Thermosolutal MHD marangoni convective flow of a nanofluid past a flat plate with viscous dissipation and radiation effects

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Abstract: The radiation and viscous effects on a steady two dimensional boundary layer flow of a nanofluid over a flat plate is analyzed. Two types of nanofluids namely Cu and Ag are considered with water as base fluid (Prandtl number 6.2). The governing non linear partial differential equations are transformed into a system of non linear ordinary differential equations by similarity transformation. A numerical solution is obtained by the help of Matlab 'bvp4c' function. The effects of volume fraction, radiation parameter, Eckert number, Schmidt number and Soret number on velocity, temperature and concentration profiles as well as on heat and mass transfer are studied. The results are presented through plots and tables.

Key–Words: Viscous dissipation, radiation, nano fluid, heat and mass transfer, MHD

1 Introduction

The suspensions of submicron solid particles (nanoparticles) in common fluids (base fluids) are called nanofluids. The nanofluids are very stable, as the particles are small in size, low weight and less chances of sedimentation [1]. Also one of the important characteristics of nanofluids is their high thermal conductivity relative to the base fluids. This conductivity can be achieved even at very low volume fraction of nano particles. This unique feature poses them as an alternative for high heat transfer applications.

Study of a convective flow in porous media has got its importance due to its wide application in engineering as post accidental heat removal in solar collectors, nuclear reactors, drying process, geothermal and oil recovery, heat exchangers, building construction etc., (Nield and Bejan [2], Ingham and Pop [3]). It is known that conventional heat transfer fluids, including oil, water and ethylene glycol mixture are poor heat transfer fluids. As the conductivity of these fluids play a vital role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. An innovative technique, which uses a mixture of nanoparticles and the base fluid was first introduced by Choi [4] in order to develop the heat transfer fluids with substantially higher conductivities. Nanofluid is visualized to describe a fluid in which nanometer sized particles are suspended in conventional heat transfer basic fluids. More number of papers has been published on numerical studies on the modeling of natural convection heat transfer in nanofluids. Congedo et al [5], Ghasemi et al [6], Ho et al [7, 8] etc. Good collection can be found by Das et al [9]. Buongiorno [10] studied the convective transport in nanofluids which contains Brownian diffusion and thermophoresis. Later Khan and Aziz[11] used this model to investigate the boundary layer flow of a nanofluid past a vertical surface with a constant heat flux. The problem of natural convection of a regular fluid over an isothermal vertical plate to the flow of a nanofluid is studied by Kuznetsov and Nield [12].

The problem of viscous flow and heat transfer over a vertical plate in moving solid surface is studied by Sakiadis [13,14]. The effect of viscous dissipation on the boundary layer flow has considerable interest, since this has a direct impact on the Nusselt number, which is the measure of heat transfer rate. Gebhart [15], later Gebhart and Mollendort [16] have studied this effect of viscous dissipation in natural convection processes. Vajravelu and Hadjini Calaou [17] explained the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. The study of magnetic field has a considerable interest in Science and Engineering applications, in particular, polymer industry and metallurgy. The applications in metallurgy include the cooling of continuous strips or filaments in thinning of copper wires. Joshi and Gebhart [18], Rahman et al [19] have analyzed the combined effect of conduction and viscous dissipation on MHD free convection flow along vertical flat plate. Alim et al.[20] discussed the combined effect of viscous dissipation and joule heating on the coupling of conduction and free convection along a vertical flat plate.

The study of marangoni convection has received great consideration in recent years in view of the industry applications. Marangoni convection is expected to be very useful in the areas like crystal growth melts and semiconductor processing. This convection induced due to surface tension gradient which is caused either by thermal convection or solutal convection or both. Pop et al.[21] studied numerically the problem of thermosolutal marangoni forced convection over a permeable surface. Later Hamid [22] extended for dual solution of the problem. Sastry et al.[23] studied the effects of heat transfer on magneto hydrodynamic marangoni convection with chemical reaction over a vertical plate and obtained the numerical solution.

In this paper, it is proposed to study the combined effects of a magnetic field, viscous dissipation, chemical reaction and Soret effects on marangoni nanofluid flow. The heat and mass transfer characteristics of this flow are presented graphically.

