

# Simple Equation Method for Nonlinear Evolution Equations in Mathematical Physics

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**Abstract:** In this article, we use a direct method namely simple equation method to construct the traveling wave solutions for some nonlinear evolution equations in mathematical physics via, the nonlinear Schrodinger equation with dual power law nonlinearity, the nonlinear variant Boussinesq differential equations, the nonlinear generalized- Zakharov equations and the nonlinear coupled Maccaris equations. In the simple equation method, we suppose the trial equation satisfies the first order Bernoulli differential equation or the first order Riccati differential equation. It has been shown that this method provides a powerful mathematical tool for solving nonlinear wave equations in mathematical physics and engineering problems.

**Key-Words:** Simple equation method, Nonlinear evaluation equations, Traveling wave solutions, Exact solutions.

## 1 Introduction

Nonlinear partial differential equations have become a rather vast subject with a long history of deep and fruitful connections with many other areas of mathematics and various sciences like physics, mechanics, chemistry, and engineering[1]. There are many methods for finding analytic solutions for nonlinear partial differential equations. Among these methods are the sine-cosine method [2], the similarity reductions method [3,4], the inverse scattering transform method [5], various tanh method [6-9], the Backlund transformation [10], the homogenous balance method [11], the exp- function method [12], the variable separation approach [13], ( $G'/G$ )-expansion method [14,15], the Darboux transformation [16], the sub-ODE method [17], the Hirota bilinear method[18], and the Jacobi elliptic function expansion method [19].

In this paper, we deduce the traveling wave solutions for some nonlinear evolution in mathematical physics namely, the nonlinear Schrodinger equation with dual power law nonlinearity, the nonlinear variant Boussinesq differential equations, the nonlinear generalized-Zakharov equations and the nonlinear coupled Maccaris equations by using the simple equation method. The simple equation method is a very powerful mathematical technique for finding exact solutions to the nonlinear evolution equations. It has been developed by Kadreyshov [20,21] and used successfully by many authors to obtain the exact solution for nonlinear evolution equations in mathematical physics [22,23].

## 2 Algorithm of the simple equation method

To describe our solution process, let us focus on a scalar (1+1)- dimensional nonlinear partial equation:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0, \quad (1)$$

where  $u(x, t)$  is an unknown function,  $P$  is a polynomial of  $u(x, t)$  and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of the simple equation method:-

**Step 1.** We suppose the traveling wave transformation

$$u(x, t) = u(\xi), \quad \xi = x - ct, \quad (2)$$

where  $c$  is an arbitrary constant. The traveling wave transformation Eq. (2) permits us to reduce Eq.(1) to the following ordinary differential equation (ODE):

$$Q(u, u', u'', \dots) = 0, \quad (3)$$

where  $Q$  is a polynomial of  $u(\xi)$  and its derivatives, with respect to  $\xi$ .

**Step 2.** We seek the solution of Eq.(3) in the following form:

$$u(\xi) = \sum_{i=0}^N a_i F^i(\xi). \quad (4)$$

where  $a_i (i = 0, 1, 2, \dots, N)$  are arbitrary constants to be determined later and  $F(\xi)$  satisfies the trial simple equation. The simple equation has two properties, the first one it is of lesser order than Eq.(3) and the second, its general solution is known.

In this paper, the simple equations we shall use are the Bernoulli and Riccati differential equations which are well known nonlinear ordinary differential equations, and their solutions can be expressed by elementary functions.

**Step 3.** The balance number  $N$  can be determined by balancing the highest order derivative and nonlinear terms in Eq. (3).

**Step 4.** We discuss the general solutions of the simple equation. For the Bernoulli equation

$$F'(\xi) = rF(\xi) + dF^2(\xi). \tag{5}$$

where  $r$  and  $d$  are arbitrary constants.

The general solution of Eq.(5) takes the following form:

$$F(\xi) = \frac{r \exp[r(\xi + \xi_0)]}{1 - d \exp[r(\xi + \xi_0)]}, d < 0, r > 0, \tag{6}$$

where  $\xi_0$  is the integration constant and

$$F(\xi) = -\frac{r \exp[r(\xi + \xi_0)]}{1 + d \exp[r(\xi + \xi_0)]}, d > 0, r < 0. \tag{7}$$

For the Riccati equation

$$F'(\xi) = \alpha F^2(\xi) + \beta. \tag{8}$$

The general solutions of Eq.(8) take the following form [23]:

(i)When  $\alpha\beta < 0$  and  $\xi$  is a positive number

$$F(\xi) = -\frac{\sqrt{-\alpha\beta}}{\alpha} \tanh(\sqrt{-\alpha\beta}\xi - \frac{\nu \ln(\xi_0)}{2}). \tag{9}$$

(ii)When  $\alpha\beta > 0$  and  $\xi$  is a real number

$$F(\xi) = \frac{\sqrt{\alpha\beta}}{\alpha} \tan(\sqrt{\alpha\beta}(\xi + \xi_0)). \tag{10}$$

**Remark 1.**

(i) When  $r = \delta$  and  $d = -1$ , Eq. (5) has another form of the Bernoulli equation

$$F'(\xi) = \delta F(\xi) - F^2(\xi), \tag{11}$$

which has the exact solutions

$$F(\xi) = \frac{\delta}{2} \left[ 1 + \tanh\left(\frac{\delta}{2}(\xi + \xi_0)\right) \right], \tag{12}$$

when  $\delta > 0$ , and

$$F(\xi) = \frac{\delta}{2} \left[ 1 - \tanh\left(\frac{\delta}{2}(\xi + \xi_0)\right) \right] \tag{13}$$

when  $\delta < 0$ .

(ii) When  $r = 1$  and  $d = -1$ , Eq. (5) has another form of the first order Riccati equation [20,21]

$$F'(\xi) - F(\xi) + F^2(\xi) = 0, \tag{14}$$

which has the logistic function as its exact solution

$$F(\xi) = \frac{1}{1 + e^{-\delta}}. \tag{15}$$

The logistic equation (15) can be presented in the hyperbolic tangent function according to the relation

$$\frac{1}{1 + e^{-\xi}} = \frac{1}{2} \left[ 1 + \tanh\left(\frac{\xi}{2}\right) \right]. \tag{16}$$

### 3 Applications of the simple equation method for NPDEs

In this section, we use the simple equation method to find the traveling wave solutions for nonlinear evolution equations in mathematical physics namely, the nonlinear Schrodinger equation with dual power law nonlinearity, the nonlinear variant Boussinesq differential equations, the nonlinear generalized- Zakharov equations and the nonlinear coupled Maccaris equations which have attracted a great deal of interest in mathematical physics and engineering research.

#### 3.1 Nonlinear Schrodinger equation with dual power law nonlinearity

In this section, we use the proposed method to study the following nonlinear Schrodinger equation with dual power law

$$i\psi_t + a\psi_{xx} + (b|\psi|^{2n} + h|\psi|^{4n})\psi + q \left\{ \frac{(|\psi|)_{xx}}{|\psi|} \right\} \psi = 0. \tag{17}$$

where  $a, b, h$  and  $q$  are arbitrary constants. The Schrodinger equation is a partial differential equation that describes how the quantum state of a quantum system changes with time. In 1926, Erwin Schrodinger found a new equation which is called the time independent Schrodinger equation. This equation has sufficiently illuminated the atomic phenomena and became the dynamical centerpiece of quantum wave mechanics [24]. This equation has been indicated to manage the evolution of a wave packet in

a weakly nonlinear and dispersive medium. Another implementation of this equation is in pattern formation, where it has been used to model some non equilibrium pattern forming systems. It is now widely used in the optics field as a good model for optical pulse propagation in nonlinear fibers[25]. Using the traveling wave transformation:-

$$\psi(x, t) = U(\xi)e^{i(-kx+\omega t+\theta)}, \xi = x + 2kat, \quad (18)$$

where  $k, \omega$  and  $\theta$  are arbitrary constants to be determined later. The traveling wave transformation (18) permits us to convert Eq. (17) to the following ordinary differential equation:

$$(a + q)U'' - (\omega + ak^2)U + bU^{2n+1} + hU^{4n+1} = 0. \quad (19)$$

We suppose the solution of Eq. (19) has the form Eq. (4). Balancing the highest order derivative  $U''$  with the nonlinear term  $U^{4n+1}$  we have,  $N = \frac{1}{2n}$ . Consequently, we take the transformation:-

$$U(\xi) = [\phi(\xi)]^{\frac{1}{2n}}, \quad (20)$$

where  $\phi$  is an arbitrary function of  $\xi$ . Equation (19) leads to write Eq. (20) in the following form:

$$(a + q) \left[ \frac{1}{2n} \left( \frac{1}{2n} - 1 \right) \phi^{-1} (\phi')^2 + \frac{1}{2n} \phi'' \right] - (\omega + ak^2)\phi + b\phi^2 + h\phi^3 = 0. \quad (21)$$

