))$ yields the existence of positive solution $(x(t), y(t))$ of model (1)-(2).

Now we estimate the eventually upper bounds of all positive solution $(x(t), y(t))$ of model (1). From the first equation of model (1), we get

$$\dot{x}(t) \leqslant x(t)\left(a_1^U - b_1^L\, x(t-\tau_1(t))\right),$$

it follows from [26, Lemma 2.3] that there exists constant $T_1 > 0$ such that

$$x(t) \leqslant \frac{a_1^U}{b_1^L}\exp\{b_1^L\,\tau_1^U\} := M_1 \ \text{ for } \ t > T_1. \qquad (4)$$

Similarly, from the second equation of model (1) we get

$$\frac{d(y^{1-m}(t))}{d\,t} \leqslant (1-m)\left(c_2^U M_1 - a_2^L\, y^{1-m}(t)\right).$$

This inequality and Lemma 3 yields

$$y(t) \leqslant \left\{\frac{c_2^U M_1}{a_2^L}\left[1+\left(\frac{c_2^U M_1}{a_2^L\, y^{1-m}(0)} - 1\right)\right.\right.$$
$$\left.\left. \times \exp\left\{(m-1)\, c_2^U M_1\, t\right\}\right]^{-1}\right\}^{\frac{1}{1-m}}.$$

Note that $\exp\left\{(m-1)\, c_2^U M_1\, t\right\} \to 0$ for $t \to +\infty$, which yields that, for any small constant $\varepsilon_0 > 0$ there exists a constant $T_2 > 0$ such that, for $t > T_2$

$$y(t) \leqslant \left(\frac{c_2^U M_1}{a_2^L} + \varepsilon_0\right)^{\frac{1}{1-m}} := M_2. \qquad (5)$$

We now estimate the eventually lower bounds of all positive solution $(x(t), y(t))$ of system (1). From (5) and the first equation of model (1) we get

$$\dot{x}(t) \geqslant x(t)\left[(a_1 - c_1 M_2^m)^L - b_1^U\, x(t-\tau_1(t))\right].$$

Condition (1) and [26, Lemma 2.4] imply that, for any small constant $\varepsilon_1 > 0$ there is a $T_3 > 0$ such that

$$x(t) \geqslant M_3 \ \text{ for } \ t > T_3, \qquad (6)$$

where $M_3 = \min\left\{\frac{K_1}{b_1^U}\, e^{(K_1 - b_1^U M_1)\tau_1^U}, \frac{K_1}{b_1^U} - \varepsilon_1\right\}$. The second equation of model (1) implies

$$\dot{y}(t) \geqslant y(t)\left[\left(c_2\, M_3\, M_2^{m-1} - a_2\right)^L - b_2^U\, y(t-\tau_3(t))\right].$$

So from [26, Lemma 2.4] and condition (2) we know that for any small constant $\varepsilon_2 > 0$ there must exist constant $T_4 > 0$ such that

$$y(t) \geqslant M_4 \ \text{ for } \ t > T_4, \qquad (7)$$

where $M_4 = \min\left\{\frac{K_2}{b_2^U}\, e^{(K_2 - b_2^U M_2)\tau_3^U}, \frac{K_2}{b_2^U} - \varepsilon_2\right\}$.

On the other hand, from the second equation of system (1) one can obtain

$$\dot{y}(t) \geqslant y(t)\left[\left(c_2\, M_3 - b_2\, M_2^{2-m}\right)^L - a_2^U\, y^{1-m}(t)\right].$$

Thus Lemma 4 and assumption (3) yield that, for any small constant $\varepsilon_3 > 0$ there is a $T_5 > 0$ such that

$$y(t) \geqslant \left(\frac{K3}{a_2^U}\right)^{\frac{1}{1-m}} - \varepsilon_3 := M_5 \ \text{ for } \ t > T_5. \qquad (8)$$

Set $T = \max_{1 \leqslant i \leqslant 5}\{T_i\}$, $M_6 = \max\{M_4, M_5\}$ and $D = \{(u, v)|\, M_3 \leqslant u \leqslant M_1,\, M_6 \leqslant v \leqslant M_2\}$, then (4)-(8) yields $(x(t), y(t)) \subseteq D$ for $t > T$. Thus $(x(t),\, y(t))$ is uniformly persistent. □

Remark 8 *If all coefficients in model (1) are continuously periodic functions, i.e., it is a periodic system, then Theorem 7 and Brouwer fixed point theorem yields the following result.*

Theorem 9 *If model (1) is a $\omega-$periodic system and condition (1)-(2) in Theorem 7 holds, and all deviating arguments $\tau_i(t)(i = 1, 2, 3)$ are continuous $\omega-$periodic functions, then model (1) has at least one positive $\omega-$periodic solution.*

3.2 Global attractivity

In this section we will present some sufficient conditions for the global asymptotic stability of model (1).