2 Problem formulation

Consider a steady two-dimensional marangoni boundary layer viscous flow past a permeable flat plate in a water based nano fluid containing two different types of nano particles Cu (Copper) and Ag (Silver). Assume that the fluid is incompressible and the flow is laminar. Also it is assumed that the base fluid and the particles are in thermal equilibrium. The thermo physical properties of nano particles are given in the Table1. Further, consider a Cartesian coordinate system (x, y), where x and y are the coordinates measured along the plate and normal to it, respectively, and the flow takes place at $y \geq 0$. Assume that the temperature of the plate is $T(x)$ and that of the ambient fluid is T_∞. Further define the surface tension as

$$\sigma = \sigma_0[1 - \gamma(T - T_\infty) - \gamma^*(C - C_\infty)] \quad (1)$$

where surface tension at the interface is σ_0 and

$$\gamma = -\frac{\partial \sigma}{\partial T} \quad , \quad \gamma^* = -\frac{\partial \sigma}{\partial C} \quad (2)$$

It is also assumed that a uniform magnetic field H_0 is imposed in the direction normal to the surface. Then the steady state boundary layer equations for a nanofluid in the Cartesian coordinates are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma^*}{\rho_{nf}}H_0^2 u \quad (4)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y} - \frac{\mu_{nf}}{(\rho C_p)_{nf}}(\frac{\partial u}{\partial y})^2 \quad (5)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D^*\frac{\partial^2 T}{\partial y^2} - K^*(C - C_\infty) \quad (6)$$

subject to the boundary conditions

$$\begin{cases} v = 0, \quad T = T_\infty + ax^2, \\ C = C_\infty + bx^2, \\ \mu_{nf}\frac{\partial u}{\partial y} = \gamma\frac{\partial T}{\partial x} + \gamma^*\frac{\partial C}{\partial x}, \quad y = 0 \\ u \to 0, T \to T_\infty, C \to C_\infty, \quad y \to \infty \end{cases} \quad (7)$$

Here u and v are the components of velocity along the x and y axes, respectively. ρ is the fluid density, T is the temperature, C is the concentration, C_∞ is the concentration of the fluid far from the surface, C_p is the specific heat at constant pressure, D is the species diffusivity, D^* is the coefficient that signifies the contribution to mass flux through temperature gradient and K^* is the chemical reaction parameter. σ^* is the electric conductivity and a, b are the coefficients of temperature and concentration gradients respectively.

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (8)$$

where ϕ is the solid particles volume fraction of nanoparticles. The quantities like effective density, thermal diffusivity, and the heat capacitance of the nanofluid are obtained as follows

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s \quad (9)$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (10)$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (11)$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2K_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (12)$$

In equations (8)-(12), the subscripts nf,f and s denote the thermo physical properties of the nanofluid, base fluid and nano-solid particles respectively. The continuity equation (3) is satisfied by introducing a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (13)$$

where

$$\psi = C_1 x f(\eta), \quad \eta = C_2 y \tag{14}$$

$$\theta(\eta) = \frac{T - T_\infty}{ax^2}, \quad h(\eta) = \frac{C - C_\infty}{bx^2} \tag{15}$$

where the constants C_1 and C_2 are given by

$$C_1 = \left(\frac{\frac{d\sigma}{dT}|_C a\mu_f}{\rho_f^2}\right)^{\frac{1}{3}}, \quad C_2 = \left(\frac{\frac{d\sigma}{dT}|Ca\rho_f}{\mu_f^2}\right)^{\frac{1}{3}} \tag{16}$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and $h(\eta)$ is the dimensionless concentration.

Using the Rosseland approximation for radiation, one can simplify the radiative heat flux as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{17}$$

where $\sigma*$ and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.

Further expand T^4 in Taylor's series about T_∞ and neglecting higher order terms, thus the radiative heat flux is modified as

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y} \tag{18}$$

from equations (16), (17) and (18), energy equation (4) is modified as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}(1 + N_r)\frac{\partial^2 T}{\partial y^2} \tag{19}$$

where the radiation parameter is defined as

$$N_r = \frac{16\sigma^* T_\infty^3}{3k^* k_{nf}} \tag{20}$$

Using equations (3)-(19) one can obtain the following non-dimensional equations of motion

$$\begin{aligned} f''' &= (1-\phi)^{2.5}[(1-\phi) + \phi\frac{\rho_s}{\rho_f}](f'^2 - ff'') \\ &\quad + M^2(1-\phi)^{2.5} f' \end{aligned} \tag{21}$$

$$\theta'' = \frac{P_r[1 - \phi + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}]}{\frac{k_{nf}(1+N_r)}{k_f}}(2f'\theta - f\theta') \tag{22}$$

$$h'' = S_c(2hf' - fh' + K^*) - S_r\theta'' \tag{23}$$

along with the boundary conditions in non dimensional form

$$\begin{cases} f(0) = 0, \quad \theta(0) = 1, \quad h(0) = 1, \\ \frac{1}{(1-\phi)^{2.5}} f''(0) = -2(1+\varepsilon) \\ f'(\infty) = 0, \\ \theta(\infty) = 0, \\ h(\infty) = 0. \end{cases} \tag{24}$$

