The complexity analysis of dynamic output game with risk-averse consideration

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Abstract: Risk aversion is prevalent behaviors of decision makers when the external environment is uncertain. In this paper, the dynamic output game strategies in two-tier supply chains under uncertain demand and competitive environment are discussed, where the supply chains include a risk-averse manufacturer and customers respectively. Two game models with a mean-variance framework are developed under decentralized and team game respectively. The complex dynamics characteristics and influences of parameters on the dynamics behaviors of two game strategies are investigated using parameter basin plots, bifurcation diagrams and the largest Lyapunov exponent et al. The results show that game strategy will affect the stability of the system, and risk reference has little effect on stability of the system while the weight parameter of the products and uncertain demand have. The instability of the system causes an unfavorable outcome and the parameter adjustment mechanism can be applied by manufacturers to eliminate the chaotic behaviors.

Key–Words: Risk-averse behavior; Team decision; Parameter basin; Mean-variance method

1 Introduction

At present, market environment is complex, manufacturers produce alternative varieties with small batch to cater for the diversity of consumers' taste. However, the manufacturers have incomplete and uncertain information about customer's demand and competitors. It is especially important to study the system stability in which manufacturers sell their alternative varieties in the uncertain environment.

Uncertain factors make manufacturers difficult to make decisions, and many scholars have studied the effect of uncertain factors on decision making. Li et al [1] considered a supply chain in which the distributor faces a known demand and orders from the producer subject to random production yield. By giving priority to the logistics service integrator, Liu et al [2] established a Stackelberg game model and investigated the fairest revenue-sharing coefficient when the logistics service integrator and the functional logistics service provider implement revenue-sharing contract under stochastic demand condition. Li et al [3] studied an incentive model for a firm who consigns the used product collection to a collector, while the firm only has incomplete information on the collector's cost. Li et al [4] explored a generalized supply chain model subject to supply uncertainty and designed coordination contracts to coordinate the supply chain with random demand. However, the above papers considered the uncertain factor from different aspects but did not investigated the effects of risk-averse behaviors which caused by the uncertain factor on the stability and the competition of supply chains.

Output decision is an important issue and its stability has been studied. Al-Nowaihi and Levine [5] examined the stability properties of the Cournot oligopoly model for the continuous adjustment process. Furth [6] studied the stability and instability in an oligopoly market. Under the circumstance of information asymmetry, Wang and Ma [7] proposed an output game model among multiple oligopolistic manufacturers and discussed the impact of system parameter on the model complexity from a perspective of complex dynamics. Gian Italo and Fabio [8] developed a dynamic model considering minimum and maximum production constraints to explore the effects on the system dynamic. Wang and Ma [9] considered a Cournot-Bertrand mixed duopoly game model, and studied the influences of the parameters on the system performance from the perspective of economics. Cnovas and Guillermo [10] explored a restricted Cournot-Puu triopoly and studied its dynamics, found conditions for removing the other firm from the market and studied the complexity of the map by means of sample permutation entropy. Ding et al [11] studied the dynamics and adaptive control of a duopoly advertising model based on heterogeneous expectations, and gave the scope of the convergent condition and control intensity. Ma and Yang [12] established a decentralized pricing game model and studied its complex dynamic characteristics of triopoly under different decision-making rule. Liu et al [13] and Liu et al [14] proposed an order allocation optimization model for logistics services integrator under rational expectations and rational pre-estimate.

In recent years, cooperation and multi-team games had become popular. Ding et al [15] studied a dynamical system of a two-team Cournot game played by a team consisting of two firms with bounded rationality and a team consisting of one firm with naive expectation. Ahmed et al [16] formulated a multi-team Bertrand game which based on Puu's incomplete information, and studied on quantum team games respectively. Elettreby and Mansour [17] studied and modified an incomplete information dynamical system and applied it to the standard multi-team dynamic Cournot game. Asker [18] mainly constructed a dynamical multi-team Cournot game model in which the enterprises exploit a renewable resource, and analyzed the asymptotic stability of the equilibrium solution of the model. Ding et al [19] studied the dynamics of a two-team Bertrand game with players having heterogeneous expectations. Liu and Simaan [20] studied the static multi-team games.

The above papers studied the output and its stability conditions of decision makers, static and dynamic multi-team under different decision rule, but they did not consider the effects of the risk preference of the decision makers on the system stability and the choice of game strategy under a competitive environment. Risk preference is the attitude and tendency of investment entities in the face of the uncertainty. The consideration of risk in decision making has gained increasing interest in supply chain studies [21-24]. Different methods have been used to study risk aversion. For example, adopting the conditional-value-at-risk (CVaR) decision criterion, Caliskan-Demirag et al.[25] analyzed the manufacturer's rebate amount decisions including the retailer's joint inventory and pricing decisions. Chiu and Choi [26] studied the classical newsvendor problem with Value-at-Risk (VaR) consideration and price-dependent demands. Eskandarzadeh and Eshghi [27] considered a sequential decision problem with risk which can be reasonably modeled by decision tree, and a new prescriptive approach was introduced for coping with risk using C-VaR.

Some other papers developed the risk aversion model in a mean-variance framework. Xu et al [28] investigated a dual-channel supply chain coordinating contract with a mean-variance framework when supply chain agents are risk aversion, and found that the price set by a risk-averse dual-channel supply chain is lower than the one set by a risk-neutral dual-channel supply chain. Li et al [29] investigated the Stackelberg equilibrium contract strategies of two competing supply chains with one risk-averse supplier and one risk-neutral retailer using mean-variance utility function.

In this paper, using mean-variance utility function and the limited rational expectations, we will study the dynamic influences of parameters on the optimal behaviors of competing supply chains under different game strategies.

In our models, for keeping the analytical model tractable, we assume that the inverse demand functions are linear with regard to self-and cross-demand sensitivities [30]. Yue et al [31] and Mukhopadhyay et al [32] also used the linear inverse demand functions for complementary products in their studies. In this paper, with a mean-variance framework and limited rational expectations, two dynamic Cournot game models using linear inverse demand functions will be constructed under decentralized and team decision respectively. By theoretical analysis and numerical simulation, the obtained results have an important theoretical and applied significance, which ca help manufacturers to formulate output strategies avoiding the market chaos and the system profit loss, and also is helpful for the government formulate relevant policies to manage the relevant resource market.

The paper is organized as follows. Section 2 presents the two dynamic game models. The existence, local stability, and bifurcation of the equilibrium points of the two models are also analyzed respectively in section 3-4. Numerical simulations are used to show the complex characteristics of the system via Lyapunov exponents, the system sensitive dependence on initial conditions and the chaotic attractor. In Section 5, the chaos control of dynamic team game model is considered with the parameters adjustment method. Finally, some conclusions are made.