Theorem 10 *Assume that all conditions in Theorem 7 hold, and further assume that the following conditions hold:*

[A_1] τ_i, $i = 1, 2, 3$ are continuously differentiable on $[0, +\infty)$ and $\inf_{t \geqslant 0}(1 - \dot{\tau}_i(t)) > 0$;

[A_2] There exist positive constants α and β such that $\liminf_{t \to +\infty}\{C_i(t),\ i = 1, 2, 3\} > 0$, where

$$\begin{aligned} C_1(t) = {} & \alpha b_1(t) - \beta\, c_2(t)\, M_2^{m-1} - \alpha(a_1(t) + M_1 b_1(t) \\ & + M_2^m c_1(t)) \int_t^{\varphi_1^{-1}(t)} b_1(l)dl \\ & - \beta M_2 M_6^{m-1} c_2(t) \int_t^{\varphi_3^{-1}(t)} b_2(l)dl \\ & - \alpha M_1 \frac{b_1(\varphi_2^{-1}(t))}{1 - \dot{\tau}_2(\varphi_2^{-1}(t))} \int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))} b_1(l)dl, \end{aligned}$$

$$C_2(t) = b_2(t) - M_3 M_6^{m-2} c_2(t) - (a_2(t) + M_2 b_2(t) + M_1 M_6^{m-1} c_2(t)) \int_t^{\varphi_3^{-1}(t)} b_2(l)dl - M_2 \frac{b_2(\varphi_3^{-1}(t))}{1 - \dot{\tau}_3(\varphi_3^{-1}(t))} \int_{\varphi_3^{-1}(t)}^{\varphi_3^{-1}(\varphi_3^{-1}(t))} b_2(l)dl,$$

$$C_3(t) = \frac{\beta M_3 c_2(t)}{M_2} - \frac{\alpha c_1(\varphi_2^{-1}(t))}{1 - \dot{\tau}_2(\varphi_2^{-1}(t))} \times \left(1 + M_1 b_1(\varphi_2^{-1}(t)) \int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))} b_1(l)dl\right).$$

Then system (1)-(2) is globally attractive.

Proof: Suppose $(x_0(t), y_0(t))$ and $(x(t), y(t))$ are any two positive solutions of model (1)-(2), by Theorem 7 one know that there exists $T > 0$ such that, for $t > T$,

$$M_3 \leqslant x_0(t), x(t) \leqslant M_1; \quad M_6 \leqslant y_0(t), y(t) \leqslant M_2.$$

Define

$$V_1(t) = \alpha \left|\ln x(t) - \ln x_0(t)\right| + \beta \left|\ln y(t) - \ln y_0(t)\right|,$$

where α, β are positive constants. Along system (1) we get its Dini derivative as follows,

$$\begin{aligned}
&D^+V_1(t)|_{(1)} \\
=&\alpha\, sgn(x(t) - x_0(t)) \Big\{ -b_1(t)[x(t) - x_0(t)] - c_1(t)[y^m(\varphi_2(t)) - y_0^m(\varphi_2(t))] + b_1(t) \\
&\times \int_{\varphi_1(t)}^t (\dot{x}(s) - \dot{x}_0(s))ds \Big\} + \beta\, sgn(y(t) - y_0(t)) \\
&\times \Big\{ -b_2(t)[y(t) - y_0(t)] + c_2(t)[x(t)y^{m-1}(t) - x_0(t)y_0^{m-1}(t)] + b_2(t) \int_{\varphi_3(t)}^t (\dot{y}(s) - \dot{y}_0(s))ds \Big\} \\
\leqslant& - \alpha b_1(t)|x(t) - x_0(t)| - \beta b_2(t)|y(t) - y_0(t)| + \alpha c_1(t)|y^m(\varphi_2(t)) - y_0^m(\varphi_2(t))| + \beta c_2(t) \\
&\times sgn(y(t) - y_0(t))[x(t)y^{m-1}(t) - x_0(t)y_0^{m-1}(t)] \\
&+ \alpha b_1(t)\, sgn(x(t) - x_0(t)) \int_{\varphi_1(t)}^t (\dot{x}(s) - \dot{x}_0(s))ds \\
&+ \beta b_2(t)\, sgn(y(t) - y_0(t)) \int_{\varphi_3(t)}^t (\dot{y}(s) - \dot{y}_0(s))ds.
\end{aligned} \tag{9}$$