The non-dimensional constants appearing in equations (21) – (24), the magnetic parameter M, Prandtl number Pr, Radiation parameter Nr, Schmidt number Sc, Soret number Sr, scaled chemical reaction parameter K^* , and thermosolutal surface tension ratio ϵ are given as follows.

$$M = \frac{\sigma_c^{1/2} H_0 \mu^{1/6}}{\rho^{1/3}\frac{d\sigma}{dT}|_c^{1/3} a^{1/3}}, \quad K^* = \frac{K_0 \mu_f^{1/3}\rho_f^{1/3}}{\frac{d\sigma}{dT}|_c^{2/3} a^{2/3}},$$
$$\epsilon = \frac{\Delta C\frac{d\sigma}{dC}|_T}{\Delta T\frac{d\sigma}{dT}|_c}, \quad Pr = \frac{\nu_f(\rho C_p)_f}{k_f},$$
$$Nr = \frac{16\sigma^* T_\infty^3}{3k^* k_{nf}}, \quad Sr = \frac{D_1(T - T_\infty)}{D(C - C_\infty)},$$
$$c = \frac{\nu_f}{D}$$

The quantities of practical interest, in this study, are skin friction coefficient, local Nusselt number Nu_x and local Sherwood number Sh_x. These parameters respectively characterize the surface drag, heat and mass transfer rates near the wall. The shearing stress at wall can be defined as

$$\tau = -\mu_{nf}\frac{\partial u}{\partial y}|_{y=0} = -\frac{\frac{d\sigma}{dT}|_c \quad axf''(0)}{(1-\phi)^{2.5}} \tag{25}$$

The coefficient of skin friction and surface velocity are given by

$$C_f = \frac{2\tau}{\rho_f \mu^2} \tag{26}$$

$$u(x) = \sqrt[3]{\frac{(\frac{d\sigma}{dT}|_c a)^2}{\rho_f \mu_f}} x f'(0) \tag{27}$$

From the equations (25)-(27) it is observed that

$$C_f Re_x^2 (1-\phi)^{2.5} = -2f''(0) \tag{28}$$

The heat transfer rate at the surface flux at the wall is given by

$$q(x) = -k_{nf}\frac{\partial T}{\partial y}|_{y=0} = -k_{nf} C_2 a x^2 \theta'(0) \tag{29}$$

The Nusselt number, a measure of heat transfer, is defined as

$$Nu_x = \frac{xq(x)}{k_f(T - T_\infty)} \tag{30}$$

Using the equations (16),(29) and (30) one can get the dimensionless wall heat transfer rate

$$\frac{Nu_x k_f}{k_{nf} C_2 x} = -\theta'(0). \tag{31}$$

The mass flux at the wall is defined as

$$m = -D\frac{\partial C}{\partial y}|_{y=0} = -DC_2 bx^2 h'(0) \tag{32}$$

Sherwood number, measure of mass transfer, is defined as

$$Sh_x = \frac{xm}{D(C - C_\infty)} \tag{33}$$

From the equations (16), (32) and (33) one can get the dimensionless mass transfer rate as

$$\frac{Sh_x}{C_2 x} = -h'(0) \tag{34}$$

where $C_2 x$ is a dimensionless quantity.

3 Problem Solution

The non-linear governing equations of motion obtained, given by equations (21)-(23) along with boundary conditions (24) are solved numerically using Matlab 'bvp4c' routine programme. In this context two different types of nanoparticles, namely Copper and Silver, with base fluid water (Prandtl number $Pr = 6.2$) are considered. The magnetic field parameter M is ranged from 0 to 100. The solid volume fraction ϕ is ranged from 0 to 0.8. The radiation parameter is taken from 0 to 10. Eckert number is ranged from 0 to 2. The Soret number is taken from 0 to 1. Thermosolutal surface tension ratio ϵ is ranged from 0 to 10. The effect of volume fraction on velocity, temperature and concentration is showed in Fig.1- Fig.3.