Balancing the highest order derivative terms  $\phi''$  with the nonlinear terms  $\phi^3$  in Eq.(21). Therefore the solution of Eq.(21) can be expressed as follows:

$$\phi(\xi) = a_0 + a_1 F(\xi), \quad (22)$$

where  $a_0$  and  $a_1$  are arbitrary constants to be determined later. Substituting Eq.(22) along with the trial simple equation (5) into Eq.(21) and cleaning the denominator, collecting all terms with the same order of  $F(\xi)$  together, the left hand side of Eq.(21) are converted into polynomial in  $F(\xi)$ . Setting each coefficient of these polynomials to be zero, we derive a set of algebraic equations for  $a_0, a_1, a, b, h, q, \omega$  and  $k$ . Solving this set of over-determined algebraic equations by using Maple software package, we get the following results:

$$\begin{aligned} a_0 &= \pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn}, \\ a_1 &= \pm \frac{\sqrt{-h(q + 2q + a + 2an)}d}{2hn}, \\ b &= \pm \frac{hr(q + 2q + a + 2an)}{\sqrt{-h(q + 2q + a + 2an)}n}, \end{aligned}$$

$$\omega = \frac{(a + q)r^2 - 4ak^2n^2}{4n^2}, \quad (23)$$

where  $a, h, d, k, q$  and  $r$  are arbitrary constants. The solution of Eq. (21) is given by

$$\begin{aligned} \phi_1(x, t) &= \pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn} \\ &\pm \frac{\sqrt{-h(q + 2q + a + 2an)}dr \exp [r\xi]}{2hn (1 - d \exp [r(x + 2kat + \xi_0)])}, \end{aligned} \quad (24)$$

when  $d < 0$  and  $r > 0$ . Substituting Eq.(24) into Eq.(20) we have:

$$\begin{aligned} U_1(x, t) &= [\pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn} \\ &\pm \frac{\sqrt{-h(q + 2q + a + 2an)}d}{2hn} \frac{r \exp [r\xi]}{(1 - d \exp [r\xi])}]^{\frac{1}{2n}} \end{aligned} \quad (25)$$

where  $\xi = x + 2kat + \xi_0$ . Therefore from Eq.(25) and Eq.(18) the solution of Eq.(17) takes the following form:

$$\begin{aligned} \psi_1(x, t) &= \exp[i(-kx + \omega t + \theta)] [\pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn} \\ &\pm \frac{\sqrt{-h(q + 2q + a + 2an)}dr \exp [r(x + 2kat + \xi_0)]}{2hn (1 - d \exp [r(x + 2kat + \xi_0)])}]^{\frac{1}{2n}}. \end{aligned} \quad (26)$$

The solution of Eq.(17) is given by:

$$\begin{aligned} \phi_2(x, t) &= \pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn} \\ &\mp \frac{\sqrt{-h(q + 2q + a + 2an)}dr \exp [r(x + 2kat + \xi_0)]}{2hn (1 + d \exp [r(x + 2kat + \xi_0)])}, \end{aligned} \quad (27)$$

when  $d > 0$  and  $r < 0$ . Substituting Eq.(27) into Eq.(20) we have:

$$\begin{aligned} U_2(x, t) &= [\pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn} \\ &\mp \frac{\sqrt{-h(q + 2q + a + 2an)}dr \exp [r(x + 2kat + \xi_0)]}{2hn (1 + d \exp [r(x + 2kat + \xi_0)])}]^{\frac{1}{2n}} \end{aligned} \quad (28)$$

Therefore from Eqs.(28) and (18) the solution of Eq.(17) takes the following form:

$$\begin{aligned} \psi_2(x, t) &= \exp[i(-kx + \omega t + \theta)] [\pm \frac{\sqrt{-h(q + 2q + a + 2an)}r}{2hn} \\ &\pm \frac{\sqrt{-h(q + 2q + a + 2an)}dr \exp [r(x + 2kat + \xi_0)]}{2hn (1 + d \exp [r(x + 2kat + \xi_0)])}]^{\frac{1}{2n}}. \end{aligned} \quad (29)$$

### 3.2 Numerical solutions for Nonlinear Schrodinger equation with dual power law nonlinearity

In this section, we present some graphs of the traveling wave solutions by taking suitable values of the parameters to visualize the mechanism of the original equation (17).

For  $h = 2, r = 1.5, \omega = 1, q = 1.25, d = -1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ . , Fig 1.a illustrates the real part of the traveling wave solution (26), Fig 1.b represents the imaginary part of the traveling wave solution , Fig 1.c represent the projection  $t = 0$ , of the real part solution (26), and Fig 1.d represents the projection  $t = 0$ , of the imaginary port solution.

Also for  $h = 2, r = -1.5, \omega = 1, q = 1.25, d = 1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ . , Fig 2.a illustrates the real part of the traveling wave solution (29), Fig 2.b represents the imaginary part of the traveling wave solution, Fig 2.c represent the projection  $t = 0$ , of the real part solution (29), and Fig 2.d represents the projection  $t = 0$ , of the imaginary port solution.

### 3.3 The variant Boussinesq nonlinear differential equations

The Boussinesq equation is a well-known model for long water waves of moderate amplitude, and describes in one dimension the weakly nonlinear internal wave which develops at the boundary between two immiscible fluids. In addition, the equation is a simplified model of the atmospheric moment equation which is applicable to mesoscale and quasi-incompressible fluids. In the present section, we focus on the variant Boussinesq equation, which was derived by Sachs [26]in 1988 as a model for water waves:

$$\begin{aligned} H_t + (Hu)_x + u_{xxx} &= 0, \\ u_t + H_x + uu_x &= 0, \end{aligned} \tag{30}$$

where  $u(x, t)$  is the velocity and  $H(x, t)$  is the height of free waves surface in the trough, and the subscripts denote partial derivatives [27]. Suppose the traveling wave transformation equation:-

$$u(x, t) = U(\xi), H(x, t) = \phi(\xi), \xi = x - ct. \tag{31}$$

The traveling wave transformation (31) permits us to reduce (30) into the following ODE:-

$$\begin{aligned} U''' + \phi'U + \phi U' - c\phi' &= 0, \\ -cU' + \phi' + UU' &= 0. \end{aligned} \tag{32}$$

By integrating the second equation of Eqs. (32) we have:

$$\phi = \frac{-U^2}{2} + cU - c_1, \tag{33}$$

where  $c_1$  is the integration constant. Substituting Eq.(33) into the first one in Eqs.(32), we get:-

$$U''' - (c_1 + c^2)U' - \frac{3}{2}U^2U' + 3cUU' = 0. \tag{34}$$

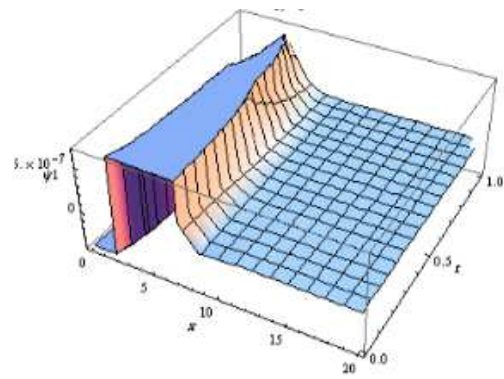


Fig 1.a The real part of the traveling wave solution (26) when the parameters take the special values  $h = 2, r = 1.5, \omega = 1, q = 1.25, d = -1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