Meanwhile, we have

$$\begin{aligned}
&sgn(y(t) - y_0(t))[x(t)y^{m-1}(t) - x_0(t)y_0^{m-1}(t)] \\
=&sgn(y(t) - y_0(t))[x(t)(y^{m-1}(t) - y_0^{m-1}(t)) + y_0^{m-1}(t)(x(t) - x_0(t))] \\
=& - x(t)|y^{m-1}(t) - y_0^{m-1}(t)| + y_0^{m-1}(t) sgn(y(t) - y_0(t))(x(t) - x_0(t)) \\
\leqslant& - x(t)|y^{m-1}(t) - y_0^{m-1}(t)| + y_0^{m-1}(t)|x(t) - x_0(t)|,
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\int_{\varphi_1(t)}^t (\dot{x}(s) - \dot{x}_0(s))ds \\
=& \int_{\varphi_1(t)}^t \{x(s)[a_1(s) - b_1(s)x(\varphi_1(s)) - c_1(s)y^m(\varphi_2(s))] - x_0(s)[a_1(s) - b_1(s)x_0(\varphi_1(s)) - c_1(s)y_0^m(\varphi_2(s))]\}ds \\
=& \int_{\varphi_1(t)}^t (x(s) - x_0(s))[a_1(s) - b_1(s)x_0(\varphi_1(s)) - c_1(s)y_0^m(\varphi_2(s))]ds + \int_{\varphi_1(t)}^t x(s)\{b_1(s)[x_0(\varphi_1(s)) \\
&- x(\varphi_1(s))] + c_1(s)[y_0^m(\varphi_2(s)) - y^m(\varphi_2(s))]\}ds
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
&\int_{\varphi_3(t)}^t (\dot{y}(s) - \dot{y}_0(s))ds \\
=& \int_{\varphi_3(s)}^t \{y(s)[-a_2(s) - b_2(s)y(\varphi_3(s)) + c_2(s)x(s)y^{m-1}(s)] - y_0(s)[-a_2(s) \\
&- b_2(s)y_0(\varphi_3(s)) + c_2(s)x_0(s)y_0^{m-1}(s)]\}ds \\
=& \int_{\varphi_3(t)}^t (y(s) - y_0(s))[-a_2(s) - b_2(s)y_0(\varphi_3(s)) + c_2(s)x_0(s)y_0^{m-1}(s)]ds + \int_{\varphi_3(t)}^t y(s)\{-b_2(s) \\
&\times [y(\varphi_3(s)) - y_0(\varphi_3(s))] + c_2(s)[x(s)y^{m-1}(s) - x_0(s)y_0^{m-1}(s)]\}ds \\
=& \int_{\varphi_3(t)}^t (y(s) - y_0(s))[-a_2(s) - b_2(s)y_0(\varphi_3(s)) + c_2(s)x_0(t)y_0^{m-1}(s)]ds - \int_{\varphi_3(t)}^t b_2(s)y(s)[y(\varphi_3(s)) \\
&- y_0(\varphi_3(s))]ds + \int_{\varphi_3(t)}^t c_2(s)y(s)[x(s)(y^{m-1}(s) - y_0^{m-1}(s)) + y_0^{m-1}(s)(x(s) - x_0(s))]ds.
\end{aligned} \tag{12}$$

Substitution of (10)-(12) into (9) yields

$$
\begin{aligned}
& D^+V_1(t)|_{(1)} \\
\leqslant & -\alpha b_1(t)|x(t)-x_0(t)| - \beta b_2(t)|y(t)-y_0(t)| \\
& + \alpha c_1(t)\,|y^m(\varphi_2(t)) - y_0^m(\varphi_2(t))| \\
& + \beta c_2(t)(-x(t)|y^{m-1}(t) - y_0^{m-1}(t)| \\
& + y_0^{m-1}(t)|x(t)-x_0(t)|) + \alpha b_1(t) \\
& \times \Big\{ \int_{\varphi_1(t)}^t |x(s)-x_0(s)|[a_1(s) - b_1(s)x_0(\varphi_1(s)) \\
& - c_1(s)y_0^m(\varphi_2(s))]ds + \int_{\varphi_1(t)}^t x(s)[b_1(s)|x_0(\varphi_1(s)) \\
& - x(\varphi_1(s))| + c_1(s)|y_0^m(\varphi_2(s)) - y^m(\varphi_2(s))|]ds \Big\} \\
& + \beta b_2(t) \Big\{ \int_{\varphi_3(t)}^t |y(s)-y_0(s)|[-a_2(s) \\
& - b_2(s)y_0(\varphi_3(s)) + c_2(s)x_0(s)y_0^{m-1}(s)]ds \\
& + \int_{\varphi_3(t)}^t b_2(s)y(s)|y(\varphi_3(s)) - y_0(\varphi_3(s))|ds \\
& + \int_{\varphi_3(t)}^t c_2(s)y(s)[-x(s)|y^{m-1}(s) - y_0^{m-1}(s)| \\
& + y_0^{m-1}(s)|x(s)-x_0(s))|ds \Big\} \\
\leqslant & \left[\beta\, c_2(t)\, M_2^{m-1} - \alpha b_1(t)\right] |x(t)-x_0(t)| \\
& - \beta b_2(t)|y(t)-y_0(t)| - \beta M_3 c_2(t)|y^{m-1}(t) \\
& - y_0^{m-1}(t)| + \alpha c_1(t)\,|y^m(\varphi_2(t)) - y_0^m(\varphi_2(t))| \\
& + \alpha b_1(t) \Big\{ \int_{\varphi_1(t)}^t |x(s)-x_0(s)|[a_1(s) + M_1 b_1(s) \\
& + M_2^m c_1(s)]ds + M_1 \int_{\varphi_1(t)}^t [b_1(s)|x_0(\varphi_1(s)) \\
& - x(\varphi_1(s))| + c_1(s)|y_0^m(\varphi_2(s)) - y^m(\varphi_2(s))|]ds \Big\} \\
& + \beta b_2(t) \Big\{ \int_{\varphi_3(t)}^t |y(s)-y_0(s)| \Big[a_2(s) + M_2 b_2(s) \\
& + \frac{M_1 c_2(s)}{M_6^{1-m}} \Big] ds + M_2 \int_{\varphi_3(t)}^t b_2(s)|y(\varphi_3(s)) \\
& - y_0(\varphi_3(s))|ds - M_6 M_3 \int_{\varphi_3(t)}^t c_2(s)|y^{m-1}(s) \\
& -y_0^{m-1}(s)|ds + \frac{M_2}{M_6^{1-m}} \int_{\varphi_3(t)}^t c_2(s)|x(s)-x_0(s)|ds \Big\}.
\end{aligned} \tag{13}
$$