The effect of magnetic field parameter on velocity, temperature and concentration is discussed in Fig.4- Fig.6. The effect of radiation parameter on temperature and concentration profiles is presented in Fig.7 and Fig.8 respectively. The contribution of Eckert number towards temperature and concentration profiles is showed in Fig.9 and Fig.10 respectively. The change in concentration along with Soret number is showed in Fig.11. The effect of thermosolutal surface tension ratio on fluid properties is showed in Fig.12- Fig.13. The impact of Schmidt number is showed in Fig.14. Also the effects of heat and mass transfer rates over different parameters like Eckert number and radiation parameter are presented from Fig.15- Fig.19.

It is observed that, when the nanoparticles volume fraction increases, the velocity and concentration of

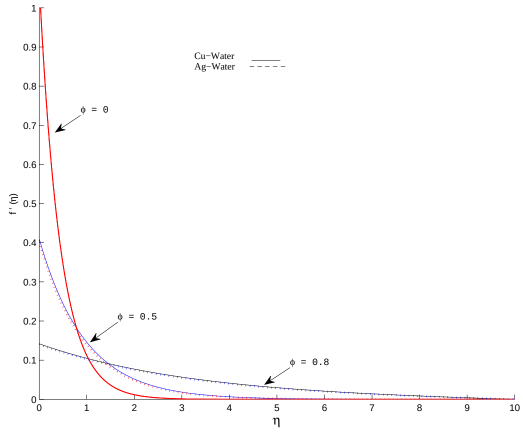

Figure 1: Effects of solid volume fraction ϕ on velocity profiles for $M = Nr = Sr = 2$,$Sc = 0.6$, $\epsilon = K^* = Ec = 0.2$

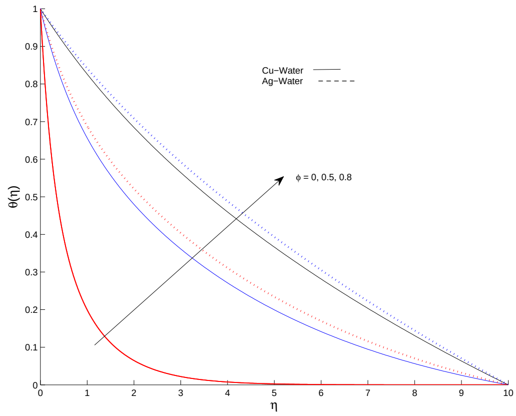

Figure 2: Effects of solid volume fraction ϕ on temperature profiles for $M = Nr = Sr = 2$,$Sc = 0.6$,$\epsilon = K^* = Ec = 0.2$

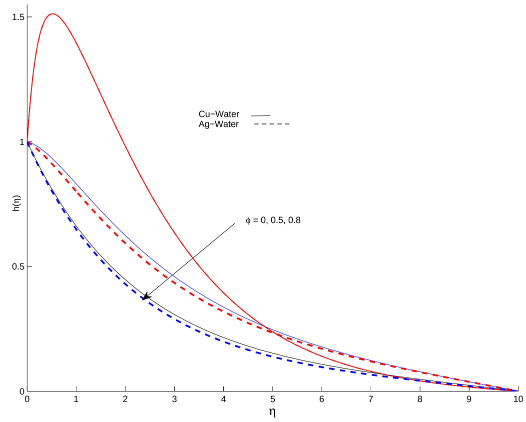

Figure 3: Effects of solid volume fraction ϕ on concentration profiles for $M = Nr = Sr = 2$,$Sc = 0.6$,$\epsilon = K^* = Ec = 0.2$

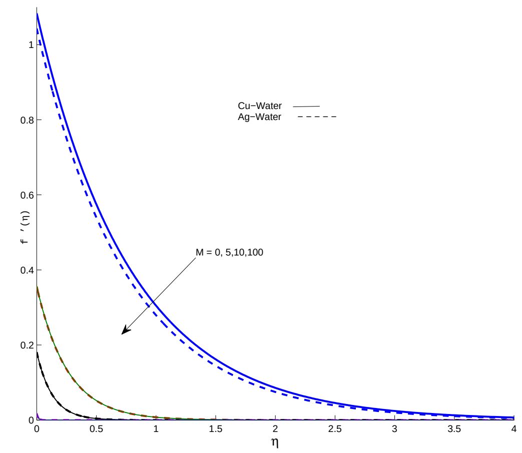

Figure 4: Effects of magnetic parameter M on velocity profiles for $Nr = Sr = 2, Sc = 0.6, \phi = \epsilon = K^* = Ec = 0.2$