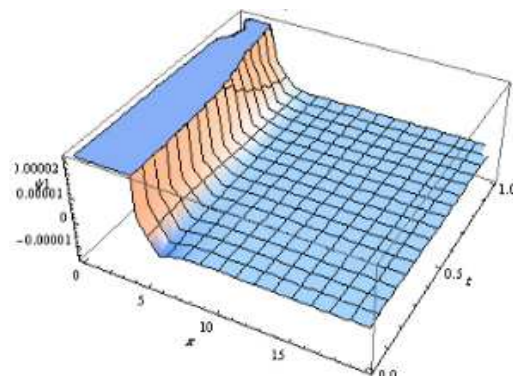


Fig 1.b The imaginary part of the traveling wave solution (26) when the parameters take the special values  $h = 2, r = 1.5, \omega = 1, q = 1.25, d = -1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

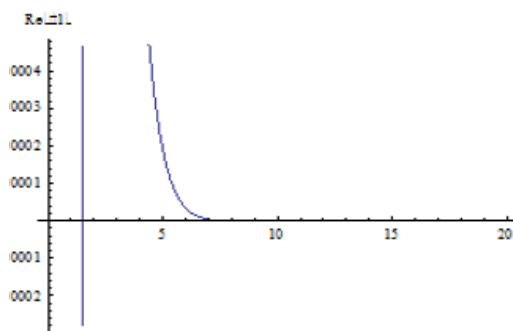


Fig 1.c The projection of real part of the solution (26) when the parameters take the special values  $h = 2, r = 1.5, \omega = 1, q = 1.25, d = -1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

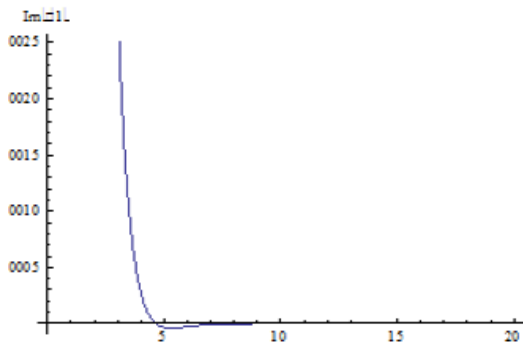


Fig 1.d The projection of imaginary part of the solution (26) when the parameters take the special values  $h = 2, r = 1.5, \omega = 1, q = 1.25, d = -1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

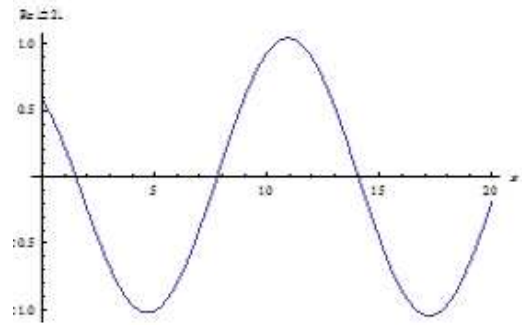


Fig 2.c The projection of the real part of the traveling wave solution (29) when the parameters take the special values  $h = 2, r = -1.5, \omega = 1, q = 1.25, d = 1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

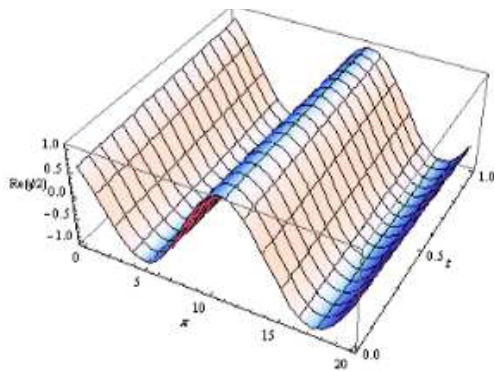


Fig 2.a The real part of the traveling wave solution (29) when the parameters take the special values  $h = 2, r = -1.5, \omega = 1, q = 1.25, d = 1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

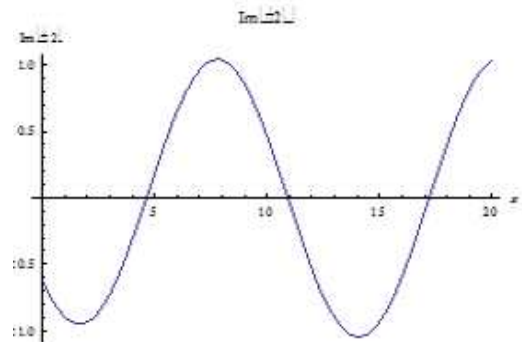


Fig 2.d The projection of the imaginary part of the traveling wave solution (29) when the parameters take the special values  $h = 2, r = -1.5, \omega = 1, q = 1.25, d = 1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

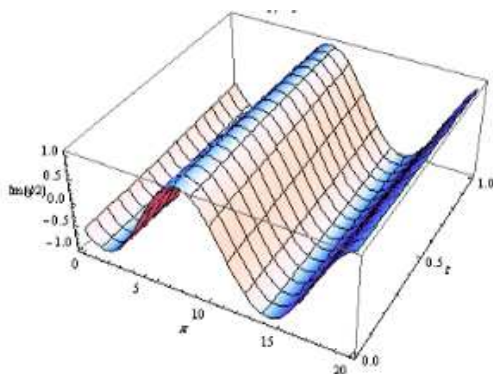


Fig 2.b The imaginary part of the traveling wave solution (29) when the parameters take the special values  $h = 2, r = -1.5, \omega = 1, q = 1.25, d = 1, k = 0.5, n = 0.5, \theta = 0.75, a = 2.5$  and  $\xi_0 = 3$ .

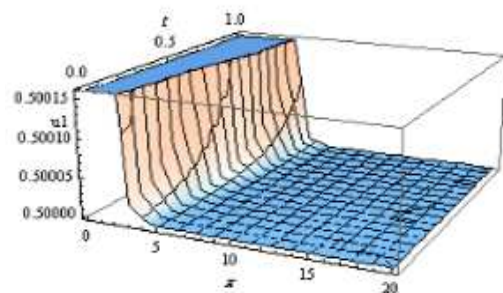


Fig 3.a The traveling wave solution (39) when the parameters take the special values  $d = -1.5, r = 2, c = 2.5$  and  $\xi_0 = 3$ .

By integrate Eq.(34), we have

$$U'' - \frac{1}{2}U^3 + \frac{3}{2}cU^2 - (c_1 + c^2)U + c_2 = 0. \quad (35)$$

Balancing the highest order derivative  $U''$  with the nonlinear term  $U^3$  in Eq. (32), the solution of Eq.(35) can be expressed as follows:

$$U(\xi) = a_0 + a_1F(\xi), \quad (36)$$

where  $a_0$  and  $a_1$  are constants to be determined later. Substituting Eq.(36) along with the simple trial equation  $F'(\xi) = rF(\xi) + dF^2(\xi)$  into Eq.(35). collecting all terms with the same order of  $F(\xi)$  together, the left hand side of Eq.(35) is converted into a polynomial in  $F(\xi)$ . Setting each coefficient of this polynomial to zero, we derive a set of algebraic equations for  $a_0, a_1, c, c_1$  and  $c_2$ . Solving the set of algebraic equations by using Maple or Mathematica software package, we get the following results:

**Case 1.**

$$a_0 = r + c, a_1 = 2d, c_1 = -\frac{1}{2}(r^2 + c^2), \quad (37)$$

$$c_2 = \frac{1}{2}(c^3 - r^2c).$$

**Case 2.**

$$a_0 = -r + c, a_1 = -2d, c_1 = \frac{1}{2}(c^2 - r^2), \quad (38)$$

$$c_2 = \frac{1}{2}(c^3 - r^2c),$$

where  $c, r$  and  $d$  are arbitrary constants.