2 The two dynamic Cournot game models with bounded rational expectations

In this paper, we investigate the stability and competition of the three supply chains which include a risk-averse manufacturer ($M_i, i = 1, 2, 3$)and customers respectively, the three risk-averse manufacturers serve three kinds of alternative products ($x_i, i = 1, 2, 3$), and the customer demand has uncertainty. The outputs and prices of the three products are represented as $q_i, p_i, i = 1, 2, 3$ respectively.

According to the literature [30-32], the inverse demand functions of three products are given by the following equations:

$$\begin{aligned} p_1(t) &= \omega - q_1(t) - a_1 q_2(t) - b_1 q_3(t), \\ p_2(t) &= \omega - q_2(t) - a_2 q_1(t) - b_2 q_3(t), \\ p_3(t) &= \omega - q_3(t) - a_3 q_2(t) - b_3 q_1(t), \end{aligned} \tag{1}$$

where the parameter $a_i, b_i, i = 1, 2$ denotes the degree of product differentiation. ω is potential intrinsic demand of three products, in order to capture the uncertainty demand from economic and business changing, we assume that ω is a random variable as follows $\omega = \varpi + \varepsilon$, where ϖ is the mean of the potential intrinsic demand and ε follows a normal distribution such that $E(\varepsilon) = 0, Var(\varepsilon) = \sigma^2$ which had been used extensively in the literature, the manufacturers know the distribution of the demand and organize the product's sale accordingly.

Facing the uncertain demand, the three manufacturers have different financial risk for their products. Therefore, we will consider the effects of the risk attitude of the manufacturer on output decision. The preference theory provides the framework for incorporating the manufacturer' financial risk propensity into their decision process, the valuation measurement we use is known as the certainty equivalent in the preference theory as the certainty equivalent, which is defined as certain value for an uncertain event which the manufacturer is just willing to accept.

One form of the utility function in both theoretical and applied work in areas of decision theory and finance is the exponential utility function which is $U(\pi_i) = -e^{-\frac{-\pi_i}{t_i}}, i = 1, 2, 3$, where t_i is the risk tolerance levels of the three manufacturers and e is the exponential constant. π_i is the profit of the firm and follows a normal distribution, the mean is $E(\pi_i)$ and the variance is $Var(\pi_i)$. The certain equivalent of π_i is expressed by the following equation:

$$E_{U_i} = E(\pi_i) - \frac{Var(\pi_i)}{2t_i}), i = 1, 2, 3, \tag{2}$$

according to equation (2), we can obtain the expected value of each product at period t:

$$\begin{aligned} &E_{U_1}(t) = (\varpi - q_1(t) - a_1 q_2(t) - b_1 q_3(t))q_1(t) \\ &-\frac{(q_1(t))^2\sigma_1^2}{2t_1}, \\ &E_{U_2}(t) = (\varpi - q_2(t) - a_2 q_1(t) - b_2 q_3(t))q_2(t) \\ &-\frac{(q_2(t))^2\sigma_2^2}{2t_2}, \\ &E_{U_3}(t) = (\varpi - q_3(t) - a_3 q_2(t) - b_3 q_1(t))q_3(t) \\ &-\frac{(q_3(t))^2\sigma_3^2}{2t_3}. \end{aligned} \tag{3}$$

However, the manufacturers' decision is a long-term and complex process under the complex external environment. Sometimes the objective of manufacturers is the maximum profit or the higher stability, and in order to the long-term development the manufacturers may cooperate with competitors to form the team. In this paper, we will analyze and compare the influences of variables and parameters on the stability of the system under two decision strategies: decentralized decision and team decision.

2.1 Decentralized decision

In this game strategy, the three manufacturers make decisions maximize their profits respectively. In fact, restricted by the decision ability, when making the output decision, manufacturers cannot completely grasp the information of the customer' demand and other manufacturers, and show characteristics of the bounded rationality. The three manufacturers make decisions on the basis of their expected marginal profits. The optimal marginal profits of the three manufacturers can be obtained by the first-order conditions of formula (3). The result is as follows:

$$\begin{aligned} \frac{\partial E_{U_1}(t)}{\partial q_1(t)} &= \varpi - 2q_1(t) - a_1 q_2(t) - b_1 q_3(t) - \frac{q_1(t)\sigma_1^2}{t_1}, \\ \frac{\partial E_{U_2}(t)}{\partial q_2(t)} &= \varpi - 2q_2(t) - a_2 q_1(t) - b_2 q_3(t) - \frac{q_2(t)\sigma_2^2}{t_2}, \\ \frac{\partial E_{U_3}(t)}{\partial q_3(t)} &= \varpi - 2q_3(t) - a_3 q_2(t) - b_3 q_1(t) - \frac{q_3(t)\sigma_3^2}{t_3}. \end{aligned} \tag{4}$$

If the marginal profit is positive (negative), the manufacturer increases (decreases) its output in the next period. Supposing the manufacturer makes decisions of period $t + 1$ based on the variables of period t, the dynamical output game model with limited rational expectations can be described as follows:

$$\begin{aligned} q_1(t+1) &= q_1(t) + k_1 q_1(t)\frac{\partial E_{U_1}(t)}{\partial q_1(t)}, \\ q_2(t+1) &= q_2(t) + k_2 q_2(t)\frac{\partial E_{U_2}(t)}{\partial q_2(t)}, \\ q_3(t+1) &= q_3(t) + k_3 q_3(t)\frac{\partial E_{U_3}(t)}{\partial q_3(t)}, \end{aligned} \tag{5}$$

where $k_i, i = 1, 2, 3$ represent the manufacturer's adjustment speed and $k_i > 0$ respectively.

2.2 Team decision

Reality suggests that it is cooperation within the team and competition between the teams. The construction of multi-team games is done as follows. Let $\prod_{iX}$ be the payoff of the player $i = (1, 2, \cdots, n_X)$ in the team $X = (1, 2, \cdots, N)$ if he played alone, where n_X is the number of players in the team X and N is the number of teams. Then by using the weighted sum of objective functions method [33], the team's payoff matrix $\prod_X$ is given by: $\prod_X = \sum_i \gamma_{iX} \prod_{iX}$, $i = 1, 2, \cdots, n_X, X = 1, 2, \cdots, N$, where γ_{iX} are the weights of the players i in the team X which satisfies: $0 \le \gamma_{iX} \le 1$ and $\sum_i \gamma_{iX} = 1$. For the case of continuous games, the NNS is obtained by solving the equations: $\frac{\partial \prod_X}{\partial U_{iX}} = 0, i = 1, 2, \cdots, n_X, X = 1, 2, \cdots, N$, where u_{iX} is the control parameter of the player$i$ in the team X, which may be the quantities q_{iX} produced (in case of Cournot game) or the prices p_{iX} of their products (in the case of Bertrand game), etc.