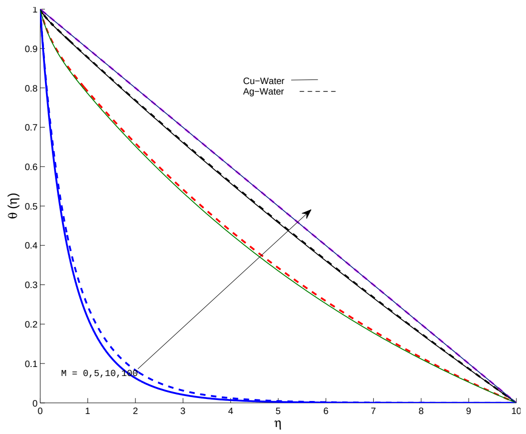

Figure 5: Effects of magnetic parameter M on velocity profiles for $Nr = Sr = 2, Sc = 0.6, \phi = \epsilon = K^* = Ec = 0.2$

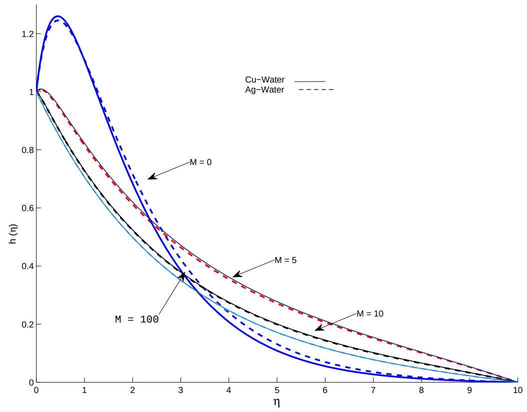

Figure 6: Effects of magnetic parameter M on velocity profiles for $Nr = Sr = 2, Sc = 0.6, \phi = \epsilon = K^* = Ec = 0.2$

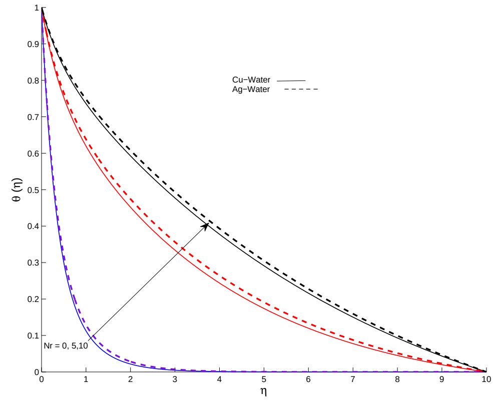

Figure 7: Effects of radiation parameter Nr on velocity profiles for $M = Sr = 2, Sc = 0.6, \phi = \epsilon = K^* = Ec = 0.2$

the nanofluid decrease while the temperature increases within the boundary layer. It is also observed that, the temperature is higher for Cu-water nanofluid particles than that of Ag-water particles. The concentration distribution is higher near the boundary and lesser far away from the boundary in the Cu-water than in Ag-water nanofluid. This is due to the high solutal conductivity of Copper (Fig.3). In addition to this, when the volume fraction of the nanoparticles increases, the thermal conductivity increases, and the thermal boundary layer increases. Cu-water nanofluid is higher velocity profile than that of Ag-water nanofluid. Also temperature distribution is more in Ag-water than Cu-water, since Silver is more thermal conductive material. With increasing nanoparticles volume fraction, the thermal boundary layer thickness increases and the velocity boundary layer thickness decreases for both types of nanofluids (Fig.1,Fig.2).

When a magnetic field is applied within the boundary layer, it produces a resistive type force, known as Lorentz force which acts to retard the fluid motion along the surface and simultaneously increase temperature and concentration values of the fluid within the boundary layer. It is also observed that the effect of magnetic parameter is more on fluid motion and temperature rather than species concentration of the nanoparticles. Also it is noted that at low magnetic number ($0 \leq M \leq 2$)Cu-water stream has more velocity distribution than that of Ag-water stream (Fig.4- Fig.6). It is shown in Fig.7 and Fig.8, how the thermal radiation parameter influences the nanofluids temperature and concentration profiles respectively. It is observed that increase in the value of radiation parameter enhances the temperature of the nanofluid profile and reduces the species concentration. The Cu-water nanoparticles exhibit lower temperature distribution than that of Ag-water nanoparti-

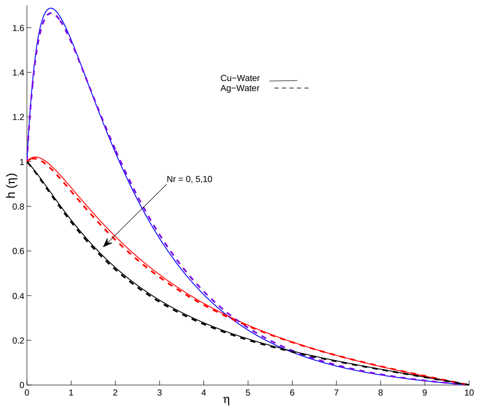

Figure 8: Effects of radiation parameter Nr on velocity profiles for $M = Sr = 2$,$Sc = 0.6$,$\phi = \epsilon = K^* = Ec = 0.2$