The solution of Eqs. (30) with using case (1) is given by

$$u_1(x, t) = r + c + \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 - d \exp [r(x - ct + \xi_0)])}, \quad (39)$$

when  $d < 0$  and  $r > 0$ . Substituting Eq.(39) into Eq.(33) to get:

$$H_1(x, t) = -\frac{1}{2}\left[r + c + \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 - d \exp [r(x - ct + \xi_0)])}\right]^2$$

$$+ c\left[r + c + \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 - d \exp [r(x - ct + \xi_0)])}\right] + \frac{1}{2}(r^2 + c^2). \quad (40)$$

Also, the solution of Eqs. (30) with using case (2) is given by

$$u_2(x, t) = c - r - \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 - d \exp [r(x - ct + \xi_0)])}. \quad (41)$$

when  $d < 0$  and  $r > 0$ . Consequently substituting Eq.(41) into Eq.(33) to get:

$$H_2(x, t) = -\frac{1}{2}\left[c - r - \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 - d \exp [r(x - ct + \xi_0)])}\right]^2$$

$$+ c\left[c - r - \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 - d \exp [r(x - ct + \xi_0)])}\right] - \frac{1}{2}(c^2 - r^2). \quad (42)$$

The solution of Eqs. (30) with using case (1) is given by

$$u_3(x, t) = r + c - \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 + d \exp [r(x - ct + \xi_0)])}, \quad (43)$$

when  $d > 0$  and  $r < 0$ . Substituting Eq.(43) into Eq.(33), we have:

$$H_3(x, t) = -\frac{1}{2}\left[r + c - \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 + d \exp [r(x - ct + \xi_0)])}\right]^2$$

$$+ c\left[r + c - \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 + d \exp [r(x - ct + \xi_0)])}\right] + \frac{1}{2}(r^2 + c^2). \quad (44)$$

Also, the solution of Eqs. (30) with using case (2) is given by

$$u_4(x, t) = c - r + \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 + d \exp [r(x - ct + \xi_0)])}, \quad (45)$$

when  $d > 0$  and  $r < 0$ . Substituting Eq.(45) into Eq.(33) to get:

$$H_4(x, t) = -\frac{1}{2}\left[c - r + \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 + d \exp [r(x - ct + \xi_0)])}\right]^2$$

$$+ c\left[c - r + \frac{2dr \exp [r(x - ct + \xi_0)]}{(1 + d \exp [r(x - ct + \xi_0)])}\right] - \frac{1}{2}(c^2 - r^2). \quad (46)$$

### 3.4 Numerical solutions for the variant Boussinesq nonlinear differential equations

In this section, we present some graphs of the traveling wave solutions by taking suitable values of the parameters to visualize the mechanism of the original equation(30).

For  $d = -1.5, r = 2, c = 2.5$  and  $\xi_0 = 3$ , Fig 3.a illustrates the traveling wave solution (39), Fig 3.b represents the traveling wave solution (40), Fig 3.c represents the projection  $t = 0$ , of the traveling wave solution (39), and Fig 3.d represents the projection  $t = 0$ , of the traveling wave solution (40).

Also, for  $d = 2.2, r = -0.5, c = 2.5$  and  $\xi_0 = 3$ . Fig 4.a illustrates the traveling wave solution (45), Fig 4.b represents the traveling wave solution (46), Fig 4.c represents the projection  $t = 0$ , of the traveling wave solution (45), and Fig 4.d represents the projection  $t = 0$ , of the traveling wave solution (46).

### 3.5 The generalized-Zakharov equations

In this section, we study the traveling wave solution of the generalized-Zakharov equations by used the simple equation method:-

$$i\psi_t + \psi_{xx} - 2\lambda|\psi|^2\psi + 2\psi v = 0, \quad (47)$$

$$v_{tt} - v_{xx} + (|\psi|^2)_{xx} = 0,$$

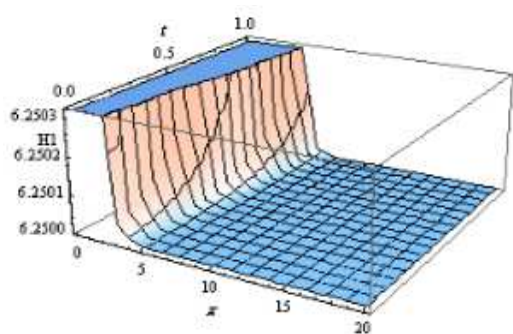


Fig 3.b The traveling wave solution (40) when the parameters take the special values  $d = -1.5, r = 2, c = 2.5$  and  $\xi_0 = 3$ .

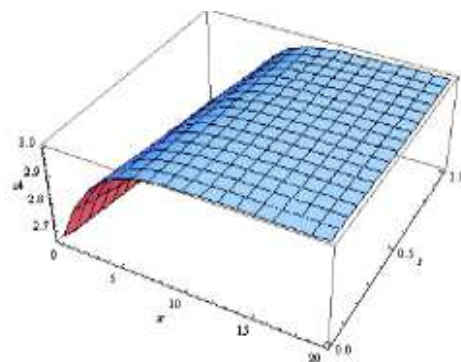


Fig 4.a The traveling wave solution (45) when the parameters take the special values  $d = 2.2, r = -0.5, c = 2.5$  and  $\xi_0 = 3$ .

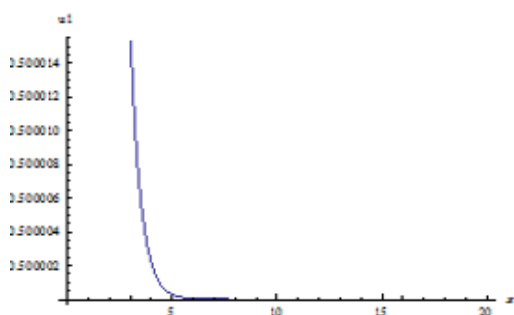


Fig 3.c The projection at  $t = 0$  of the traveling wave solution (39) when the parameters take the special values  $d = -1.5, r = 2, c = 2.5$  and  $\xi_0 = 3$ .

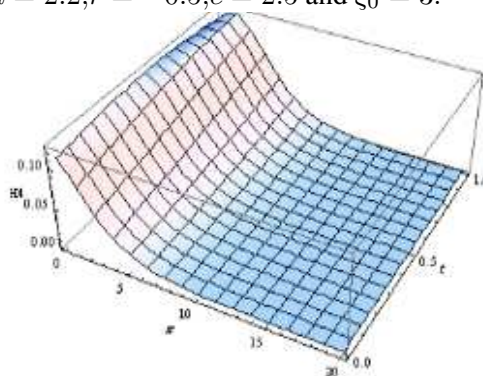


Fig 4.b The traveling wave solution (46) when the parameters take the special values  $d = 2.2, r = -0.5, c = 2.5$  and  $\xi_0 = 3$ .

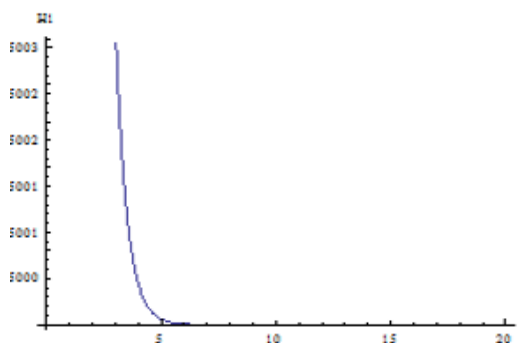


Fig 3.d The projection at  $t = 0$  of the traveling wave solution (40) when the parameters take the special values  $d = -1.5, r = 2, c = 2.5$  and  $\xi_0 = 3$ .