The idea of construction team is an important contribution to the game theory, which is relevant to the real cases, and can be applied to many realistic systems (economic, biological, evolutionary systems). Considering of the idea of team, the total expected value of three manufacturers in a team at time t is as follows:

$$\begin{aligned}
&E_{U_T}(t)\\
&= \gamma[E_{U_1}(t)] + \eta[E_{U_1}(t)] + (1-\gamma-\eta)[E_{U_3}(t)]\\
&= \gamma[(\varpi - q_1(t) - a_1 q_2(t) - b_1 q_3(t))q_1(t) - \tfrac{(q_1(t))^2\sigma_1^2}{2t_1}]\\
&+ \eta[(\varpi - q_2(t) - a_2 q_1(t) - b_2 q_3(t))q_2(t) - \tfrac{(q_2(t))^2\sigma_2^2}{2t_2}]\\
&+ (1-\gamma-\eta)[(\varpi - q_3(t) - a_3 q_2(t) - b_3 q_1(t))q_3(t)\\
&- \tfrac{(q_3(t))^2\sigma_3^2}{2t_3}],
\end{aligned} \tag{6}$$

where γ and η are the weight parameter of the product in the team which satisfy: $0 \le \gamma \le 1, 0 \le \eta \le 1$ and $0 \le \gamma + \eta \le 1$, Since the game among three manufacturers is a continuous and long-term repeated dynamical process, using the standard approach of bounded rational strategy, the dynamic team game model can be generalized:

$$\begin{aligned}
q_1(t+1) &= q_1(t) + \beta_1 q_1(t)\frac{\partial E_{U_T}(t)}{\partial q_1(t)},\\
q_2(t+1) &= q_2(t) + \beta_2 q_2(t)\frac{\partial E_{U_T}(t)}{\partial q_2(t)},\\
q_3(t+1) &= q_3(t) + \beta_3 q_3(t)\frac{\partial E_{U_T}(t)}{\partial q_3(t)},
\end{aligned} \tag{7}$$

where $\beta_i, i = 1, 2, 3$ is the output adjustment speed parameter.

3 The complex dynamic characteristics of the system (5)

3.1 Equilibrium points and local stability

By Eq. (4), we can get the fixed points of the system (5) and only more interest to the Nash equilibrium point from the view of economics, $E^{D*} = (q_1^{D*}, q_2^{D*}, q_3^{D*})$, the superscripts $i*, (i = D, T)$ represents decentralized and team decision respectively.

$$\begin{aligned}
q_1(t) + \beta_1 q_1(t)\frac{\partial E_{U_T}(t)}{\partial q_1(t)} &= 0,\\
q_2(t) + \beta_2 q_2(t)\frac{\partial E_{U_T}(t)}{\partial q_2(t)} &= 0,\\
q_3(t) + \beta_3 q_3(t)\frac{\partial E_{U_T}(t)}{\partial q_3(t)} &= 0.
\end{aligned} \tag{8}$$

The local stability of equilibrium points can be determined by the nature of the eigenvalues of Jacobian matrix evaluated at the corresponding equilibrium points. To study the stability of the fixed points, the Jacobian matrix of the system (5) corresponding to the state variables (q_1, q_2, q_3) is calculated as follows:

$$J(q_1, q_2, q_3) = \begin{pmatrix} u^* & -k_1 a_1 q_1 & -k_1 b_1 q_1 \\ -k_2 a_2 q_2 & u^{**} & -k_2 b_2 q_2 \\ -k_3 b_3 q_3 & -k_3 a_3 q_3 & u^{***} \end{pmatrix}, \tag{9}$$

where

$$\begin{aligned}
&u^* = 1 + k_1 q_1(-2 - \tfrac{\sigma_1^2}{t_1})\\
&+ k_1(\varpi - 2q_1(t) - a_1 q_2(t) - b_1 q_3(t) - \tfrac{q_1(t)\sigma_1^2}{t_1}),\\
&u^{**} = 1 + k_2 q_2(t)(-2 - \tfrac{\sigma_2^2}{t_2})\\
&+ k_2(\varpi - 2q_2(t) - a_2 q_2(t) - b_2 q_3(t) - \tfrac{q_2(t)\sigma_2^2}{t_2}),\\
&u^{***} = 1 + k_3 q_3(t)(-2 - \tfrac{\sigma_3^2}{t_3})\\
&+ k_3(\varpi - 2q_3(t) - a_3 q_2(t) - b_3 q_1(t) - \tfrac{q_3(t)\sigma_3^2}{t_3}).
\end{aligned}$$

With respect to Nash equilibrium point, it is more difficult to explicitly calculate the expression of eigenvalues, but it still possible to evaluate its stability by using the Jury conditions [34]. According to the actual market situation, we get the parameter values as follows: $\varpi = 100$,$a_1 = 0.4$, $a_2 = 0.4$, $a_3 = 0.4$, $b_1 = 0.4$, $b_2 = 0.4$, $b_3 = 0.4$, $\sigma_1 = 8$, $\sigma_2 = 8$, $\sigma_3 = 8$, $t_1 = 50$, $t_2 = 60$, $t_3 = 70$. So the Nash equilibrium point of the system (5) is E^{D*}=(23.98, 25.9, 27.47), the characteristic polynomial of Jacobian matrix of E^{D*} is

$$f(\lambda) = \lambda^3 - A\lambda^2 - B\lambda - C, \tag{10}$$

where

$$\begin{aligned}
&A = 3 - 78.65k_1 - 79.42k_2 - 80.05k_3;\\
&B = -3 + 157.31k_1 + 158.84k_2 - 6147.39k_1k_2\\
&+ 160.2k_3 - 6190.75_1k_3 - 6243.74k_2k_3;\\
&C = 1 - 78.65k_1 - 79.42k_2 + 6147.39k_1k_2\\
&- 80.05k_3 + 6190.75k_1k_3 + 6243.74k_2k_3\\
&- 476952k_1k_2k_3.
\end{aligned}$$

The necessary and sufficient conditions for the locally stability of E^{D*} are as follows:

$$\begin{array}{l} f(1) = 1 - A - B - C > 0, \\ f(-1) = -1 - A + B - C < 0, \\ |(C^2 - A^2)^2 - (AC - B)^2| \\ > |(BC - A)(C^2 - AC + B - 1)|, \\ |C^2 - 1| > |AC - B|, \\ |C| < 1. \end{array} \quad (11)$$

Condition (12) gives the necessary and sufficient conditions for the stable region of E^{D*} in system (5). In stable region, whatever the initial outputs of the three manufacturers, the final outputs of the three products will stay stable at the Nash equilibrium after a limited number game. What is noticeable is that the three manufacturers may accelerate output adjustment speeds in order to maximum profit, once one of the output adjustment speeds out of stable region for whatever purpose, the stability of system at the Nash equilibrium will be broken and the bifurcations, even chaos phenomena, will appear.