Let

$$
\begin{aligned}
& V_2(t) \\
=& \alpha \int_{\varphi_2(t)}^t \frac{c_1(\varphi_2^{-1}(s))}{1-\dot{\tau}_2(\varphi_2^{-1}(s))} |y^m(s) - y_0^m(s)|ds \\
& + \beta M_2 M_4^{m-1} \int_t^{\varphi_3^{-1}(t)} \int_{\varphi_3(l)}^t b_2(l)c_2(s)|x(s) \\
& - x_0(s)|dsdl + \alpha \int_t^{\varphi_1^{-1}(t)} \int_{\varphi_1(l)}^t b_1(l)[a_1(s) \\
& + M_1 b_1(s) + M_2^m c_1(s)]|x(s)-x_0(s)|dsdl \\
& + \alpha M_1 \int_t^{\varphi_1^{-1}(t)} \int_{\varphi_1(l)}^t b_1(l)[b_1(s)|x(\varphi_2(s)) \\
& -x_0(\varphi_2(s))| + c_1(s)|y^m(\varphi_2(s)) - y_0^m(\varphi_2(s))|]dsdl \\
& + \beta \int_t^{\varphi_3^{-1}(t)} \int_{\varphi_3(l)}^t b_2(l)|y(s)-y_0(s)|[a_2(s) \\
& + M_2 b_2(s) + M_1 M_6^{m-1} c_2(s)]dsdl + \beta M_2 \\
& \times \int_t^{\varphi_3^{-1}(t)} \int_{\varphi_3(l)}^t b_2(l)b_2(s)|y(\varphi_3(s)) - y_0(\varphi_3(s))|dsdl,
\end{aligned}
$$

then its derivative is as follows,

$$
\begin{aligned}
& V_2'(t) \\
=& \frac{\alpha c_1(\varphi_2^{-1}(t))}{1-\dot{\tau}_2(\varphi_2^{-1}(t))} |y^m(s) - y_0^m(t)| - \alpha c_1(t) \\
& \times |y^m(\varphi_2(t)) - y_0^m(\varphi_2(t))| + \alpha[a_1(t) + M_1 b_1(t) \\
& + M_2^m c_1(t)] \int_t^{\varphi_1^{-1}(t)} b_1(l)dl\, |x(t)-x_0(t)| \\
& - \alpha b_1(t) \int_{\varphi_1(t)}^t [a_1(s) + M_1 b_1(s) + M_2^m c_1(s)] \\
& \times |x(s)-x_0(s)|ds + \alpha M_1 b_1(t) \int_t^{\varphi_1^{-1}(t)} b_1(l)dl \\
& \times \Big[|x(\varphi_2(t)) - x_0(\varphi_2(t))| + c_1(t)|y^m(\varphi_2(s)) \\
& - y_0^m(\varphi_2(t))| \Big] - \alpha M_1 b_1(t) \int_{\varphi_1(t)}^t [b_1(s)|x(\varphi_2(s)) \\
& - x_0(\varphi_2(s))| + c_1(s)|y^m(\varphi_2(s)) - y_0^m(\varphi_2(s))|]ds \\
& + \beta \left[a_2(t) + M_2 b_2(t) + M_1 M_6^{m-1} c_2(t)\right] \\
& \times \int_t^{\varphi_3^{-1}(t)} b_2(l)dl\, |y(t)-y_0(t)| - \beta b_2(t)
\end{aligned}
$$

$$\times \int_{\varphi_3(t)}^{t} |y(s)-y_0(s)|[a_2(s)+M_2b_2(s) + M_1M_6^{m-1}c_2(s)]ds + \beta M_2 b_2(t)\int_t^{\varphi_3^{-1}(t)} b_2(l)dl \times |y(\varphi_3(t))-y_0(\varphi_3(t))| - \beta M_2 b_2(t)\int_{\varphi_3(t)}^{t} b_2(s) \times |y(\varphi_3(s))-y_0(\varphi_3(s))|ds + \beta M_2 M_6^{m-1}c_2(t) \times \int_t^{\varphi_3^{-1}(t)} b_2(l)dl\,|x(t)-x_0(t)| - \beta M_2 M_6^{m-1} b_2(t)\int_{\varphi_3(t)}^{t} c_2(s)|x(s)-x_0(s)|ds. \tag{14}$$

Define

$$V_3(t) = \alpha M_1 \int_{\varphi_2(t)}^{t}\int_{\varphi_2^{-1}(s)}^{\varphi_1^{-1}(\varphi_2^{-1}(s))} b_1(l)\frac{b_1(\varphi_2^{-1}(s))}{1-\dot\tau_2(\varphi_2^{-1}(s))} \times \left[|x(s)-x_0(s)| + c_1(\varphi_2^{-1}(s))|y^m(s)-y_0^m(s)|\right] dlds + \beta M_2 \int_{\varphi_3(t)}^{t}\int_{\varphi_3^{-1}(s)}^{\varphi_3^{-1}(\varphi_3^{-1}(s))} b_2(l)\frac{b_2(\varphi_3^{-1}(s))}{1-\dot\tau_3(\varphi_3^{-1}(s))} \times |y(s)-y_0(s)|dlds.$$