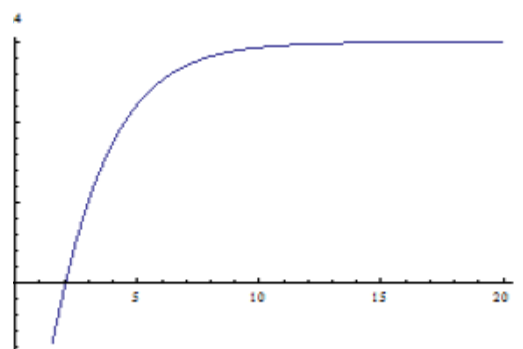


Fig 4.c The projection at  $t = 0$  of the traveling wave solution (45) when the parameters take the special values  $d = 2.2, r = -0.5, c = 2.5$  and  $\xi_0 = 3$ .

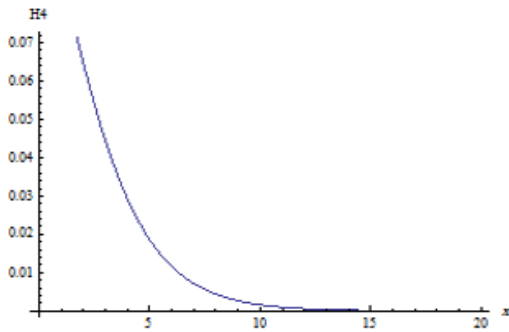


Fig 4.d The projection at  $t = 0$  of the traveling wave solution (46) when the parameters take the special values  $d = 2.2, r = -0.5, c = 2.5$  and  $\xi_0 = 3$ .

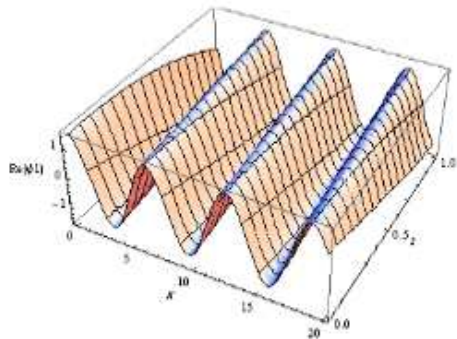


Fig 5.a real part of the traveling wave solution (55) when the parameters take the special values  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

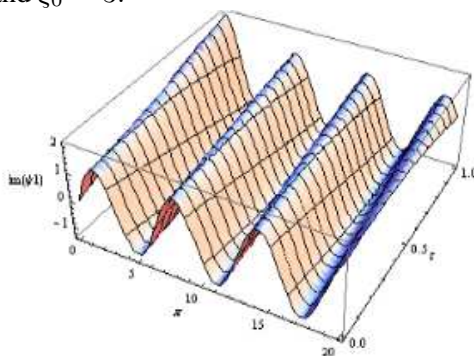


Fig 5.b The imaginary part of the traveling wave solution (55) when the parameters take the special values  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

The generalized-Zakharov system are one of the fundamental models governing dynamics of nonlinear waves in one-dimensional systems. It describes the interaction between high-frequency and low frequency waves. The physically most important example involves the interaction between the Langmuir and ion-acoustic waves in plasmas [28]. Let the traveling wave solutions of Eqs.(47) is the form:

$$\psi(x, t) = U(\xi)e^{i(px+qt)}, v(x, t) = V(\xi), \xi = kx - 2kpt. \tag{48}$$

The traveling wave transformation (48) permits us to reduce (47) into the following ODE:-

$$k^2U'' - 2\lambda U^3 - (q + p^2)U + UV = 0, \tag{49}$$

$$V'' = \frac{-2}{(4p^2 - 1)} [(U')^2 + UU''].$$

By integrate the second equation of Eqs.(49) two times we get:

$$V = \frac{-U^2}{4p^2 - 1} + c_2, \tag{50}$$

where  $c_2$  is the integration constant and substituting Eq.(50) into the first one in Eqs.(42) we have:-

$$k^2U'' - (q + p^2 - c_2)U - (2\lambda + \frac{1}{4p^2 - 1})U^3 = 0 \tag{51}$$

Balancing the highest order derivative  $U''$  with the nonlinear term  $U^3$  in Eq. (43). Therefore the solution of Eq.(51) can be expressed as follow form :

$$U(\xi) = a_0 + a_1F(\xi), \tag{52}$$

where  $a_1$  and  $a_0$  are constants to be determined later. Substituting Eq.(52) along with the simple trial equation  $F'(\xi) = rF(\xi) + dF^2(\xi)$  into Eq.(51). Collecting all terms with the same order of  $F(\xi)$  together, the left hand side of Eq.(51) are converted into polynomial in  $F(\xi)$ . Setting each coefficient of these polynomials to be zero, we derive a set of algebraic equations for  $a_0, a_1, p, q, \lambda$  and  $c_2$ . Solving the set of algebraic equations by using Maple or Mathematica, software package to get the following results:

$$a_0 = \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}}$$

$$a_1 = \pm \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda}} dk, \tag{53}$$

$$c_2 = p^2 + \frac{k^2r^2}{2} + q,$$

where  $k, p, q, \lambda, r$  and  $d$  are arbitrary constants. When  $d < 0$  and  $r > 0$  the solution of Eq.(51) is given by



$$U_1(x, t) = \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}} \pm \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda} \frac{dkr \exp[r(kx - 2kpt + \xi_0)]}{(1 - d \exp[r(kx - 2kpt + \xi_0)])}}. \tag{54}$$

Therefore substituting Eq.(54) into Eqs.(48) and into Eq.(50) to get the solution of Eqs.(47) as the following form:

$$\psi_1(x, t) = \exp[i(px + qt)] \left[ \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}} \pm \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda} \frac{dkr \exp[r(kx - 2kpt + \xi_0)]}{(1 - d \exp[r(kx - 2kpt + \xi_0)])}} \right]. \tag{55}$$

$$v_1(x, t) = \frac{-1}{4p^2 - 1} \left[ \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}} \pm \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda} \frac{dkr \exp[r(kx - 2kpt + \xi_0)]}{(1 - d \exp[r(kx - 2kpt + \xi_0)])}} \right]^2 + p^2 + \frac{k^2 r^2}{2} + q. \tag{56}$$

when  $d > 0$  and  $r < 0$  the solution of Eq. (51) is given by

$$U_2(x, t) = \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}} \mp \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda} \frac{dkr \exp[r(kx - 2kpt + \xi_0)]}{(1 + d \exp[r(kx - 2kpt + \xi_0)])}}. \tag{57}$$

Therefore substituting Eq.(57) into Eqs.(48) and into Eq.(50) to get the solution of Eqs.(47) as the following form:

$$\psi_2(x, t) = \exp[i(px + qt)] \left\{ \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}} \mp \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda} \frac{dkr \exp[r(kx - 2kpt + \xi_0)]}{(1 + d \exp[r(kx - 2kpt + \xi_0)])}} \right\}. \tag{58}$$

$$v_2(x, t) = \frac{-1}{4p^2 - 1} \left[ \pm \frac{kr(4p^2 - 1)\sqrt{1 + 8\lambda p^2 - 2\lambda}}{1 + 8\lambda p^2 - 2\lambda\sqrt{2(4p^2 - 1)}} \mp \sqrt{\frac{2(4p^2 - 1)}{1 + 8\lambda p^2 - 2\lambda} \frac{dkr \exp[r(kx - 2kpt + \xi_0)]}{(1 + d \exp[r(kx - 2kpt + \xi_0)])}} \right]^2 + p^2 + \frac{k^2 r^2}{2} + q. \tag{59}$$

### 3.6 Numerical solutions for the generalized-Zakharov equations

In this section, we present some graphs of the traveling wave solutions by taking suitable values of the parameters to visualize the mechanism of generalized-Zakharov equations (47).