Figure 1 gives the stability and instability region of the system (5) in $(k_1, k_{2,3} k)$ and k_1, k_3 planes, we can see that: (a) when $k_1 < 0.024, k_2 < 0.0238$ and $k_3 < 0.0236$, the system (5) will stable in the Nash equilibrium point, otherwise the system (5) will lose stability; (b) the larger the risk preference, the worse the stability of the system (5) is.

3.2 The effects of uncertain demand and risk preference on the stability of the system (5)

In this section, we will investigate the influences of the uncertain demand and risk preference on the system stability through 2-D parameter basin which is more powerful to describe the complexity of dynamic system [35].

Figure 2 gives the parameter basins in (k_1, k_2) and (k_1, k_3) planes for $\sigma_1 = 8, 10$ respectively, in which different colors represent different states of system (5). The red represents stable state, blue for cycles of period 2, pink for period 4, light blue for period 8, yellow for chaos. In figure 2, we can see clearly the variation trend of stable region: the increase of σ_1 only affects the system behavior in the direction of k_1 and not in the direction of k_2 and k_3, that is, the behaviors of M_1 will be affected while the behaviors of M_2 and M_3 will not with the change of variance of x_1. Figure 3 gives the parameter basins for $t_1 = 80, 150$ respectively. We can see that the stability of system (5) changes little with the change of risk preference. So, the risk preference is insufficient to arouse chaos. The yellow regions in period 2, period 4 and period 8 in figures 2 and 3 do not represent the system (5) in a chaos state, which can be proved by the 1-D bifurcation diagrams in next section.

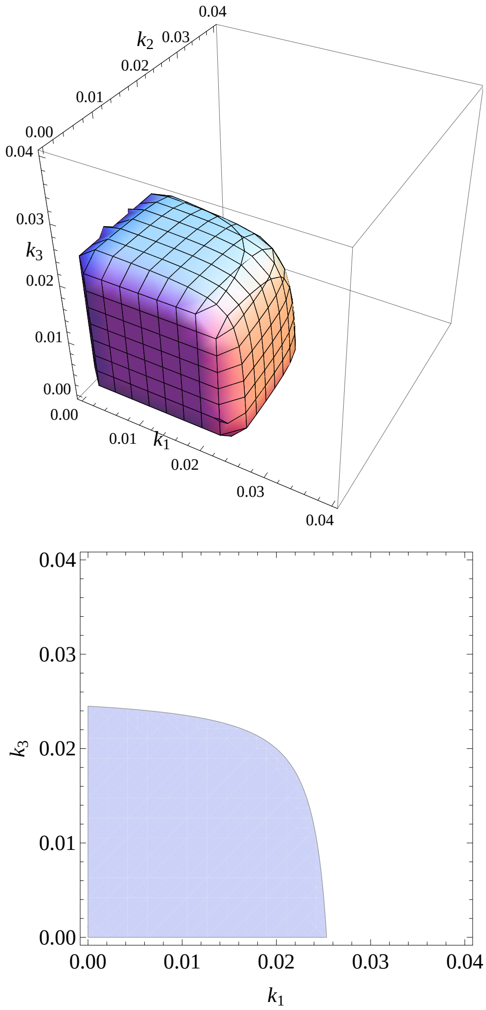

Figure 1: The 3D and 2D stable region of the Nash equilibrium point of the system (5)

3.3 The effects of output adjustment parameter on the system (5)

In figure 2(a), when the adjustment parameters (k_1, k_2) pass from red area through blue, pink, light blue and yellow in turn, the system (5) enters into chaos from slip bifurcation. Figure 4 shows the bifurcation and the maximum Lyapunov exponents for $k_2 = 0.02, k_3 = 0.02$. If the adjustment parameters are big enough, we can see that cycles and chaos occur which can prove the yellow regions in period 2, period 4 and period 8 in figures 2 and 3 do not represent the system (5) in chaos state. Strange attractors are shown in figure 5 with different viewing angles. Figures 6-7 show evolution processes of the system (5), which is according to the figures 2(b) and 3(a).

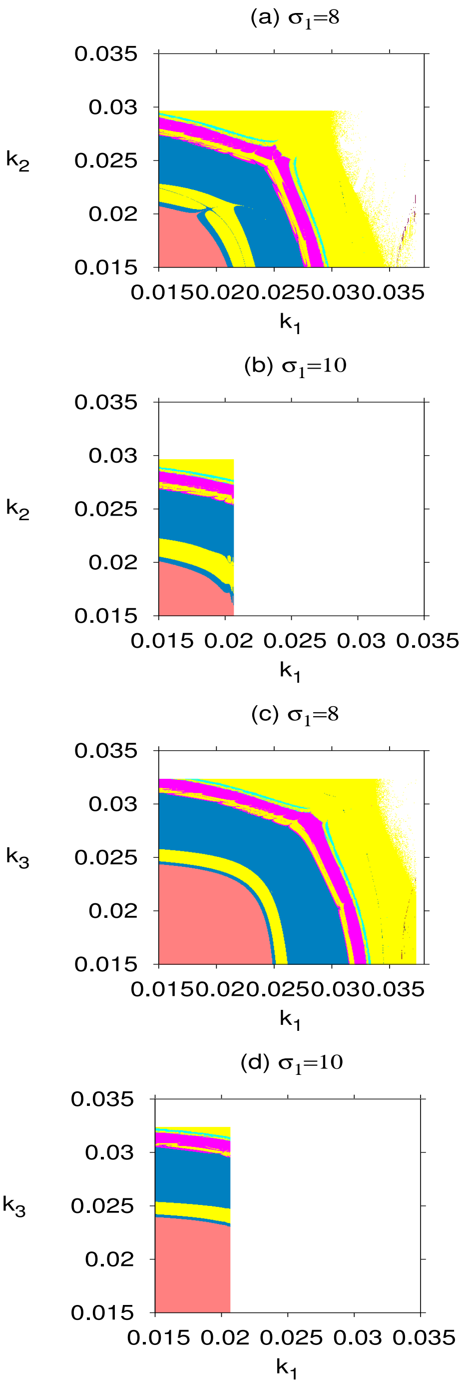

Figure 2: The parameter basin in (k_1, k_2) and (k_1, k_3) plans with the change of σ_1