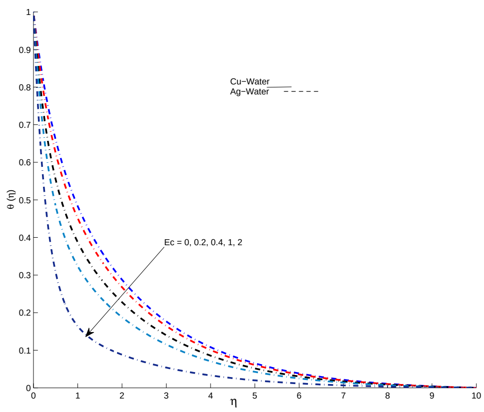

Figure 9: Effects of Eckert number Ec on velocity profiles for $M = Nr = Sr = 2$,$Sc = 0.6$,$\phi = \epsilon = K^* = 0.2$

cles. This is because Silver is more conductive metal than Copper. From the Fig.9, it is observed that temperature decreases as Eckert number increases. Also it is found from Fig.10 that increase in Eckert number enhances the species concentration. □

Fig.11 depicts the effect of Soret number on the species concentration of the nanoparticles. It is noticed that increase in Soret number enhances the concentration of the nanofluid particles. This increase is more in Cu-water nanofluid particles. Also from Fig.12 and Fig.13 it is noticed that thermosolutal surface tension ratio significantly decreases the fluid temperature and concentration. This finding is obtained due to the increase in the values of ϵ demand the increase in the marangoni convection which produces more induced flows within the boundary layer. As a consequence, the resulting flows will propagate within the boundary layers causing the maximum velocity obtained at the wall which reduces the temperature boundary layer. Fig.14 depicts the change of species concentration over the Schmidt number. Concentration for different values of Sc is plotted. Schmidt number represents the relationship between mass diffusivity and thermal diffusivity which physically relates the hydrodynamic boundary layer and mass transfer boundary layer. As Schmidt number depends on mass diffusion rate, concentration should decrease for increasing Schmidt number values. It shows that the mass transfer boundary layer decreases for increasing values of Schmidt number. □

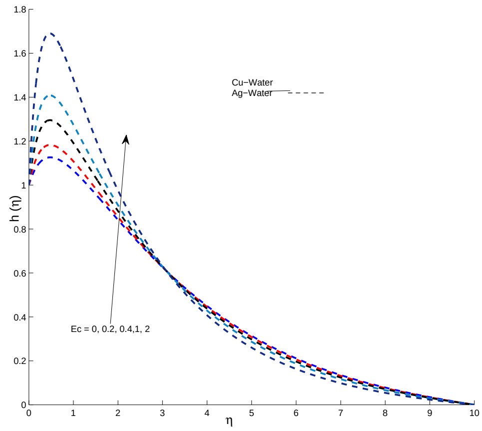

Figure 10: Effects of Eckert number Ec on velocity profiles for $M = Nr = Sr = 2$,$Sc = 0.6$,$\phi = \epsilon = K^* = 0.2$

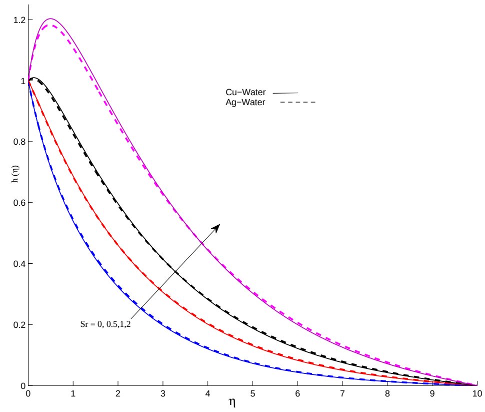

Figure 11: Effects of Soret number Sr on concentration profiles for $M = Nr = 2$,$Sc = 0.6$,$\phi = \epsilon = K^* = Ec = 0.2$

It is known that the Nusselt number and Sherwood number are the measures of heat and mass transfer gradients respectively, Fig.15- Fig.19 show how these vary against viscous dissipation and radiation. From Fig.15, it is clear to note that heat transfer rate increases with increase in the viscous dissipation parameter. This is minimized by applying the magnetic field. Also in non magnetic case, there is a sig-