For  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ , Fig 5.a illustrates the real part of the traveling wave solution (55), Fig 5.b represents the imaginary part of the traveling wave solution(55), Fig 5.c represents the traveling wave solution (56), Fig 5.d the represents the projection  $t = 0$ , of the real part solution (55), Fig 5.e represents the projection  $t = 0$ , of the imaginary part solution (55) and Fig 5.f represents the projection  $t = 0$ , of the traveling wave solution (56).

Also for  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ , Fig 6.a illustrates the real part of the traveling wave solution(58), Fig 6.b represents the imaginary part of the traveling wave solution(58), Fig 6.c represents the behavior of the traveling wave solution (59), Fig 6.d the represents the projection  $t = 0$ , of the real part solution (58), Fig 6.e represents the projection  $t = 0$ , of the imaginary part solution (58) and Fig 6.f represents the projection  $t = 0$ , of the traveling wave solution (56).

### 3.7 The coupled Maccaris equations

In this section, we use the simple equation method to solve the coupled Maccaris equations in the following form:-

$$\begin{aligned} iQ_t + Q_{xx} + QR &= 0, \\ R_t + R_y + (|Q|^2)_x &= 0. \end{aligned} \tag{60}$$

This system brings nonlinear evolution equations that are frequently used to describe location in a small part of space, and motion of the isolated waves in varied fields, such as, nonlinear optics, fluid mechanics, quantum field theory, and plasma physics [28].Let the traveling wave solutions of Eqs.(60) is the form:

$$Q(x, y, t) = U(\xi)e^{i(px+qy+kt)}, R(x, y, t) = V(\xi), \xi = x+y-ct. \tag{61}$$

The traveling wave transformation (61) permits us to reduce (60) into the following system of NODE's:-

$$\begin{aligned} U'' - (k + p^2)U + UV &= 0, \\ (c + 1)V' &= -2UU'. \end{aligned} \tag{62}$$

By integrate the second equation of Eqs.(62), we have:

$$V = \frac{-U^2}{c + 1} + c_1. \tag{63}$$

where  $c_1$  is the integration constant. Substituting Eq.(63) into the first one in Eqs.(62), we get:-

$$U'' - (k + p^2 - c_1)U - \frac{1}{c + 1}U^3 = 0. \tag{64}$$

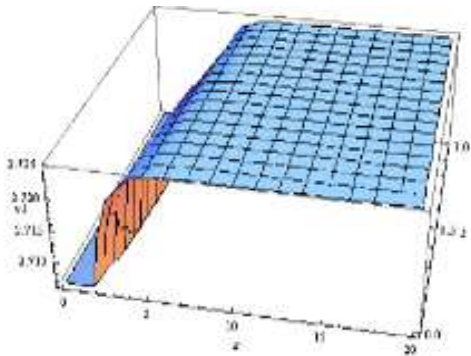


Fig 5.c The traveling wave solution (56) when the parameters take the special values  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

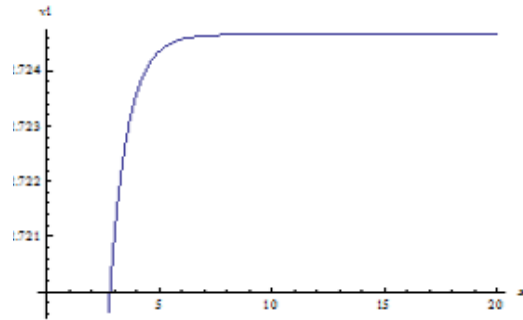


Fig 5.f The projection at  $t = 0$  of the traveling wave solution (56) when the parameters take the special values  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

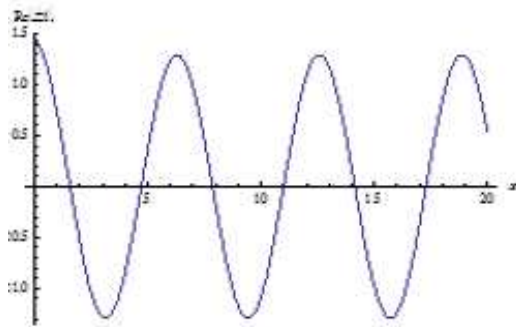


Fig 5.d The projection at  $t = 0$  of the real part from traveling wave solution (55) when the parameters take the special values  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

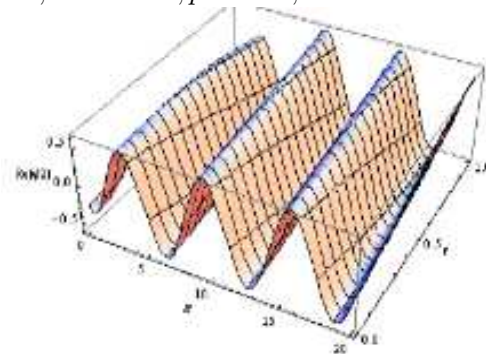


Fig 6.a The real part of traveling wave solution (58) when the parameters take the special values  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

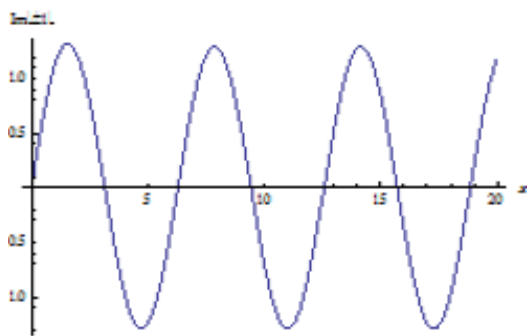


Fig 5.e The projection at  $t = 0$  of the real part from traveling wave solution (55) when the parameters take the special values  $d = -1.5, r = 0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

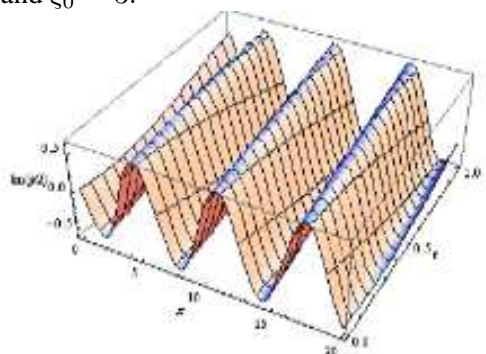


Fig 6.b The imaginary part of traveling wave solution (58) when the parameters take the special values  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

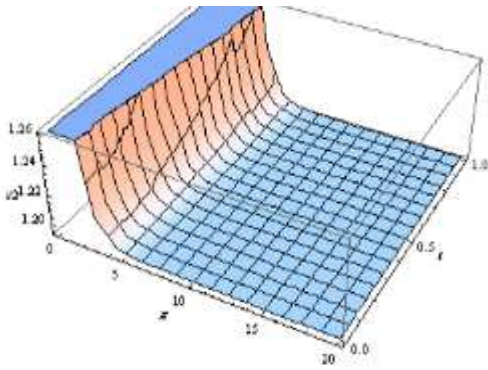


Fig 6.c The traveling wave solution (59) when the parameters take the special values  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

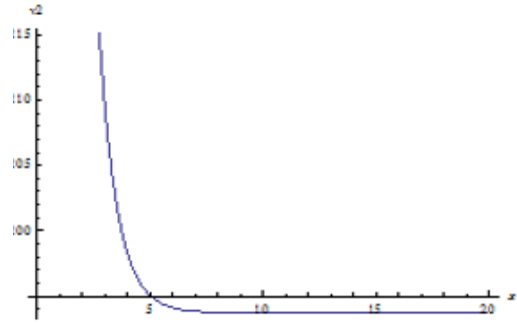


Fig 6.f The projection  $t = 0$ , of the traveling wave solution (59) when the parameters take the special values  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