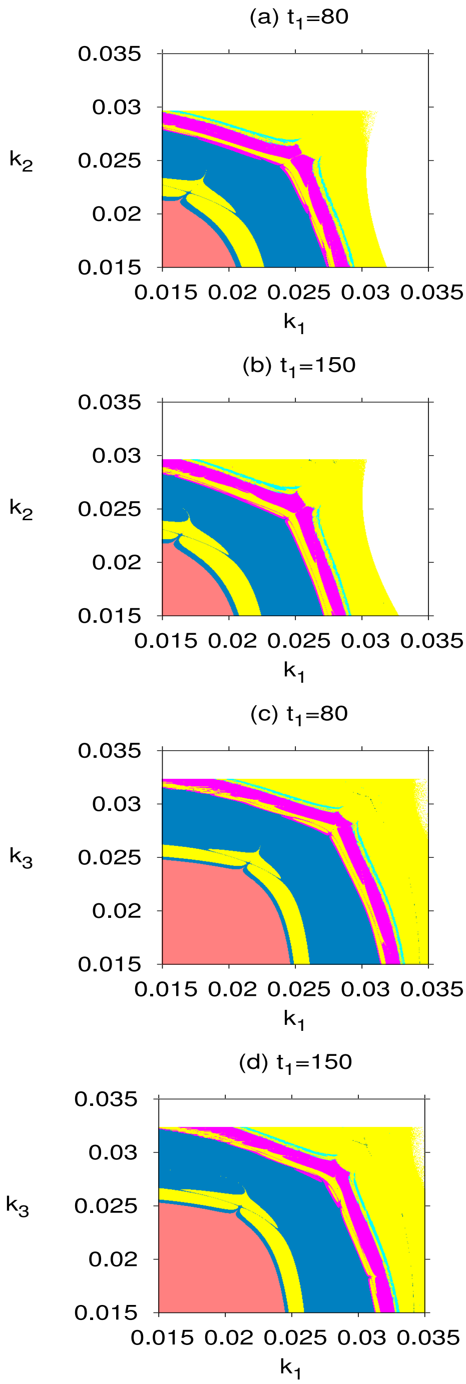

Figure 3: The parameter basin in (k_1, k_2) and (k_1, k_3) plans with the change of t_1

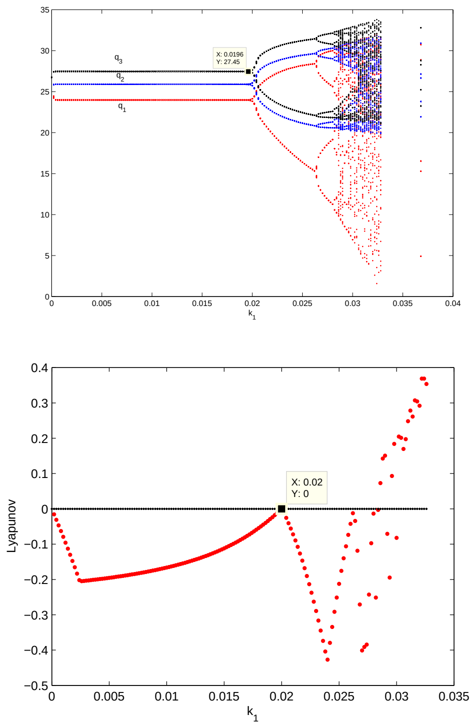

Figure 4: The bifurcation diagram and Lyapunov exponent diagram for $k_2 = 0.02, k_3 = 0.02$

The bifurcation diagram with the change of other adjustment parameters has similar characteristics that have been mentioned above. Here, no discussions are made.

From the comparison with figures 4, 6 and 7, we can obtain that the outputs of manufacturers can be affected by the change of uncertain demand and risk reference. When σ_1 increases, M_1 decreases its output while the M_2 and M_3 increase their outputs, which may reduce the occurrence of bullwhip effect of x_1 while increase occurrence of bullwhip effect of x_2, x_3. When t_1 increases, M_1 increases its output while the M_2 and M_3 decrease their outputs, which may increase the occurrence of bullwhip effect of x_1 while reduce occurrence of bullwhip effect of x_2 and x_3. So the three manufacturers should control demand uncertainty and risk reference so as to make the system in stable state and reduce the occurrence of bullwhip effect as much as possible.

4 The complex dynamic character of the system (7)

4.1 Equilibrium points and local stability

In the same way, we can calculate the only Nash equilibrium point of the system (7) and simulate the stable

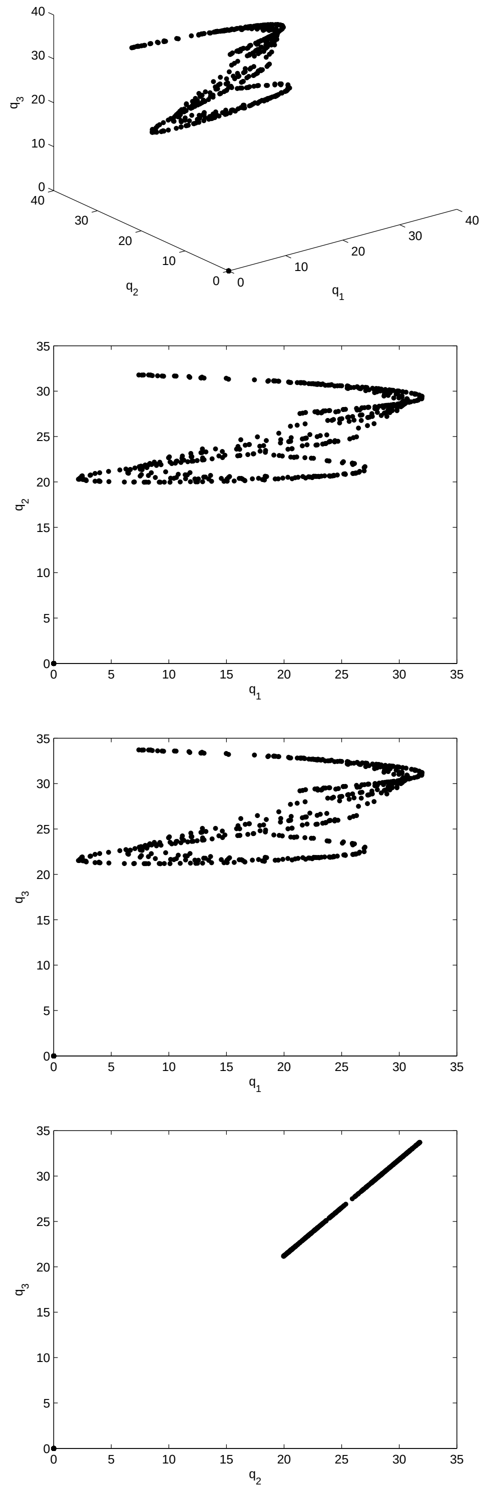

Figure 5: The phase plots of system (5) for $k_1 = 0.032$, $k_2 = 0.02$ and $k_3 = 0.02$