By simply calculating we get its derivative in the following form,

$$V_3'(t) = \alpha M_1 \int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))} b_1(l)dl\,\frac{b_1(\varphi_2^{-1}(t))}{1-\dot\tau_2(\varphi_2^{-1}(t))} \times \left[|x(t)-x_0(t)| + c_1(\varphi_2^{-1}(t))|y^m(t)-y_0^m(t)|\right] -\alpha M_1 b_1(t)\int_t^{\varphi_1^{-1}(t)} b_1(l)dl[|x(\varphi_2(t))-x_0(\varphi_2(t))| +c_1(t)|y^m(\varphi_2(t))-y_0^m(\varphi_2(t))|] + \beta M_2|y(t)-y_0(t)| \times \int_{\varphi_3^{-1}(t)}^{\varphi_3^{-1}(\varphi_3^{-1}(t))} b_2(l)dl\,\frac{b_2(\varphi_3^{-1}(t))}{1-\dot\tau_3(\varphi_3^{-1}(t))} - \beta M_2 b_2(t)\int_t^{\varphi_3^{-1}(t)} b_2(l)dl\,|y(\varphi_3(t))-y_0(\varphi_3(t))|. \tag{15}$$

Define the Lyapunov functional by

$$V(t) = V_1(t)+V_2(t)+V_3(t),$$

then

$$D^+V(t) = D^+V_1(t) + V_2'(t) + V_3'(t). \tag{16}$$

Substituting (13)-(15) into (16), we obtain

$$D^+V(t)|_{(1)} \leqslant \left[\beta\, c_2(t)\, M_2^{m-1} - \alpha b_1(t)\right]|x(t)-x_0(t)| - \beta b_2(t) \times |y(t)-y_0(t)| - \beta M_3 c_2(t)|y^{m-1}(t)-y_0^{m-1}(t)| -\beta M_6 M_3 b_2(t)\int_{\varphi_3(t)}^{t} c_2(s)|y^{m-1}(s)-y_0^{m-1}(s)|ds + \frac{\alpha c_1(\varphi_2^{-1}(t))}{1-\dot\tau_2(\varphi_2^{-1}(t))}|y^m(t)-y_0^m(t)| + \alpha[a_1(t) +M_1b_1(t)+M_2^m c_1(t)]\int_t^{\varphi_1^{-1}(t)} b_1(l)dl\,|x(t)-x_0(t)| + \beta\left[a_2(t)+M_2b_2(t)+M_1M_6^{m-1}c_2(t)\right] \times \int_t^{\varphi_3^{-1}(t)} b_2(l)dl\,|y(t)-y_0(t)| + \beta M_2 M_6^{m-1}c_2(t) \times \int_t^{\varphi_3^{-1}(t)} b_2(l)dl\,|x(t)-x_0(t)| + \frac{\alpha M_1 b_1(\varphi_2^{-1}(t))}{1-\dot\tau_2(\varphi_2^{-1}(t))} \times \int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))} b_1(l)dl[|x(t)-x_0(t)| + c_1(\varphi_2^{-1}(t)) \times |y^m(t)-y_0^m(t)|] + \beta M_2\int_{\varphi_3^{-1}(t)}^{\varphi_3^{-1}(\varphi_3^{-1}(t))} b_2(l)dl \times \frac{b_2(\varphi_3^{-1}(t))}{1-\dot\tau_3(\varphi_3^{-1}(t))}|y(t)-y_0(t)|$$

$$\leqslant -\Bigg[\alpha b_1(t) - \beta\, c_2(t)\, M_2^{m-1} - \alpha\Big(a_1(t)+M_1b_1(t) + M_2^m c_1(t)\Big)\int_t^{\varphi_1^{-1}(t)} b_1(l)dl - \beta M_2 M_6^{m-1}c_2(t) \times \int_t^{\varphi_3^{-1}(t)} b_2(l)dl - \alpha M_1 \frac{b_1(\varphi_2^{-1}(t))}{1-\dot\tau_2(\varphi_2^{-1}(t))} \times \int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))} b_1(l)dl\Bigg]|x(t)-x_0(t)| - \beta\Bigg[b_2(t) - \Big(a_2(t)+M_2b_2(t)+\frac{M_1c_2(t)}{M_6^{1-m}}\Big)\int_t^{\varphi_3^{-1}(t)} b_2(l)dl -\frac{M_2b_2(\varphi_3^{-1}(t))}{1-\dot\tau_3(\varphi_3^{-1}(t))}\int_{\varphi_3^{-1}(t)}^{\varphi_3^{-1}(\varphi_3^{-1}(t))} b_2(l)dl\Bigg]|y(t)-y_0(t)| -\beta M_3 c_2(t)|y^{m-1}(t)-y_0^{m-1}(t)| + \frac{\alpha c_1(\varphi_2^{-1}(t))}{1-\dot\tau_2(\varphi_2^{-1}(t))} \times \left[1 + M_1 b_1(\varphi_2^{-1}(t))\int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))} b_1(l)dl\right] \times |y^m(t)-y_0^m(t)|. \tag{17}$$

In view of

$$a|a^{m-1}-b^{m-1}| \geqslant |a^m-b^m|-b^{m-1}|b-a|$$

for $a>0, b>0$, which yields

$$\begin{aligned}&-\beta M_3 c_2(t)|y^{m-1}(t)-y_0^{m-1}(t)|\\ \leqslant&\beta M_3 M_6^{m-2} c_2(t)|y(t)-y_0(t)|\\ &-\frac{\beta M_3 c_2(t)}{M_2}|y^m(t)-y_0^m(t)|.\end{aligned} \tag{18}$$