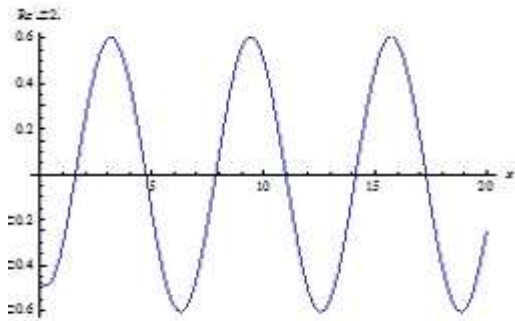


Fig 6.d The projection  $t = 0$ , of real part from traveling wave solution (58) when the parameters take the special values  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

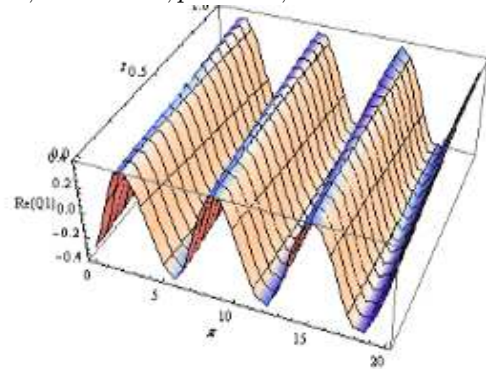


Fig 7.a The real part of traveling wave solution (66) when the parameters take the special values  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

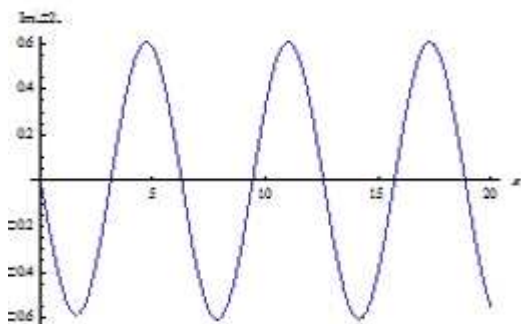


Fig 6.e The projection  $t = 0$ , of imaginary part from traveling wave solution (58) when the parameters take the special values  $d = 1.5, r = -0.5, q = 1.5, k = 2.5, p = 1, \lambda = 0.9$  and  $\xi_0 = 3$ .

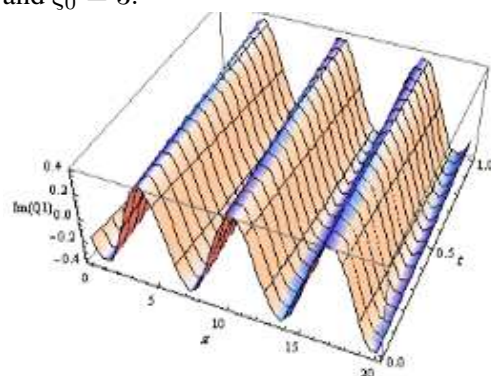


Fig 7.b The imaginary part of traveling wave solution (66) when the parameters take the special values  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

Balancing the highest order derivative  $U''$  with nonlinear term  $U^3$  in Eq. (64), hence  $N = 1$ . Therefore the solution of Eq.(64) can be expressed as follows:

$$U(\xi) = a_0 + a_1 F(\xi). \tag{65}$$

where  $a_0$  and  $a_1$  are arbitrary constants to be determined later. Substituting Eq.(65) along with simple trial equation  $F'(\xi) = rF(\xi) + dF^2(\xi)$  into Eq.(64). Collecting all terms with the same order of  $F(\xi)$  together, the left hand side of Eq. (64) are converted into polynomial in  $F(\xi)$ . Setting each coefficient of these polynomials to zero. we derive a set of algebraic equations for  $a_0, a_1, p, k, c_1$  and  $c$ . Solving the set of algebraic equations by using Maple or Mathematica, software package to get the following results:

$$a_0 = \pm \frac{r(c+1)}{\sqrt{2(c+1)}}, a_1 = \pm \sqrt{2(c+1)}d, c_1 = \frac{r^2}{2} + p^2 + k, \tag{66}$$

where  $p, k, r, c$  and  $d$  are arbitrary constants. When  $d < 0$  and  $r > 0$  the solution of Eq. (64) is given by

$$U_1(x, t) = \pm \frac{r(c+1)}{\sqrt{2(c+1)}} \pm \frac{\sqrt{2(c+1)}dr \exp [r(x+y-ct+\xi_0)]}{(1-d \exp [r(x+y-ct+\xi_0)])}. \tag{67}$$

Therefore substituting Eq.(67) into Eqs.(61) and (63) to get the solution of Eqs.(60) as the following form:

$$Q_1(x, t) = \exp[i(px + qy + kt)] \left[ \pm \frac{r(c+1)}{\sqrt{2(c+1)}} \pm \frac{\sqrt{2(c+1)}dr \exp [r(x+y-ct+\xi_0)]}{(1-d \exp [r(x+y-ct+\xi_0)])} \right], \tag{68}$$

$$V_1 = R_1(x, t) = \frac{-1}{c+1} \left[ \pm \frac{r(c+1)}{\sqrt{2(c+1)}} \pm \frac{\sqrt{2(c+1)}dr \exp [r(x+y-ct+\xi_0)]}{(1-d \exp [r(x+y-ct+\xi_0)])} \right]^2 + \frac{r^2}{2} + p^2 + k. \tag{69}$$

When  $d > 0$  and  $r < 0$  the solution of Eq. (64) is given by

$$U_2(x, t) = \pm \frac{r(c+1)}{\sqrt{2(c+1)}} \mp \frac{\sqrt{2(c+1)}dr \exp [r(x+y-ct+\xi_0)]}{(1+d \exp [r(x+y-ct+\xi_0)])}. \tag{70}$$

Therefore substituting Eq.(70) into Eqs.(61) and (63) to get the solution of Eqs.(60) as the following form:

$$Q_2(x, t) = \exp[i(px + qy + kt)] \left[ \pm \frac{r(c+1)}{\sqrt{2(c+1)}} \mp \frac{\sqrt{2(c+1)}dr \exp [r(x+y-ct+\xi_0)]}{(1+d \exp [r(x+y-ct+\xi_0)])} \right]. \tag{71}$$

$$V_2 = R_2(x, t) = \frac{-1}{c+1} \left[ \pm \frac{r(c+1)}{\sqrt{2(c+1)}} \mp \frac{\sqrt{2(c+1)}dr \exp [r(x+y-ct+\xi_0)]}{(1+d \exp [r(x+y-ct+\xi_0)])} \right]^2 + \frac{r^2}{2} + p^2 + k. \tag{72}$$

### 3.8 Numerical solutions for The coupled Maccaris equations

In this section, we present some graphs of the traveling wave solutions by taking suitable values of the parameters to visualize the mechanism of coupled Maccaris equations (60). For  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ , Fig 7.a illustrates the real part traveling wave solution (68), Fig 7.b represents the imaginary part of the traveling wave solution(68), Fig 7.c represents the behavior of the traveling wave solution (69), Fig 7.d the represented the projection  $t = 0$ , of the real part of the traveling solution (68), Fig 7.e represents the projection  $t = 0$ , of the imaginary part of the traveling solution (68)and Fig 7.f represents the projection  $t = 0$ , of the traveling wave solution (69).

Also for  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ , Fig 8.a illustrates the real part of the traveling wave solution (71), Fig 8.b represents the imaginary part of the traveling wave solution(71), Fig 8.c represents the behavior of the traveling wave solution (72), Fig 8.d the represents the projection  $t = 0$ , of the real part traveling wave solution (71), Fig 8.e represented the projection  $t = 0$ , of the imaginary part traveling solution (71)and Fig 8.f represented the projection  $t = 0$ , of the traveling wave solution (72).