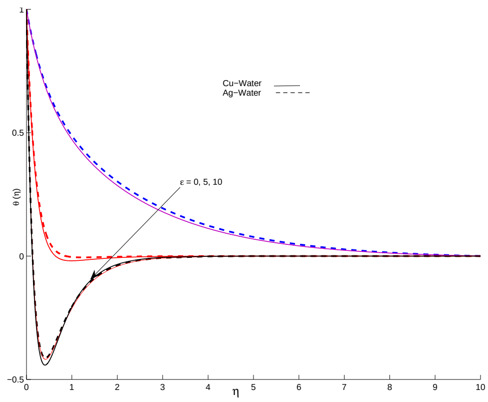

Figure 12: Effects of thermosolutal surface tension ratio ϵ on temperature profiles for $Nr = Sr = M = 2$,$Sc = 0.6$,$\phi = K^* = Ec = 0.2$

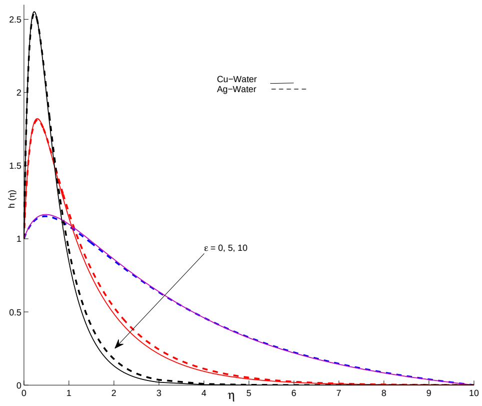

Figure 13: Effects of thermosolutal surface tension ratio ϵ on temperature profiles for $Nr = Sr = M = 2$,$Sc = 0.6$,$\phi = K^* = Ec = 0.2$

nificant temperature gradient exist between the two nanofluid particles. But this is nullified in the case of MHD flow. From Fig.16, it is noticed that mass transfer rate decreases with the increase of viscous dissipation. From Fig.17 and Fig.18 respectively, one can observe that Nusselt number decreases and Sherwood number increases with the increase in radiation parameter. Fig.19 explains about the nature of mass transfer against Soret number for different magnetic numbers. Mass transfer rate decreases with increase in the Soret number. Also it is observed that magnetic number is playing a vital role over the change of Sherwood number. It is noticed that near the wall, where the friction is more, increase in magnetic number reduces the transfer rate and away from the wall this effect is reversed. Also it is noted from the Table2 that skin friction increases with increase in the value of thermosolutal surface tension ratio and decreases with the increase of volume fraction.

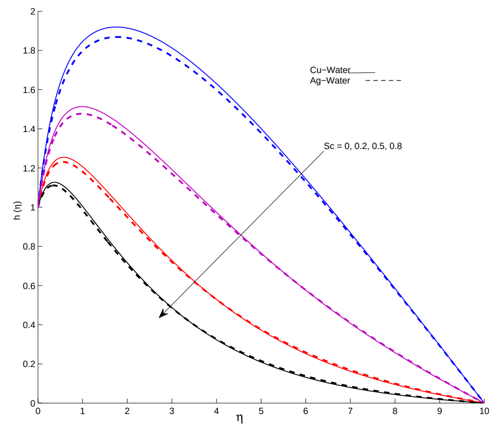

Figure 14: Effects of Schmidt number Sc on concentration profiles for $M = Nr = Sr = 2$, $\phi = K^* = Ec = 0.2$

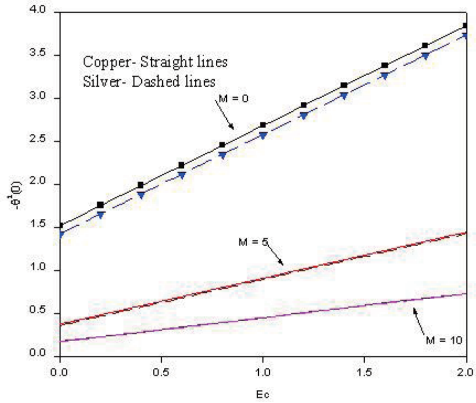

Figure 15: Effect of Eckert number Ec on Heat transfer coefficient for different Magnetic parameter values $Nr = Sr = 2$,$Sc = 0.6$,$\phi = K^* = 0.2$