Thus, combination of (18) and (17) yields

$$\begin{aligned}D^+V(t)\leqslant&-C_1(t)|x(t)-x_0(t)|-\beta C_2(t)\\ &\times|y(t)-y_0(t)|-C_3(t)|y^m(t)-y_0^m(t)|,\end{aligned}$$

which together with $[A_1]$ and $[A_2]$ yield there must exist three positive constants λ_1, λ_2 and λ_3 such that

$$\begin{aligned}D^+V(t)\leqslant&-\lambda_1\,|x(t)-x_0(t)|-\lambda_2\,|y(t)-y_0(t)|\\ &-\lambda_3\,|y^m(t)-y_0^m(t)|\end{aligned}$$

for $t>T$. Thus, $V(t)$ is non-increasing on $[0,+\infty)$. Integrating the above inequality from T to t we obtain

$$\begin{aligned}&\lambda_1\int_T^t|x(t)-x_0(t)|dt+\lambda_2\int_T^t|y(t)-y_0(t)|\,dt\\ &+\lambda_3\int_T^t|y^m(t)-y_0^m(t)|\,dt<+\infty\ \text{ for }\ t>T.\end{aligned}$$

By Barbalat's Lemma [27], we get the result. □

Theorem 11 *Assume that all conditions in Theorem 7 and $[A_1]$ hold, and further assume that there exist mutual prime positive integers p,q with $p>q$ such that $[A_3]$: $m=\frac{q}{p}$ and $[A_4]$: There exist positive constants α and β such that $\liminf_{t\to+\infty}\{C_1(t),B(t)\}>0$, where*

$$\begin{aligned}B(t):=&\beta B_1(t)\sum_{i=1}^{p}u^{\frac{i-1}{p}}v^{\frac{p-i}{p}}-B_2(t)\sum_{i=1}^{q}u^{\frac{i-1}{p}}v^{\frac{q-i}{p}},\\ &\text{with } M_6\leqslant u,v\leqslant M_2,\\ B_1(t)=&b_2(t)+M_3M_2^{m-2}c_2(t)-(a_2(t)+M_2b_2(t)\\ &+M_1M_6^{m-1}c_2(t))\int_t^{\varphi_3^{-1}(t)}b_2(l)dl\\ &-M_2\frac{b_2(\varphi_3^{-1}(t))}{1-\dot\tau_3(\varphi_3^{-1}(t))}\int_{\varphi_3^{-1}(t)}^{\varphi_3^{-1}(\varphi_3^{-1}(t))}b_2(l)dl,\\ B_2(t)=&\frac{\beta M_3c_2(t)}{M_6}+\frac{\alpha c_1(\varphi_2^{-1}(t))}{1-\dot\tau_2(\varphi_2^{-1}(t))}\\ &\times\left(1+M_1b_1(\varphi_2^{-1}(t))\int_{\varphi_2^{-1}(t)}^{\varphi_1^{-1}(\varphi_2^{-1}(t))}b_1(l)dl\right),\end{aligned}$$

where $C_1(t)$ is defined in Theorem 10, M_2, M_6 is defined in the proof of Theorem 7 Then model (1) is globally attractive.

Proof: One see

$$a|a^{m-1}-b^{m-1}|\geqslant b^{m-1}|b-a|-|a^m-b^m|$$

for $a>0$, $b>0$, thus

$$\begin{aligned}&-\beta M_3c_2(t)|y^{m-1}(t)-y_0^{m-1}(t)|\\ \leqslant&\frac{\beta M_3c_2(t)}{M_6}|y^m(t)-y_0^m(t)|\\ &-\beta M_3c_2(t)M_2^{m-2}(t)|y(t)-y_0(t)|.\end{aligned} \tag{19}$$

Substituting (19) into (17), one has

$$\begin{aligned}D^+V(t)\leqslant&-C_1(t)|x(t)-x_0(t)|-\beta B_1(t)\\ &\times|y(t)-y_0(t)|+B_2(t)|y^m(t)-y_0^m(t)|.\end{aligned} \tag{20}$$

Assumption $[A_3]$ yields

$$y(t)-y_0(t)=\left[y^{\frac1p}(t)-y_0^{\frac1p}(t)\right]\sum_{i=1}^{p}y^{\frac{i-1}{p}}(t)y_0^{\frac{p-i}{p}}(t),$$

$$y^m(t)-y_0^m(t)=\left[y^{\frac1p}(t)-y_0^{\frac1p}(t)\right]\sum_{i=1}^{q}y^{\frac{i-1}{p}}(t)y_0^{\frac{q-i}{p}}(t),$$

which together with (20) give

$$\begin{aligned}&D^+V(t)\\ \leqslant&-C_1(t)|x(t)-x_0(t)|-w(t)\left|y^{\frac1p}(t)-y_0^{\frac1p}(t)\right|,\end{aligned}$$

where

$$\begin{aligned}w(t)=&\beta B_1(t)\sum_{i=1}^{p}y^{\frac{i-1}{p}}(t)y_0^{\frac{p-i}{p}}(t)\\ &-B_2(t)\sum_{i=1}^{q}y^{\frac{i-1}{p}}(t)y_0^{\frac{q-i}{p}}(t),\end{aligned}$$

which together with assumption $[A_4]$ implies

$$\lim_{t\to+\infty}|x(t)-x_0(t)|=\lim_{t\to+\infty}\left|y^{\frac1p}(t)-y_0^{\frac1p}(t)\right|=0.$$