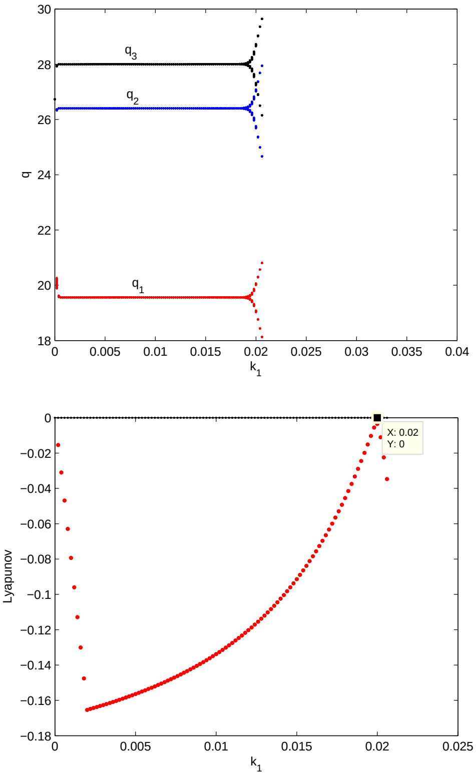

Figure 6: The bifurcation diagram and Lyapunov diagram for $k_2 = 0.02$, $k_3 = 0.02$ and $\sigma_1 = 10$

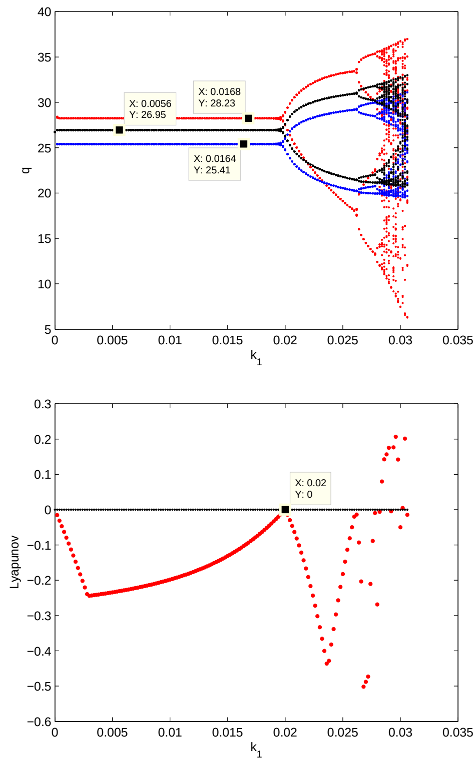

Figure 7: The bifurcation diagram and Lyapunov diagram for $k_2 = 0.02$, $k_3 = 0.02$ and $t_1 = 80$

region of the system (7) in the Nash equilibrium point which is shown in figures 8 and 9. Comparing figures 8 and 9 with figure 1, we can see the stability of the system (7) is better than the one of the system (5). That means more competition among the three manufacturers in team decision.

We can see that when the weight factor of product increases, the stable region of its output adjustment will decrease, others will increase. So the three manufacturers should adjust the weight factor of products to make the system (7) stable as soon as possible according to the parameters' values.

In team decision, with the change of demand variance and risk preference, the evolution characteristics of the system (7) is similar with the one of the system (5) that have been mentioned above. Here, no discussions are made. We will analyze the influence of weight parameter and adjustment parameter on the stability of the system (7) in the next section.

4.2 The influence of weight parameter on the stability of the system (7)

For the research of weight factor of product on the system stability, the figures 10-12 give 2D parameter attract basin in (γ, η) planes with different adjustment parameters. We can find some new results:

(1) With the increase of adjustment parameter, the red (stable region), the blue (period-2 region) and the total evolution region of the system (7) become smaller, which means the manufacturer should choose the product weight more cautious to avoid chaotic risk and maximize the its profit.

(2) The product weight that the manufacturer may choose becomes smaller with the increase of adjustment parameter. That means the manufacturer is easy to be out of the market if the manufacturer adjusts its output faster. For stronger competition of economies, the adjustment parameter the manufacturer chooses is of great influence on the stability of the system (7).

(3) The yellow regions in period 1, period 2, period 4 and period 8 in figures 10-12 do not represent the system (7) is in a chaos state, which can be proved in next section.

4.3 The effects of weight parameter of the product on the system (5)

In figures 10-12, when three adjustment parameter values determined, the behaviors of the system (7) first pass from yellow area through light blue, pink, blue, and red in turn, after that go in the opposite direction. Figure 13 shows the output bifurcation and the corresponding Lyapunov exponent diagram which

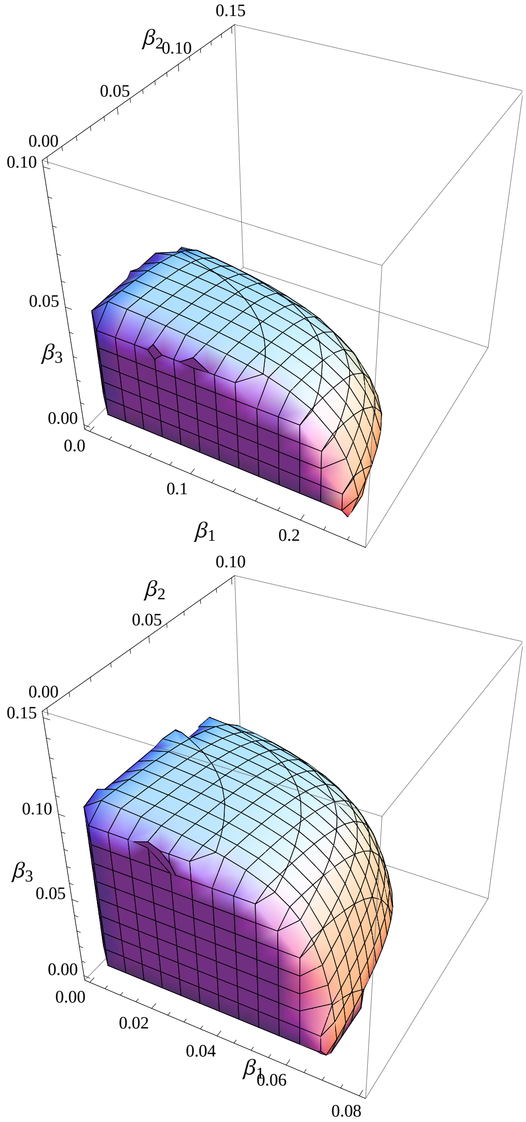

Figure 8: The 3D stable region of the system (7), left($\gamma = 0.2, \eta = 0.3$), right ($\gamma = 0.4, \eta = 0.3$)