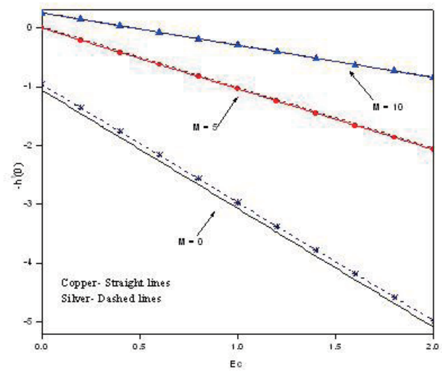

Figure 16: Effect of Eckert number Ec on Mass transfer coefficient for different Magnetic parameter values: $Nr = Sr = 2$,$Sc = 0.6$,$\phi = K^* = 0.2$

4 Conclusions

The problem of marangoni MHD convective nanofluid flow subject to radiation, chemical reaction and

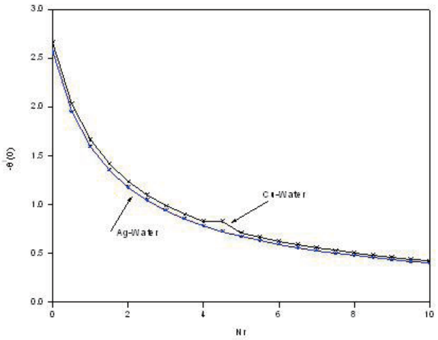

Figure 17: Effect of radiation parameter on Heat transfer coefficient for $Sr = 2$,$Sc = 0.6$,$\phi = K^* = Ec = 0.2$

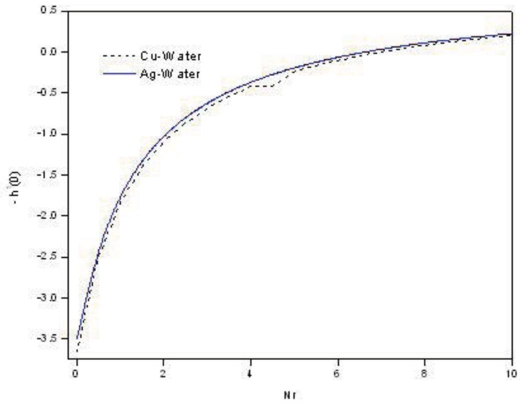

Figure 18: Effect of radiation parameter on Mass transfer coefficient for $Sr = 2$,$Sc = 0.6$,$\phi = K^* = Ec = 0.2$

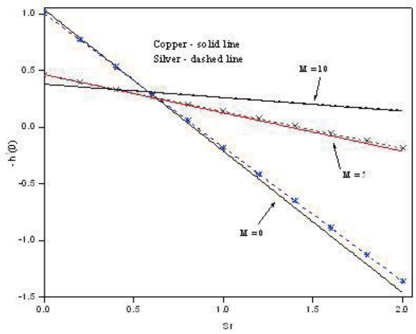

Figure 19: Effect of Soret number on Heat transfer coefficient for $Nr = 2$,$Sc = 0.6$,$\phi = Ec = K^* = 0.2$

Soret effects past a flat porous plate is considered. Two types of nanofluids, namely Cu-water and Ag-

Table 1: Thermo physical properties of nano particles.

Physical property	Pure Water	Cu	Ag
$\rho(kg/m^3)$	997.1	8933	10500
$C_p(J/kgK)$	4179	385	235
$k(W/mK)$	0.613	401	429

Table 2: Skin friction against thermosolutal surface tension ratio

ϵ	Skin friction $f''(0)$	ϕ	Skin friction $f''(0)$
0	1.144867	0	2.4
0.1	1.259353	0.1	1.84424
0.2	1.37384	0.2	1.37384
0.3	1.488327	0.3	0.983912
0.4	1.602814	0.4	0.669252
0.5	1.7173	0.5	0.424264
0.6	1.831787	0.6	0.242863
0.7	1.946274	0.7	0.118308
0.8	2.06076	0.8	0.042933
0.9	2.175247	0.9	0.007589

Thermo-solutal surface tension ratio ϵ, Volume fraction ϕ

water are considered. Governing equations of motion are solved numerically using MATLAB ”bvp4c” routine. Results for the velocity, temperature, concentration, Nusselt number and Sherwood number for selected values of the governing parameters are showed graphically. A good agreement is found with the previous works that are presented in the literature. It is observed that Cu-water nanofluid exhibits higher wall heat and mass transfer rates as compared to Ag-water nanofluid. It is observed that Eckert number increases the heat transfer rate and decreases the mass transfer rate. Also it is observed that increase in Soret number results in decrease the mass transfer rate. When the solid particles increase in the fluid, it leads to the decrease in the skin friction.

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