## 4 Conclusions

In this paper, the simple equation method has been successfully used to obtain the exact solution of nonlinear Schrodinger equation with dual power law nonlinearity, the variant Boussinesq nonlinear differential equations, the generalized-Zakharov equations, the coupled Maccaris equations. As the simple equation, we have used the Bernoulli and Riccati equations. For the simple equation, we have obtained a balance equation. By means of balance equation, we obtained exact solutions of the studied class of nonlinear PDEs, we have also verified that solutions we have found are indeed solutions to the original equations. Finally, we point out of either integrable or non-integrable nonlinear coupled systems.

### References:

[1] P. Lions, On some recent methods for nonlinear partial differential equations, ceremade, University Paris-Dauphine, Place de Lattre de Tassigny, F-75775 Paris Cedex 16, France.

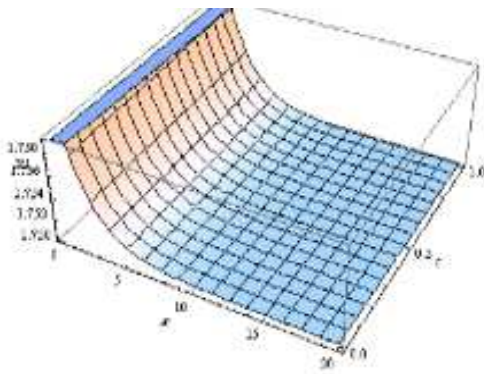


Fig 7.c The traveling wave solution (66) when the parameters take the special values  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

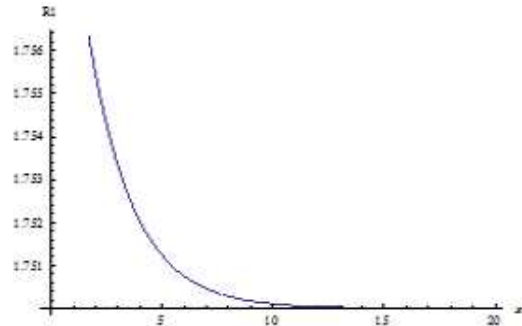


Fig 7.f The projection  $t = 0$ , of the traveling wave solution (66) when the parameters take the special values  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

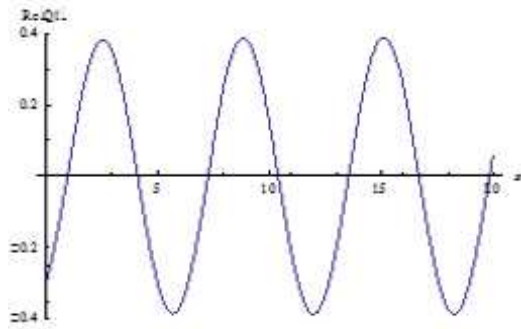


Fig 7.d The projection  $t = 0$ , of real part from traveling wave solution (66) when the parameters take the special values  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

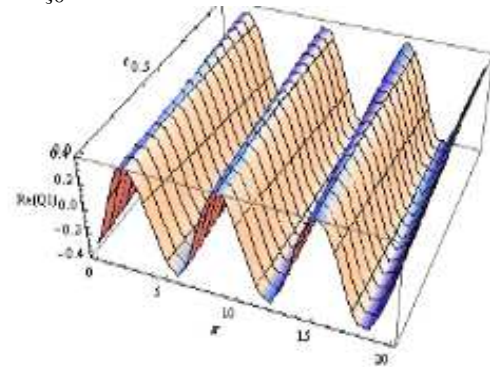


Fig 8.a The real part of traveling wave solution (68) when the parameters take the special values  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

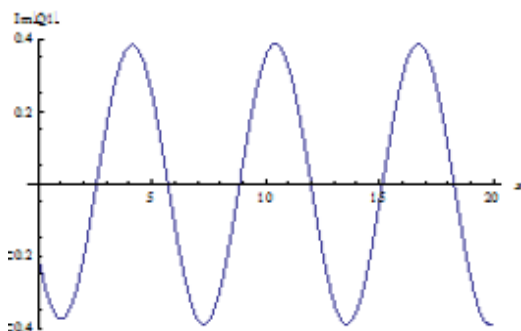


Fig 7.e The projection  $t = 0$ , of real part from traveling wave solution (66) when the parameters take the special values  $d = -1.5, r = 0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

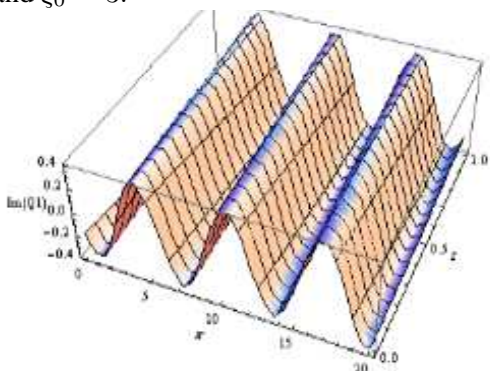


Fig 8.b The imaginary part of traveling wave solution (68) when the parameters take the special values  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

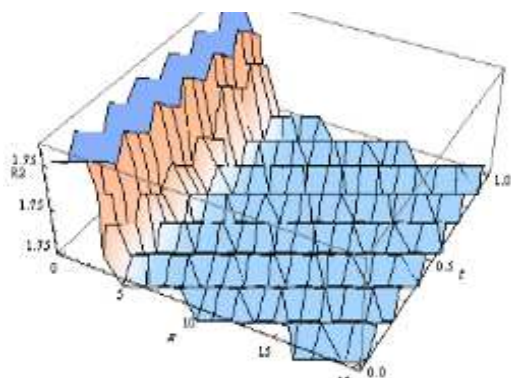


Fig 8.c The traveling wave solution (68) when the parameters take the special values  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

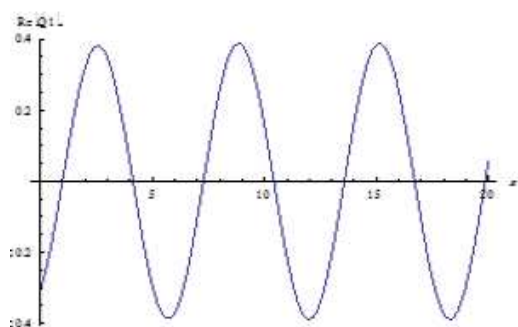


Fig 8.d The projection  $t = 0$ , of real part from traveling wave solution (68) when the parameters take the special values  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

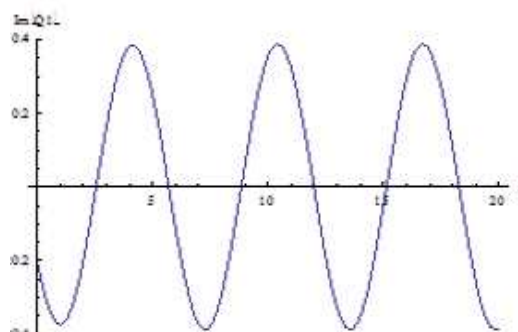


Fig 8.e The projection  $t = 0$ , of real part from traveling wave solution (68) when the parameters take the special values  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

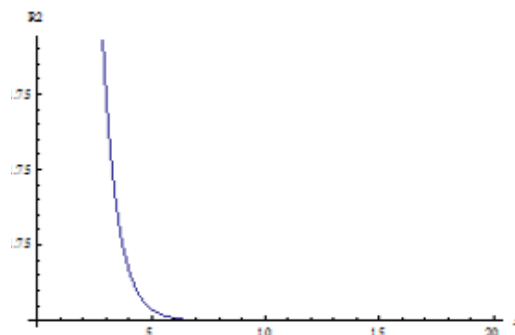


Fig 8.f The projection  $t = 0$ , of the traveling wave solution (68) when the parameters take the special values  $d = 1.5, r = -0.5, k = 0.75, p = 1, y = 4, c = 0.2$  and  $\xi_0 = 3$ .

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