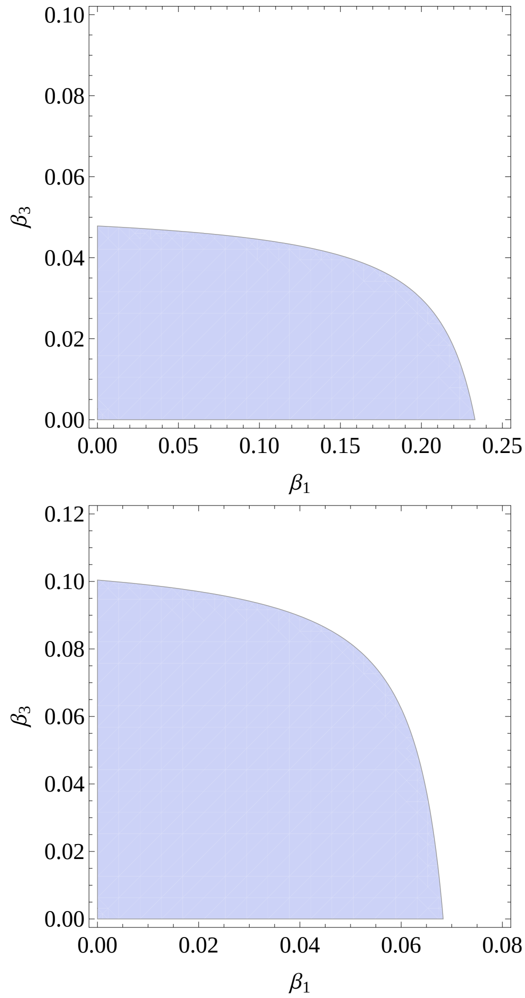

Figure 9: The stable region of the system (7) in (β_1, β_3) plane, left ($\gamma = 0.2, \eta = 0.3, \beta_2 = 0.04$), right($\gamma = 0.4, \eta = 0.3, \beta_2 = 0.04$)

is agreement with evolution process of the system (7) in the figure 11 for $\eta = 0.3$, $\beta_1 = 0.07, \beta_2 = 0.04, \beta_3 = 0.04$. We can see that when $0.145 \leq \gamma \leq 0.34$, the outputs of the three products is in stable s-tate. Figure 14 shows the output bifurcation and the corresponding Lyapunov exponent diagram which is agreement with evolution process of the system (7) in the figure 12 for $\gamma = 0.3, \beta_1 = 0.04, \beta_2 = 0.07, \beta_3 = 0.04$. We can see that when $0.25 \leq \eta \leq 0.35$, the outputs of the three products is stable. Figures 13 and 14 confirm the yellow regions in period 1, period 2, period 4 and period 8 in figures 10-12 do not represent the system (7) in chaos state.

From the analysis above, we can see that the weight parameter γ, η not only affect the stability of the system (7), but also changes the value of the Nash equilibrium of the system (7). When the manufacturer determines its weight value, the other manufacturers must choose weight values in a certain range so as to make the system (7) stable. When determined the value of adjustment speed, the three manufacturers must allocate appropriate weight values for the three products in order to obtain the maximum profit.

5 Chaos control

Our study find that once the behaviors of manufacturers is in chaos, the total profit of the manufacturers is less than the one in the equilibrium state. So the chaos state is not expected to appear. However, the current situation is that the manufacturers often maximize their profits by any kind of means in the process of marketization considering. So the market will be out of order and finally fall in-

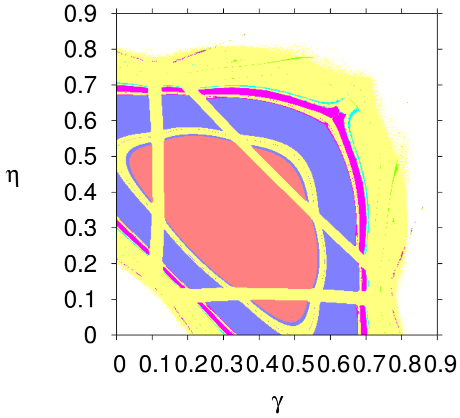

Figure 10: The parameter basin for $\beta_1 = 0.04, \beta_2 = 0.04, \beta_3 = 0.04$

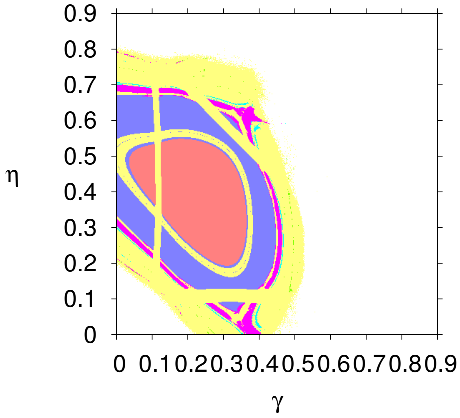

Figure 11: The parameter basin for $\beta_1 = 0.07, \beta_2 = 0.04, \beta_3 = 0.04$

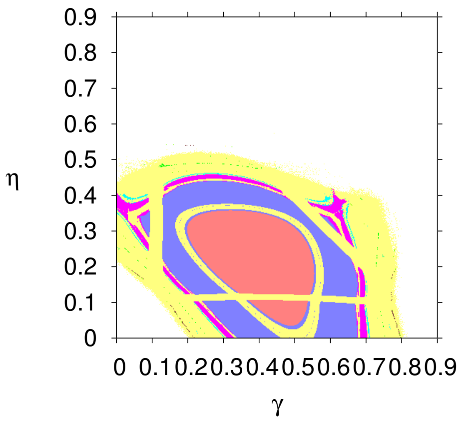

Figure 12: The parameter basin for $\beta_1 = 0.04, \beta_2 = 0.07, \beta_3 = 0.04$

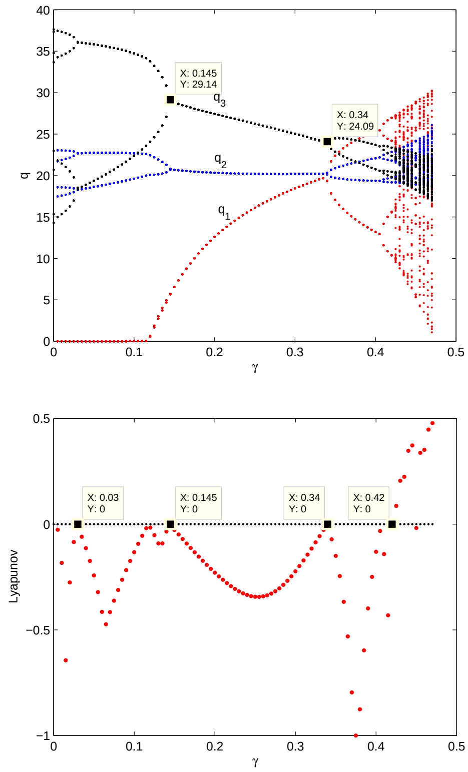

Figure 13: The bifurcation diagram of output and the corresponding Lyapunov exponent for $\eta = 0.3, \beta_1 = 0.07, \beta_2 = 0.04, \beta_3 = 0.04$

to chaos. It is particularly important for manufacturers that some control measures should be adopted in a timely manner, in order to make the system return to the stable equilibrium. Therefore, we use the parameter adjustment method to control the effect of parameter on the system (7). Assume the system (7) is $q_i(t+1) = f_i(q_1(t), q_3(t), q_3(t)), i = 1, 2, 3$, the model under control is as follows:

$$\begin{aligned} q_1(t+m) &= (1-\mu) f_1^m(q_1(t), q_3(t), q_3(t)) + \mu q_1(t), \\ q_2(t+m) &= (1-\mu) f_2^m(q_1(t), q_3(t), q_3(t)) + \mu q_2(t), \\ q_3(t+m) &= (1-\mu) f_3^m(q_1(t), q_3(t), q_3(t)) + \mu q_3(t). \end{aligned} \tag{12}$$