□

Remark 12 *Compare Theorem 10 with Theorem 11, we find that the assumptions in the former is more simple than those in the later. But we will show that the assumptions in Theorem 11 is more weak than Theorem 10. In fact, if $q=1$, then $B(t)=\beta B_1(t)\sum_{i=1}^{p}u^{\frac{i-1}{p}}v^{\frac{p-i}{p}}-B_2(t)\geqslant p\,\beta M_6^{\frac{p-1}{p}}B_1(t)-B_2(t)$, which is more simpler than that of $q\neq1$, and thus can be checked easily.*

If $\tau_i(t) = \tau_i > 0,\ i = 1, 2, 3$, that is, system (1) reduces to the following system:

$$\begin{cases} \dot{x}(t) = x(t)(a_1(t) - b_1(t)x(t-\tau_1) \\ \qquad - c_1(t)y^m(t-\tau_2)), \\ \dot{y}(t) = y(t)(-a_2(t) - b_2(t)y(t-\tau_3) \\ \qquad + c_2(t)x(t)y^{m-1}(t)), \end{cases} \tag{21}$$

then corresponding to Theorems 10-11 we get the following results.

Theorem 13 *Assume all conditions in Theorem 7 hold, and there exist positive constants* α, β *such that* $\lim\inf_{t\to+\infty}\{C_i(t),\ i = 4, 5, 6\} > 0$, *where*

$$\begin{aligned} C_4(t) =& \alpha b_1(t) - \beta c_2(t) M_2^{m-1} - \alpha(a_1(t) + M_1 b_1(t) \\ &+ M_2^m c_1(t)) \int_t^{t+\tau_1} b_1(l)dl - \beta M_2 M_6^{m-1} c_2(t) \\ &\times \int_t^{t+\tau_3} b_2(l)dl - \alpha M_1 b_1(t+\tau_2) \int_{t+\tau_2}^{t+\tau_1+\tau_2} b_1(l)dl, \\ C_5(t) =& b_2(t) - M_3 M_6^{m-2} c_2(t) - M_2 b_2(t+\tau_3) \\ &\times \int_{t+\tau_3}^{t+2\tau_3} b_2(l)dl - (a_2(t) + M_2 b_2(t) \\ &+ M_1 M_6^{m-1} c_2(t)) \int_t^{t+\tau_3} b_2(l)dl, \\ C_6(t) =& \frac{\beta M_3 c_2(t)}{M_2} - \alpha c_1(t+\tau_2) \\ &\times \left(1 + M_1 b_1(t+\tau_2) \int_{t+\tau_2}^{t+\tau_1+\tau_2} b_1(l)dl\right). \end{aligned}$$

Then model (21) is globally attractive.

Theorem 14 *Assume that all conditions in Theorem 7 and [A_1] hold, and there exist mutual prime positive integers* p, q *with* $p > q$ *such that [A_3] hold, and there exist positive constants* α *and* β *such that* $\lim\inf_{t\to+\infty}\{C_4(t), B_0(t)\} > 0$, *where*

$$\begin{aligned} B_0(t) =& \beta B_3(t) \sum_{i=1}^{p} u^{\frac{i-1}{p}} v^{\frac{p-i}{p}} - B_4(t) \sum_{i=1}^{q} u^{\frac{i-1}{p}} v^{\frac{q-i}{p}}, \\ & M_6 \leqslant u, v \leqslant M_2, \\ B_3(t) =& b_2(t) + M_3 M_2^{m-2} c_2(t) - M_2 b_2(t+\tau_3) \\ &\times \int_{t+\tau_3}^{t+2\tau_3} b_2(l)dl - (2(t) + M_2 b_2(t) \\ &+ M_1 M_6^{m-1} c_2(t)) \int_t^{t+\tau_3} b_2(l)dl, \\ B_4(t) =& \frac{\beta M_3 c_2(t)}{M_6} + \alpha c_1(t+\tau_2) \\ &\times \left(1 + M_1 b_1(t+\tau_2) \int_{t+\tau_2}^{t+\tau_1+\tau_2} b_1(l)dl\right). \end{aligned}$$

Then model (21) is globally attractive.

Corollary 15 *Assume that all conditions in Theorem 7 and [A_1] hold, and further assume that there exists a positive integer* p *with* $p > 1$ *such that* $m = \frac{1}{p}$, *and there exist positive constants* α *and* β *such that*

$$\lim\inf_{t\to+\infty}\{C_4(t), B_0(t)\} > 0,$$

where $B_0(t) := p\,\beta M_6^{\frac{p-1}{p}} B_3(t) - B_4(t)$. *Then model (21) is globally attractive.*

4 Application and simulation

Example: Let

$$\begin{aligned} & a_1(t) = 12 + 0.01\sin t,\ a_2(t) = 5 - 0.01\sin t, \\ & b_1(t) = 6,\ b_2(t) = 3.4,\ c_1(t) = 0.3 + 0.29\sin t, \\ & c_2(t) = 1.2 + 0.1\sin t,\ \tau_1 = 0.005,\ \tau_2 = 0.03, \\ & \tau_3 = 0,\ m = 1/3. \end{aligned}$$

Choose $\varepsilon_0 = \varepsilon_1 = 1 \times 10^{-10}$, $\varepsilon_2 = \varepsilon_3 = 2 \times 10^{-10}$, $\alpha = 1.2$, $\beta = 0.005$.

By calculating we obtain

$$\begin{aligned} & M_1 = 2.062627,\ M_2 = 0.3939083,\ K_1 = 11.5775, \\ & M_3 = 1.921898,\ K_2 = -1.07577,\ K_3 = 1.394411, \\ & M_4 = -0.316404,\ M_5 = 0.146835,\ M_6 = 0.146835, \\ & \liminf_{t\to+\infty} C_4(t) = 5.84892,\ \liminf_{t\to+\infty} C_5(t) = -58.1291, \\ & \liminf_{t\to+\infty} C_6(t) = 0.939147,\ \liminf_{t\to+\infty} B_0(t) = 0.082385. \end{aligned}$$

According to Theorems 7,10, this system is uniformly persistent and has 2π-periodic positive solution. Meanwhile, according to Corollary 15 we assert that the system is globally attractive, see Figs.1-2 for more details.

Remark 16 *Example shows that* $\liminf_{t\to+\infty} B_0(t) > 0$ *is weak than* $\liminf_{t\to+\infty}\{C_5(t), C_6(t)\} > 0$, *which verifies Remark 12.*

Remark 17 *Obviously, if* $m = 1$, *then there is no mutual inference between preys and predators, then from Figs.3-4. one can easily see that the prey is uniformly persistent but the predator is extinct eventually. So the mutual interference can effect the population of the prey and predator. In the real world we must consider some PP models under the influence of mutual interference.*

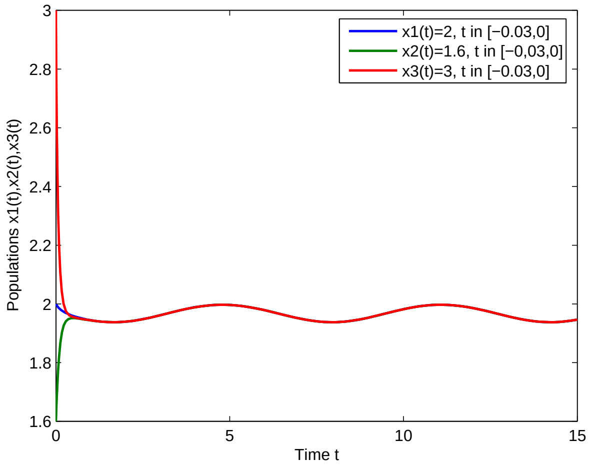

Fig.1. The integral curves of example with $m = 1/3$.

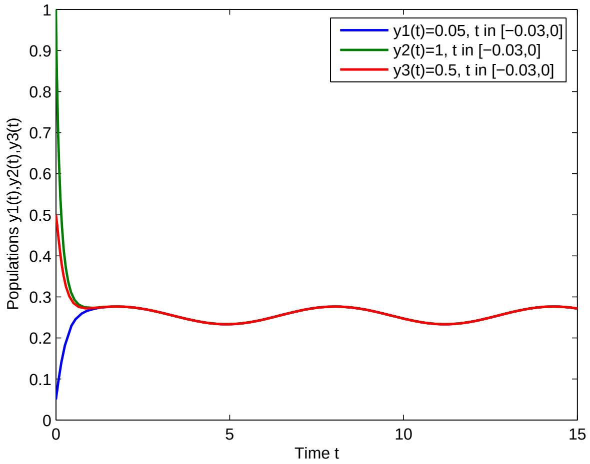

Fig.2. The integral curves of example with $m = 1/3$.

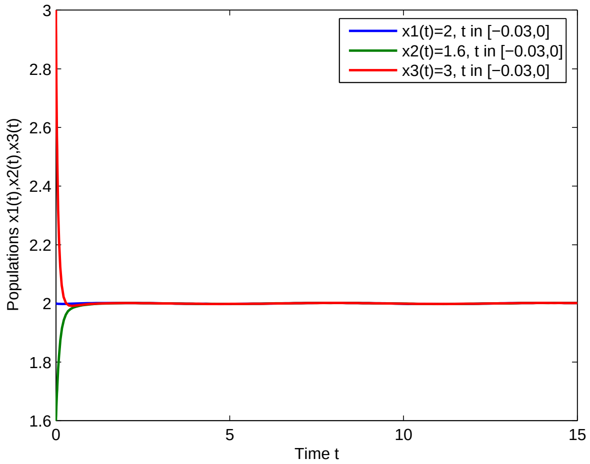

Fig.3. The integral curves of example with $m = 1$.

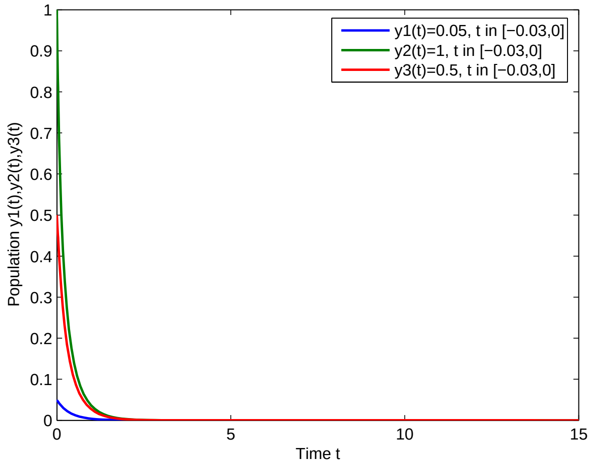

Fig.4. The integral curves of example with $m = 1$.

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