Here, μ is an adjustment parameter, when $\mu = 0$, the controlled system (7) degrades into original the system (7), they have the same period orbits. Figure 15 shows that the chaos can be delayed and even eliminated with the proper μ. Figure 16 shows that with the control parameter μ increasing, the controlled system (12) is gradually controlled from the chaotic state, 4-period bifurcation, 2-period bifurcation to stable state. When $\mu > 0.28$, the controlled system stabilizes at the Nash equilibrium point. In a real market, when manufacturers pursuit its maximum profit, we can consider as the output adjustment regulation for

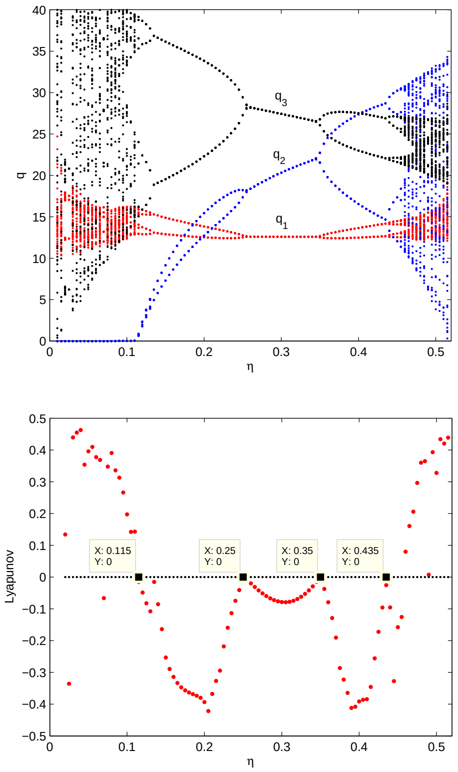

Figure 14: The bifurcation diagram of output and the corresponding Lyapunov exponent for $\gamma = 0.3$, $\beta_1 = 0.04, \beta_2 = 0.07, \beta_3 = 0.04$

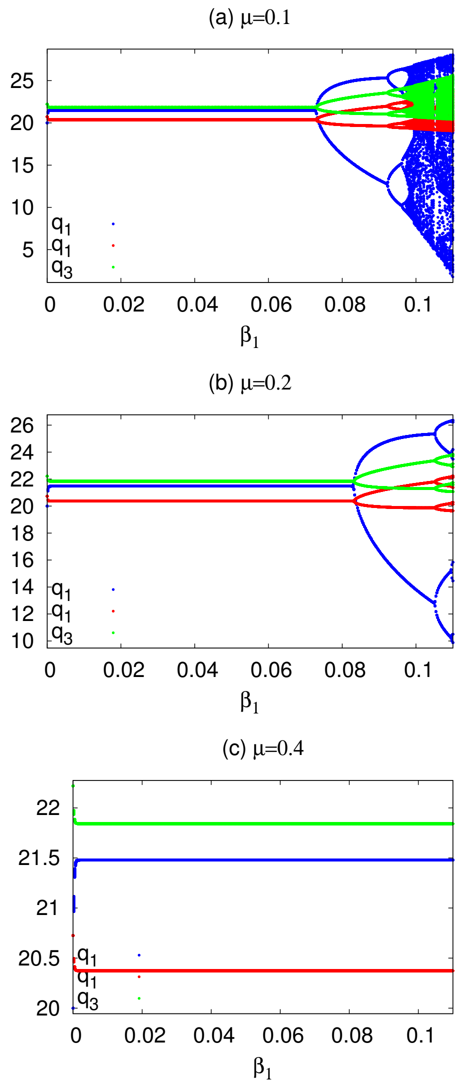

Figure 15: The output bifurcation for $\beta_2 = 0.07, \beta_3 = 0.04$

the manufacturer to avoid market chaos. We can also consider μ as the learning ability or adaptability of the market. Simulation results show that the bigger the μ, the larger the stability area of the system is, the faster the speed of reaching the equilibrium point is. So using the parameter μ, the system is under control, chaos is delayed or eliminated completely.

6 Conclusions

In this paper, we propose dynamic game models of the supply chains in decentralized and team decision which include a risk-averse manufacturer and customers respectively, where the customer demand for each product is uncertainty. The stability of the two dynamic game models were investigated using parameter basin, bifurcation diagram, and attractors with different parameter situation. The simulation shows that: (1) The risk preference has little effect on the stability of the system but affected the outputs of the three manufacturers, uncertain demand of the product can affect the behaviors of its manufacturer, but has not effects on the others, the influence on the output will expand or shrink the products' outputs which maybe cause the occurrence of the bullwhip effect; (2) The stability under team decision is better than the one of decentralized decision when the weight parameters make a certain value, the weight parameters also affect the stable region of the system and the Nash equilibrium value; (3) The manufacturers can control or delay the occurrence of chaos using the parameters adjustment method; (4) The predict method can be used to accurately forecast the customer demand so as to reduce the influence of uncertain demand on the complex behaviors of the manufacturers.

In this paper, there exist the yellow region in 2-D parameter basins in 2-period, 4-period, 8-period, why the chaos area appears will be the focus of our future research.

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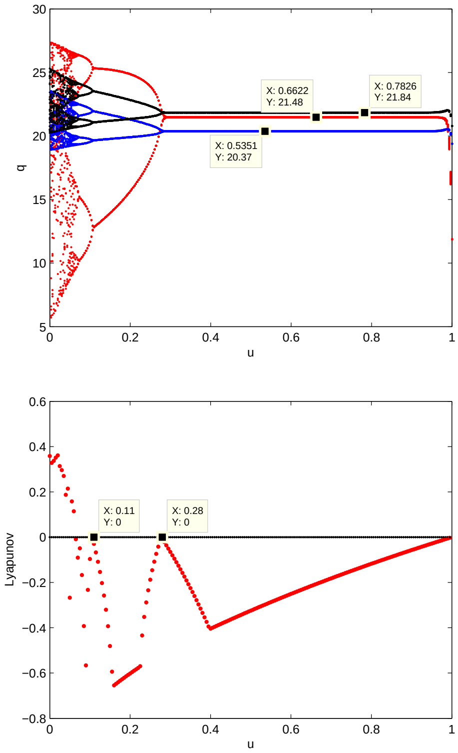

Figure 16: The output bifurcation for $\beta_1 = 0.093, \beta_2 = 0.04, \beta_3 = 0.04$

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